SUSY Mass Determination using Kinematic Fits

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Introduction

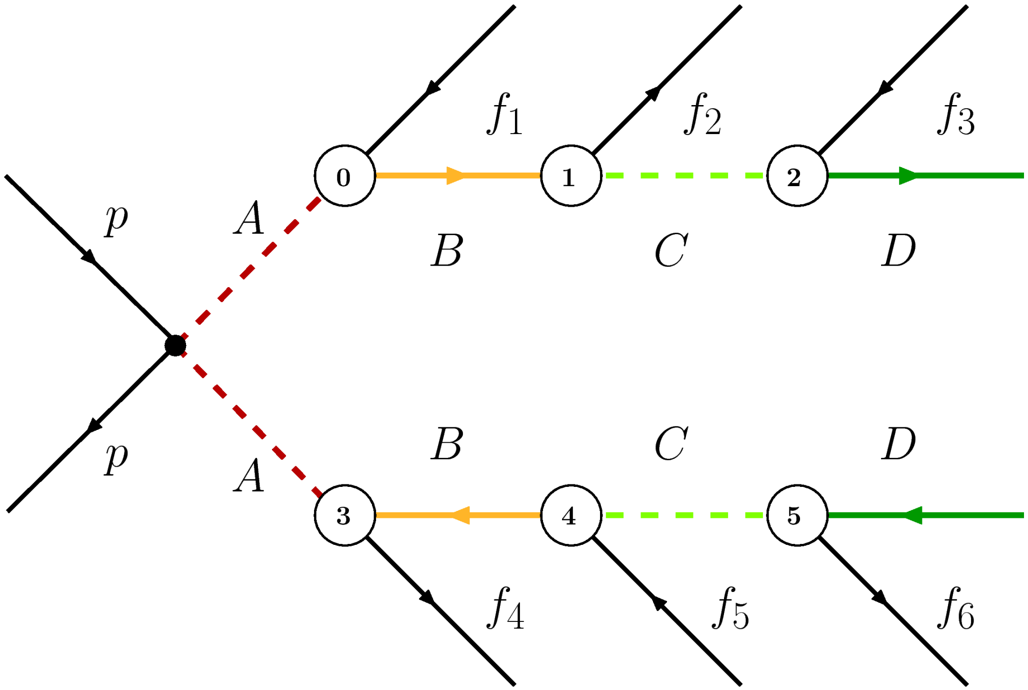
- Possible discovery of supersymmetry at the LHC
- Input for model determination / fitting:
  - Cross section, branching ratios, **masses of new particles**, ...
- Major approaches to mass determination:

A) Mass differences from endpoints in kinematic distributions

B) Reconstruction of entire decay chains, including
- LSP momenta
- All particle masses
- Combinatorial problem

- Combined approach to mass determination:
  - A scan over the mass space
  - A kinematic fit for event reconstruction

Event Reconstruction



Parameters

- *Measured:*
  f_i momenta 6x3=18
- *Unmeasured:*
  LSPs momenta 2x3= 6

Constraints

- *Masses (2x A,B,C) 6*
- *P_T-balance (p_x,p_y) 2*

→ Overconstrained if masses are given/known:

$N_{constr} - N_{unmeas} > 0$

- Reconstruct entire event using a kinematic fit
  - Need a certain topology
  - Constrain kinematics of final state, incl. LSPs
  - Takes into account experimental uncertainties

- Potential Problems
  - Combinatorics
  - Detector acceptance & resolutions
  - Initial/final state radiation

Kinematic Fit

- Method of lagrangian multipliers (λ_j).
  - The minimum of the squared sum of residuals

$$S = \chi^2 = \sum_{i=1}^{N_{par}} \left(\frac{y_i - f_i}{\sigma_i} \right)^2$$

  subject to constraints c_j is given by extremum of

$$L = S + 2 \cdot \sum_{j=1}^{M_{constr}} \lambda_j \cdot c_j$$

- Non-linear constraints:
  - Linearization
  - Iterative solution

- Treatment of combinatorics:
  - Run fit for each possible combination and **select smallest χ^2 value**

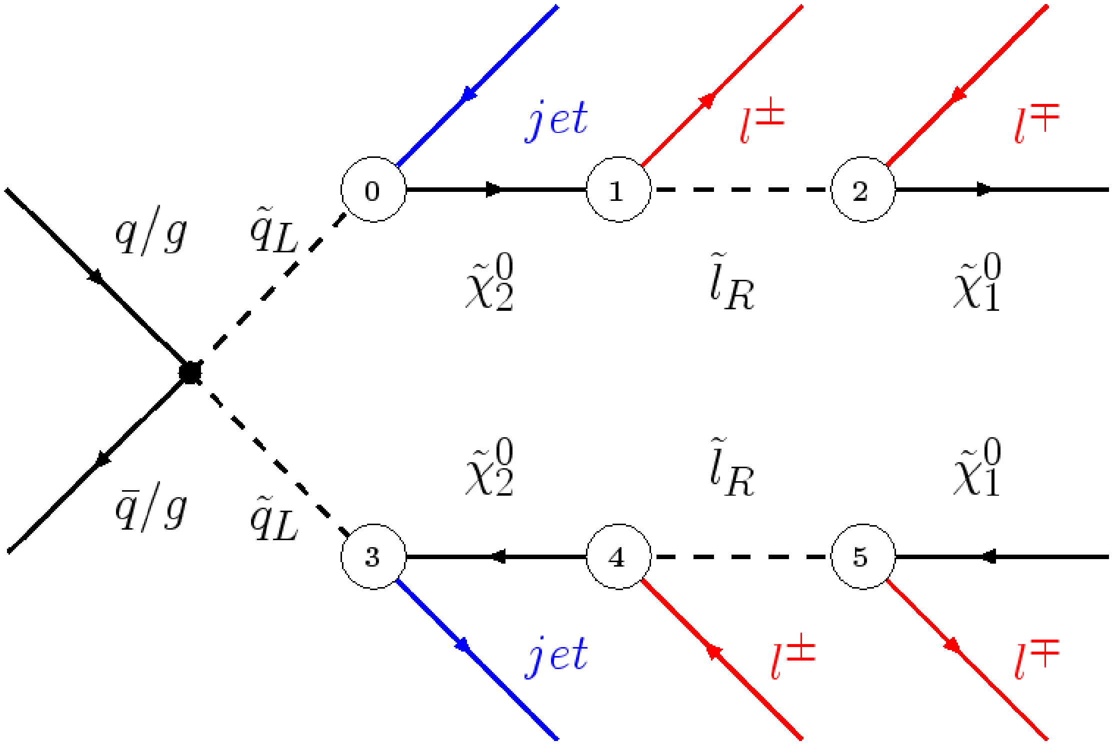
Definition of convergence:

- Change in χ^2<0.01
- Constraints: $\sum_j |c_j| < 0.1 * M_{constr}$

Implementation: *KinFitter*

- [CMS Note-2006/23]
- Additional modifications (step scaling and scaling of constraints)

Scenario with Leptonic Decays



- mSUGRA benchmark point (SPS1a)

SPS1a Parameter	
m_0	100 GeV
$m_{1/2}$	250 GeV
A_0	-100 GeV
tan β	10
sign μ	+

Particle	Mass [GeV]
Squark	565
Neutralino 2	180
Slepton	147
Neutralino 1	97

  - LO X-section @14 TeV: 36 pb

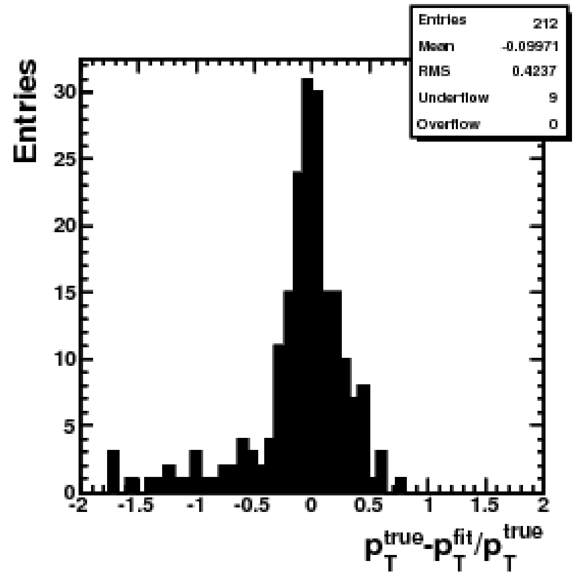
- Signature: 2 jets + 2x2 OSSF leptons
- Max. 32 possible permutations
- Total branching ratio of $1.7 \cdot 10^{-3}$

- Signal: all events with exactly these decay branches
- Background: all other SUSY processes

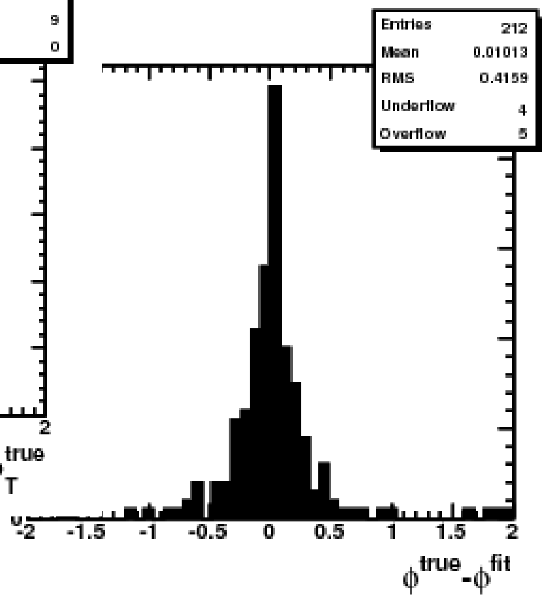
Event Selection and Testing

- Monte Carlo Sample:
  - Suspect+Pythia generated events with $\sqrt{s}=14$ TeV
  - Incl. initial/final state radiation (ISR/FSR)
  - Final state smearing according to typical detector resolutions at CMS
- Event Selection:
  - 2 jets, $p_T>30$ GeV, $|\eta|<3.0$
  - 2x2 OSSF leptons, $p_T>10$ GeV, $|\eta|<2.0$
  - Efficiency: 45%
  - S/B ≈ 1/1 w.r.t other SUSY processes

- Test event reconstructing with kinematic fit on signal events
  - Using true masses in the fit
  - Full combinatorics
- LSP momenta well reconstructed

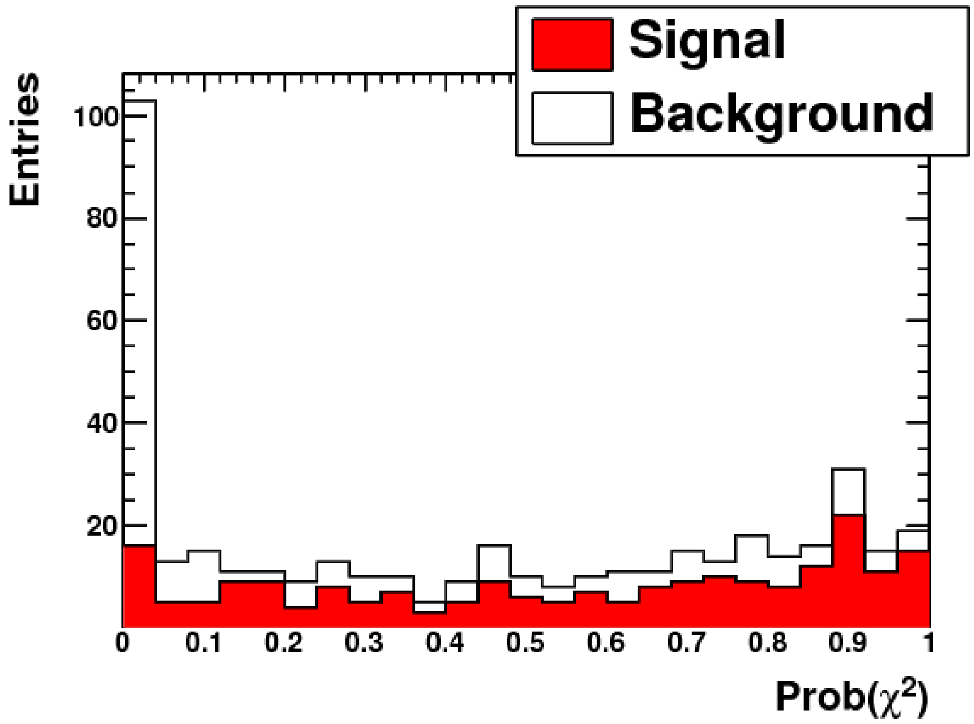


LSP p_T-resolution



LSP ϕ-resolution

Fit using True Mass Values



- Signal: Fit probability distribution flat as expected
- Background: Peak at zero
  - Some "signal-like" events distributed over entire range

- Signal combinatorics:
  - For 42% of the events correct combination has smallest χ^2
  - Same number of events has smallest χ^2 if leptons are exchanged on the same branch

- Use events' combined fit probability to quantify how good the assumed masses fit.

$$\log \mathcal{P} = \sum_i^N \log P(\chi_i^2)$$

$$\text{with } P_i = P_{\text{cut}} \text{ for} P_i < P_{\text{cut}}$$

  - Cut-off to avoid numerical fluctutations

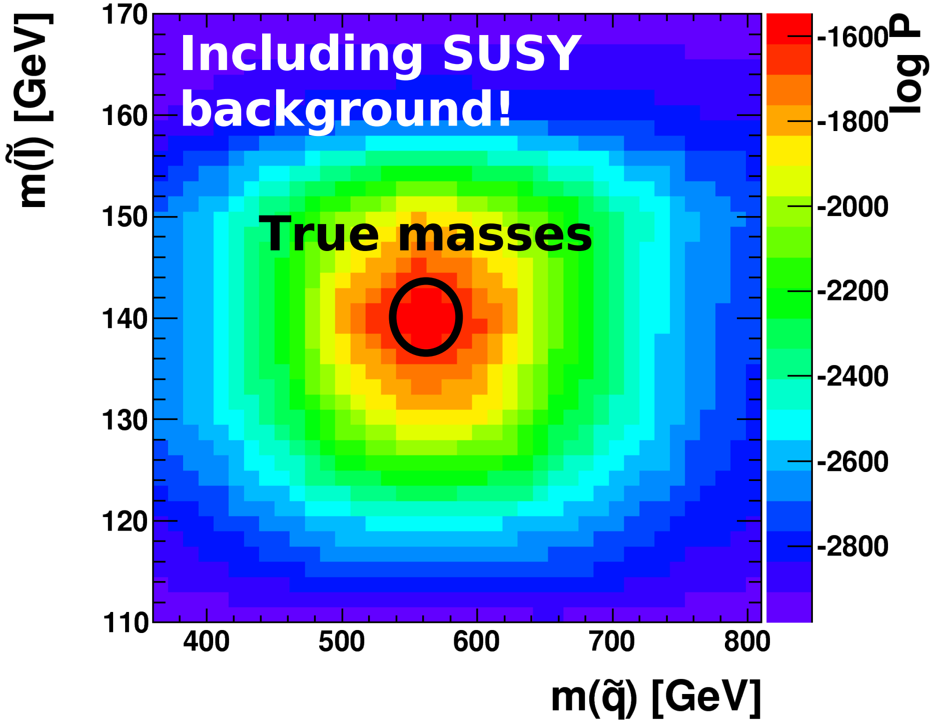
Mass Scan I

- Scan over possible mass values
  - Test each mass hypothesis with the fit on each event
  - Find best fitting hypothesis
- For a **set of events**
  - Hypotheses close to true masses fit on average better
  - Problem becomes overconstrained
- Likelihood-Estimator:

$$\log \mathcal{P} = \sum_{i}^{N} \log P(\chi_i^2)$$

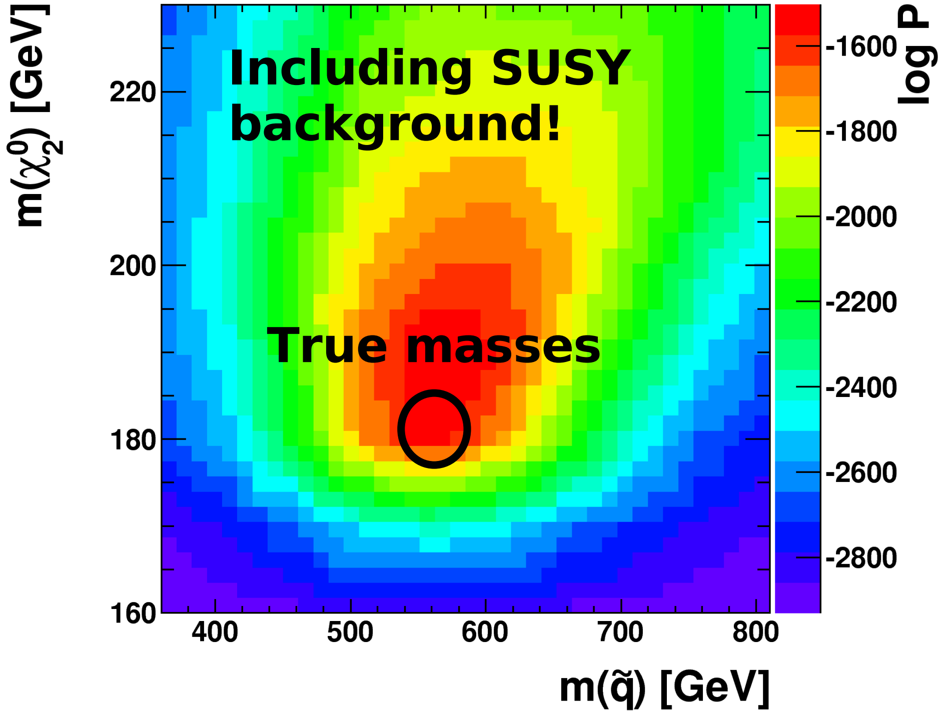
$$\text{with } P_i = P_{\text{cut}} \text{ for} P_i < P_{\text{cut}}$$

- Visualization in 2D mass plane:
  - Fix neutralino and LSP masses to true values
  - Vary squark & slepton masses
- Very good agreement of maximum with true masses



Mass Scan II

- Visualization in 2D mass plane:
  - Fix slepton and LSP masses to true values
  - Vary squark & neutralino masses



- Good correspondance between minimum and squark mass
- Systematic shift w.r.t to true neutralino mass
  - Under investigation

- Similar behaviour in all mass hyperplanes
- Next steps:
  - Determination of maximum in 4-dimensional mass space
  - Statistical interpretation: Estimation of errors and confidence intervals

Summary

- Showed reconstruction of entire SUSY cascade decays using a kinematic fit.
- Application to scenario with leptonic decays
  - Good reconstruction of LSP momenta
  - Strong reduction of combinatorial problem
- Scan over mass space
  - Demonstrated derival of probability map
  - Mass determination seems feasible

Outlook:

- Determine confidence regions
- Study systematic shifts
- Suppress combinatorial background using angular decay information
- Extend to more event categories

BACKUP SLIDES

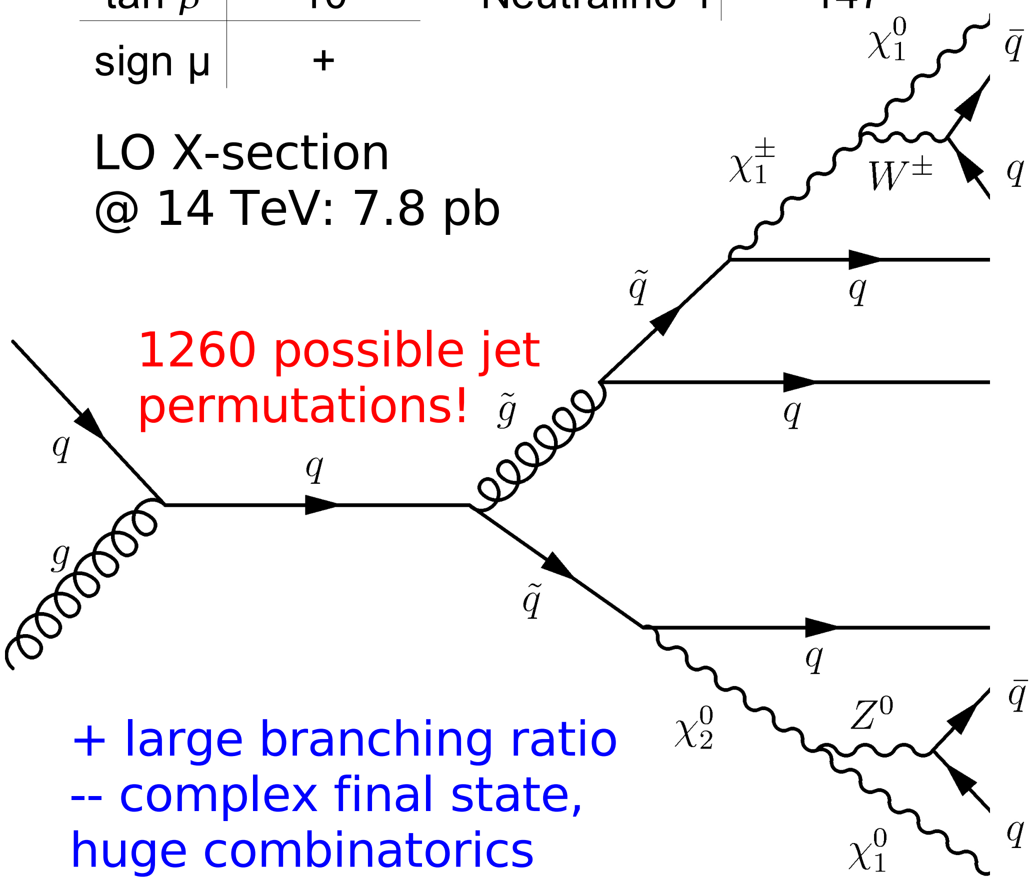
Scenario with Hadronic Decays

Parameter	
m_0	230 GeV
$m_{1/2}$	360 GeV
A_0	0
tan β	10
sign μ	+

Particle	Mass [GeV]
Squark	860
Gluino	810
Chargino 1	273
Neutralino 1	147

LO X-section
@ 14 TeV: 7.8 pb

1260 possible jet permutations!



+ large branching ratio
-- complex final state, huge combinatorics

- mSUGRA benchmark point (CMS point LM5)
- Pythia generated sample
  - Incl. initial/final state radiation (ISR/FSR)
- Each final state parton smeared according to typical jet resolutions at ATLAS/CMS
- Event Selection:
  - 7 jets, p_T>30 GeV, $|\eta|$<3.0
  - Efficiency 30%
  - S/B ~1/33 w.r.t other SUSY processes

Fit Alternative

- Low convergence rate of Lagrange multiplier method in this topology
- Alternative approach: formulate constraints as additional χ^2 term to

$$S = \chi^2 = \sum_{i=1}^{N_{par}} \left(\frac{y_i - f_i}{\sigma_i} \right)^2$$

  - Mass constraints:

$$S_m = \frac{\left(\left(\sum_{i=1}^{R} \mathbf{p}_i\right)^2 - m^2 \right)^2}{(2m\Gamma)^2}$$

  - p_T-balance constraints

$$S_T = \frac{\left(\left(\sum_{i=1}^{\text{allparticles}} p_{x/y}^i\right)^2 - 0 \right)^2}{\sigma_T^2}$$

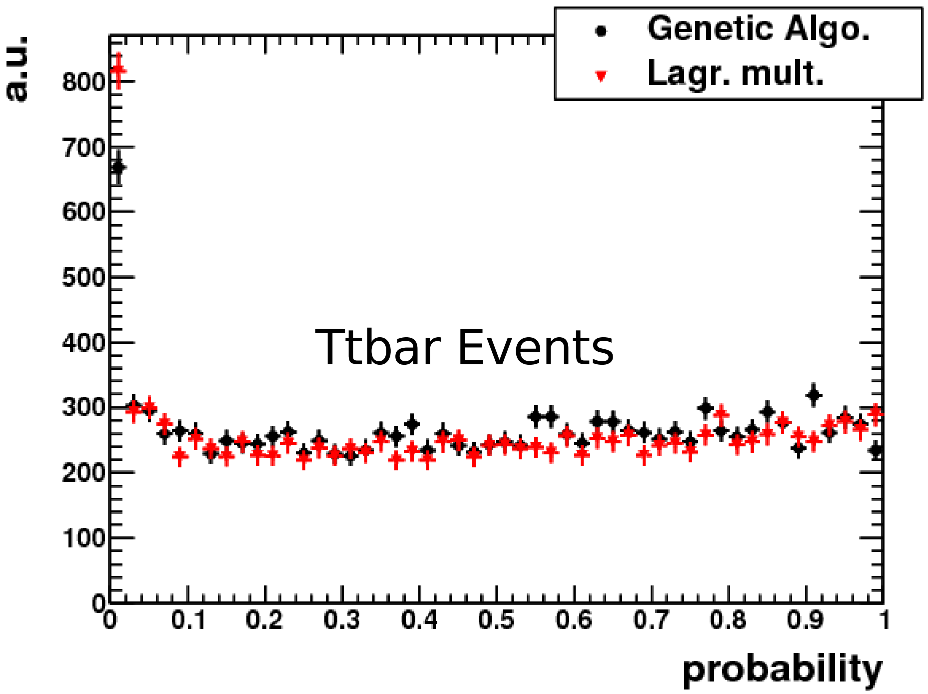
- Find minimum of the combined expression

$$S' = S + S_m + S_T$$

- Using a genetic algorithm for minimization
  - Natural treatment of combinatorics as additional 'gene'
  - No linearisation necessary
  - Very CPU intensive
- Tests show comparable performance to LM method on semi-leptonic ttbar events.

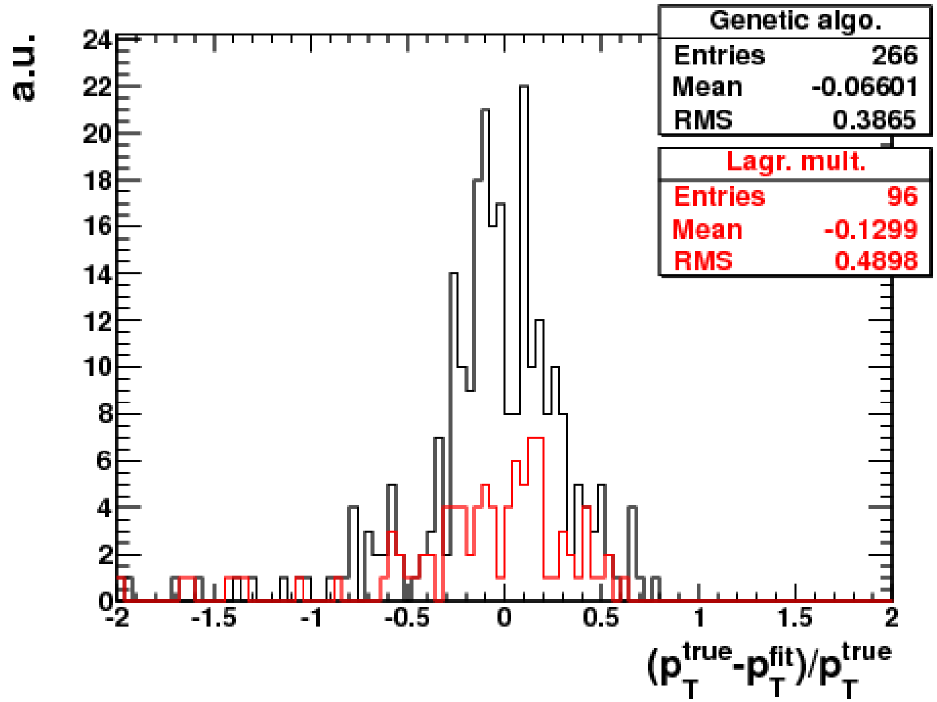
Test of Genetic Algorithm

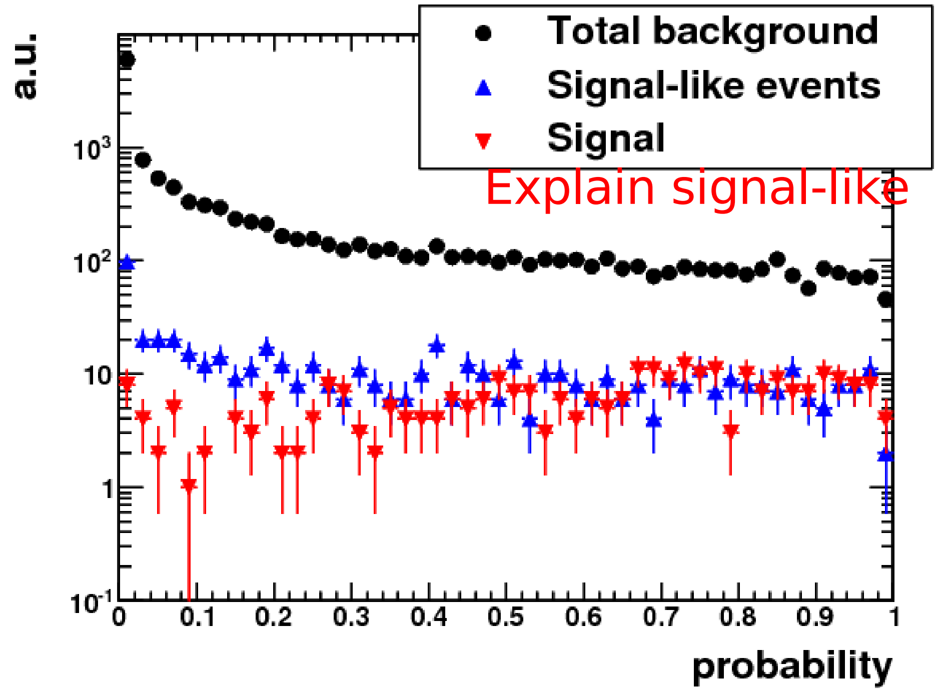
- Test on semileptonic ttbar events
  - Pythia generated
  - Each final state parton smeared according to typical detector resolutions at ATLAS/CMS
  - No b-tagging → 12 possible jet configurations
  - 1 neutrino = 3 unmeasured parameters
  - 6 constraints (p_x, p_y, 2xM_W and 2xM_{top})
- Comparable to fit using Lagrangain Multiplier



Fit of Correct Mass Hypothesis

- Test fit with correct mass hypotheses
  - LSP momenta well reconstructed
- Correct jet combination has best χ^2 in 45% of the events





- Fit probability distribution
  - flat for signal (slighty shifted toward higher values by combinatorics)
  - Background peaks at lower values: cut on 0.1(0.3) improves B/S from 33 to 11(8)

Angular Distributions

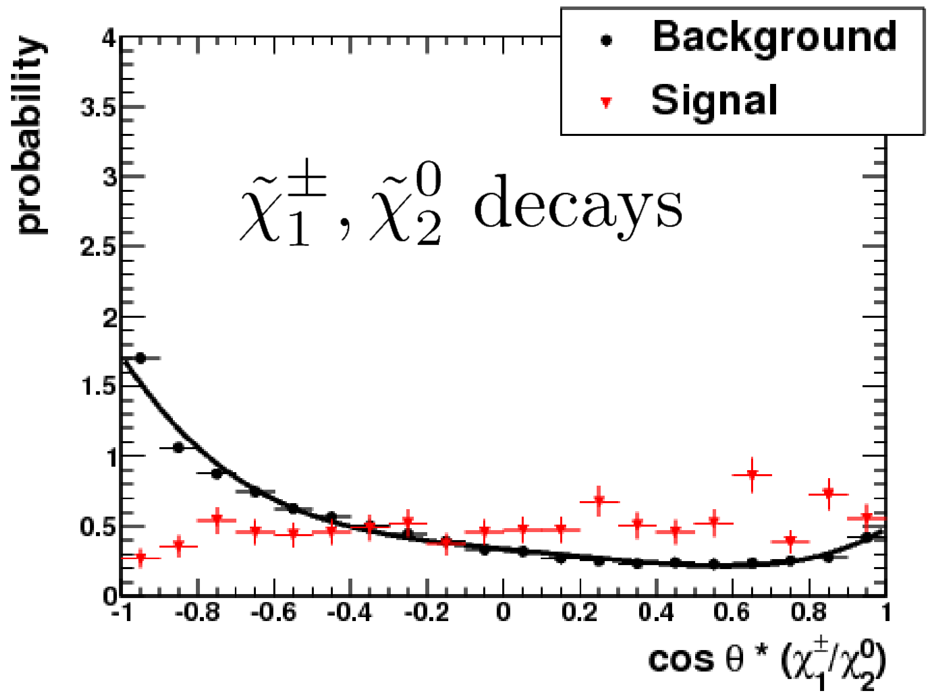
- Use more kinematic information to reduce backgrounds
- Angular distribution in rest frame of SUSY particles:
  - cos θ* of decay products w.r.tr flight direction of decaying particle should be ~isotropic (for spin 0)
  - cos θ* for typical background 4-vector configurations are not uniformly distributed smaller angles preferred

• Use likelihood ratio to incorporate angular information:

$$LR = \frac{1}{C} \prod_{k=1}^{N_{decays}} \frac{f_k^{signal}}{f_k^{signal} + f_k^{bg}}$$

• Use combined likelihood for classification

$$\mathcal{L} = p \cdot LR$$



Mass Scan Hadronic Channel

- Fixed gluino and neutralino mass
- Vary two remaining mass (squark, chargino)

$$\log \mathcal{L} = \sum_i^N \log \mathcal{L}_i$$

$$\text{with } \mathcal{L}_i = \mathcal{L}_{\text{cut}} \text{ for} \mathcal{L}_i < \mathcal{L}_{\text{cut}}$$

- Good correspondance between likelihood and true squark mass
- Bias in chargino mass
  - Reduces with improved momentum balance