



Constraints on Quantum Gravity

Hirosi Ooguri

Caltech & Kavli IPMU

Hamburg Colloquium, 6 November 2018

First, an advertisement:



Last month, I became the Director of the Kavli IPMU, and we started a long term strategic planning exercise. We welcome new ideas to address fundamental questions on the universe.

What I would like to tell you today:

- ☆ Why has the unification of general relativity and quantum mechanics been difficult?
- \Leftrightarrow Why is superstring theory important?
- ☆ What is the holographic principle?
- ☆ What are known and not known about quantum gravity ?

Why has the unification of general reativity and quamtum mechanics been difficult?

It is often said that, since the Einstein theory,

$$S = \int d^4x \sqrt{-g} \left(\Lambda + \frac{1}{G_N} R + \text{matter} \right)$$

is *not renormalizable*, we cannot use it to compute quantum gravity effects reliably.

This is not the whole story.

For example, the pion theory in nuclear physics,

$$S = \int d^{4}x \ \frac{(2\pi(x))^{2}}{1 + \pi(x)^{2}/F^{2}}$$

is also not renormalizable.

Neverthelss, we can used it to study phenomena whose energy scale is less than $F [\sim 184 \text{ MeV}]$, and compute their quantum effects reliably.

In the pion theory,

$$S = \int d^{4}x \ \frac{(2\pi(x))^{2}}{1 + \pi(x)^{2}/F^{2}}$$

we can expand observable quantities in powers of [*energy/F*] and [*momenta/F*].

Each term in the perturbative expansion can be calculated systematically by renomalizing finite number of parameters.

Wilsonian View:

The pion theory is a low energy approximation to QCD. It can be derived by performing QCD functional integral while freezing the low energy pion degrees of freedom.

\Rightarrow Effective Theory

Despite being non-renormalizable, low energy predictions including quantum effects can be made; they have been verified experimentally.

Einstein gravity is also an effective theory

As in the pion theory, the Einstein gravity can be used to make reliable predictions including quantum effects, provided energy and momenta are much less than its cutoff scale (threshold above which a more fundamental theory is required).

For example : ☆ Hawking radiation from a black hole

☆ Cosmic microwave background fluctions caused by quantum effects during the inflation

 \Leftrightarrow Corrections to the newton potential

$$V = -\frac{G_{N}m_{1}m_{2}}{r} \left(1 - \frac{G_{N}(m_{1} + m_{2})}{r} - \frac{135}{30\pi^{2}} \frac{G_{N}h}{r^{2}} + \cdots\right)$$

relativity effect

one-loop

Issues:

[1] In relativity, energy cutoff depents on observers.Does the Wilsonian approach really work in gravity?

⇐ Black hole firewall paradox, Swampland

[2] The pion theory can be UV-completed by QCD, which is a consistent quantum theory.

Is the Einstein gravity with any matter fields in low energy guaranteed to have a UV completion?

[3] The asymptotic freedom of strong interaction was not expected in the pion theory.

Many interesting phenomena specific to quantum gravity, such as a fate of an evaporating black hole, top-down derivation of inflation models, and the initial singularity of the Universe, cannot be addressed by the Einstein gravity. [2] The pion theory can be UV-completed by QCD.

Is the Einstein gravity guaranteed to have a UV completion?

Existence Theorem of Consistent Quantum Gravity

☆ In superstring theory, one can compute quantum effects without UV cuttoff to all order in the perturbative expansion. The AdS/CFT gives its non-pertrubative completion.

 \bigstar Its low energy effective theory contains the Einstein gravity.

 \Rightarrow By compactifying the spacetime dimensions to 4, we find:

- several generations of quarks and leptons
- gauge interactions including SU(3) x SU(2) x U(1)
- Higgs mechanism

The theory contains all ingredients for the Standard Model.

[2] The pion theory can be UV-completed by QCD.

Is the Einstein gravity guaranteed to have a UV completion?

However, we will see later in this colloquium that **not all low energy theories of gravity have UV-completion.**

Swampland

This is contrary to what we know about theories without gravity.

Another special feature of quantum gravity:

The physical world is hierarchical. Historically, exploration of shorter distances led us to more fundamental laws of nature.

This hierarchy of scales will terminates once we complete quantum gravity.

Quantum gravity will lead us to the ultimate unification of elementary particles and forces.

Black Hole Paradox and Holographic Principle



Stephen Hawking (1942 - 2018)





 $S = \frac{1}{4G_{N}} \begin{pmatrix} Area & of \\ Event Horizon \end{pmatrix}$ Why is it proportional to the area?

Entropy is extensive.

Holographic Principle

Fundamental degrees of freedom for a region of spacetime are defined on the surface surrounding it.



The holographic principle is realized in string theory.

To explain how the holographic principle is realized in string theory, we need some preparation:

To explain how the holographic principle is realized in string theory, we need some preparation:

D-Branes



Joseph Polchinski (1954 - 2018) To explain how the holographic principle is realized in string theory, we need some preparation:

There are closed strings $\left(\right)$ and open strings $\left(\right)$



To explain how the holographic principle is realized in string theory, we need some preparation:



We need to specify a sub-space on which open strings can end.





When open string end-points are located in a sub-space,

the sub-space can emit and absorb closed strings.





The graviton is also a closed string state. The fact that a sub-space can emit and absorb closed strings means that mass/energy is localized on the sub-space.



Since mass/energy is localized on the D-brane, it becomes a black hole/brane if the gravity is strong.

By quantizing open strings on the D-brane and by analyzing its Hilbert space, one can count black hole microstates.

In cases when this calculation can be done exactly,

$$S = \frac{1}{4G_N} \left(\begin{array}{c} Area & of \\ Event & Horizon \end{array} \right)$$
 has been reproduced.





Physical phenomena on the event horizon of a black hole can be described by quantum theory of **open strings** on the corresponding D-brane.

- \Rightarrow Quantum theory of open strings does not contain gravity.
- ⇒ Gravitational phenomena can be described by the non-gravitational theory, localized on the horizon.

AdS/CFT correpsondence:

Gravitational theory in anti-de Sitter space (AdS) is equivalent to conformal field theory (CFT) at the boundary.





The evaporation of a black hole by the Hawking radiation can be described by a unitary time evolution in CFT. (*In principle,* it provides a solution to the information paradox.) The AdS/CFT correspondence defines a consistent quantum theory of gravity including non-perturbative effects.

HOLOGRAPHIC QUANTUM MATTER

SEAN A. HARTNOLL, ANDREW LUCAS, AND SUBIR SACHDEV





There are important applications to condensed matter physics and hadron physics, but we will not discuss them today.

Instead, let me discuss new insights into quantum gravity provided by the holographic principle.

Quantum Entanglement and Emergence of Spacetime

MAY 15, 1935

PHYSICAL REVIEW

Quantum Entanglement

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, Institute for Advanced Study, Princeton, New Jersey (Received March 25, 1935)

$$\mathcal{H}_{A} \otimes \mathcal{H}_{B} \text{ with } \mathcal{H}_{A} = \{ 10 \rangle_{A}, 11 \rangle_{A} \}, \mathcal{H}_{B} = \{ 10 \rangle_{B}, 11 \rangle_{B} \}$$

$$\left\{ \begin{array}{c} 10 \rangle_{A} \mid 0 \rangle_{B} \quad : \text{ no entanglement} \\ 1EPR \rangle = \frac{1}{\sqrt{2}} \left(10 \rangle_{A} \mid 0 \rangle_{B} + 11 \rangle_{A} \mid 1 \rangle_{B} \right) \quad : \underset{entangled}{\text{maximally}}$$

Quantum
Entanglement

$$\mathcal{H}_{A} \otimes \mathcal{H}_{B} \qquad \text{with} \qquad \mathcal{H}_{A} = \{ 10\rangle_{A}, 11\rangle_{A} \}, \quad \mathcal{H}_{B} = \{ 10\rangle_{B}, 11\rangle_{B} \}$$

 $\left\{ 10\rangle_{A}, 10\rangle_{B} : \text{ no entanglement} \\ |EPR\rangle = \frac{1}{\sqrt{2}} (10\rangle_{A}, 10\rangle_{B} + 11\rangle_{A} |1\rangle_{B}) : \begin{array}{c} \text{maximally} \\ \text{entangled} \end{array} \right\}$

Entanglement entropy: quantifying the entanglement

$$\begin{aligned} \Psi > \in \mathcal{H}_{A} \otimes \mathcal{H}_{B} , \text{ partial trace } \mathcal{P}_{A} = \operatorname{tr}_{\mathcal{H}_{B}} (14 > < \Psi 1) \\ S(1\Psi >) = -\operatorname{tr}_{\mathcal{H}_{A}} (\mathcal{P}_{A} \log_{2} \mathcal{P}_{A}) \\ \uparrow \\ \\ \underset{\text{in A and B.}}{\uparrow} = \begin{cases} 0 & (1\Psi > = 10 >_{A} 10 >_{B}) \\ 1 & (1\Psi > = 1 \in \mathbb{PR} >) \end{cases} \end{aligned}$$

MAY 15, 1935

PHYSICAL REVIEW

Quantum Entanglement

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, Institute for Advanced Study, Princeton, New Jersey (Received March 25, 1935)

Entanglement entropy: quantifying the entanglement

$$\begin{split} | \Psi \rangle \in \mathcal{H}_{A} \otimes \mathcal{H}_{B} , \text{ partial trace } \mathcal{P}_{A} &= \operatorname{tr}_{\mathcal{H}_{B}} (|\Psi \rangle \langle \Psi|) \\ S(|\Psi \rangle) &= -\operatorname{tr}_{\mathcal{H}_{A}} (\mathcal{P}_{A} \log_{2} \mathcal{P}_{A}) \\ &= \left\{ \begin{array}{c} 0 & (|\Psi \rangle = |0\rangle_{A} |0\rangle_{B} \\ 1 & (|\Psi \rangle = |EPR\rangle \end{array} \right\} \end{split}$$

How many EPR pairs can be extracted from $|\psi
angle$

AdS/CFT correspondence:

Gravitational theory in AdS is equivalent to CFT at the boundary.



AdS/CFT correspondence:

Gravitational theory in AdS is equivalent to CFT at the boundary.





For a given choice of a sub-region A and its complement \overline{A} of the Cauchy surface, we can decompose the total Hilbert space of CFT into a direct product of Hilbert spaces associate to A and \overline{A} .

AdS/CFT correspondence:

Gravitational theory in AdS is equivalent to CFT at the boundary.





For a given choice of a sub-region **A** and its complex of the Cauchy surface, we can device the total Hilbert space of CFT into a cirect product of Hilbert spaces associate to **A** and **A**.

We need to say the right set of words such as Tomita-Takesaki (富田-竹崎) .

Split Property

QFT has the split property on $\int_{-\infty}^{\infty}$ if there is a type I factor \mathcal{N} (von Neumann algebra with trivial center) for every nested open subregions \mathcal{R} and \mathcal{R} 'such that: $\mathcal{A}[\mathcal{R}] \subset \mathcal{N} \subset \mathcal{A}[\mathcal{R}']$

with a trivial center

This does not hold for the pure Maxwell theory with $\int = \int x \int d^{-2}$




For a given choice of a sub-region A and its complement A of the Cauchy surface, we can decompose the total Hilbert space of CFT into a direct product of Hilbert spaces associate to A and A.

To measure entanglement between

$$A$$
 and \overline{A} for 14>,
 $P(14>) = t_{H_{\overline{A}}}(14><41)$ partial trace
 $S(14>) = -t_{H_{\overline{A}}}(P\log_{e} P)$ entanglement
 $entropy$

Ryu-Takayanagi Formula for Entanglement Entropy

PRL 96, 181602 (2006)

PHYSICAL REVIEW LETTERS

week ending 12 MAY 2006

Holographic Derivation of Entanglement Entropy from the anti-de Sitter Space/Conformal Field Theory Correspondence

Shinsei Ryu and Tadashi Takayanagi Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA (Received 8 March 2006; published 9 May 2006)

$$g(|\psi\rangle) = t_{\mathcal{H}_{\overline{A}}}(|\psi\rangle\langle\psi|)$$

$$S(14>) = -t_{\mathcal{H}_{A}}(Plog_{e}P)$$



 $= \frac{1}{4G_N} \left(\begin{array}{c} Area & of minimum \\ surface & subtending \\ \end{array} \right)$



Finite temparature state can be regarded as an entangled state:

Thermo Field Double:
$$|TFD\rangle \sim \sum_{i} e^{-\frac{E_{i}}{2kT}} |i\rangle_{A} |i\rangle_{B}$$

 $m_{\mathcal{H}_{B}}|TFD\rangle\langle TFD| \sim \sum_{i} e^{-\frac{E_{i}}{kT}} |i\rangle_{A} \langle i|_{A}$

Finite temperature state can be regarded as an entangled state.

$$\sum_{i} e^{\frac{E_{i}}{2kT} |i\rangle_{A} |i\rangle_{B}}$$



In AdS gravity, a finite temperature state can be interpreted as an eternal black hole (with two asymptotic AdS regions).

The strength of the entanglement (i.e., the **number of EPR pairs**) is proportional to the **size of the Einstein-Rosen bridge**.

The strength of the entanglement (i.e., the **number of EPR pairs**) is proportional to the **size of the Einstein-Rosen bridge**.

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47 Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? A. EINSTEIN, B. PODOLSKY AND N. ROSEN, Institute for Advanced Study, Princeton, New Jersey (Received March 25, 1935)

JULY 1, 1935

PHYSICAL REVIEW

VOLUME 48

The Particle Problem in the General Theory of Relativity

A. EINSTEIN AND N. ROSEN, Institute for Advanced Study, Princeton (Received May 8, 1935)

Fortschr. Phys. 61, No. 9, 781-811 (2013) / DOI 10.1002/prop.201300020

$\mathbf{ER} = \mathbf{EPR}$?

Cool horizons for entangled black holes

Juan Maldacena 1,* and Leonard Susskind 2

¹ Institute for Advanced Study, Princeton, NJ 08540, USA

² Stanford Institute for Theoretical Physics and Department of Physics, Stanford University, Stanford, CA 94305-4060, USA

Consider the shaded sub-region bounded by **A** on the boundary and the Ryu-Takayanagi surface **RT** subtending it.



Quantum gravity operator localized in the **shaded region in AdS** can be represented by an operator acting on the sub-region **A of CFT**.

Hamilton, Kabat, Lifschytz, Lowe: hep-th/0606141 Papadodimas, Raju: 1310.6335 Headrick, Hubeny, Lawrence, Rangamani: 1408.6300 Almheiri, Dong, Harlow: 1411.7041, Dong, Harlow, Wall: 1601.05416



Quantum gravity operator localized in the **shaded region in AdS** can be represented by an operator acting on the sub-region **A of CFT**.

 \Rightarrow Reconstruction Paradox



Different operators acting on different sub-spaces of CFT correspond to the same operator in AdS: **uniqueness?**



A local operator in AdS commutes with every local operator in CFT: **contradicting its basic axioms?**

Relation to Quantum Error Correcting Codes



Almheiri, Dong, Harlow: 1411.7041 Harlow: 1607.03901

Local excitations of the gravitational theory in AdS correspond to states with a special type of entanglement in CFT similar to the one used for **quantum error correcting codes**, where different sub-spaces of CFT share **quantum secret keys**.

Applications of Quantum Information to Gravitational Theory

Swampland Question

Given an effective theory of gravity, how can one judge whether it is realized as a low energy appropximation to a consistent quantum theory with **ultra-violet completion**, such as string theory?

> Vafa: hep-th/0509212; Vafa + HO: hep-th/0605264

Constraints on Symmetry

Symmetry has played important roles in physics: in identifying and formulating fundamental laws of nature and

in using these laws to understand and predict dynamics and phases of matters.

Symmetry can be deceiving:

Two seemingly different microscopic Lagrangians with **different gauge symmetries** and different matter contents **can describe the same quantum system.**

"Duality"

Equivalencen can be between full Hilbert spaces or about their low energy limits (such as in the Seiberg dualities). Symmetry can be deceiving:

Global symmetry is well-defined and is independent of which Lagrangian description you use.

Symmetry can be deceiving:

Global symmetry is well-defined and is independent of which Lagrangian description you use.

However, it has been argued that a **consistent quantum theory of gravity does not have global symmetry**.

Standard argument for



No global symmetry in quantum gravity:

If there is a continuous global symmetry G, we can combine a large number of G-charge matters to make a **black hole in an arbitrary large representations of G**.

Let it Hawking-radiate, keeping its mass > the Planck mass.

Since the Hawking radiation is G-blind, the black hole still contains the large representation of G with the number of states **exceeding the Bekenstein-Hawking entropy** formula.

(1) Any **global symmetry** in AdS is **inconsistent** with locality of CFT.

(2) A compact (discrete or continuous) symmetry G in CFT corresponds to a **gauge symmetry** with the same G in AdS.

(3) In a gravitational theory with gauge group G, there must be physical states in every finite dimensional irreducible unitary representation in G.

+ with some additional assumption:(4) Internal global symmetry of CFT is compact.

We need to define what we mean by symmetry.

Global Symmetry

Standard definition:

For every element g of group G, there is a unitary operator U(g) on the Hilbert space such that,

$$U(g_1) U(g_2) = U(g_1 \cdot g_2),$$

 $[U(g), Hamiltonian] = 0.$

How about the projection operator onto the 42th eigenstates?

We would like to make additional **locality** assumptions.



Global Symmetry

We sharpen our requirements:

- (1) Symmetry should map a local operator to a local operator.
- (2) Symmetry action should be faithful on the set of local operators.
- (3) Symmetry should commute with the energy-momentum tensor.

Global Symmetry

We sharpen our requirements:

- (1) Symmetry should map a local operator to a local operator.
- (2) Symmetry action should be faithful on the set of local operators.
- (3) Symmetry should commute with the energy-momentum tensor.
- (4) For a set of open disjoint subspaces of the Cauchy surface:

(4)
$$U(g, \bigcup_{i} R_{i}) = \prod_{i} U(g, R_{i})$$

For continuous symmetry, $(1) + (2) + (3) \Rightarrow (4)$ by the Noether theorem. More generally, (4) hold unders the split property assumption.

[Buchholz-Duplicher-Lungo: Ann. Phys. 170 (1989) 1]

Basic idea : If U : unitary on
$$\mathcal{H} = \bigotimes_{i} \mathcal{H}_{i}^{\perp}$$

s.t. for $\forall \mathcal{O}$ acting non-trivially only on \mathcal{H}_{i}^{\perp} ,
 $U^{\dagger} \mathcal{O} U$ also acts non-trivially only on \mathcal{H}_{i}^{\perp} ,
 R_{z}

$$R_{z}$$

$$R_{z}$$

$$R_{z}$$

$$R_{z}$$

$$Gauge theory can also have the split property
by adding degrees of freedom in UV.$$

In the following, we will apply the entangement wedge reconstruction.



Hamilton, Kabat, Lifschytz, Lowe: hep-th/0606141 Papadodimas, Raju: 1310.6335 Headrick, Hubeny, Lawrence, Rangamani: 1408.6300 Almheiri, Dong, Harlow: 1411.7041, Dong, Harlow, Wall: 1601.05416 Global symmetry in AdS is inconsistent with local structure of CFT.

If a gravitational theory in AdS has global symmetry G, there must be a bulk local operator that transforms faithfully into another local operator at the same point. Global symmetry in AdS is inconsistent with local structure of CFT.

If a gravitational theory in AdS has global symmetry G, there must be a bulk local operator that transforms faithfully into another local operator at the same point.



Symmetry generator,

$$U(g) = \pi U(g, \mathcal{R}_{i})$$

commute with the local operator at x in the bulk.

Contradiction

Global symmetry in AdS is inconsistent with local structure of CFT.



The argumetn also **works** for discrete spacetime symmetry, *P* & *T*.

This means that we need to sum over non-orientable manifolds.

The argument does **not work** for 2d gravity since the holographic dual is CFT defined on a point.

For example, the string worldsheet is a 2d gravity. It can have global symmetry, and we can choose not to sum over non-orientable surfaces (e.g., type II string theory).

Weak Gravity Conjecture

In any low energy theory described by the Einstein gravity + Maxwell field + finite number of matters, if it has an UV completion as a consistent quantum theory, there must be a particle with charge Q and mass $m \ll M$ _Planck, such that:

$$m \leq \frac{|Q|}{\sqrt{G}}$$
, G: Newton constant

Arkani-Hamed, Motl, Nicolis, Vafa: hep-th/0601001

 $a = (m, Q) \quad s.t. \quad m \leq \frac{|Q|}{\sqrt{G}}$

Motivated by:

(1) Black Hole Physics: Extremal black holes should decay unless protected by supersymmetry.

Otherwise, charged black holes can decay to Planck-size remnants with entropies, exceeding the Bekenstein-Hawking bound.

(2) True in all known constructions from string theory.

(3) Holography

In all cases,

$$m < \frac{\Omega}{\sqrt{G}}$$
 (no "=") unless BPS

If this sharpened weak gravity conjecture is true, **non-SUSY AdS** supported by fluxed **must be unstable**.

Vafa + H.O.: 1610.1533

All known non-SUSY AdS's are marginally stable at best, and some of them are unstable in interesting ways.

Example: AdS5 x S5 / Γ in IIB:

Supersymmetry is broken when Γ does not fit in SU(3).

★ If *I* has a fixed point or S5 is small, there is a tachyon violating the BF bound. Dymarsky, Klebanov, Roiban: 0509132

★ If I has no fixed point and S5 is large, there is Witten's instanton, creating a bubble of nothing.



Witten (1982) Horowitz, Orgera, Polchinski: 0709.4262

The bulk geometry terminates with S1 collapsing.

Supersymmetry is broken.

Though the fundamental group of CP3 is trivial (and thus, there is no Witten's instanton), the geometry allows a generalization of Witten's instanton where a 2-sphere collapses.



The bulk geometry terminates with S2 collapsing.

Spodyneiko + H.O.: 1703.03105



Standard Model of Particle Physics gives rise to a rich landscape of stable dS and AdS vacua in 2 and 3 dimensions upon compactification, depending on types (Majorana or Dirac) of neutrinos and their masses.

Arkani-Hamed Dubovsky, Nicolis, Villadoro: hep-th/0703067

We pointed out that the sharpened weak gravity conjecture would **rule out certain types and masses of neutrinos** if they give rise to stable non-supersymmetric AdS_3.

Vafa + H.O.: 1610.1533

Our idea has been explored further in recent papers, leading to constraints on particle physics models beyond the Standard Model.

Ibanez, Martin-Lozano, Valenzuela: 1706.05392,1707.05811; Hamada, Shiu: 1707.06326; Gonzalo, Herraez, Ibanez: 1803.08455

Distance Conjecture

- (1) The space of scalar fields has finite volume but infinite diameter. You can start at any point and go indefinitely long distance.
- (2) If you go the order of M_planck away from the starting point, a tower of new particles with exponentially small masses emerge, and the original low energy effective theory breaks down.



Vafa + H.O.: hep-th/0605264

Distance Conjecture

For a circle compactification, large radius => KK modes, small radius => winding. The distance conjecture requires extended objects.

It has been tested extensively in Calabi-Yau compactifications.

It is proven when with N > 2 supersymmetry in 4d.

It is false for non-gravitational systems.

In some cases, related to the weak gravity conjecture for p = -1 form charge.

Landscape of Swampland Conditions



The UV/IR connection may imply surprising IR predictions on observable phenomena from UV completion of quantum gravity.



Collaborations with quantum information may provide a key to quantum gravity.



Thank you