Bounds on a Fourth Generation of Quarks Using Unitarity Constraints

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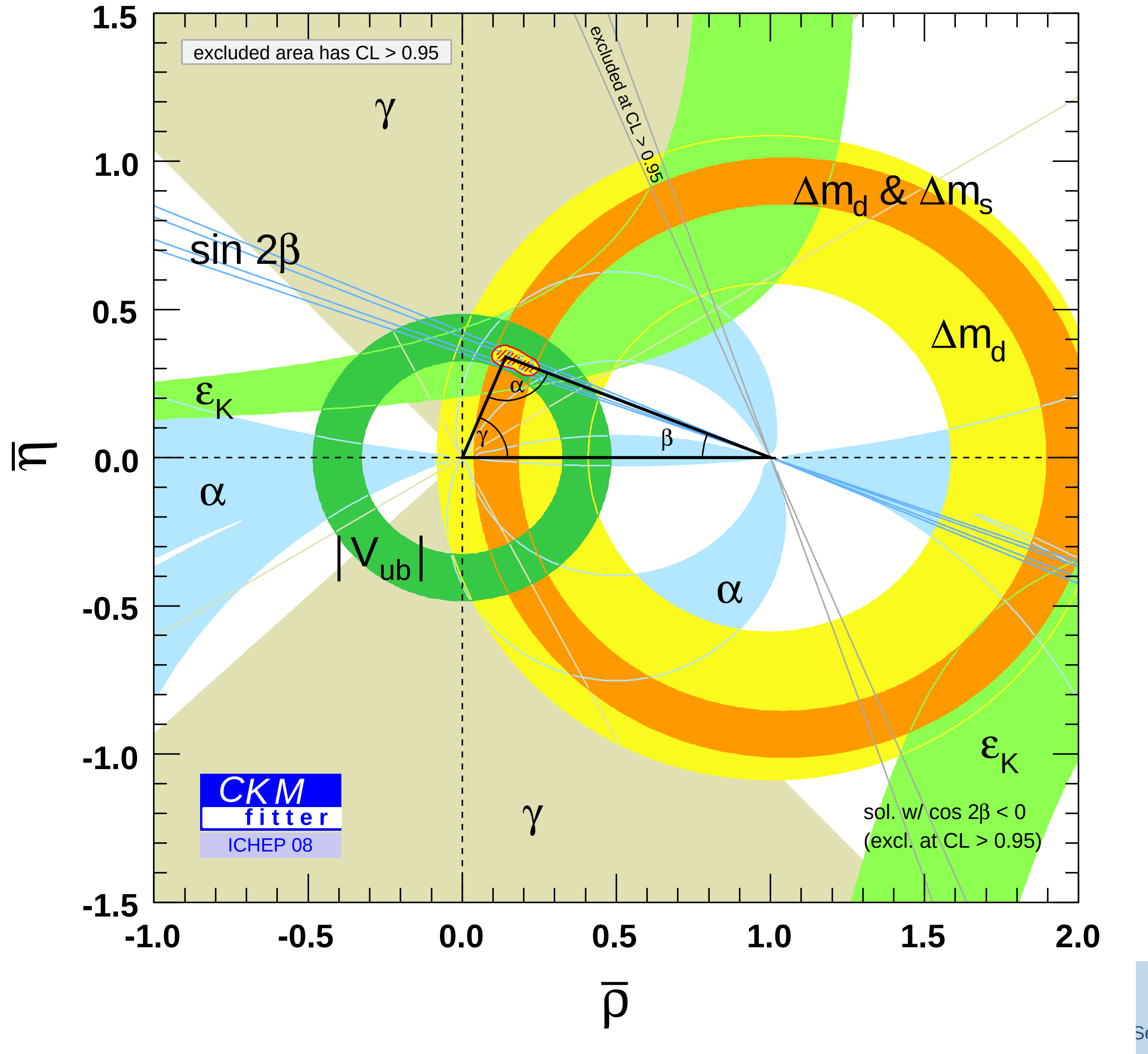
Success of the SM - CKM mechanism

Elegant description of Nature at per mille precision

- **CKM-Mechanism NP 2008**

How can CP-violation be incorporated in the SM?
1972 only u,d and s-Quarks were known, Kobayashi and Maskawa postulated six quarks!

Success of the SM - CKM mechanism



Sexy Extensions of the Standard Model

SUPERSYMMETRY

SUPERSTRING

Warped **Extra** **Dimensions**

Grand Unified Theories

Lee-Wick

Technicolor

Unparticles

Quantum Gravity

Little Higgs **Littlest Higgs**

The most boring Extension of the SM I

Add one generation of fermions: **SM3 $\longrightarrow$ SM4**

The most boring Extension of the SM II

What Wikipedia tells us:

“Possibility of a fourth generation:

Within the Standard Model, fourth and further generations have been ruled out by theoretical considerations. Some of the arguments against the possibility of a fourth generation are based on the subtle modifications of precision electroweak observables that extra generations would induce; such modifications are strongly disfavored by measurements. Furthermore, a fourth generation with a light neutrino (one with a mass less than about 40 GeV/c2) has been ruled out by measurements of the widths of the Z boson (LEP, CERN). Nonetheless, searches at high-energy colliders for particles from a fourth generation continue, but as yet no evidence has been observed. In such searches, fourth-generation particles are denoted by the same symbols as third-generation ones with an added prime (e.g. b' and t'). Given the unlikeliness of any such particles being discovered, no other names have been seriously proposed.”

What Constraints exist really for SM4?

- Anomaly cancellation: full family
- LEP: 4th Neutrino heavier than 45 GeV
- Cosmology/DM: Heavy particles should not be stable
- Direct search at TeVatron:

$$m_{b'} > 268\,\text{GeV} \qquad\qquad m_{t'} > 311\,\text{GeV}$$

- Electroweak precision: Kribs, Plen, Spannowsky, Tait; PRD 76, 075016 (2007)
  A lot of space, e.g. $m_{t'} - m_{b'} \geq 50$ GeV for $M_H > 115$ GeV
- CKM-Fits agree well $\Rightarrow$ small couplings to 4th family?

$\Rightarrow$ **Seems not yet ruled out!!!**

Mixing with the 4th Family I

in collaboration with

Markus Bobrowski, Johann Riedl, Jürgen Rohrwild;

arXiv:0902.4883, PRD; arXiv:0904.3971

As in any extension of the SM:
more parameters appear

Add a complete 4th family (b', t', l^{-}', ν') $\Rightarrow$ new parameters:

- Quark masses: 2
- Lepton masses: 2
- V_{CKM4}: 3 angles + 2 phases
- V_{PMNS4}: 3 angles + 2 phases + Majorana-phases

$\Rightarrow$ at least 14 new parameters

"Rome was not built in a day" : Start with bounds on V_{CKM4}

Mixing with the 4th Family II

The general form of V_{CKM4} reads

$$V_{CKM4} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix}$$

- What tree-level constraints do we have?
- What can we say about the mixing with the 4th family, if we assume V_{CKM4} to be unitary?

Mixing with the 4th Family III

Tree-level constraints

$$
\begin{array}{lllll}
V_{ud} & = & 0.97418 & \pm & 0.00027 & \text{Nuclear Beta decay} \\
V_{us} & = & 0.2255 & \pm & 0.0019 & \text{Semileptonic K-decay} \\
V_{ub} & = & 0.00393 & \pm & 0.00036 & \text{Semileptonic B-decay} \\
 & & & & & \\
V_{cd} & = & 0.230 & \pm & 0.011 & \text{Semileptonic D-decay} \\
V_{cs} & = & 1.04 & \pm & 0.06 & \text{Semi- /Leptonic D-decay} \\
V_{cb} & = & 0.0412 & \pm & 0.0011 & \text{Semileptonic B-decay} \\
 & & & & & \\
V_{tb} & > & 0.74 & & & \text{Single Top-production}
\end{array}
$$

Mixing with the 4th Family IV

From the unitarity of V_{CKM4} one gets ($\lambda := V_{us} = 0.2255$)

$$
\begin{aligned}
|V_{ub'}|^2 &= 0.0001 \pm 0.0014 \\
&\Rightarrow \text{Error: } 0.037 \approx 0.74 \cdot \lambda^2 \approx 3.3 \cdot \lambda^3 \\
|V_{td}|^2 + |V_{t'd}|^2 &= -0.0020 \pm 0.0055 \\
&\Rightarrow \text{Error: } 0.074 \propto 1.5 \cdot \lambda^2 \\
|V_{ts}|^2 + |V_{t's}|^2 &= -0.13 \pm 0.13 \\
&\Rightarrow \text{Error: } 0.36 \approx 1.6 \cdot \lambda^1 \\
|V_{cb'}|^2 &= -0.14 \pm 0.18 \\
&\Rightarrow \text{Error: } 0.42 \approx 1.9 \cdot \lambda^1 \\
|V_{t'b}|^2 &< 0.45 \\
&\Rightarrow |V_{t'b}| < 0.67 = 0.67 \cdot \lambda^0
\end{aligned}
$$

Mixing with the 4th Family V

Strategy: Follow Wolfenstein and Buras, Lautenbacher, Ostermaier

- Start with a suitable general parametrization of V_{CKM}
- Perform a Taylor expansion in λ
- Get more than 1861/292 citations

There are many exact parametrizations of V_{CKM4} in the literature

We use the one by Fritzsch and Plankl

- First line easy - well measured
- Last row easy - Taylor expansion
- For $4 \to 3$ this reduces to the familiar standard parametrization of V_{CKM3}

Mixing with the 4th Family VI

We have now the following parameters:

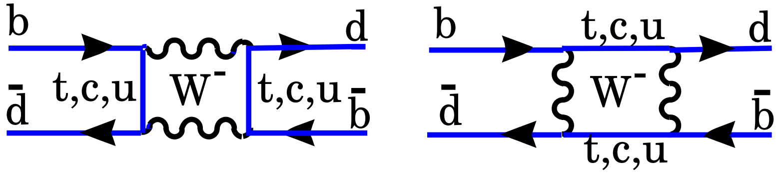
- The angles $\theta_{12}, \theta_{13}, \theta_{23}, \theta_{14}, \theta_{24}, \theta_{34}$ with $s_{ij} := \sin\theta_{ij}$, $c_{ij} := \cos\theta_{ij}$
- The CP-violating phases $\delta_{13}, \delta_{14}, \delta_{24}$

$$V_{CKM4} =$$

$$\begin{pmatrix}
c_{12}c_{13}c_{14} & c_{13}c_{14}s_{12} & c_{14}s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\
\begin{matrix} -c_{23}c_{24}s_{12} - c_{12}c_{24}s_{13}s_{23}e^{i\delta_{13}} \\ -c_{12}c_{13}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} \end{matrix} & \begin{matrix} c_{12}c_{23}c_{24} - c_{24}s_{12}s_{13}s_{23}e^{i\delta_{13}} \\ -c_{13}s_{12}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} \end{matrix} & \begin{matrix} c_{13}c_{24}s_{23} \\ -s_{13}s_{14}s_{24}e^{-i(\delta_{13}+\delta_{24}-\delta_{14})} \end{matrix} & c_{14}s_{24}e^{-i\delta_{24}} \\
\begin{matrix} -c_{12}c_{23}c_{34}s_{13}e^{i\delta_{13}} + c_{34}s_{12}s_{23} \\ -c_{12}c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}} \\ +c_{23}s_{12}s_{24}s_{34}e^{i\delta_{24}} \\ +c_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} \end{matrix} & \begin{matrix} -c_{12}c_{34}s_{23} - c_{23}c_{34}s_{12}s_{13}e^{i\delta_{13}} \\ -c_{12}c_{23}s_{24}s_{34}e^{i\delta_{24}} \\ -c_{13}c_{24}s_{12}s_{14}s_{34}e^{i\delta_{14}} \\ +s_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} \end{matrix} & \begin{matrix} c_{13}c_{23}c_{34} \\ -c_{13}s_{23}s_{24}s_{34}e^{i\delta_{24}} \\ -c_{24}s_{13}s_{14}s_{34}e^{i(\delta_{14}-\delta_{13})} \end{matrix} & c_{14}c_{24}s_{34} \\
\begin{matrix} -c_{12}c_{13}c_{24}c_{34}s_{14}e^{i\delta_{14}} \\ +c_{12}c_{23}s_{13}s_{34}e^{i\delta_{13}} \\ +c_{23}c_{34}s_{12}s_{24}e^{i\delta_{24}} - s_{12}s_{23}s_{34} \\ +c_{12}c_{34}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} \end{matrix} & \begin{matrix} -c_{12}c_{23}c_{34}s_{24}e^{i\delta_{24}} + c_{12}s_{23}s_{34} \\ -c_{13}c_{24}c_{34}s_{12}s_{14}e^{i\delta_{14}} \\ +c_{23}s_{12}s_{13}s_{34}e^{i\delta_{13}} \\ +c_{34}s_{12}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} \end{matrix} & \begin{matrix} -c_{13}c_{23}s_{34} \\ -c_{13}c_{34}s_{23}s_{24}e^{i\delta_{24}} \\ -c_{24}c_{34}s_{13}s_{14}e^{i(\delta_{14}-\delta_{13})} \end{matrix} & c_{14}c_{24}c_{34}
\end{pmatrix}$$

Mixing with the 4th Family VII

Bounds from FCNC:



$$\Delta_X := \frac{X_{SM4}}{X_{SM3}}$$

- K-Mixing: $Re(\Delta_K) = 1 \pm 0.5\,(0.25)$ $Im(\Delta_K) = 0 \pm 0.3\,(0.15)$
- B_d-Mixing: $|\Delta_{B_d}| = 1 \pm 0.3\,(0.1)$ $Arg(\Delta_{B_d}) = 0 \pm 10^\circ\,(5^\circ)$
- B_s-Mixing: $|\Delta_{B_s}| = 1 \pm 0.3\,(0.1)$ $Arg(\Delta_{B_s}) =$ free
- $b \longrightarrow s\gamma$ $\Delta_{b\to s\gamma} = 1 \pm 0.15\,(0.07)$

Mixing with the 4th Family VIII

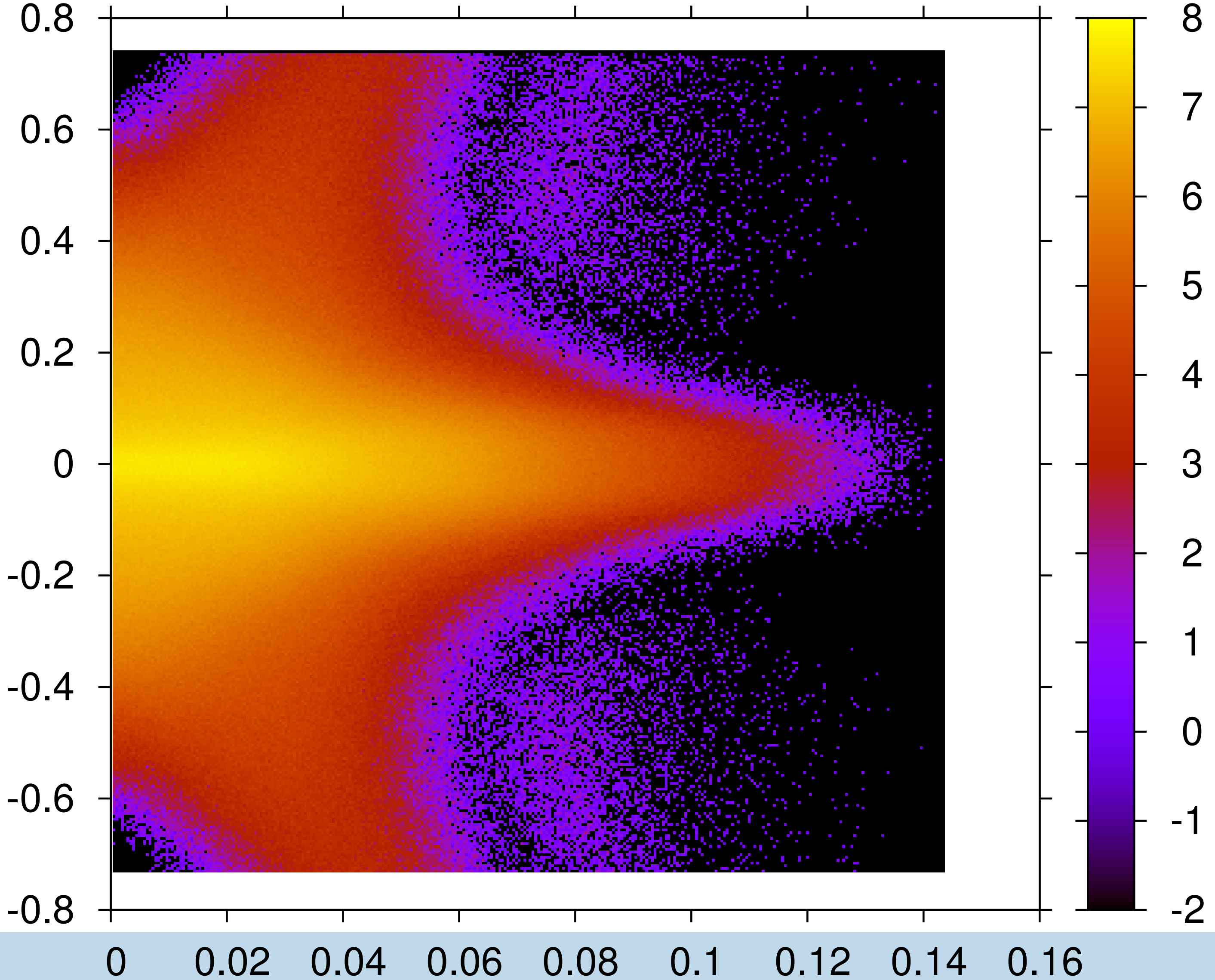
Strategy

1. Create randomly 10^{10} data points for V_{CKM4}
2. Check if tree level constraints are full-filled
3. Check if FCNC constraints are full-filled

$\Rightarrow 10^7 (10^5)$ data points survive

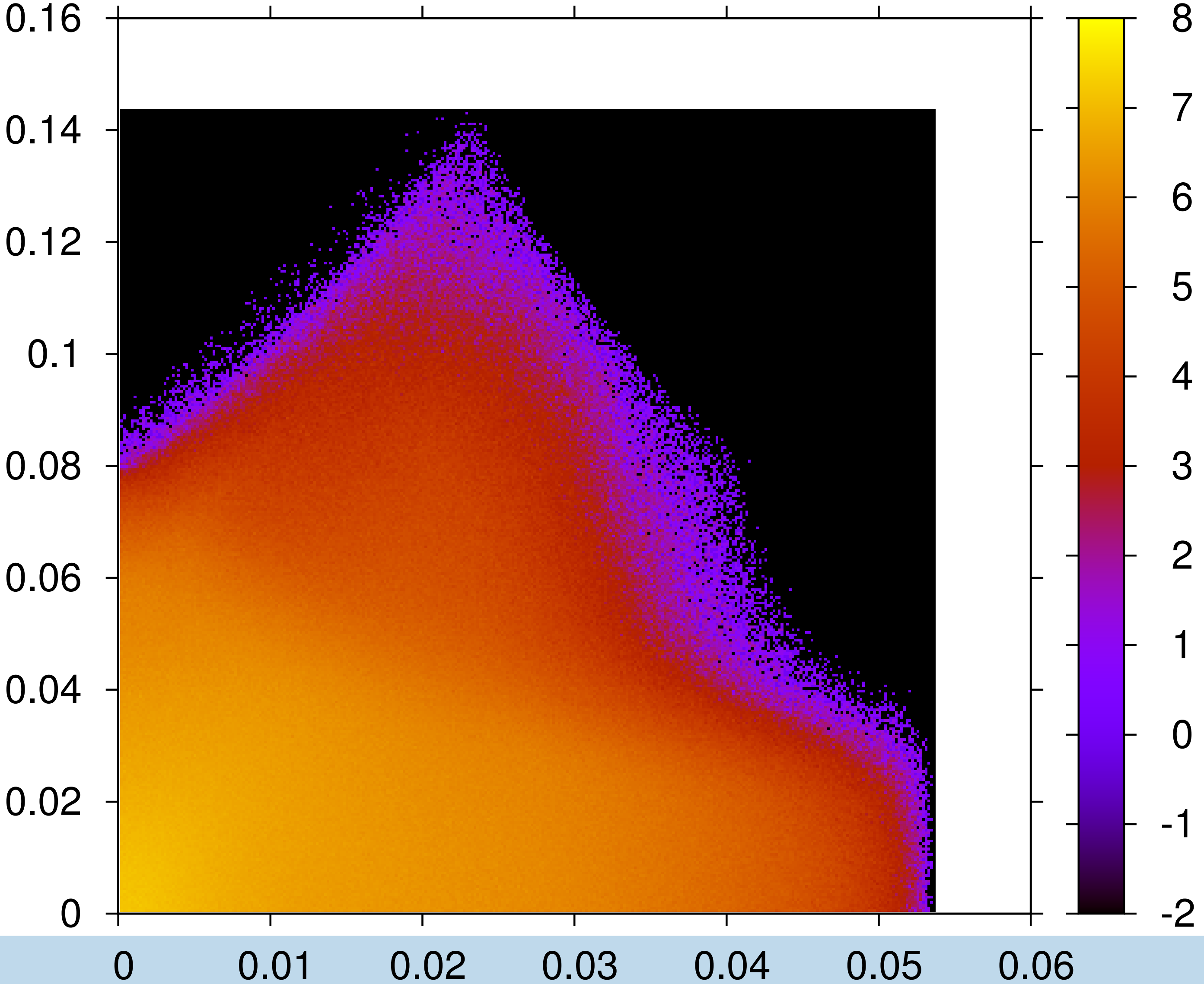
Mixing with the 4th Family IX

Allowed Parameters: θ_{34} vs. θ_{24}



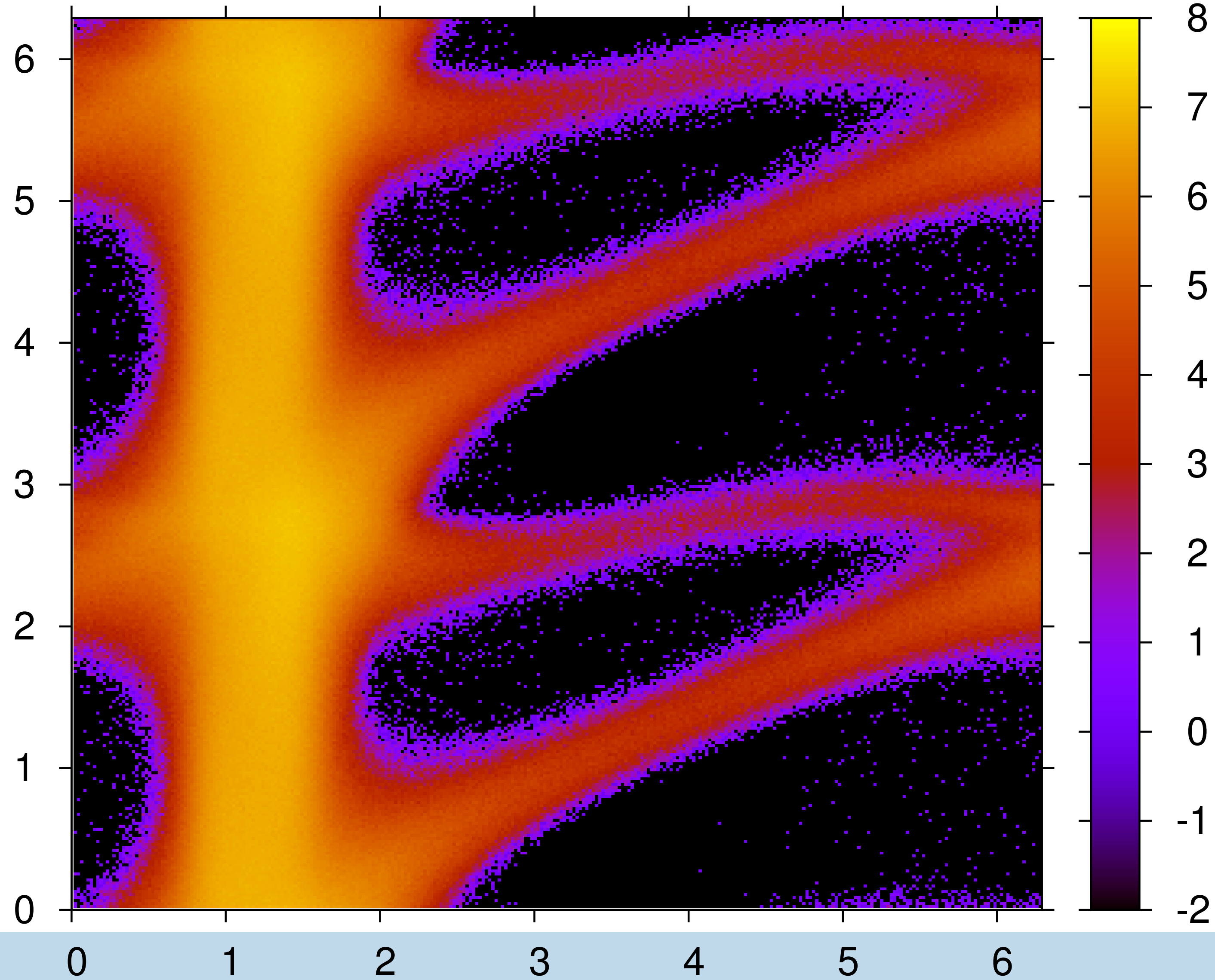
Mixing with the 4th Family X

Allowed Parameters: θ_{24} vs. θ_{14}



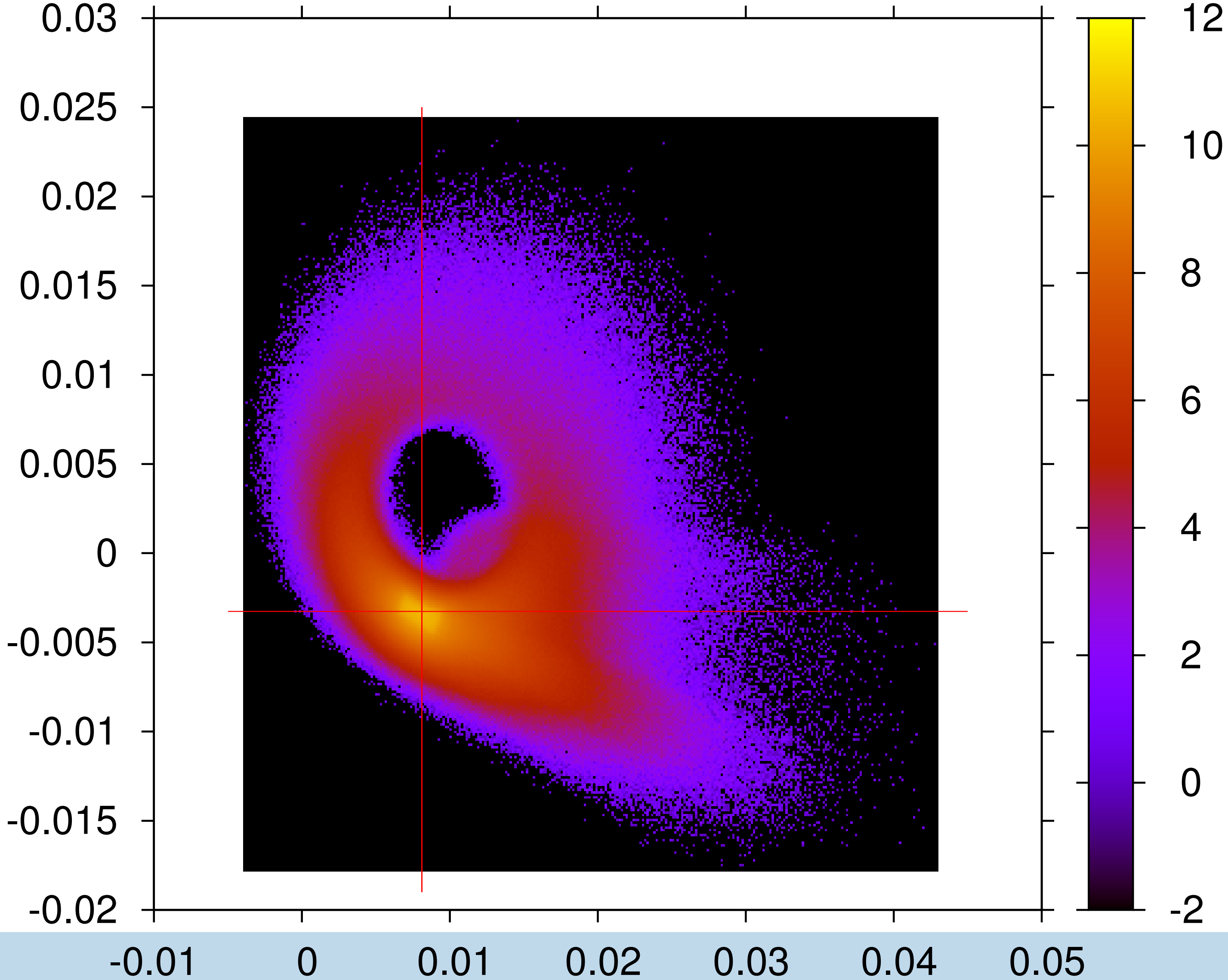
Mixing with the 4th Family XI

Allowed Parameters: δ_{14} vs. δ_{13}



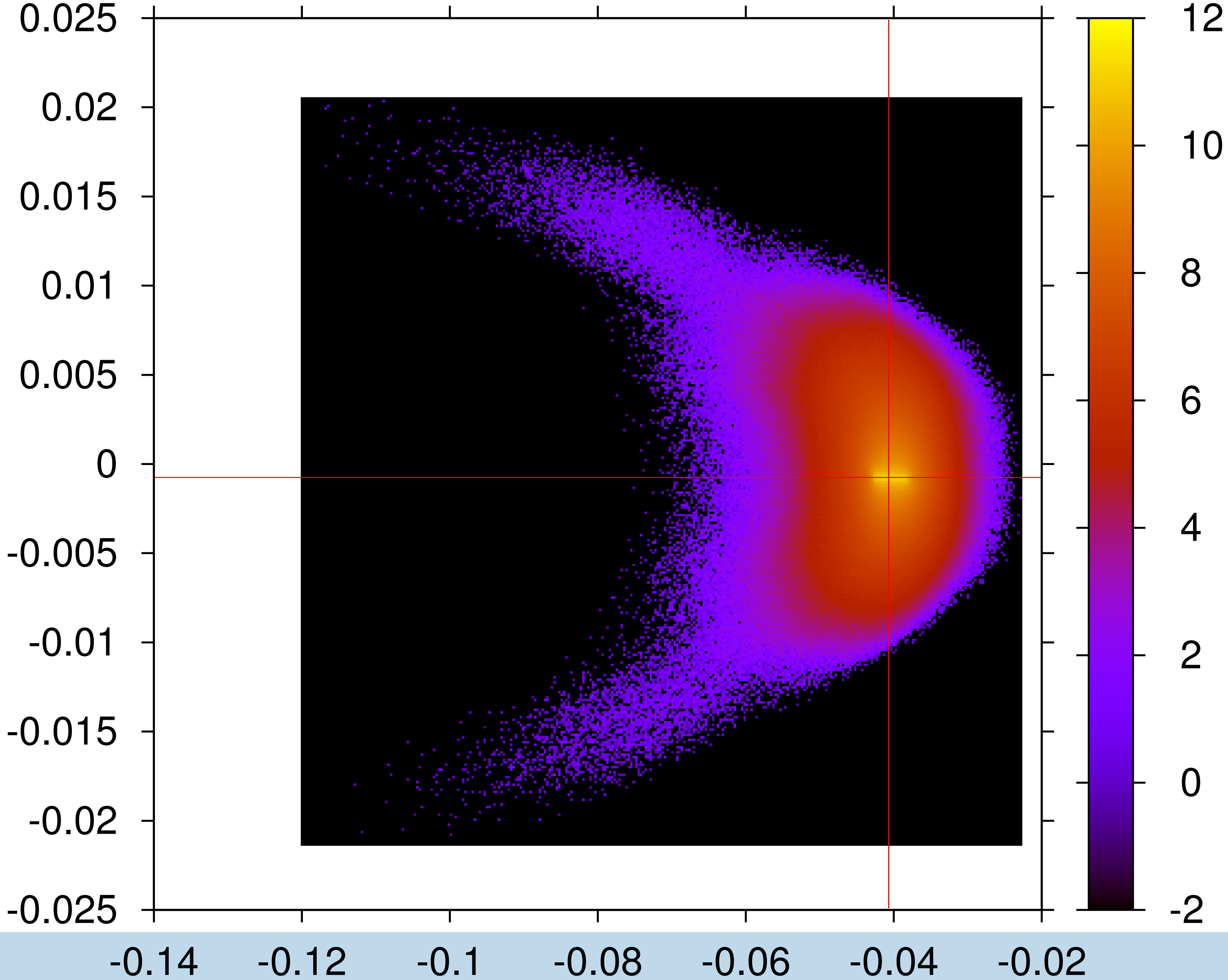
Mixing with the 4th Family XII

Allowed Parameters: Im V_{td} vs. Re V_{td}



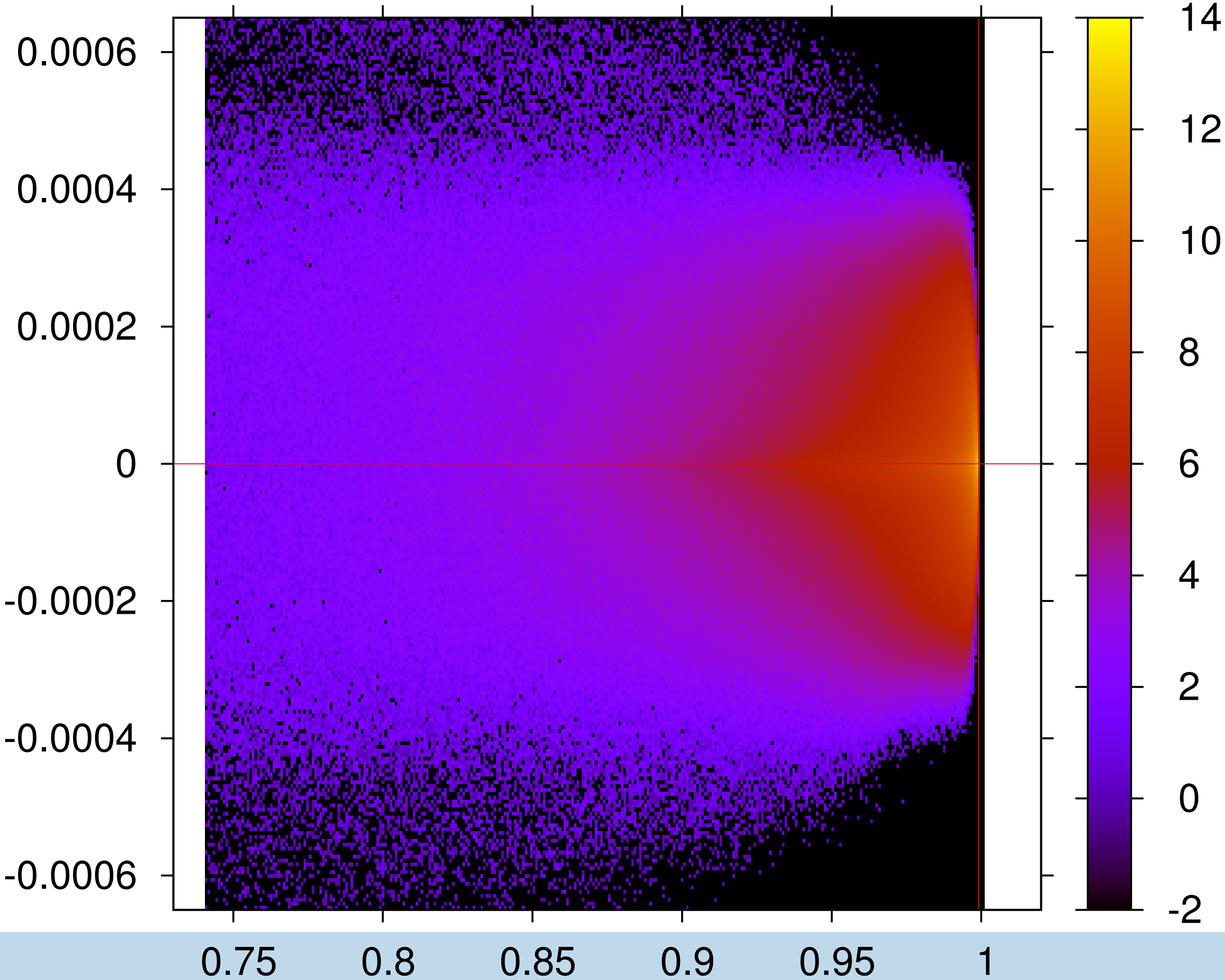
Mixing with the 4th Family XIII

Allowed Parameters: Im V_{ts} vs. Re V_{ts}



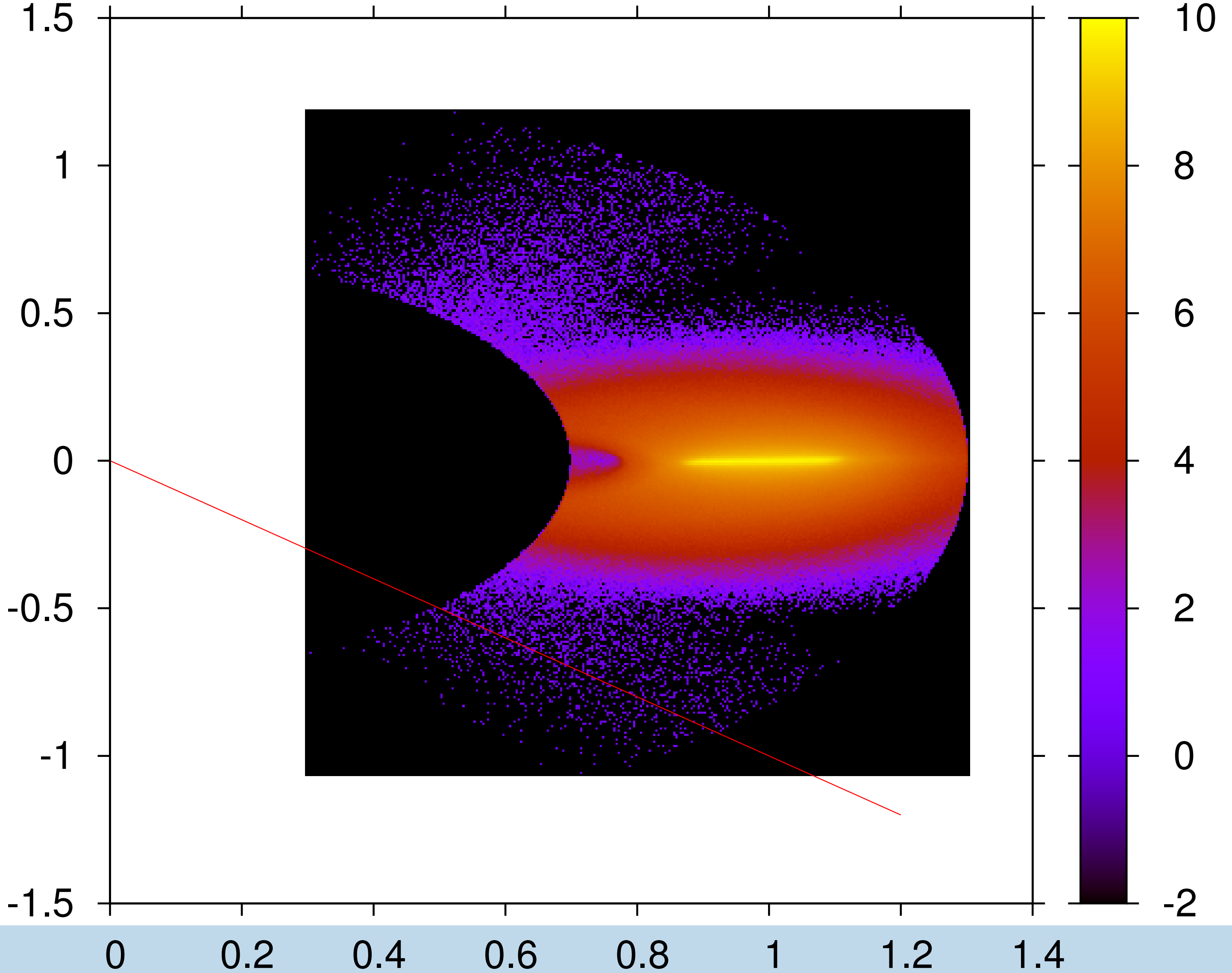
Mixing with the 4th Family XIV

Allowed Parameters: Im V_{tb} vs. Re V_{tb}



Mixing with the 4th Family XV

Allowed Parameters: Im Δ_{B_s} vs. Re Δ_{B_s}



Mixing with the 4th Family XVI

Allowed ranges

Ultraconservative

$$
\begin{aligned}
\theta_{14} &\leq 0.04 \approx 1.27\lambda^2 \\
\theta_{24} &\leq 0.25 \approx 0.9\lambda^1 \\
\theta_{34} &\leq 0.8 \approx 0.8\lambda^0 \\
\delta_{14} &\leq 2\pi \\
\delta_{24} &\leq 2\pi
\end{aligned}
$$

FCNC: no further constraint !

Mixing with the 4th Family XVII

Final Parametrization, Taylor Expansion

$$\begin{pmatrix}
1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A(\rho-i\eta)\lambda^4 & (x_{14}-iy_{14})\lambda^3 \\
-\lambda & \begin{array}{l} 1-\frac{\lambda^2}{2} \\ -\left(\frac{A^2}{2}+\frac{{x_{24}}^2+{y_{24}}^2}{2}+\frac{1}{8}\right)\lambda^4 \end{array} & A\lambda^2 & (x_{24}-iy_{24})\lambda^2 \\
\begin{array}{l} Ac_{34}\lambda^3 \\ -\left[x_{14}-x_{24}+i(y_{14}-y_{24})\right]s_{34}\lambda^3 \\ -Ac_{34}(i\eta+\rho)\lambda^4 \end{array} & \begin{array}{l} -Ac_{34}\tilde{\lambda}^2 \\ -(x_{24}+iy_{24})s_{34}\tilde{\lambda}^2 \\ -(x_{14}+iy_{14})s_{34}\lambda^4 \end{array} & \begin{array}{l} c_{34}\left(1-\frac{A^2}{2}\lambda^4\right) \\ -A\lambda^2(x_{24}+iy_{24})s_{34} \end{array} & s_{34}-\frac{s_{34}\lambda^4\left({x_{24}}^2+{y_{24}}^2\right)}{2} \\
\begin{array}{l} (-x_{14}-iy_{14}+x_{24}+iy_{24})c_{34}\lambda^3 \\ -As_{34}\lambda^3+A\lambda^4(\rho+i\eta)s_{34} \end{array} & \begin{array}{l} \left[As_{34}-(x_{24}+iy_{24})c_{34}\right]\tilde{\lambda}^2 \\ -(x_{14}+iy_{14})c_{34}\lambda^4 \end{array} & \begin{array}{l} -s_{34}\left(1-\frac{A^2\lambda^4}{2}\right) \\ -(x_{24}+iy_{24})c_{34}A\lambda^4 \end{array} & c_{34}-\frac{c_{34}\lambda^4\left({x_{24}}^2+{y_{24}}^2\right)}{2}
\end{pmatrix}$$

with

$$\tilde{\lambda}^2 := \lambda^2\left(1-\frac{\lambda^2}{2}\right)$$

Mixing with the 4th Family XVIII

Unexpected regions in the allowed parameter space

$$x_{14} = 0.8617, \qquad y_{14} = 0.8838$$
$$\theta_{24} = 0.08367, \qquad \theta_{34} = 0.5574, \quad \delta_{24} = 0.3149,$$
$$m_t = 160 \text{ GeV}, \qquad m_{t'} = 503.3 \text{ GeV}, \quad m_c = 1.2 \text{ GeV}$$

leads to

$$\begin{aligned} \Delta_K &= 1.012 + 0.139i \\ \Delta_{B_d} &= 0.718 - 0.040i = 0.72\, e^{i3.2^\circ} \\ \Delta_{B_s} &= 0.6393 - 0.5353i = 0.834\, e^{-i39.9^\circ} \\ \Delta_{b\to s\gamma} &= 1.041 \end{aligned}$$

and

$$\begin{aligned} |V_{td}| &= 0.012 & \text{vs.} \quad & 0.00874 \pm 0.0004 \\ |V_{ts}| &= 0.08 & \text{vs.} \quad & 0.0407 \pm 0.0010 \\ |V_{tb}| &= 0.84 & \text{vs.} \quad & 0.99913 \pm 0.0004 \end{aligned}$$

Mixing with the 4th Family XIX

Why is this not seen in CKM-Fits?

Mixing with the 4th Family XIX

Why is this not seen in CKM-Fits?

Nature might be nasty
Large Effects cancel and imitate the SM3 result

Mixing with the 4th Family XX

Mixing with the 4th Family XXI

Why is this not seen in CKM-Fits?

Split up the contributions as

$$\frac{M_{12}^{SM4}}{M_{12}^{SM3}} = 1 + \left(\frac{M_{12}^{t,VCKM4}}{M_{12}^{t,VCKM3}} - 1 \right) + \frac{M_{12}^{t',VCKM4}}{M_{12}^{t,VCKM3}}$$

With our previous example we obtain

$$\begin{aligned} \frac{M_{B_s,12}^{SM4}}{M_{B_s12}^{SM3}} &= 1 + (1.48304 - 0.986885I) + (-1.84369 + 0.451341I) \\ &= 0.6393 - 0.5353i = 0.834\, e^{-i39.9^\circ} \end{aligned}$$

Nature might be nasty
Large Effects cancel and imitate the SM3 result

SM predictions for Γ_{12} in D-mixing I

in collaboration with M. Bobrowski, J. Riedl, J. Rohrwild; arXiv:0904.3971

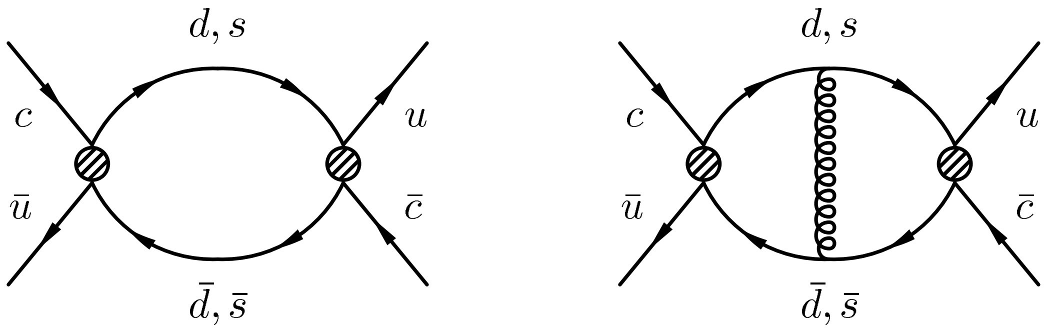
$$\Gamma_{12} = -\lambda_s^2 \Gamma_{ss} - \lambda_s \lambda_d \Gamma_{sd} - \lambda_d^2 \Gamma_{dd}$$

with $\lambda_d + \lambda_s + \lambda_b = 0$ and $\lambda_x = V_{cx} V_{ux}^*$.

If $\lambda_b \approx 0 \Rightarrow \Gamma_{12}$ real and vanishes in the SU(3)$_F$ limit

$$GIM cancellations: \Gamma_{12} = -\lambda_s^2 \left(\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}\right) + 2\lambda_s \lambda_b \left(\Gamma_{sd} - \Gamma_{dd}\right) - \lambda_b^2 \Gamma_{dd}$$

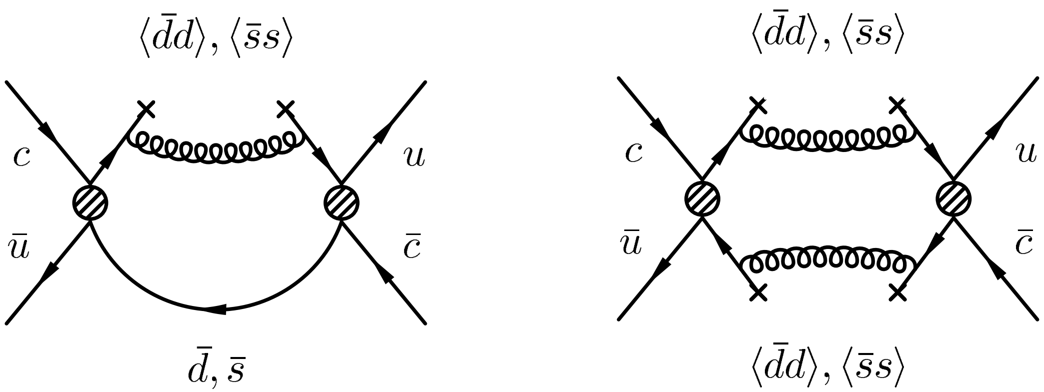
- not zero in SU(3)$_F$ limit



- $\Gamma_{12} \propto \lambda_s^2 m_s^6 / m_c^6 + 2\lambda_s \lambda_b m_s^2 / m_c^2 - \lambda_b^2 \; 1$
- $y_D \in [0.3, 1.5] \cdot 10^{-6} \Rightarrow$ much smaller than experiment ($7 \cdot 10^{-3}$), but large phase ($\mathcal{O}(1)$) possible

SM predictions for Γ_{12} in D-mixing II

Idea: higher orders in HQE might be dominant if GIM is less pronounced



naive expectation for a single diagram:

y_D	no GIM	with GIM
$D = 6, 7$	$2 \cdot 10^{-2}$	$5 \cdot 10^{-7}$
$D = 9$	$5 \cdot 10^{-4}$	???
$D = 12$	$2 \cdot 10^{-5}$	???

? Can one obtain $y_D^{Exp.}$?

?How big can ϕ be?

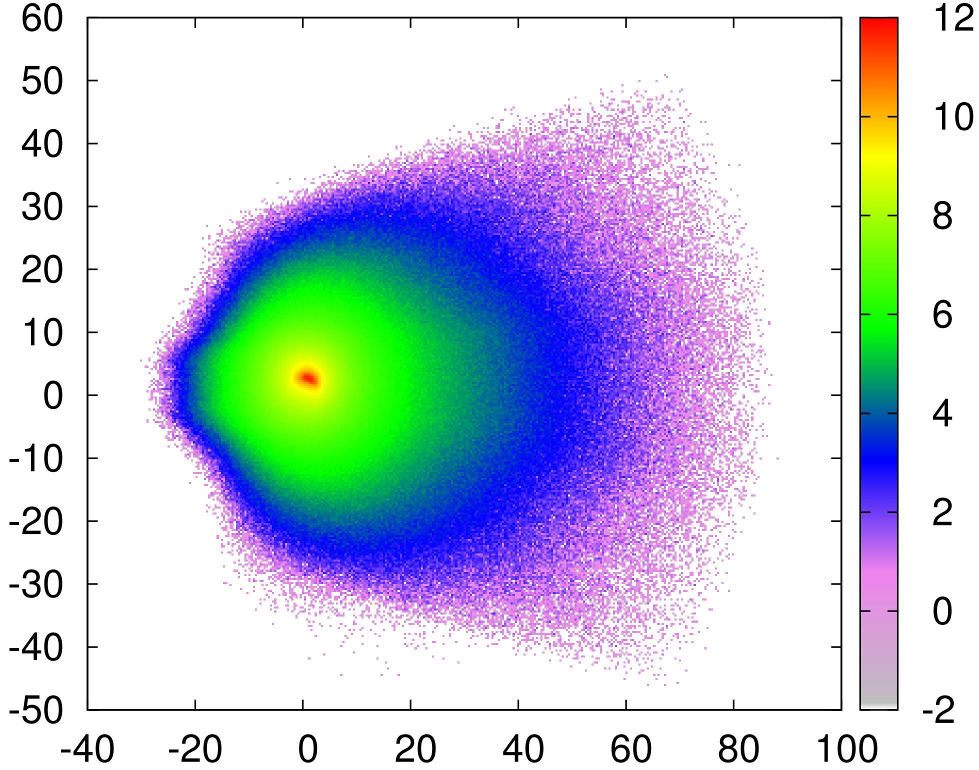
SM4 predictions for Γ_{12} in D-mixing

Overseen: Large Effects in Γ_{12} in D-mixing due to NP possible!

$$\Gamma_{12} = -\lambda_s^2 \left(\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}\right) + 2\lambda_s(\lambda_b + \lambda_{b'}) \left(\Gamma_{sd} - \Gamma_{dd}\right) - (\lambda_b + \lambda_{b'})^2 \Gamma_{dd}$$

$\lambda_b \propto \lambda^{5\ldots 6}$ - still possible $\lambda_{b'} \propto \lambda^3$

Im $\left(\Gamma_{12}^{SM4}/\Gamma_{12}^{SM3}\right)$



Re $\left(\Gamma_{12}^{SM4}/\Gamma_{12}^{SM3}\right)$

Conclusion

Main Messages:

1) “Manchmal ist mehr drin, als man denkt”

2) There is still a lot of space

for a

new generation

3) Nature might be nasty

Wishlist

What do we need to constrain V_{CKM4} further?

- More precise determination of V_{cd} and V_{cs} — Need e.g. df_{D_s}, $D \to K$ form factor
- More precise determination of V_{tb} — Single top production at TeVatron
- Tree level determination of V_{td} and V_{ts} possible?
- Precise determination of all mixing quantities

Theoretical work

- Include also e-weak precision observables
- Include NLO-QCD "exact"
- Investigate more flavor observables
- Investigate non-perturbative effects of large Yukawa couplings
- Investigate Unification of couplings
- Investigate Baryogenesis

Final Experimental work

- Find fourth family