Electroweak Sudakov Logarithms and Real Gauge Boson Radiation in the TeV Region

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in collaboration with

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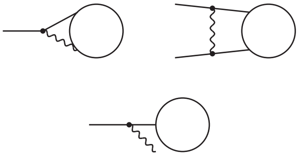
Sudakov Logarithms

If a theory contains particles with **zero** mass:
Propagators go **on-shell** if the momentum of the gauge boson is

- **collinear** to the momentum of a external particle
- **soft**



and causes **divergences**.

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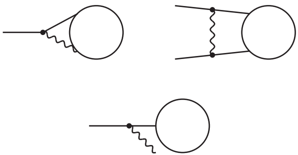

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divergences $\Rightarrow$ $\underbrace{\frac{\alpha}{4\pi \sin^2\theta_W} \log^2\left(\frac{s}{M^2_{W/Z}}\right)}_{\sim 7\% \quad \text{at 1 TeV}}$ for each external particle

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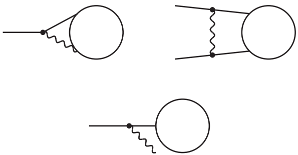

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Electroweak corrections can become large (20-30%)!

Questions

Mainly the **virtual** corrections are calculated (no need for real corrections)

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First Question

Can real Z, W radiation be separated?

- **collinear** radiation into the beam pipe
- **collinear** radiation into a jet
- **soft** gauge boson decays into particles similar to the background

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Second Question

QED / QCD	
virtual cor.:	- divergence
real cor.:	+ divergence
observable:	finite

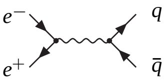
$$M \neq 0 \quad \longrightarrow \quad M^2 \ll s$$

EW

$$-C^{(V)} \log^2(s/M^2)$$
$$+C^{(R)} \log^2(s/M^2)$$

no logarithms?

Born cross section:



- inclusive final state $(\Sigma_{u,d})$
- EW: $SU(2)_L \left(\times U(1)_Y \right)$

4 Fermion Scattering

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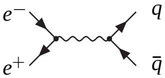

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Virtual correction:

$$\sigma^{(V)} = \frac{\alpha}{4\pi}\left[\mathrm{v}_{sc} \underbrace{\log^2 \frac{s}{M^2}}_{\text{soft + collinear}} + \mathrm{v}_c \underbrace{\log \frac{s}{M^2}}_{\text{collinear}} + \mathrm{v}_s \underbrace{\log \frac{s}{M^2}}_{\text{soft}} + \mathrm{v}_0 + \mathcal{O}\left(\frac{M^2}{s}\right)\right] \sigma_B(s)$$

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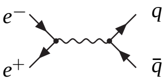

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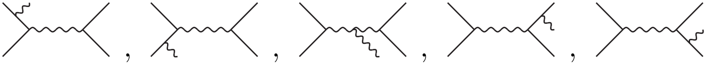

$$\sigma^{(R)} = \frac{\alpha}{4\pi}\left[\mathrm{r}_{sc} \underbrace{\log^2 \frac{s}{M^2}}_{\text{soft + collinear}} + \mathrm{r}_c \underbrace{\log \frac{s}{M^2}}_{\text{collinear}} + \mathrm{r}_s \underbrace{\log \frac{s}{M^2}}_{\text{soft}} + \mathrm{r}_0 + \mathcal{O}\left(\frac{M^2}{s}\right)\right] \sigma_B(s)$$

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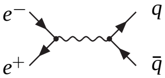

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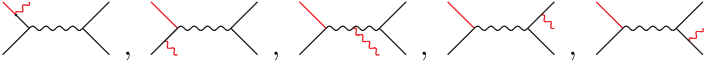

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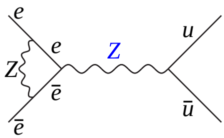
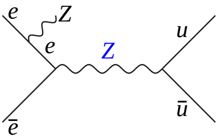
$$+\mathrm{r}_{\text{initial}}\log\frac{s}{M^2}$$

Collinear Logarithms - Initial State Radiation

virtual correction | **real correction**

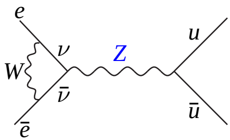


(or $u \longrightarrow d$)



(or $u \longrightarrow d$)

$\Rightarrow$ cancellation



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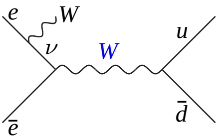
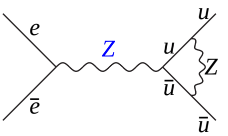


$\Rightarrow$ **no** cancellation
Bloch-Nordsieck Violation

Ciafaloni, Comelli

Collinear Logarithms - Final State Radiation

virtual correction | **real correction**



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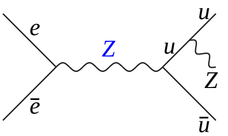

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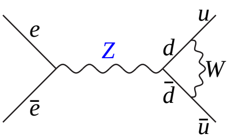

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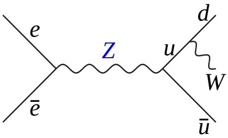

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4 Fermion Scattering - Logarithms

$$-2\Bigg[\underbrace{\left(\frac{1}{4}+\frac{1}{2}\right)\sigma_B^Z}_{initial}\Bigg] - 2\Bigg[\underbrace{\left(\frac{1}{4}+\frac{1}{2}\right)\sigma_B^Z}_{final}\Bigg] = -3\cdot\sigma_B^Z$$

$$\sigma^{(V)} = \frac{\alpha}{4\pi}\left[-3\left(\log^2\frac{s}{M^2} - 3\log\frac{s}{M^2}\right) - \frac{26}{3}\log\frac{s}{M^2}\right]\sigma_B^Z(s)$$

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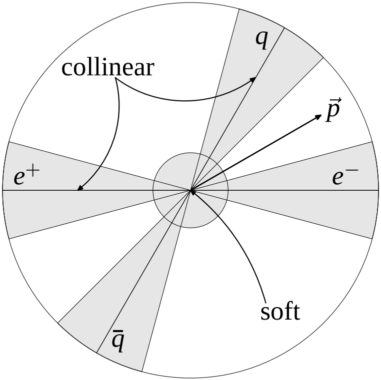
short summary

Real corrections with full phase space can **overcompensate** the virtual corrections!

Cuts on Phase Space



Three integration variables are of interest

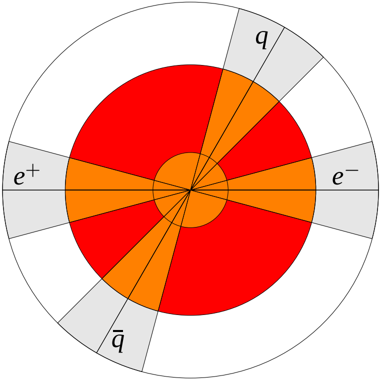
- Energy E of the gauge boson $\cong$ invariant mass Q of final fermion pair
- Angle θ_e or $\theta_{\bar{e}}$ between gauge boson and initial fermions (beam pipe)
- Angle θ_q or $\theta_{\bar{q}}$ between gauge boson and final fermions

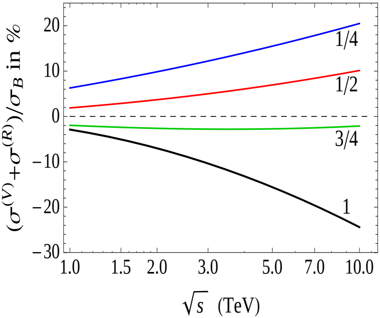
Soft Region

high energetic fermion pair

$$\text{Definition: } z = Q^2/s = 1 + M^2/s - 2\,E/\sqrt{s}$$

$$\Rightarrow \quad E_{cut} \sim (1-z)E_{max} \quad \Rightarrow \quad z \sim 1\text{: soft region}$$



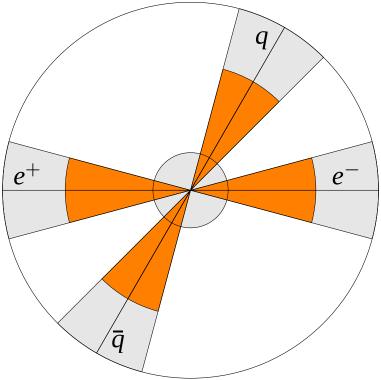


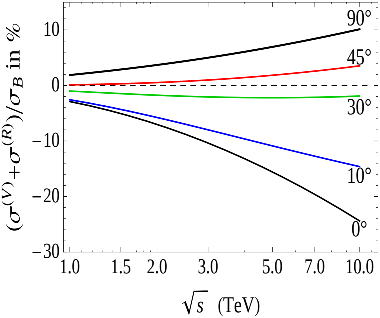
Collinear Region

only collinear radiation

$$z = 1/2$$

$\theta_e, \theta_{\bar{e}}, \theta_q$ and $\theta_{\bar{q}} \leq \theta_{cut} \quad \Rightarrow \quad$ cones around fermion direction



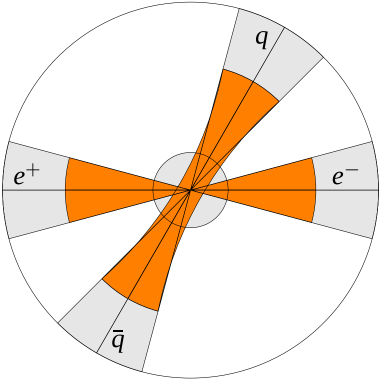


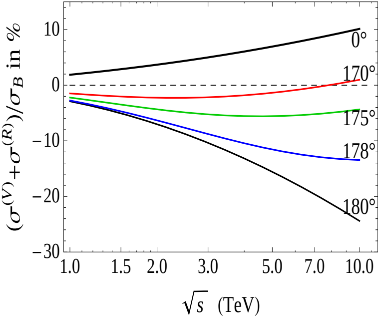
Collinear and Soft Region

collinear and soft radiation

$$z = 1/2 \quad \text{and} \quad \theta_e, \theta_{\bar{e}} \leq 5^\circ$$

$$\theta_{q\bar{q}} \geq \theta_{cut} \quad \Rightarrow \quad \text{also soft region allowed}$$





Summary

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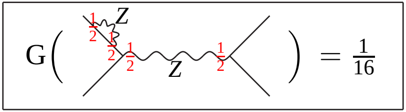
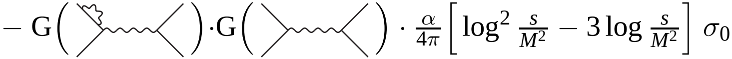
- we studied the structure of the Sudakov Logarithms in the four-fermion process (with full phase space)
- we analyzed different restrictions on the phase space (soft, collinear and collinear+soft)
- we used naive cuts but gained an idea of importance of real radiation

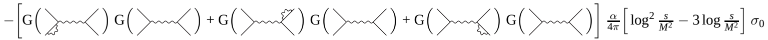
Backup Files

Collinear Logarithms

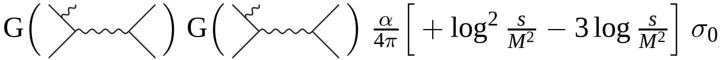


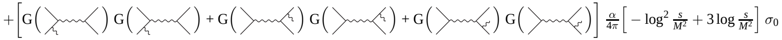
Virtual correction:





Real correction:





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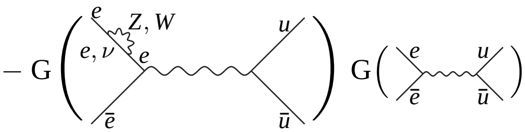

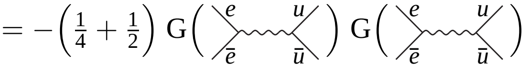

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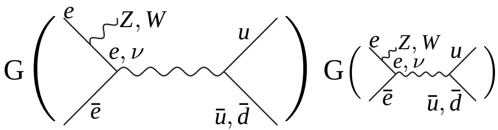

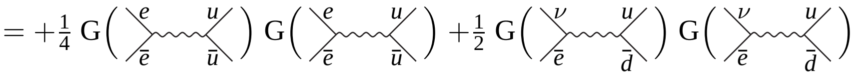

Collinear Logarithms - Final State Radiation

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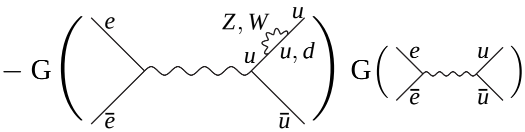

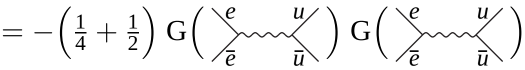

Real correction: