Supersymmetry Enhancement

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General Things

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Where I come from

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My random walk

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Research Interests.

1. Swampland constraints from Quantum Gravity
2. F-Theory
3. Exact results in rigid supersymmetric QFTs

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2. F-Theory
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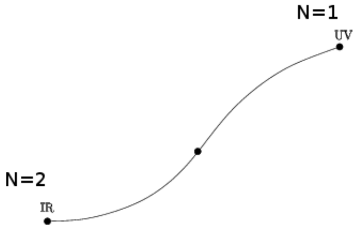
Supersymmetry enhancement.

- A UV QFT follows an RG flow to a IR QFT with more explicit supersymmetry.

A schematic picture.



Different approaches.

1. MS flows. $4d\ \mathcal{N}=1 \to \mathcal{N}=2$
2. BG flows. $3d\ \mathcal{N}=2 \to \mathcal{N}=4$
3. Ungauging quiver tails. (Gadde-Rastelli-Razamat).
4. $3d\ \mathcal{N}=1 \to \mathcal{N}=2$. (Komargodski).
5. $3d$ CS-Maxwell. (Yamazaki et al.)

Why do we care?

- Interesting phenomenon in QFT, a priori.
- Find new and unexpected RG flows.
- If the UV $\mathcal{N} = 1$ is lagrangian and the IR $\mathcal{N} = 2$ is not, you really have lagrangianized a non-lagrangian theory.
- Many studied 4d $\mathcal{N} = 2$ (like class-S) are believed to *never* admit a lagrangian. Is it true? Or people look under the lamppost of flows preserving the amount of SUSY?
- Use the UV description to compute quantities conserved along the RG flow, like the superconformal index.
- SUSY Enhancement $\Longleftrightarrow$ T-branes and geometry.

Maruyoshi-Song flows.

- Start in UV with a $4d\ \mathcal{N}=2$ SCFT $\mathcal{T}$ with flavor symmetry F.
- Add by hand a $\mathcal{N}=1$ chiral M.
- M is gauge singlet and in the adjoint of the F.
- Turn on a superpotential term $W_{def} = TrM\mu$ where μ is the moment map operator. ($\mu \simeq q\tilde{q}$ if $\mathcal{T}$ is lagrangian).
- Give a *special kind of vev* to M. This triggers a RG flow.
- Depending on the choice of $\mathcal{T}$ and $\langle M \rangle$ sometimes we find that $\mathcal{T}[\langle M \rangle]$ flows in the IR a *new* $\mathcal{N}=2$ SCFT: call it $\mathcal{T}[\langle M \rangle]_{IR}$.
- “New" means $\mathcal{T}[\langle M \rangle]_{IR} \neq \mathcal{T}$

Nilpotent orbit vev.

- M is a matrix in the adjoint of F.
- We choose $\langle M \rangle$ to be along a nilpotent orbit of $\mathfrak{f}_{\mathbb{C}}$
- Nilpotent orbit is defined as ghg^{-1} for $g \in F$ and $h \in \mathfrak{f}_{\mathcal{C}}$ nilpotent.
- Nilpotent orbits are *completely classified* by mathematicians.
- For a fixed $\mathcal{T}$, finite number of possible MS deformations.

Example:

$$\langle M \rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{1}$$

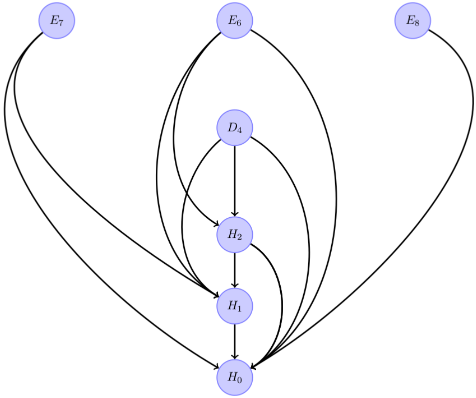
is nilpotent vev in the maximal nilpotent orbit of $\mathfrak{sl}_2$.

Evidence for the enhancement.

- a-maximization.
  1. $\mathcal{N}=2\ U(1)_r \times SU(2)_R$ R-symmetry is broken to $U(1)^{UV}_{\mathcal{N}=1} \times U(1)_{\mathcal{F}}$
  2. $U(1)_{\mathcal{N}=1} = U(1)^{UV}_{\mathcal{N}=1} + \epsilon\, U(1)_{\mathcal{F}}$ mixing of R-symmetry and flavor.
  3. $a(\epsilon)$ and $c(\epsilon)$ central charges depend on ϵ.
  4. $a(\epsilon)$ is extremal at the IR fixed point. $\implies$ find ϵ^*
  5. If a and c are rational, good evidence for enhancement.
  6. Often find a and c of previously known AD theories.

- The full Superconformal Index.
  1. Compute the UV $\mathcal{N}=1$ Index by SUSY localization.
  2. Upon finding the extremizing ϵ^*, shift the index fugacities
  3. Find the $\mathcal{N}=2$ index
  4. Check limits of the index against previously computed cases. (Shur, McDonald, Coulomb, HL, HS of 3d mirror CB)
  5. Huge evidence for enhancement.

Enhancing flows connecting rank 1 theories



A figure summarizing all the existent MS connecting rank one theories. Multiple flows happen for different nilpotent orbit deformations.

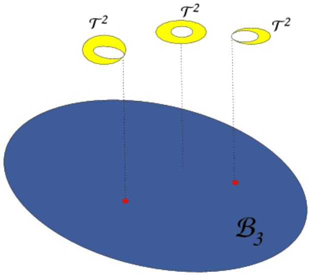
Open questions.

1. Why only some specific nilpotent vevs for M give Susy enhancement?
2. Can one find more examples of SCTs showing these features?
3. Can one understand in a more direct way why the enhancement happens, from a physical point of view?
4. Is there a way to see this phenomenon from a String Theory engineering of the QFT?

F-Theory introduction.

Vafa '96

- IIB Axiodilaton $\tau = C_0 + ie^{-\phi}$ as the complex structure of an elliptic curve, fibered over a 3-fold kahler base.
- Fibration is singular at the location of 7-branes.
- Codim $1 \implies$ gauge group.
- Codim $2 \implies$ matter curves. Where two 7-branes intersect.



We can put a D3 brane to probe the 7-brane.

In F-Theory this means the D3 brane probes the singularity of the fibration.

The geometrical picture. Part 1

- Consider a F-theory setup, with a D3 probing the elliptic fibration.
- Write the Weierstrass model for the elliptic fibration. The theory on the D3 will be the theory $\mathcal{T}$ in the UV. In particular the Weiestrass model fixes the flavor group F.
- Elliptic fiber $\simeq$ Seiberg-Witten curve of the QFT.
- The chiral M is geometrized by a T-brane deformation of the 7-brane stack. $\implies$ Nilpotent orbit + fluctuation.
- Ex. For the case in which $\langle M \rangle$ is in the maximal orbit of $\mathfrak{sl}_2$, we take a T-brane profile given by:

$$\langle \varphi \rangle = \begin{pmatrix} 0 & 1 \\ x & 0 \end{pmatrix} \tag{2}$$

The geometrical picture. Part 2

Singularity	Curve	Flavor group
II^*	$u^2 = v^3 + v(M_2 z^3 + M_8 z^2 + M_{14} z + M_{20}) + (z^5 + M_{12} z^3 + M_{18} z^2 + M_{24} z + M_{30})$	E_8
III^*	$u^2 = v^3 + v(z^3 + M_8 z + M_{12}) + (M_2 z^4 + M_6 z^3 + M_{10} z^2 + M_{14} z + M_{18})$	E_7
IV^*	$u^2 = v^3 + v(M_2 z^2 + M_5 z + M_8) + (z^4 + M_6 z^2 + M_9 z + M_{12})$	E_6
I_0^*	$u^2 = v^3 + v(\tau z^2 + M_2 z + M_4) + (z^3 + \tilde{M}_4 z + M_6)$	$SO(8)$
IV	$u^2 = v^3 + v(M_{1/2} z + M_2) + (z^2 + M_3)$	$SU(3)$
III	$u^2 = v^3 + vz + (M_{2/3} v + M_2)$	$SU(2)$
II	$u^2 = v^3 + v M_{4/5} + z$	no

Table: Maximally deformed Weierstrass models

- The parameters M_i are the versal deformations of the model
- They correspond to casimir operators of the Higgs field in the F-theory picture, which we take to be $\langle \varphi \rangle = \langle M \rangle + \delta M$
- $\langle M \rangle$ is fixed by the chosen nilpotent orbit. δM will be the highest-spin singlet appearing in the decomposition of Adj.

The geometrical picture. Part 3

- When the D3 probes the deformed Weiestrass model, the theory is $\mathcal{N} = 1$, due to the T-brane presence. Original K3 $\rightarrow$ CY3.
- RG flow is a local zoom at the singularity. In the IR the probe does not have enough energy to resolve global aspects of the singularity.
- In the IR, some terms in the Weiestrass become subleading. We throw them away and recover the Weierstrass for $\mathcal{T}^{IR}$
- Assume there is enhancement. Then CY3 $\sim$ K3 $\times \mathbb{C}$. Make an ansatz for which operator decouple, drop it from the Weierstrass. Check the ansatz. When consistent, gives the Weierstrass of $\mathcal{N} = 2$ IR theory. **Always**.

Thank you for your attention.