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NEW DEVELOPMENTS AND THEIR IMPACT ON PDF DETERMINATION

DESY Hamburg

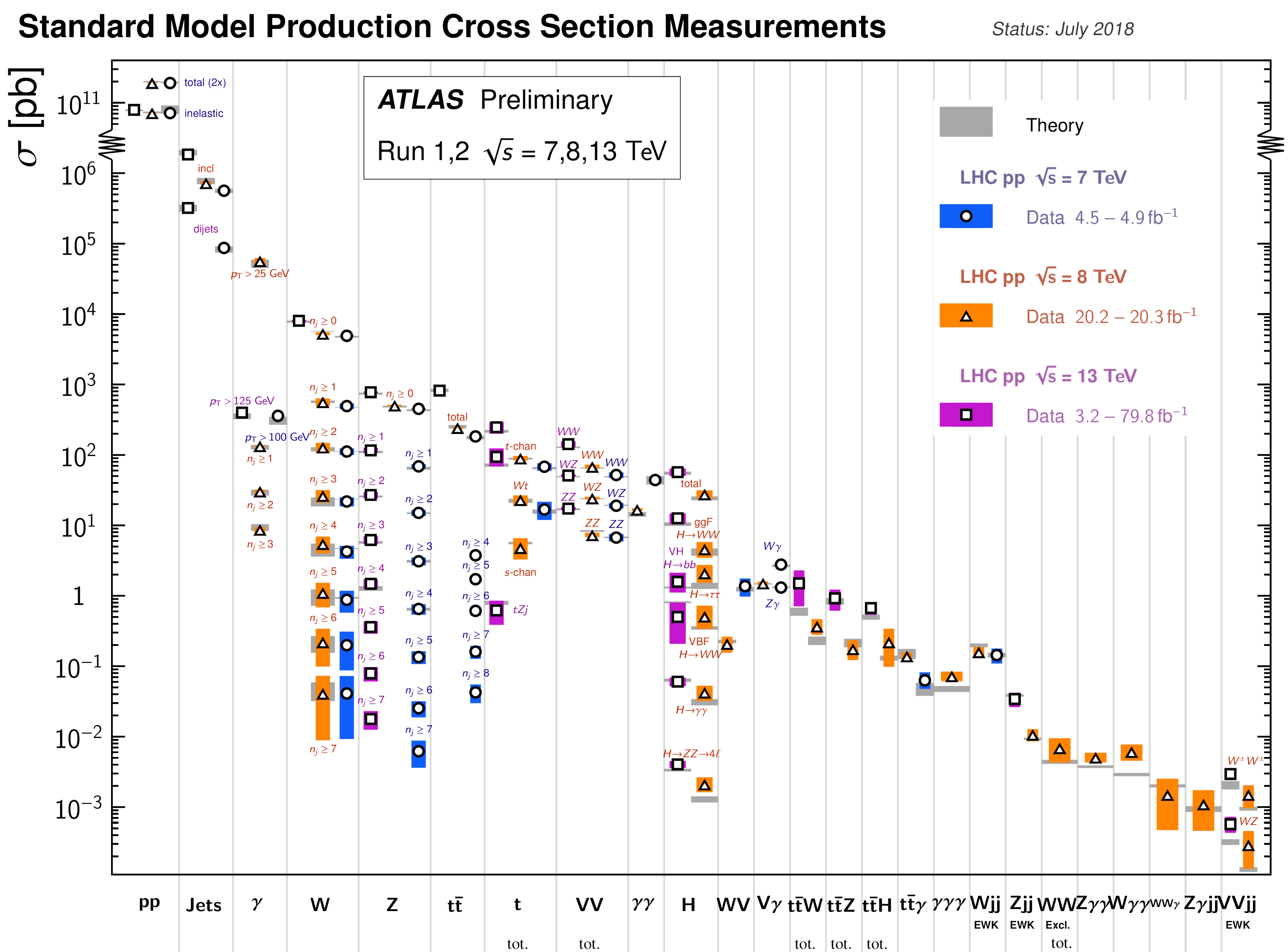
Wednesday 14th November 2018

OUTLINE

- An introduction to resummation
- Small-x resummation
- PDFs with small-*x* resummation:
  - Evidence for BFKL dynamics in inclusive HERA data
  - Small-x physics at the LHC and beyond
- Conclusion and Outlook

AN INTRODUCTION TO RESUMMATION

WHERE TO START?



- All theoretical expectations were calculated at NLO or higher

PERTURBATIVE QCD CALCULATIONS

- High-precision theoretical predictions needed for the LHC
- NLO calculations in QCD are now standard
- NNLO exists for an increasing number of processes
- NNNLO has been completed for Higgs

$$\sigma(x,Q^2) = \sigma_{\mathrm{LO}}(x,Q^2) + \alpha_s \sigma_{\mathrm{NLO}}(x,Q^2) + \alpha_s^2 \sigma_{\mathrm{NNLO}}(x,Q^2) + \alpha_s^3 \sigma_{\mathrm{NNNLO}}(x,Q^2) + \dots$$

$$x = \frac{Q^2}{S}$$

- Many observables at LHC characterised by multiple scales Q_i
- Multi-scale problems are affected by logarithmic corrections

$$\alpha_s^n \log^m \frac{Q_i^2}{Q_j^2}$$

WHERE DO LOGARITHMS COME FROM?

- Real emissions diagrams are singular for soft/collinear emissions
- These singularities are cancelled by virtual counterparts
- Finite logarithmic pieces are left over, e.g.

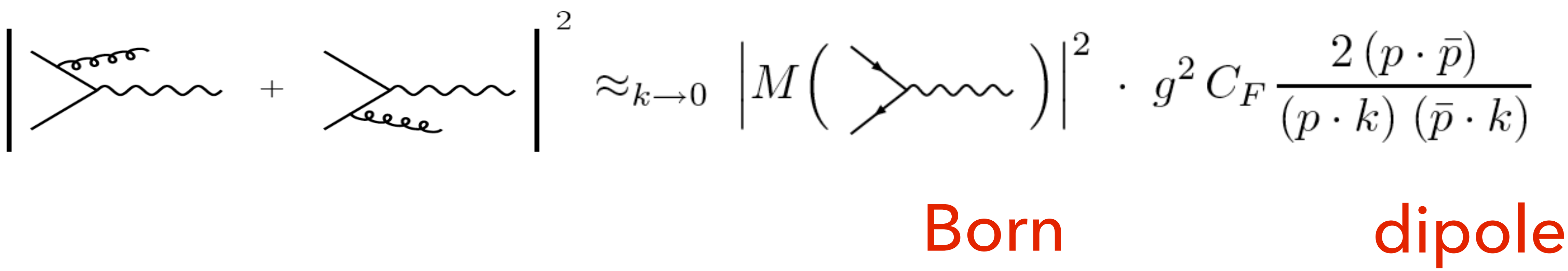
$$\alpha_s \int \frac{d\theta}{\theta}\frac{dz}{z}\Theta\left(\kappa - z\theta\right) - \alpha_s \int \frac{d\theta}{\theta}\frac{dz}{z} = -\alpha_s \int \frac{d\theta}{\theta}\frac{dz}{z}\left[\Theta\left(z\theta - \kappa\right)\right]$$

$$= -\frac{1}{2}\alpha_s \ln^2 \kappa$$

- These corrections are important for observables that probe small deviations from lowest order kinematics
- Real radiation is constrained to a small corner of phase space and the logarithms are large
- Examples: event (jet) shapes (thrust, jet mass), Z transverse momentum distribution, production at threshold

RESUMMATION: A SKETCH

- All-order calculations are based on factorisation
- Matrix elements factorises in the soft/collinear limit



$$\left| \ldots + \ldots \right|^2 \approx_{k \to 0} \left| M\left(\ldots \right) \right|^2 \cdot g^2 C_F \frac{2\,(p \cdot \bar{p})}{(p \cdot k)\,(\bar{p} \cdot k)}$$

- this can be generalised to the multi-gluon case
- factorisation often leads to exponentiation

$$\sigma_{res} = g_0 \exp[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots]$$

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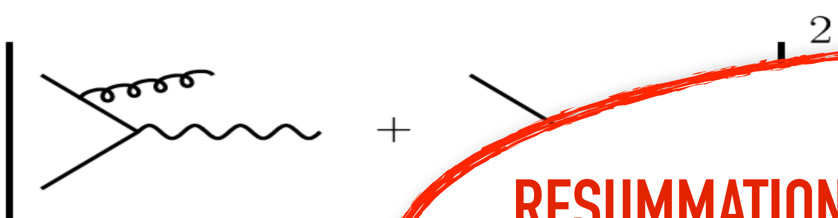


$$\simeq g^2 C_F \frac{2\,(p\cdot\bar{p})}{(p\cdot k)\,(\bar{p}\cdot k)}$$

dipole

RESUMMATION IS A SYSTEMATIC RE-ARRANGEMENT OF PERTURBATION THEORY

- this can be generalised to the multi-gluon case
- factorisation often leads to exponentiation

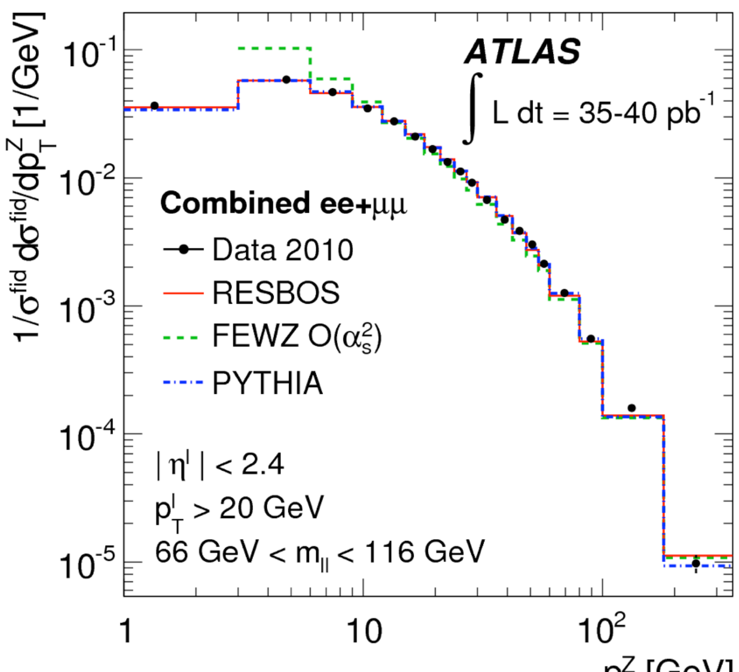
LL

NLL

NNLL

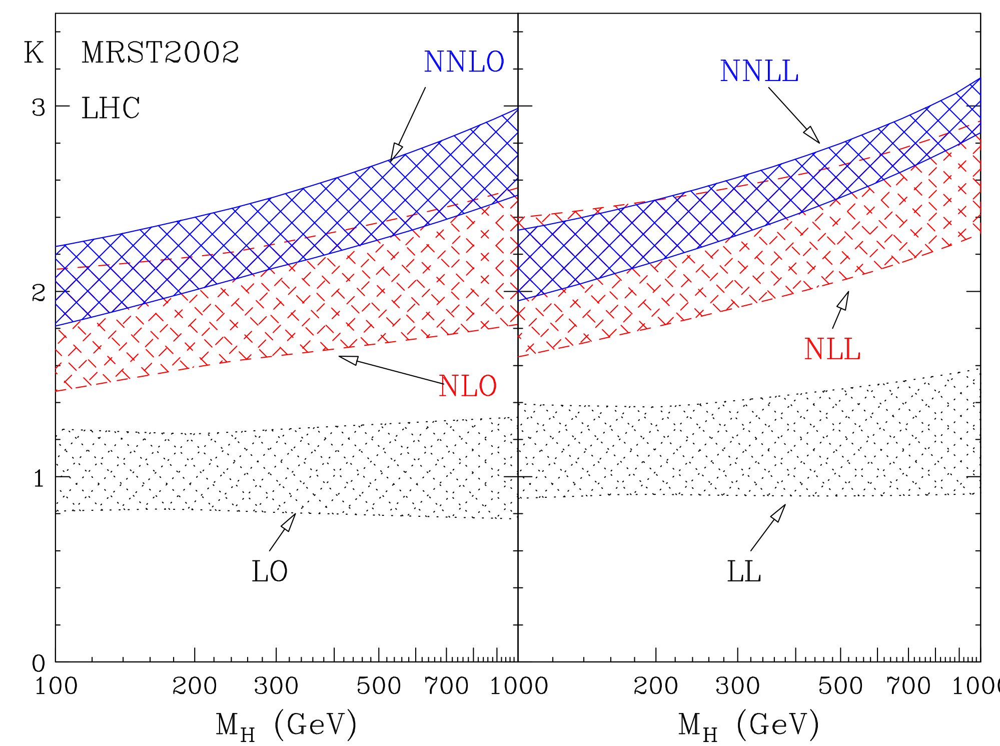
$$\sigma_{res} = g_0 \exp[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots]$$

RESUMMATION IN ACTION

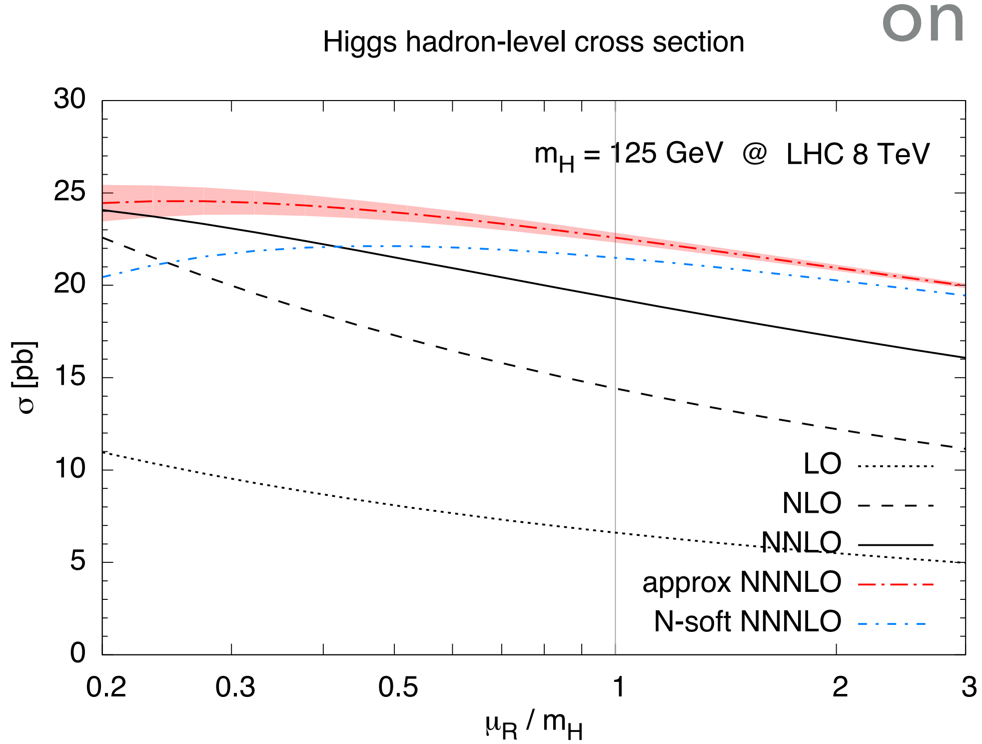


Phys.Lett. B705 (2011) 415-434

- it's necessary for describing data in particular kinematic limits
- it reduces theoretical uncertainties (and offers more handles on them)



Catani, de Florian, Grazzini, Nason (2003)



Ball, Bonvini, Forte, SM, Ridolfi (2013)

- it can be used to approximate (yet) unknown higher orders

A WORD ON PDFs

- Parton distribution functions describe the non-perturbative structure of the colliding protons
- collinear factorisation implies their universality (up to power corrections)

$$\sigma(x,Q) = \sigma_0 C\left(\frac{x}{x_1 x_2}, \alpha_s(\mu)\right) \otimes f_1(x_1,\mu) \otimes f_2(x_2,\mu)$$

- coefficient functions (NLO,NNLO, N^3LO)
- parton evolution (NLO,NNLO)
- electro-weak corrections
- quark mass effects
- target-mass corrections
- …

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measure

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measure compute

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measure compute extract

- coefficient functions (NLO,NNLO, N^{3}LO)
- parton evolution (NLO,NNLO)
- electro-weak corrections
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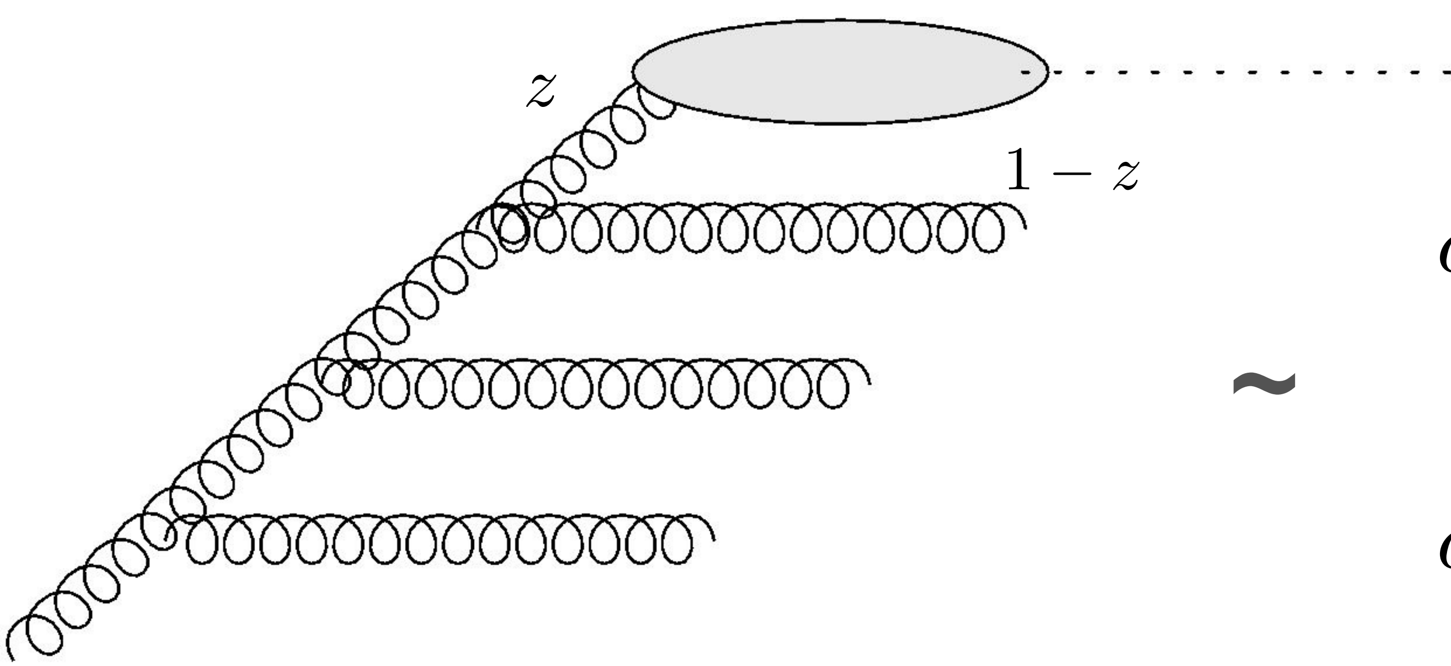
- …

HIGHER-ORDER CORRECTIONS

- Higher-order QCD corrections correspond to emission of extra partons or virtual corrections
- these corrections are enhanced in particular regions of phase-space

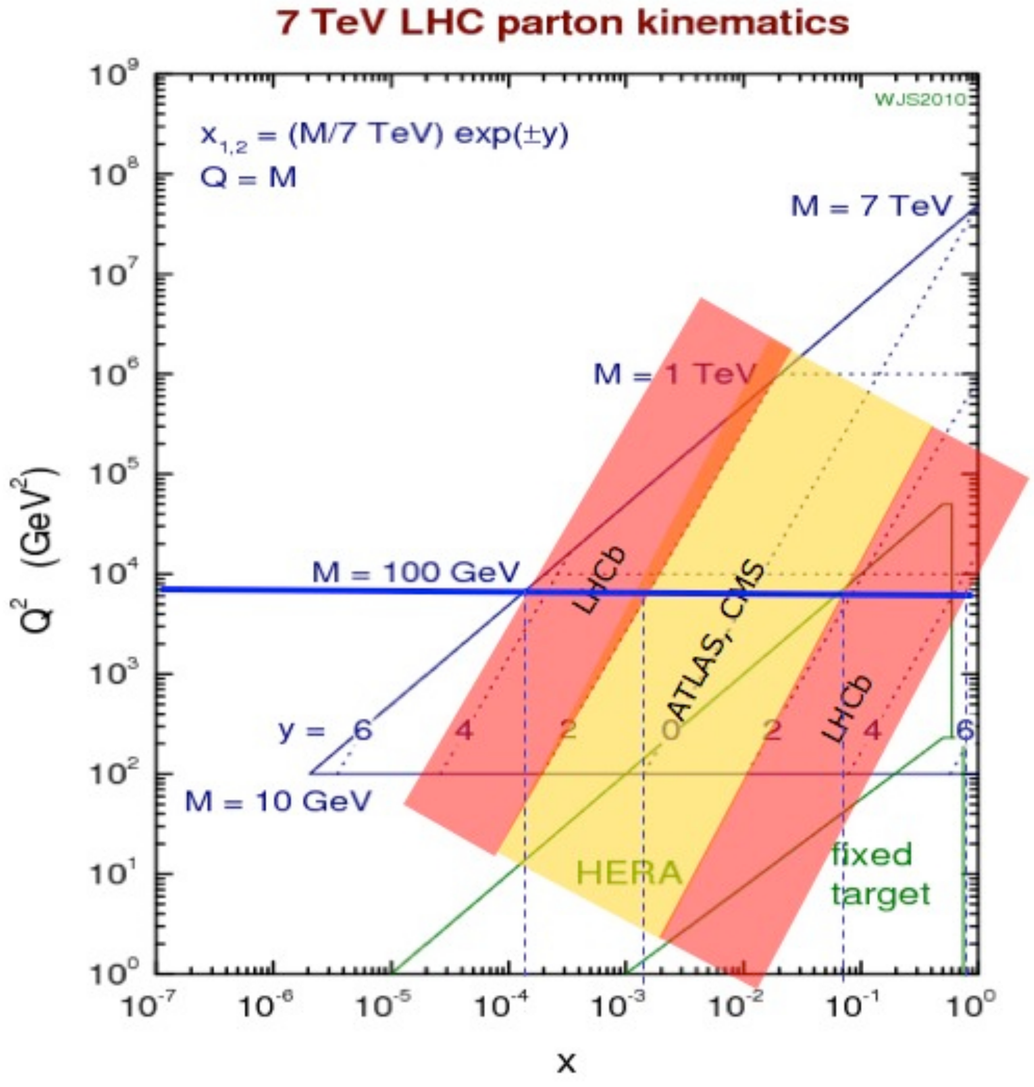


$$\sim \quad \alpha_s^k \left[\frac{\log^{2k-1}(1-z)}{1-z}\right]_+ , \; z \to 1$$

$$\alpha_s^k \frac{\log^{k-1} z}{z} , \; z \to 0$$

LHC KINEMATICS

- LHC probes a vast region of the (x,Q^2) plane
- New hierarchy of scales appear and resummation might me needed (not necessarily Sudakov-like)



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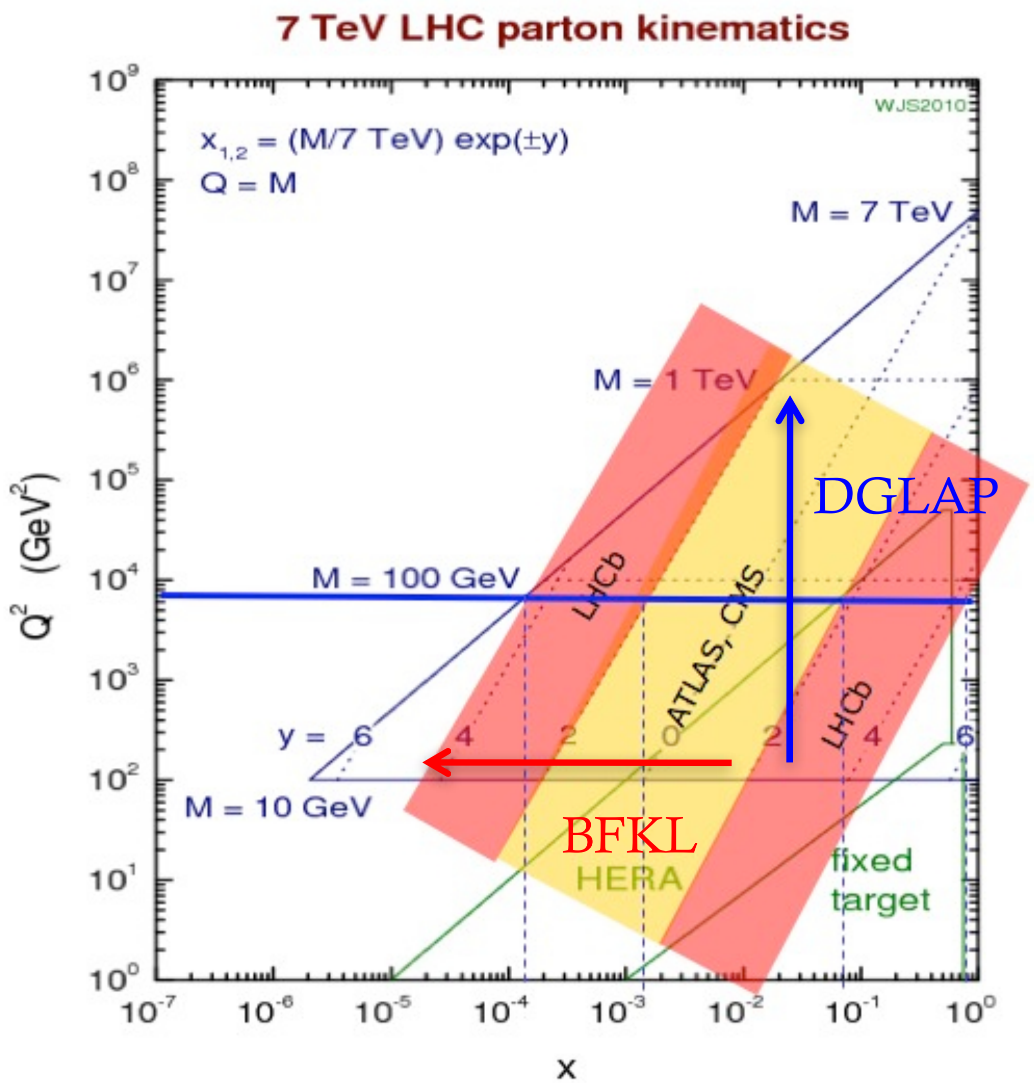


DGLAP: Q^2 evolution for N moments of the parton density

$$\frac{d}{d\ln(Q^2/\mu^2)}G(N,Q^2)=\gamma(N,\alpha_s)G(N,Q^2)$$

BFKL: small-x evolution for M moments of the parton density

$$\frac{d}{d\ln(1/x)}G(x,M)=\chi(M,\alpha_s)G(x,M)$$

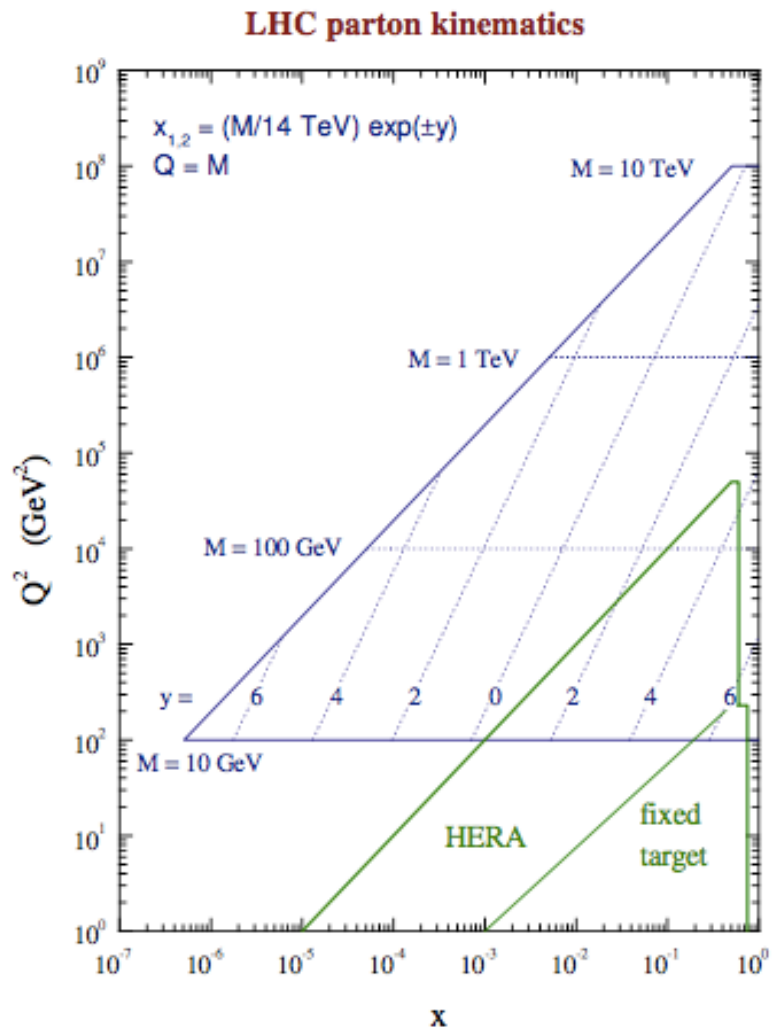
Mellin moments:

logs ↔ poles

$$\ln^k\frac{Q^2}{\mu^2}\leftrightarrow\frac{1}{M^{k+1}}$$

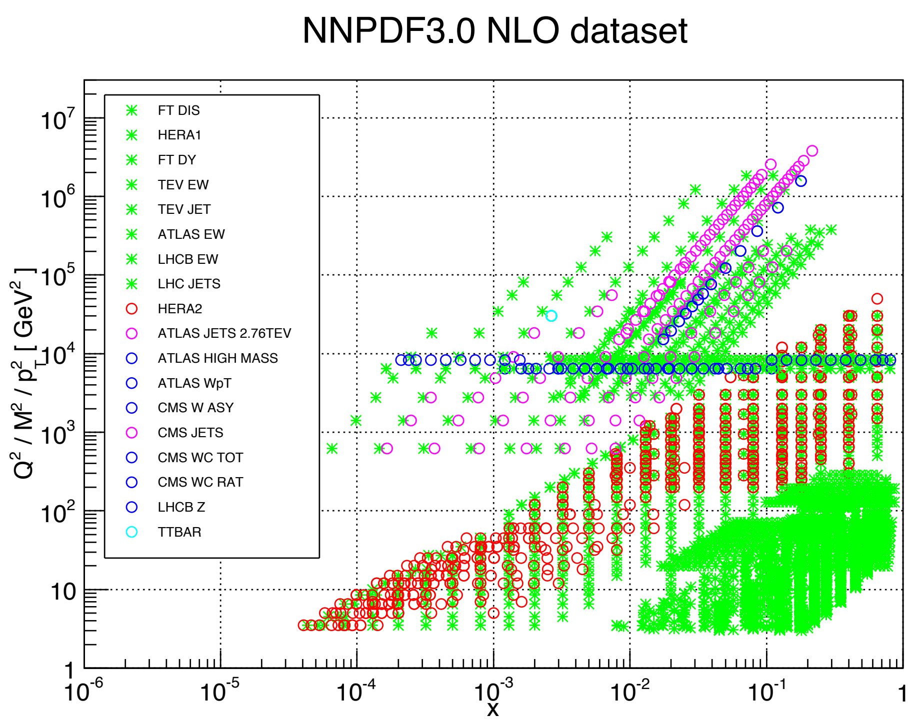
$$\ln^k\frac{1}{x}\leftrightarrow\frac{1}{N^{k+1}}$$

PROF JAMES STIRLING (1953–2018)



- James had a profound impact on particle physics and parton distribution functions in particular
- Besides his scientific merits, he has strongly influenced and shaped the UK community as founding director of the IPPP in Durham and as Provost of the Imperial College

DATASET OF A GLOBAL FIT



- Standard PDFs fits rely on NLO and NNLO calculations of coefficient functions and evolution
- current datasets span several orders of magnitude in Q^2 and x

QUESTIONS THAT COME TO MIND

- Do we trust FO everywhere?
- Do we see evidence of all-order effects in the data?
- Is it ok to use standard PDFs with resummed calculation?

SMALL-X RESUMMATION

DGLAP EVOLUTION AT SMALL-X

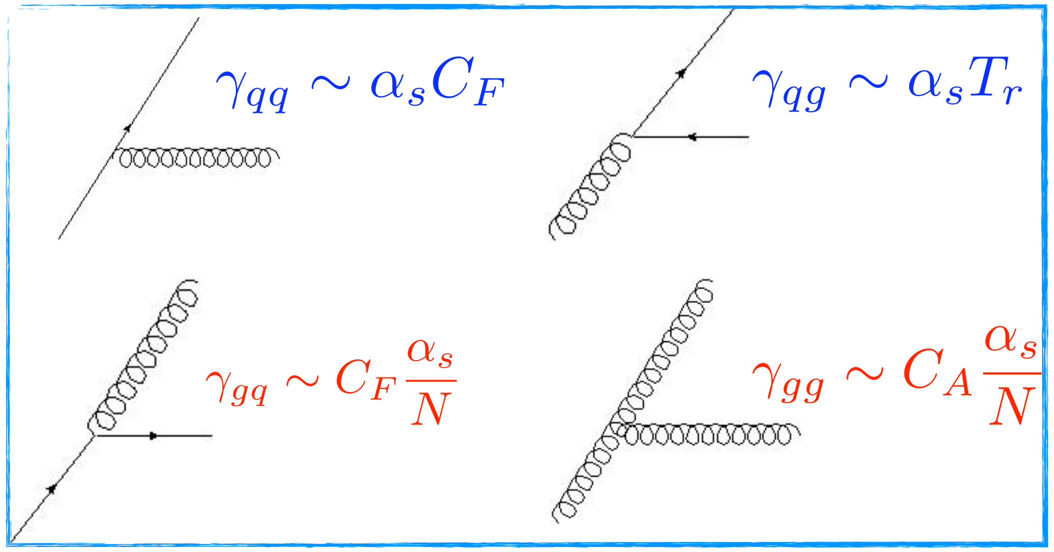
- DGLAP evolution in the singlet sector

$$Q^2 \frac{d}{dQ^2} \begin{pmatrix} f_g \\ f_q \end{pmatrix} = \Gamma(N, \alpha_s(Q^2)) \begin{pmatrix} f_g \\ f_q \end{pmatrix}, \qquad \Gamma(N, \alpha_s) \equiv \begin{pmatrix} \gamma_{gg} & \gamma_{gq} \\ \gamma_{qg} & \gamma_{qq} \end{pmatrix}$$

- the gluon splitting functions are enhanced at small-*x* (LL*x*)

$$\gamma_{gg} \sim c_1 \frac{\alpha_s}{N} + c_2 \left(\frac{\alpha_s}{N}\right)^2 + \dots$$

$$\gamma_{gq} \sim \frac{C_F}{C_A} \gamma_{gg}$$



- so are the quark ones but at NLL*x*

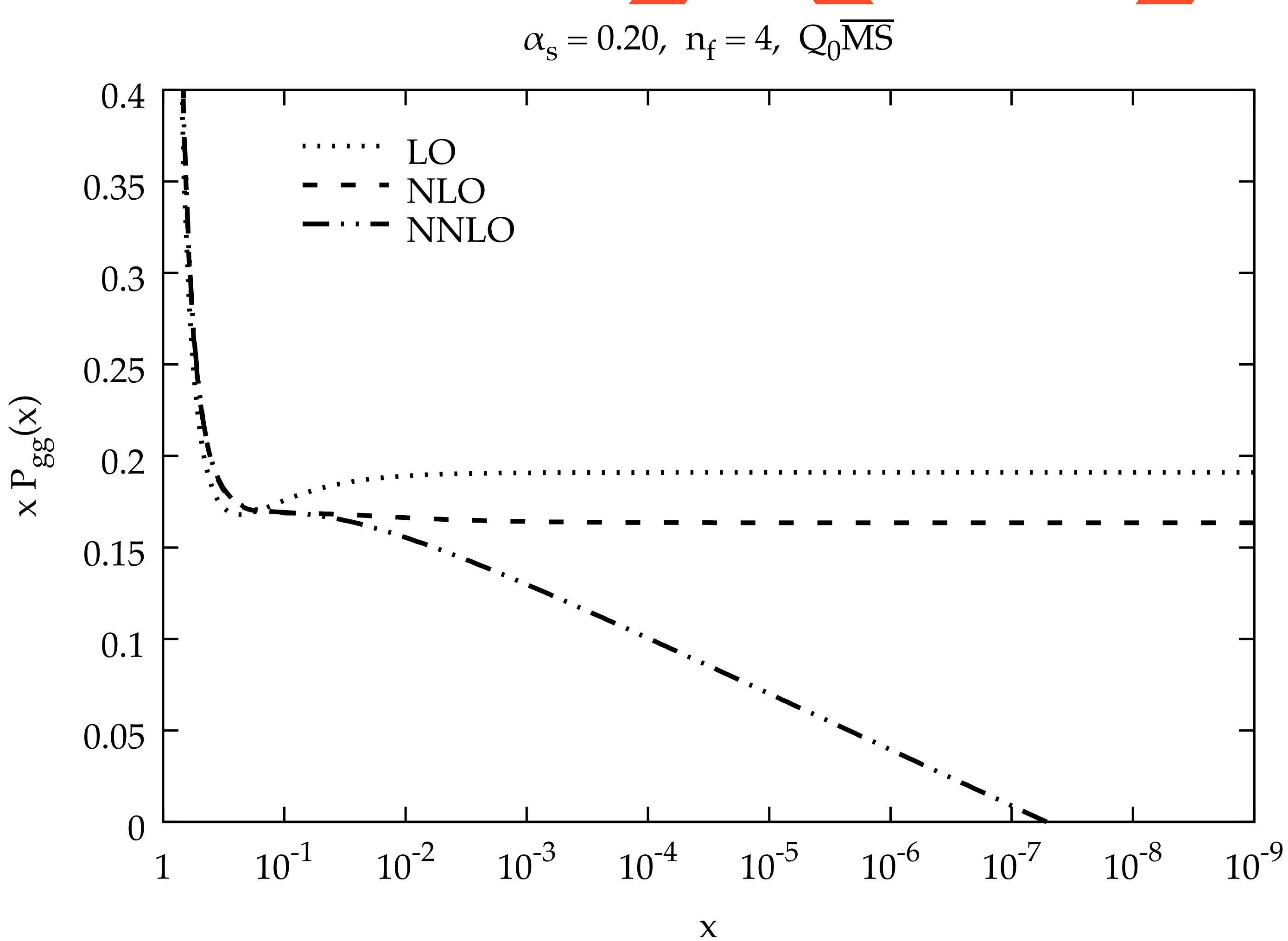
$$\gamma_{qg} \sim \alpha_s d_0 + d_1 \alpha_s \frac{\alpha_s}{N} + c_2 \alpha_s \left(\frac{\alpha_s}{N}\right)^2 + \dots$$

$$\gamma_{qq}^{(\mathrm{PS})} \sim \frac{C_F}{C_A} \left(\gamma_{qg} - \alpha_s d_0\right)$$

FIXED-ORDER CONSIDERATIONS

- Note that some of the coefficients can be zero because of accidental cancellations: most notably c_2 and c_3 in MS-like schemes

$$xPgg \sim c_1\alpha_s + c_2\alpha_s^2 \log x + c_3\alpha_s^3 \log^2 x + c_4\alpha_s^4 \log^3 x + \dots$$



- NNLO is less stable than NLO (subleading logs survive)
- N3LO (calculations underway) is likely to exhibit stronger instabilities

DGLAP-BFLK DUALITY

- Consistency of DGLAP and BFKL requires that

$$G(N,M) = \underset{\text{DGLAP}}{\frac{G_0(N)}{M-\gamma(\alpha_s,N)}} \quad \underset{\text{and}}{=} \quad \underset{\text{BFKL}}{\frac{\bar{G}_0(M)}{N-\chi(\alpha_s,M)}}$$

- we now expand the last equality around $M=M_0$ such that $\chi(\alpha_s,M_0)=N$

$$\frac{\bar{G}_0(M)}{N-\chi(\alpha_s,M)} = -\frac{\bar{G}_0(M_0)/\chi'(M_0)}{N-M_0} + \dots$$

Jaroszewicz (1982)

- we have obtained a DGLAP-type equation, provided that the following duality relations hold

$$\longrightarrow \quad \begin{aligned} \chi(\gamma(N,\alpha_s),\alpha_s) &= N \\ \gamma(\chi(M,\alpha_s),\alpha_s) &= M \end{aligned}$$

DGLAP-BFLK DUALITY

- (N)LLx behaviour can be determined from the (N)LO BFKL kernel

$$\chi(\alpha_s, M) = \alpha_s \chi_0(M) \;\Rightarrow\; \chi_0(\gamma_s) = N/\alpha_s$$

$$\Rightarrow\; \gamma_s = \sum_{n=1}^{\infty} c_n \left(\frac{\alpha_s}{N}\right)^n$$

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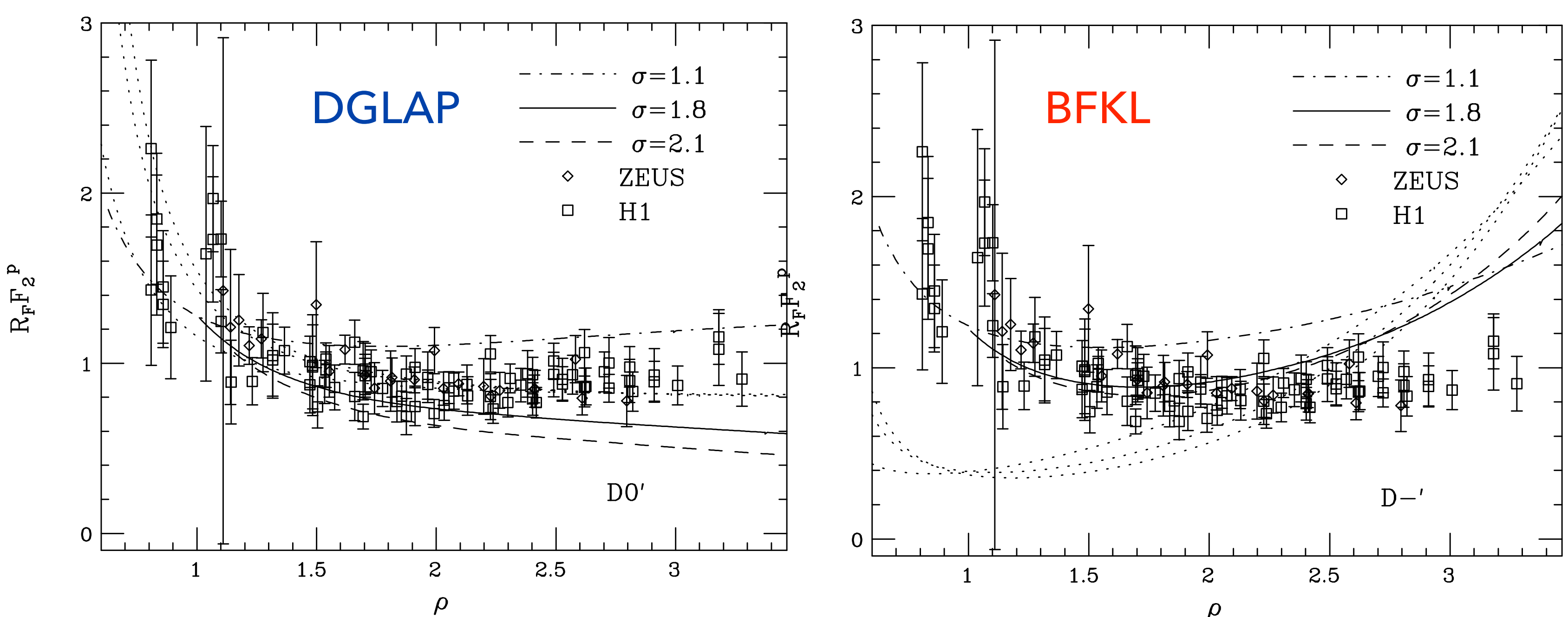
$$\Rightarrow\; \gamma_s = \sum_{n=1}^{\infty} c_n \left(\frac{\alpha_s}{N}\right)^n$$

- however: naive implementation of BFKL leads to results not supported by HERA data (too strong, too soon)



double-scaling variables

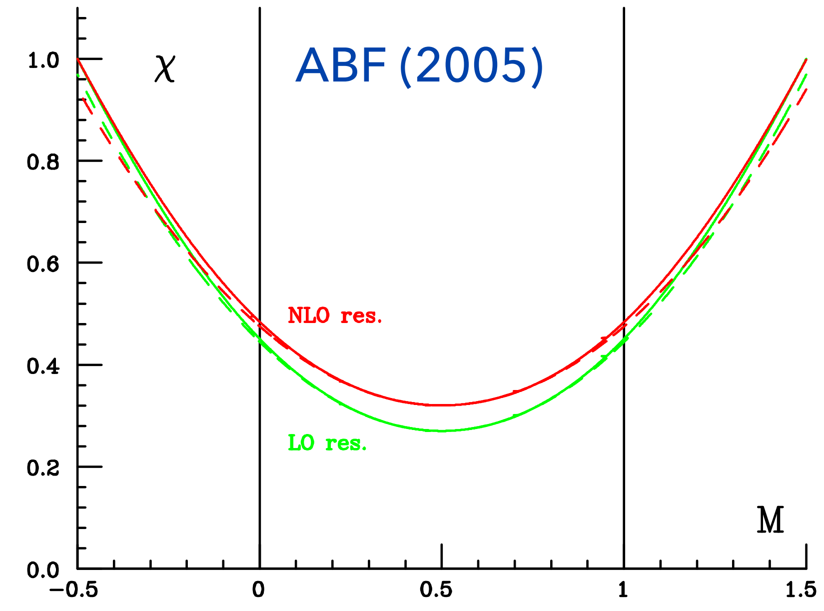
$$\sigma \equiv \sqrt{\ln\frac{x_0}{x}\ln\frac{t}{t_0}}, \qquad \rho \equiv \sqrt{\ln\frac{x_0}{x}\Big/\ln\frac{t}{t_0}}$$

models that naively implement BFKL are disfavoured by HERA data

Ball, Forte (1994)

RESUMMATION OF DGLAP EVOLUTION

- Problem studied by different groups in late '90s /early '00s: Altarelli, Ball, Forte; Ciafaloni, Colferai, Salam, Stasto; Thorne, White
- for a comparative review see HERA-LHC Proc. arXiv:0903.3861
- recent progress in SCET Rothstein, Stewart (2016)
- we mostly follow the approach by ABF
- *key ingredients:*
  - duality between DGLAP and BFKL kernels
  - stable solution of the running coupling BFKL equation (important subleading effects)
  - match to standard DGLAP at large *N (x)*



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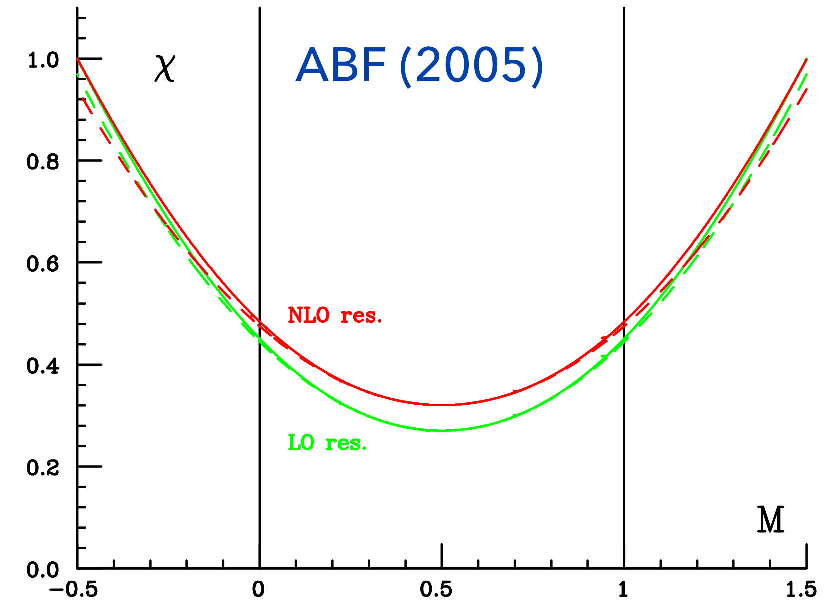
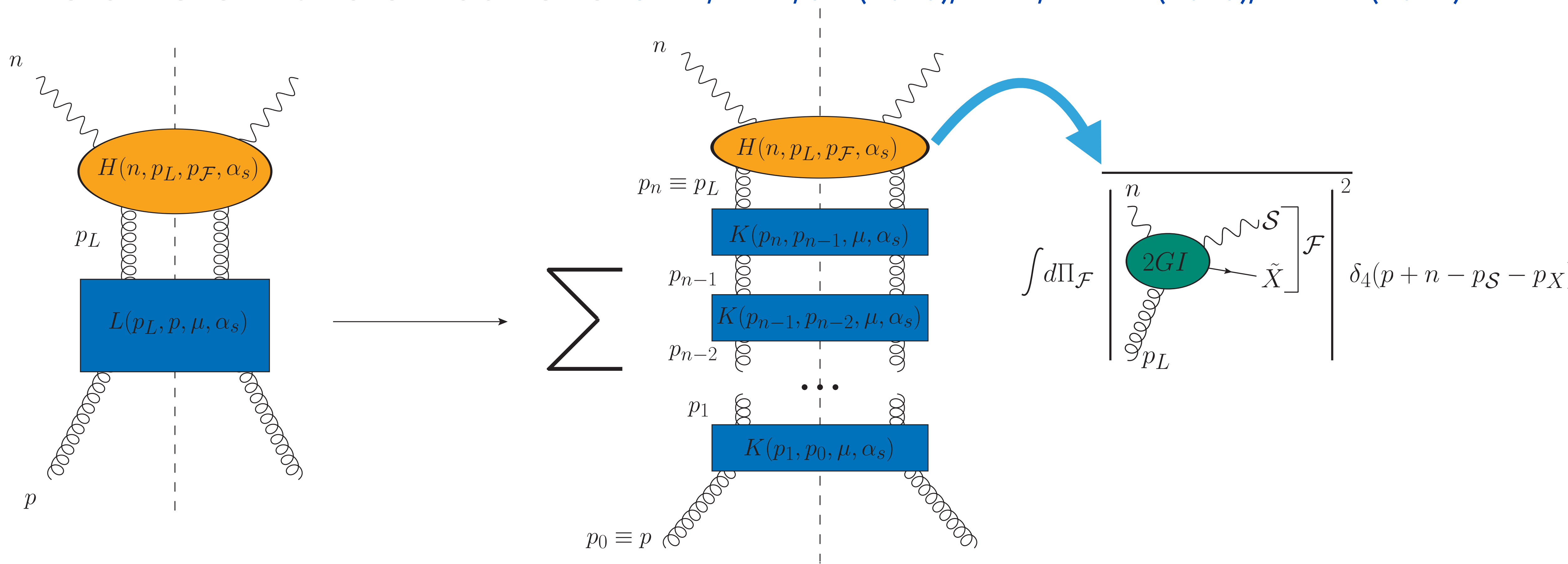


three resummations at work!

COEFFICIENT FUNCTIONS AT SMALL X

- the high-energy behaviour of coefficient function is obtained using k_t-factorisation Catani, Ciafaloni, Hautmann (1991); Collins, Ellis (1991)
- derivation in terms of ladder expansion allowed for its generalisation to differential distributions Caola, Forte, SM (2010); Forte, Muselli (2016); Muselli (2017)



- the building blocks *K* are essentially the resummed anomalous dimensions

RESUMMATION OF COEFFICIENT FUNCTIONS

- naive (i.e. fixed-log counting) resummation has same issues as evolution
- running coupling corrections are crucial
- elegant but complex treatment in Mellin space Ball (2008)
- our approach in a nutshell: resummation in momentum space

High-energy (k_T) factorization:

$$\sigma \propto \int \frac{dz}{z} \int d^2\boldsymbol{k}\, \hat{\sigma}_g\left(\frac{x}{z}, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s(Q^2)\right) \mathcal{F}_g(z, \boldsymbol{k})$$

$$\begin{cases} \mathcal{F}_g(x, \boldsymbol{k}) : \text{unintegrated PDF} \\ \hat{\sigma}_g\left(z, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) : \text{off-shell xs} \end{cases}$$

Defining

$$\mathcal{F}_g(N, \boldsymbol{k}) = U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right) f_g(N, \mu^2)$$

we get

$$C_g(N, \alpha_s) = \int d^2\boldsymbol{k}\, \hat{\sigma}_g\left(N, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right)$$

At LLx accuracy, U has a simple form, in terms of small-x resummed anom dim γ

$$U\left(N, \frac{\boldsymbol{k}^2}{\mu^2}\right) \approx \boldsymbol{k}^2 \frac{d}{d\boldsymbol{k}^2} \exp \int_{\mu^2}^{\boldsymbol{k}^2} \frac{d\nu^2}{\nu^2} \gamma(N, \alpha_s(\nu^2))$$

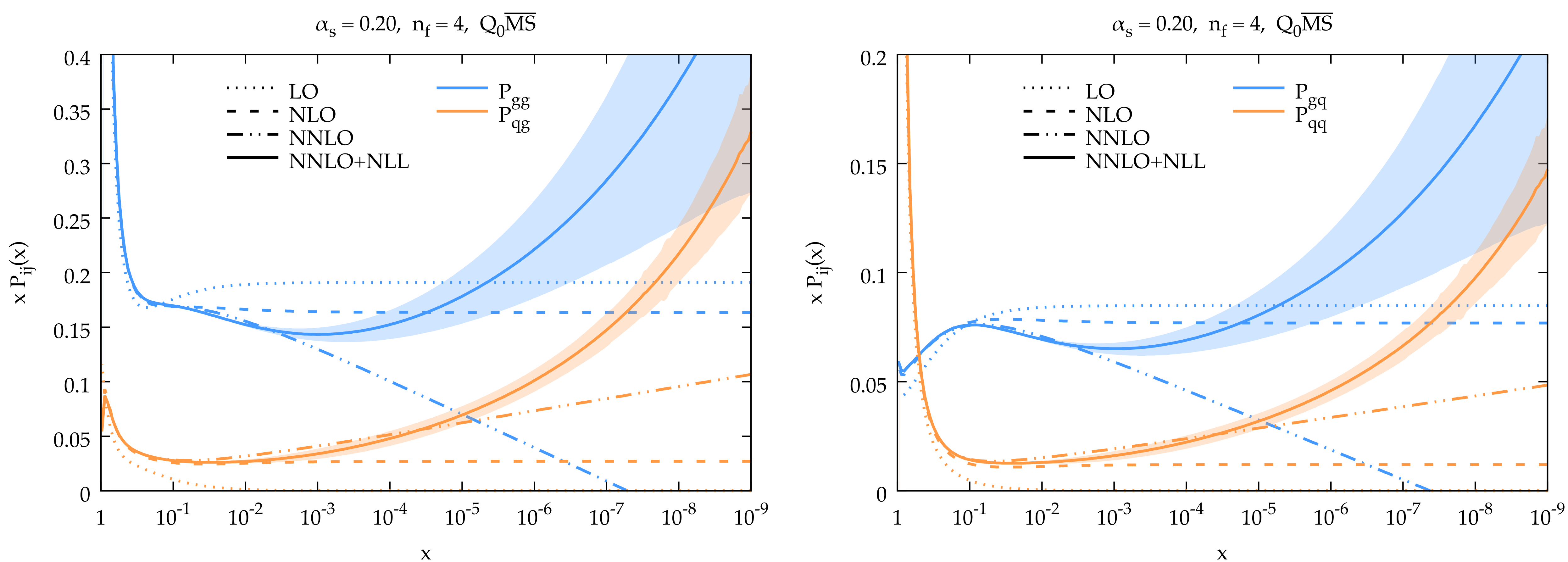
- until recent: very little phenomenology because a comprehensive code was missing

HIGH ENERGY LARGE LOGARITHMS

- public code that computes resummed splitting functions and perturbative coefficient functions
- HELL-x: pheno tool with pre-tabulated results, interfaced with evolution code APFEL
- in current HELL 3.x version
  - DIS (both NC and CC)
  - heavy-quark matching conditions
  - Higgs in gluon fusion
- implementation of DY is work in progress

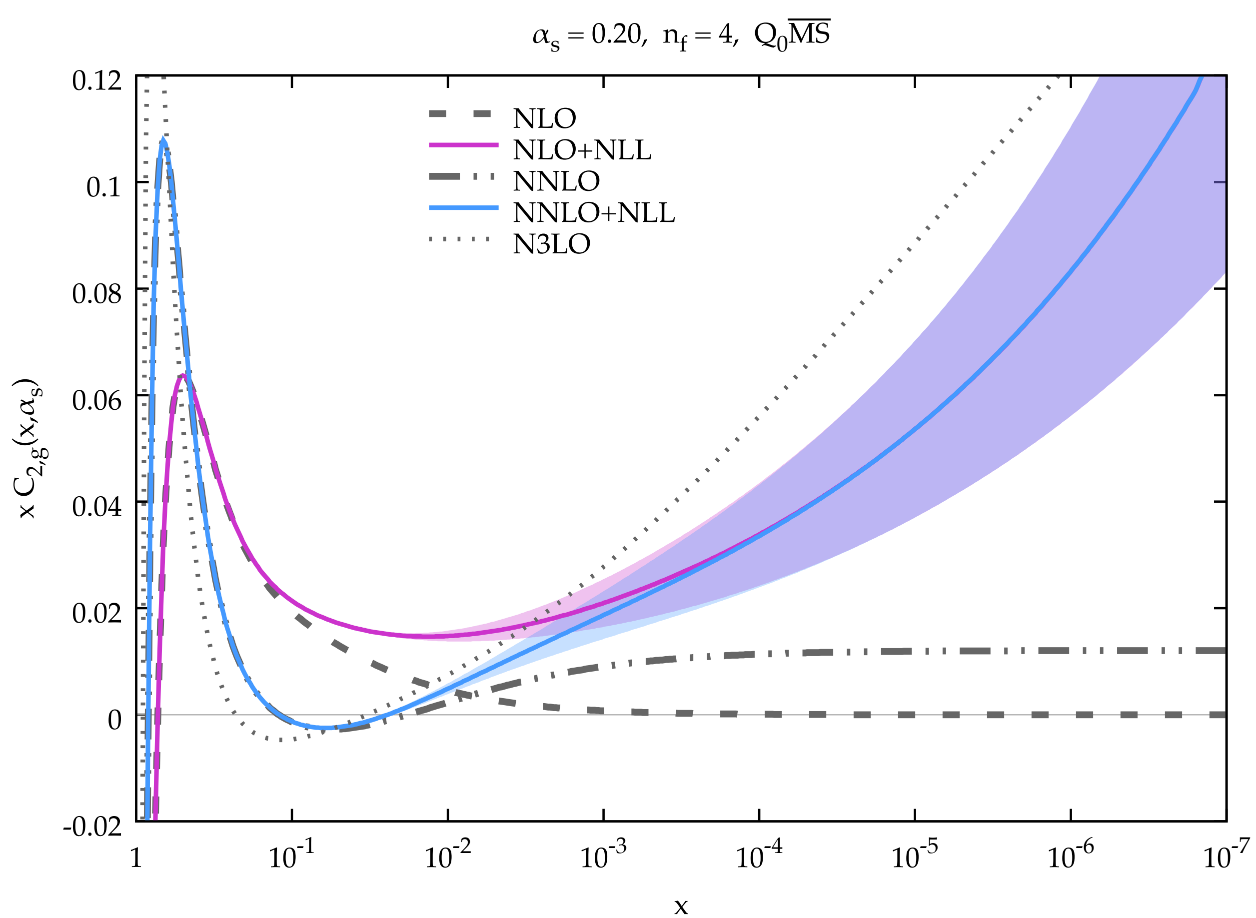
https://www.ge.infn.it/~bonvini/hell/

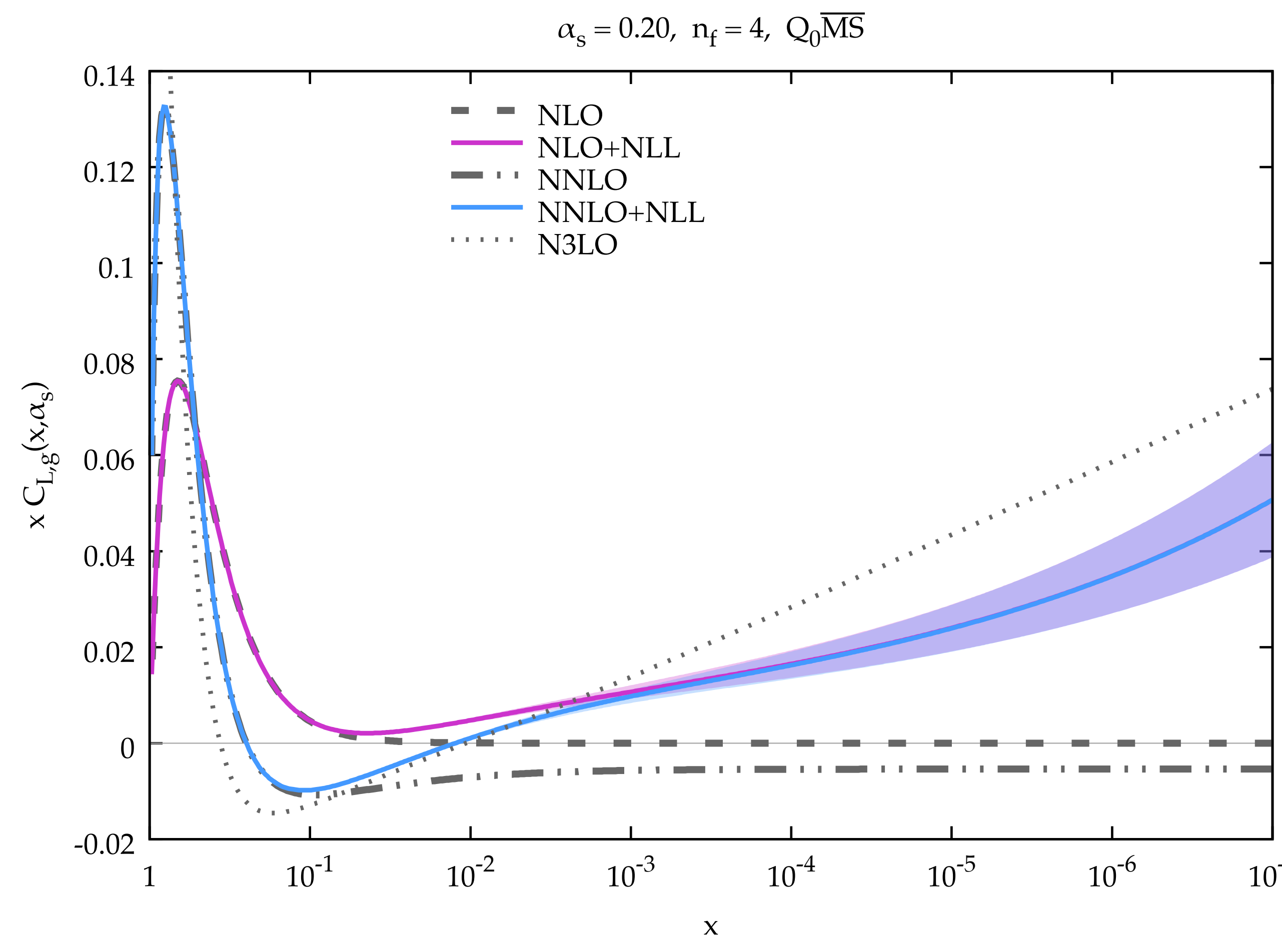
RESULTS FROM HELL: SPLITTING FUNCTIONS



- resummation matched up to NNLO
- uncertainty bands obtained by varying subleading corrections
- quark splitting functions under less control (they start at NLL)

RESULTS FROM HELL: DIS COEFFICIENT FUNCTIONS





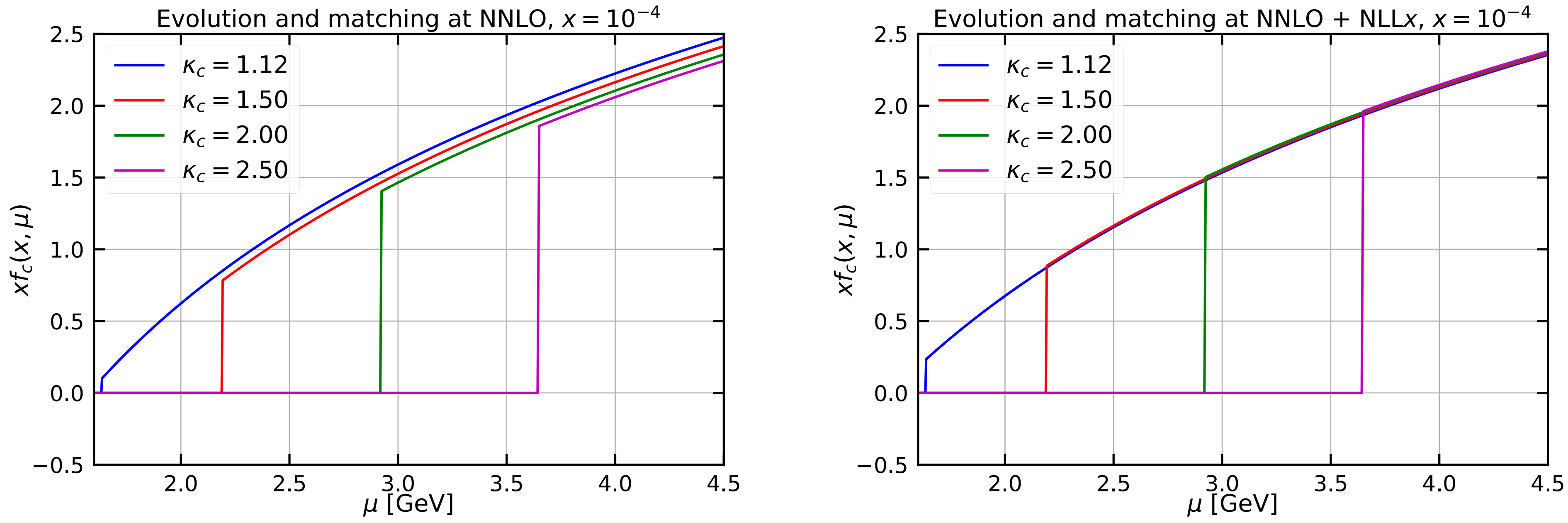
- parton level results
- instability shows up at N^{3}LO
- large theoretical uncertainty (they start at NLL)

RESULTS FROM HELL: HEAVY-QUARK MATCHING

- description of DIS data across several decades in Q^2 requires the matching of factorisation schemes with different active flavours

$$f_i^{[n_f+1]} = \sum_{j=-n_f}^{n_f} K_{ij}^{[n_f]} f_j^{[n_f]}$$

- matching coefficients are evaluated order by order in perturbation theory
- similarly to coefficients functions they are affects by small-x logs
- inclusion of resummation reduces matching uncertainties



xFitter Developers (2018)

PDFs WITH SMALL-X RESUMMATION

A FIT WITH SMALL-X RESUMMATION: THE DATASET

- exploit NNPDF state-of-art technology to perform fits with small-x resummation
- for DIS with have a consistent implementation of small-x resummation (both evolution and coefficient functions)
- similar dataset as standard NNLO analysis (NNPDF 3.1)
- lower the initial scale of the fit to Q_0=1.64 GeV to include an extra bin of the HERA data (Q^2=2.7 GeV^2)
- what about hadronic data?

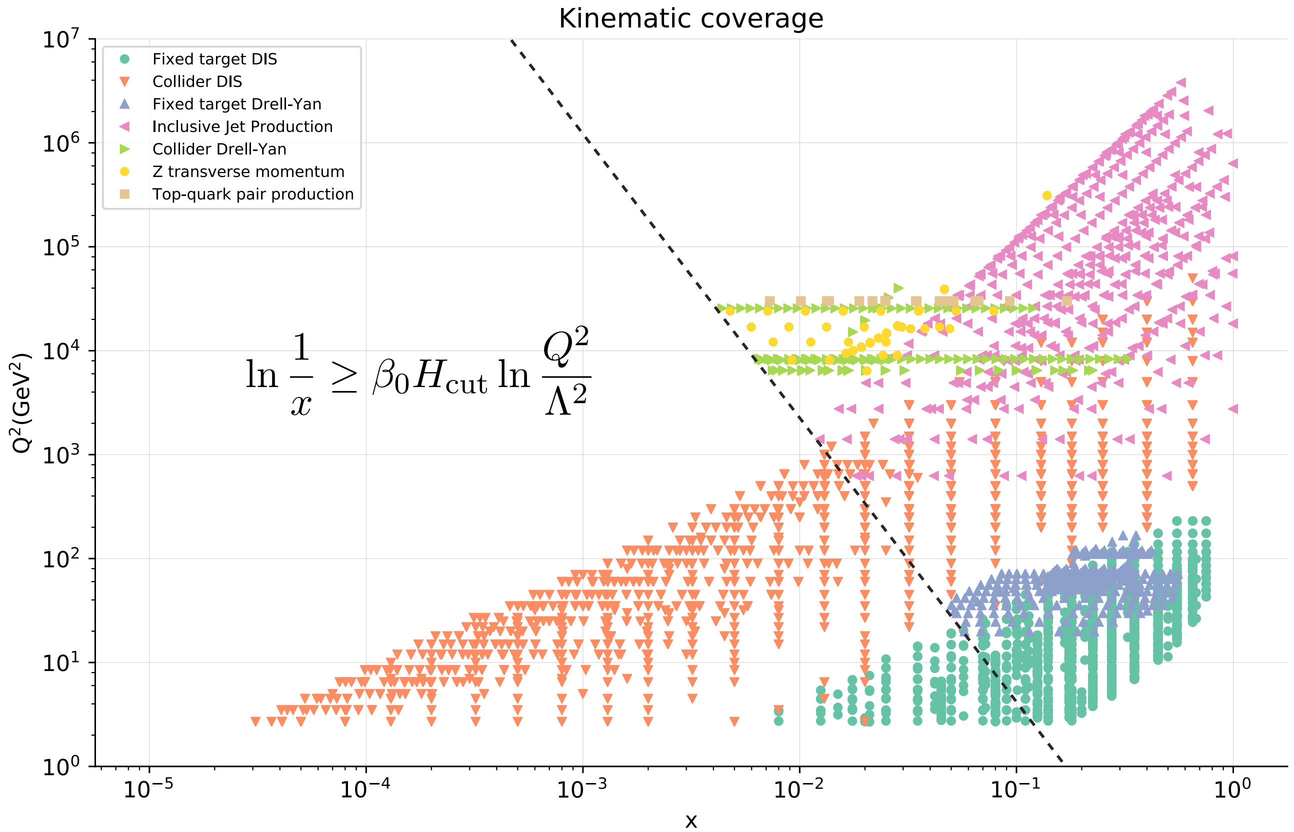
Experiment	$N_{\rm dat}$
NMC	367
SLAC	80
BCDMS	581
CHORUS	886
NuTeV dimuon	79
HERA I+II incl. NC	1081
HERA I+II incl. CC	81
HERA $\sigma_c^{\rm NC}$	47
HERA F_2^b	29
Total	**3231**

THE ISSUE WITH HADRONIC DATA

- resummation for coefficient functions in pp collisions is known but not yet implemented in HELL
- resummation only included in the evolution
- to avoid biases we cut away *hadronic* low-*x* data (mostly LHCb DY)
- we discard points for which (based on LO kinematics)

$$\alpha_s(Q^2)\ln\frac{1}{x} \geq H_{\rm cut}$$

- the smaller $H_{\rm cut}$, the tighter the cut
- we find $H_{\rm cut}$=0.6 to be a good compromise
- we keep ~70% of hadronic data



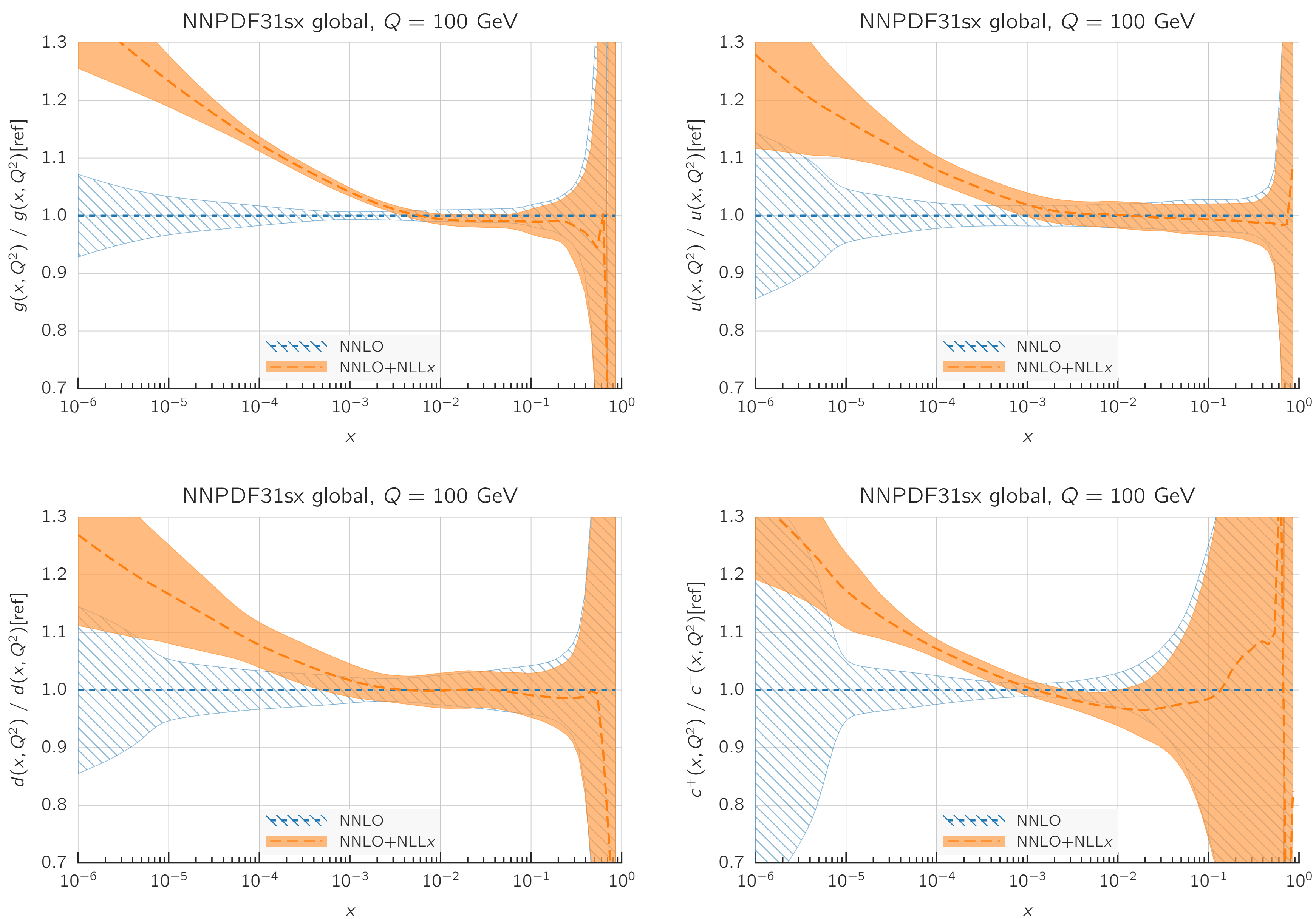
FIT RESULTS

	$\chi^2/N_{\rm dat}$ NLO	$\chi^2/N_{\rm dat}$ NLO+NLLx	$\Delta\chi^2$	$\chi^2/N_{\rm dat}$ NNLO	$\chi^2/N_{\rm dat}$ NNLO+NLLx	$\Delta\chi^2$
NMC	1.35	1.35	+1	1.30	1.33	+9
SLAC	1.16	1.14	−1	0.92	0.95	+2
BCDMS	1.13	1.15	+12	1.18	1.18	+3
CHORUS	1.07	1.10	+20	1.07	1.07	−2
NuTeV dimuon	0.90	0.84	−5	0.97	0.88	−7
HERA I+II incl. NC	1.12	1.12	-2	1.17	1.11	−62
HERA I+II incl. CC	1.24	1.24	-	1.25	1.24	−1
HERA $\sigma_c^{\rm NC}$	1.21	1.19	−1	2.33	1.14	−56
HERA F_2^b	1.07	1.16	+3	1.11	1.17	+2
DY E866 $\sigma^d_{\rm DY}/\sigma^p_{\rm DY}$	0.37	0.37	-	0.32	0.30	-
DY E886 σ^p	1.06	1.10	+3	1.31	1.32	-
DY E605 σ^p	0.89	0.92	+3	1.10	1.10	-
CDF Z rap	1.28	1.30	-	1.24	1.23	-
CDF Run II k_t jets	0.89	0.87	−2	0.85	0.80	−4
D0 Z rap	0.54	0.53	-	0.54	0.53	-
D0 $W \to e\nu$ asy	1.45	1.47	-	3.00	3.10	+1
D0 $W \to \mu\nu$ asy	1.46	1.42	-	1.59	1.56	-
ATLAS total	1.18	1.16	−7	0.99	0.98	−2
ATLAS W, Z 7 TeV 2010	1.52	1.47	-	1.36	1.21	−1
ATLAS HM DY 7 TeV	2.02	1.99	-	1.70	1.70	-
ATLAS W, Z 7 TeV 2011	3.80	3.73	−1	1.43	1.29	−1
ATLAS jets 2010 7 TeV	0.92	0.87	−4	0.86	0.83	−2
ATLAS jets 2.76 TeV	1.07	0.96	−6	0.96	0.96	-
ATLAS jets 2011 7 TeV	1.17	1.18	-	1.10	1.09	−1
ATLAS Z p_T 8 TeV (p_T^{ll}, M_{ll})	1.21	1.24	+2	0.94	0.98	+2
ATLAS Z p_T 8 TeV (p_T^{ll}, y_{ll})	3.89	4.26	+2	0.79	1.07	+2
ATLAS σ_{tt}^{tot}	2.11	2.79	+2	0.85	1.15	+1
ATLAS $t\bar{t}$ rap	1.48	1.49	-	1.61	1.64	-
CMS total	0.97	0.92	−13	0.86	0.85	−3
CMS Drell-Yan 2D 2011	0.77	0.77	-	0.58	0.57	-
CMS jets 7 TeV 2011	0.88	0.82	−9	0.84	0.81	−3
CMS jets 2.76 TeV	1.07	0.98	−7	1.00	1.00	-
CMS Z p_T 8 TeV (p_T^{ll}, y_{ll})	1.49	1.57	+1	0.73	0.77	-
CMS σ_{tt}^{tot}	0.74	1.28	+2	0.23	0.24	-
CMS $t\bar{t}$ rap	1.16	1.19	-	1.08	1.10	-
Total	**1.117**	**1.120**	**+11**	**1.130**	**1.100**	**−121**

- the quality of NLO+NLL*x* and NLO fits is comparable
- it's expected because the two theories are rather similar
- situation changes dramatically at NNLO
- NNLO+NLL*x* provides the best fit
- the bulk of the improvement comes from HERA data

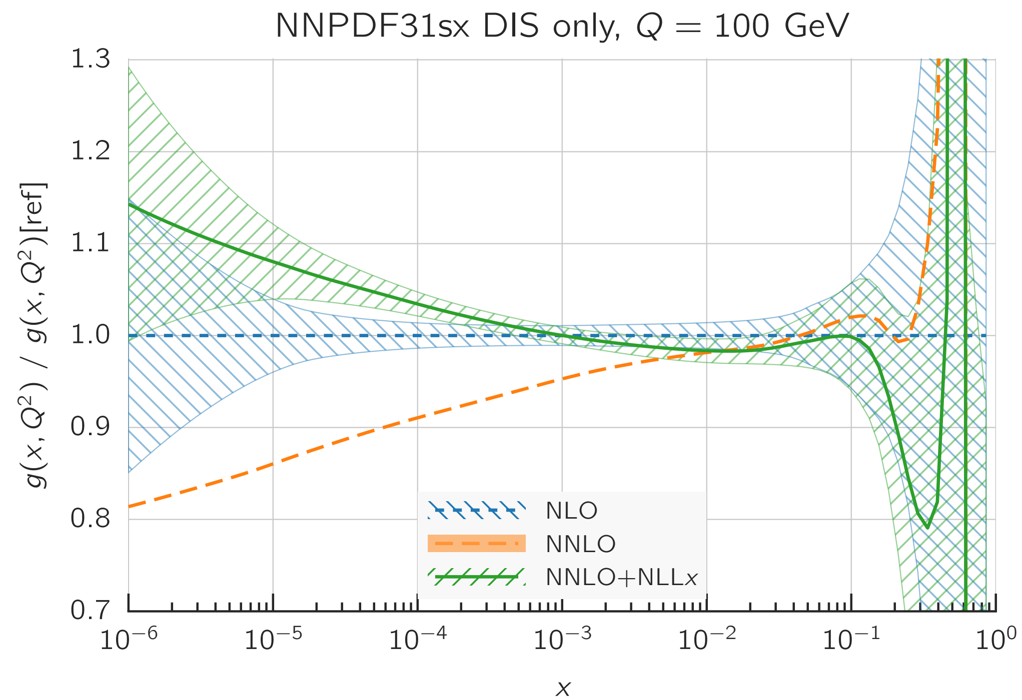
PARTON DENSITIES WITH SMALL-X RESUMMATION

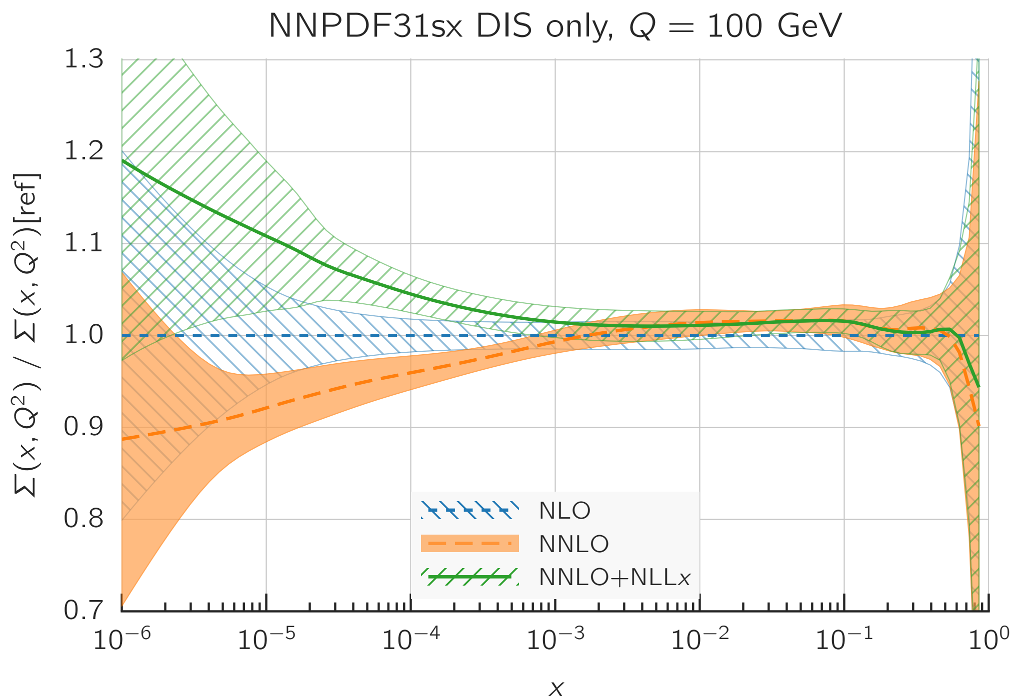
- resulting PDFs show interesting features
- agreement at large *x* but they're much steeper at low *x*



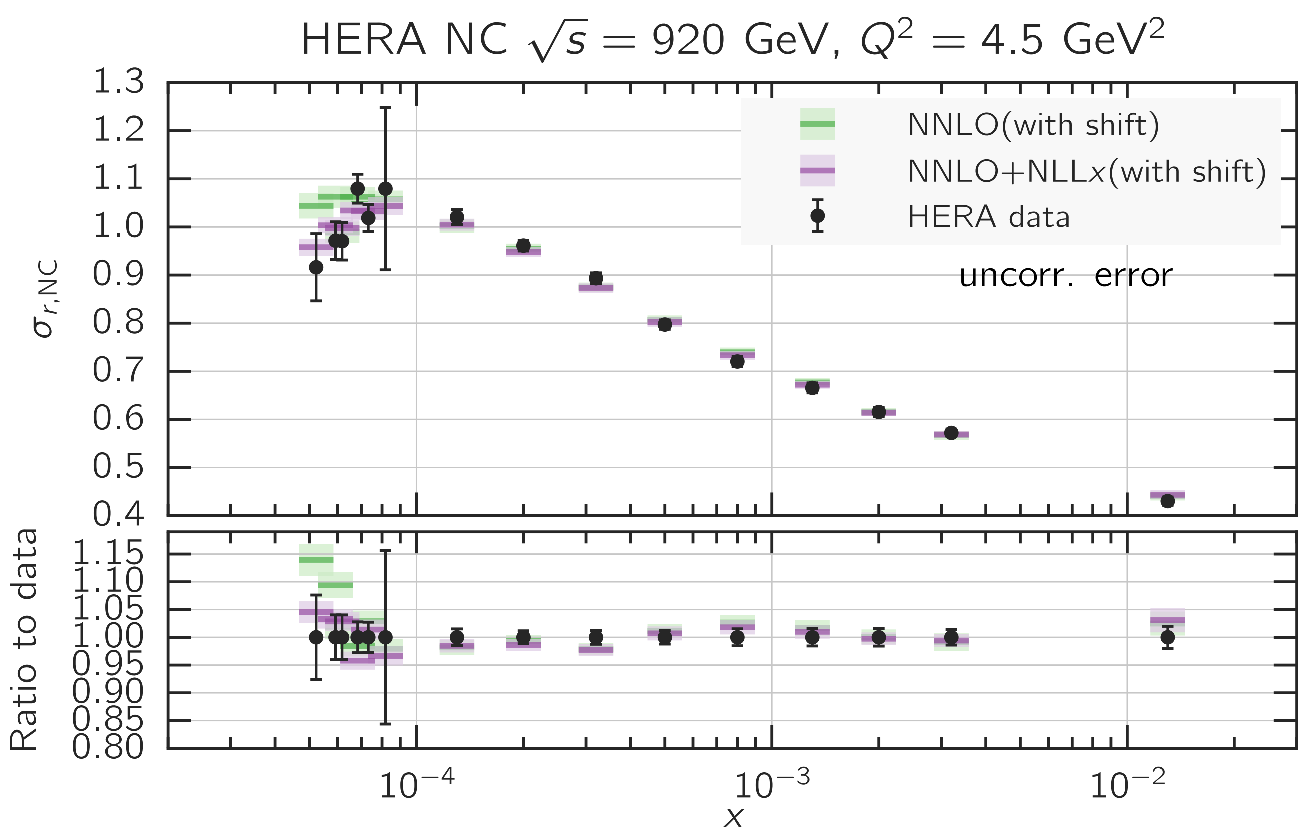
PERTURBATIVE STABILITY

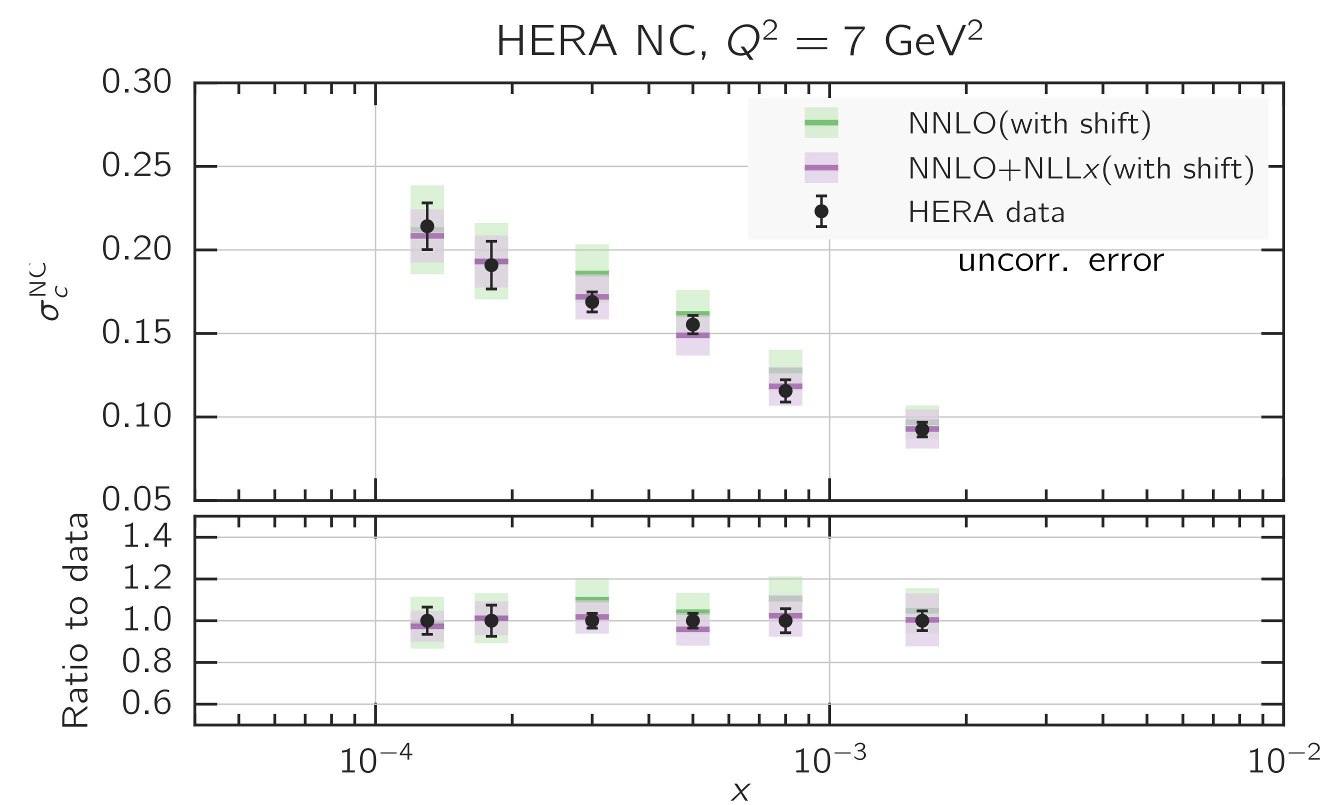
- NNLO and NNLO+NLLx differ quite dramatically
- one could question the reliability of the resummed procedure
- what gives us confidence we're not talking rubbish?
- resummation cures perturbative instability of NNLO

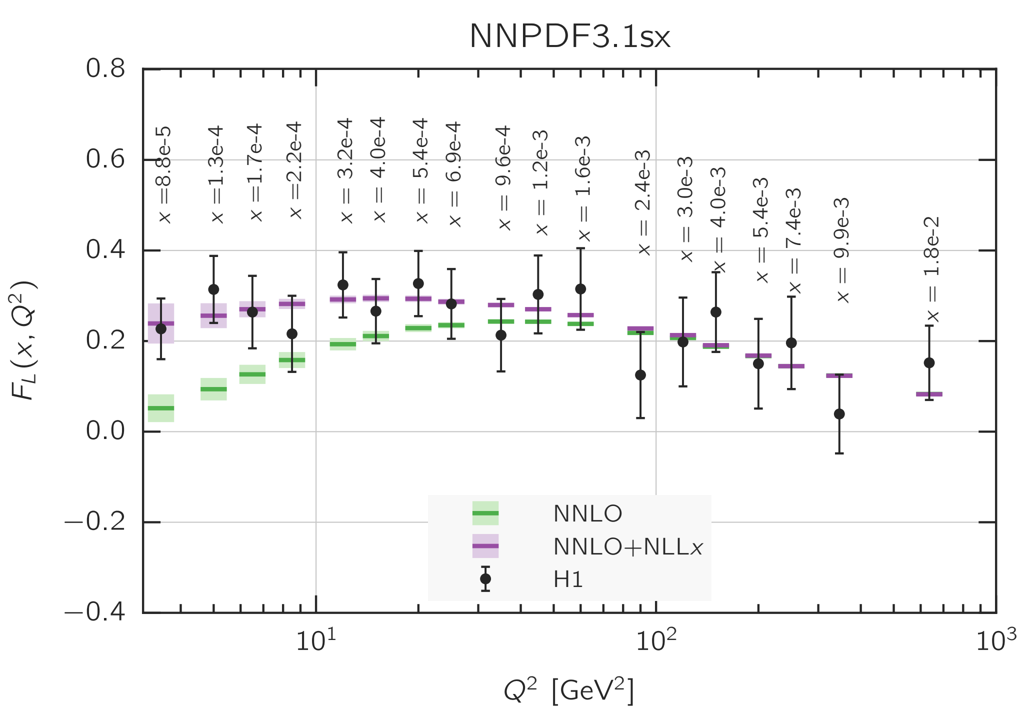




HERA STRUCTURE FUNCTIONS







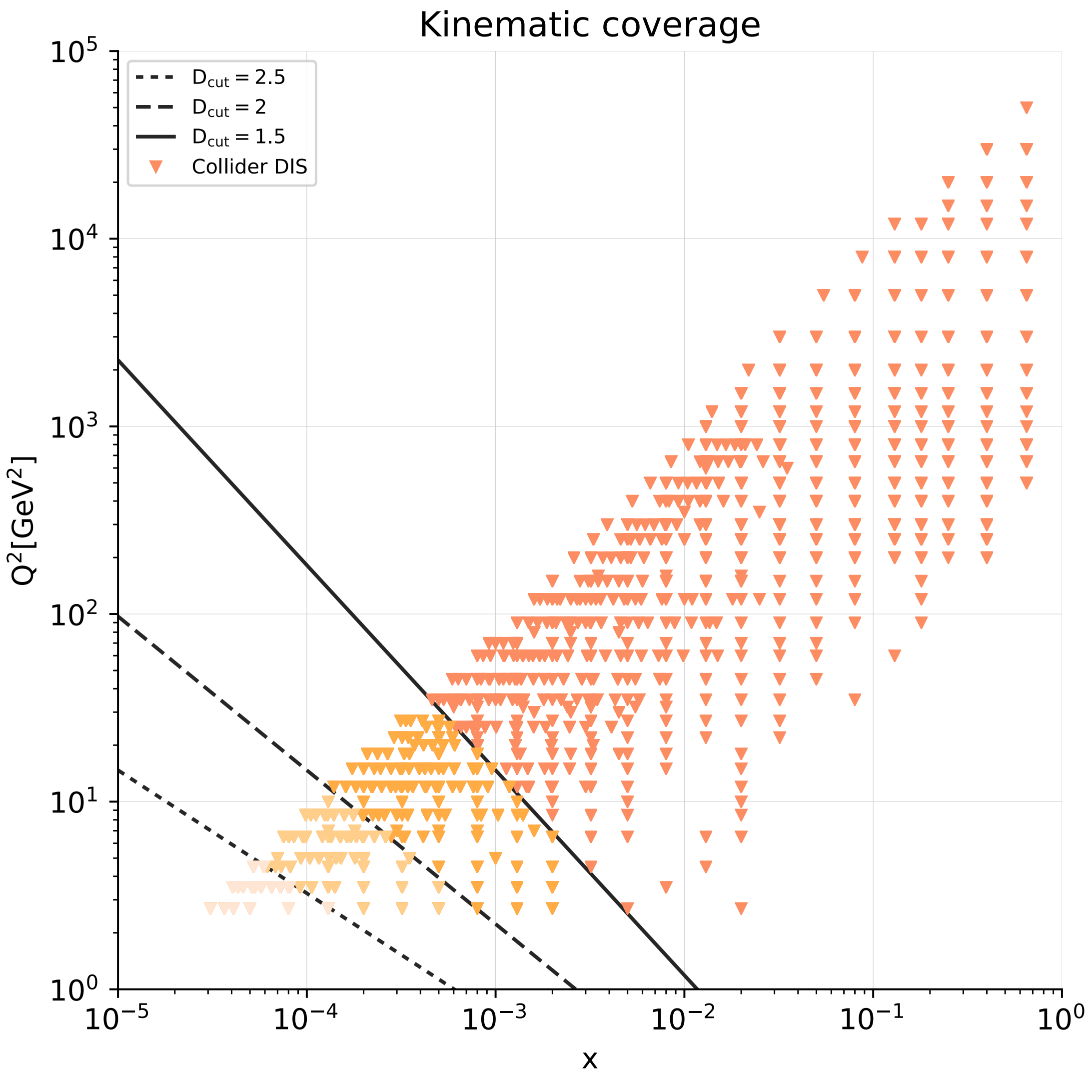
- the improved description of DIS structure functions is clearly visible
- this is particularly true for F_L where resummation effects starts at its LO

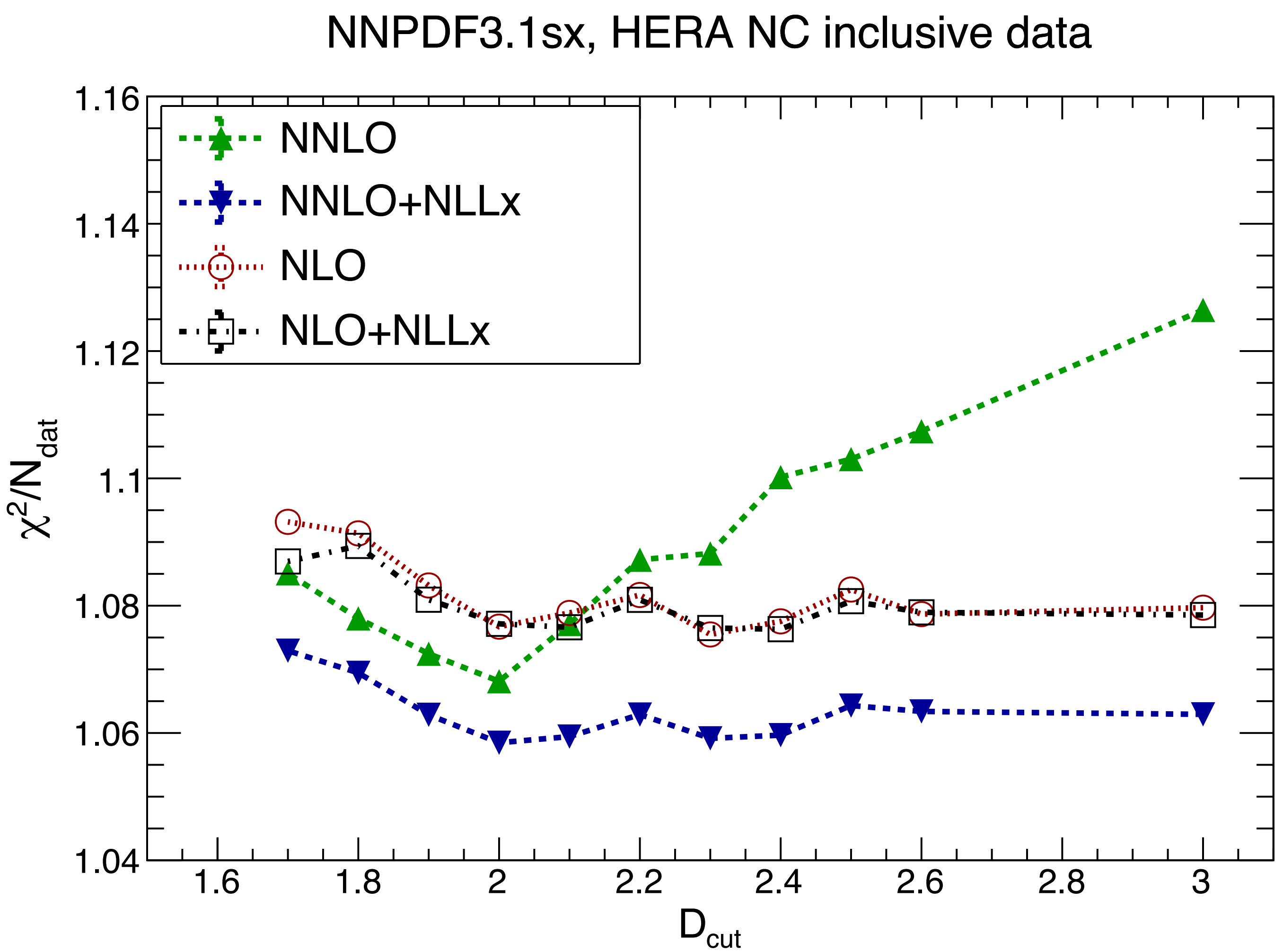

A CHRISTMAS CAROL.
IN PROSE.
BEING
BFKL
A Ghost Story of Christmas.
BY
CHARLES DICKENS.
WITH ILLUSTRATIONS BY JOHN LEECH.
LONDON:
CHAPMAN & HALL, 186, STRAND.
MDCCCXLIII.

BFKL: THE GHOST OF CHRISTMAS PAST

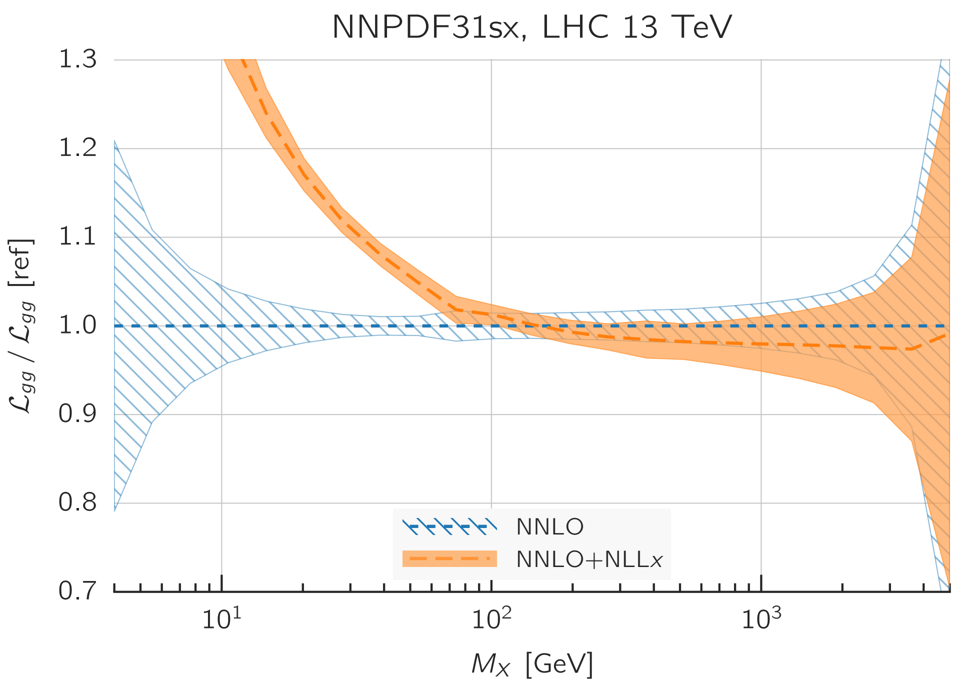
- How does the fit-quality change if we include data at smaller and smaller *x*?
- similar strategy as for hadronic data

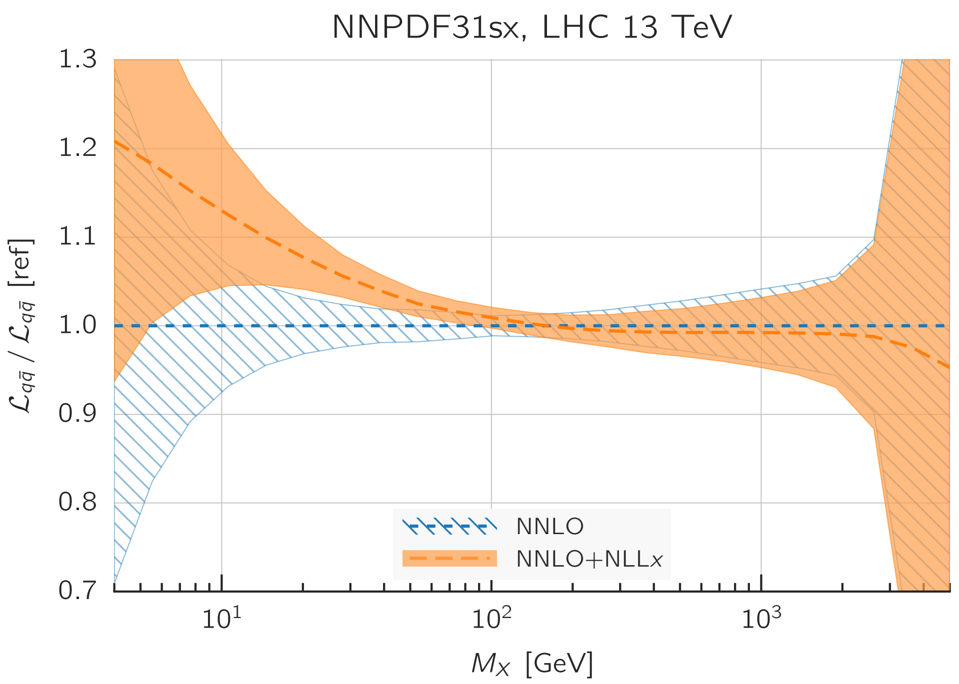
$$\alpha_s(Q^2)\ln\frac{1}{x} \geq D_{\rm cut}$$



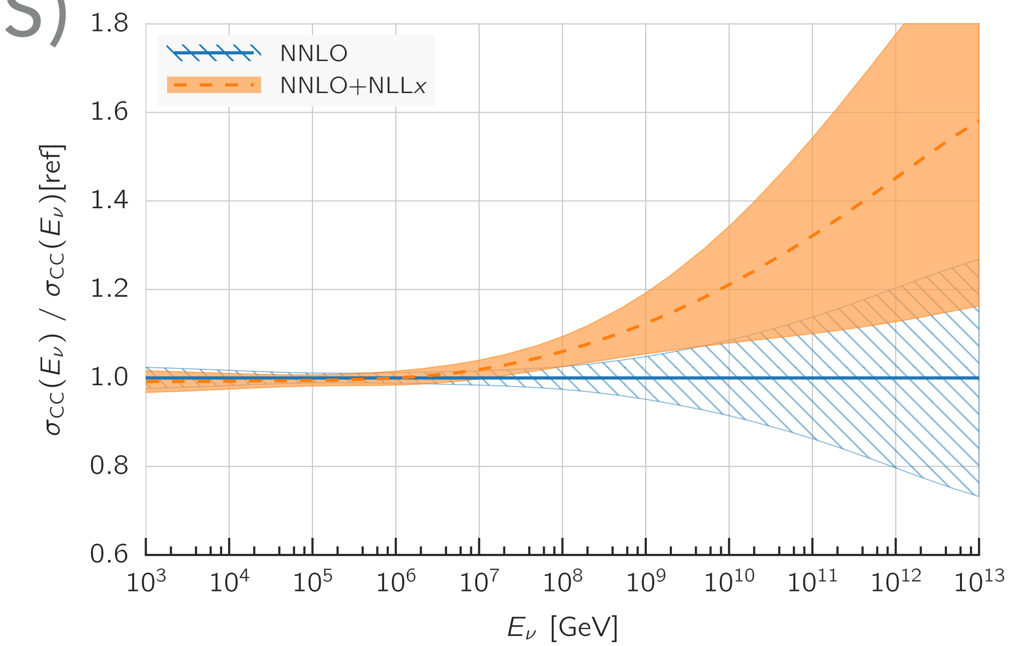


BFKL: THE GHOST OF CHRISTMAS PRESENT

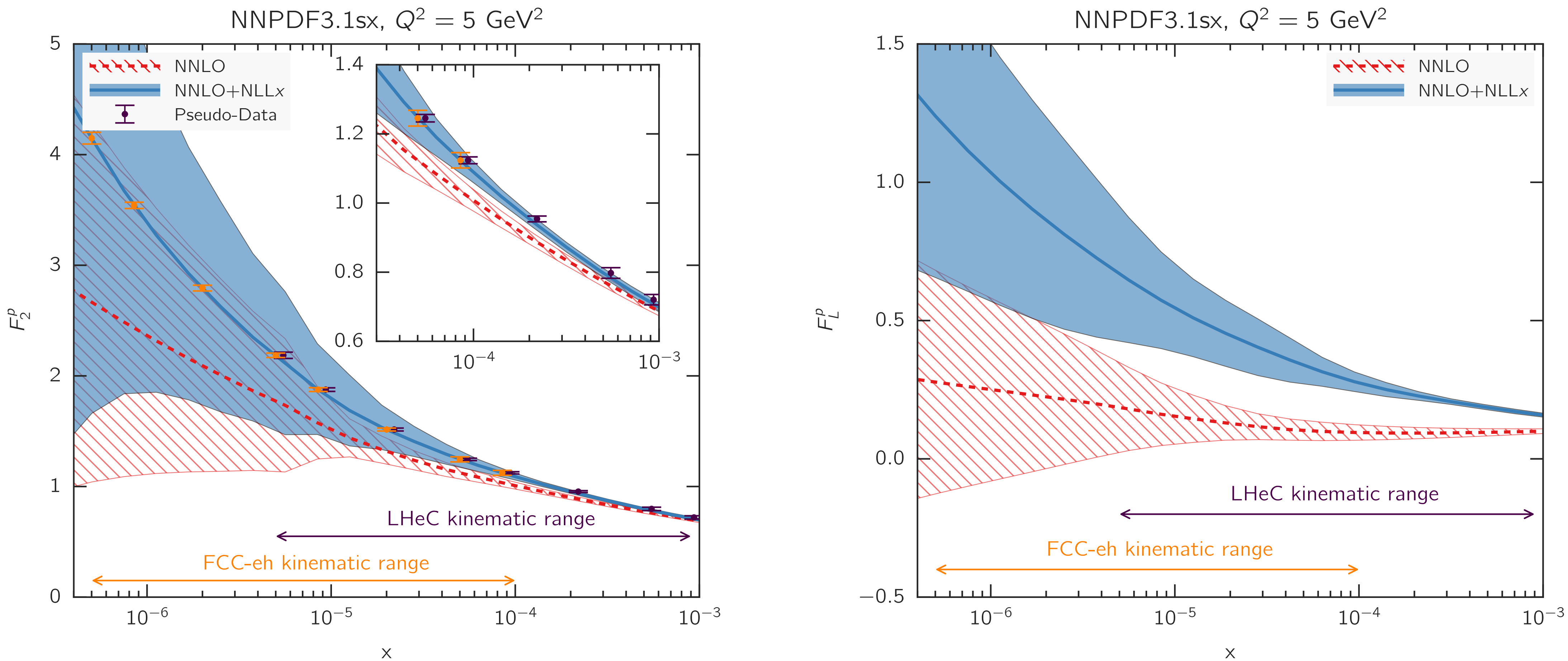




- to investigate LHC phenomenology we need resummed coefficient functions
- we can have a look at parton luminosities: qqbar doesn't change much but the change in gg is striking!
- consistent phenomenology for cosmic ray neutrinos (CC-DIS)
- unique "lab" for low-x physics



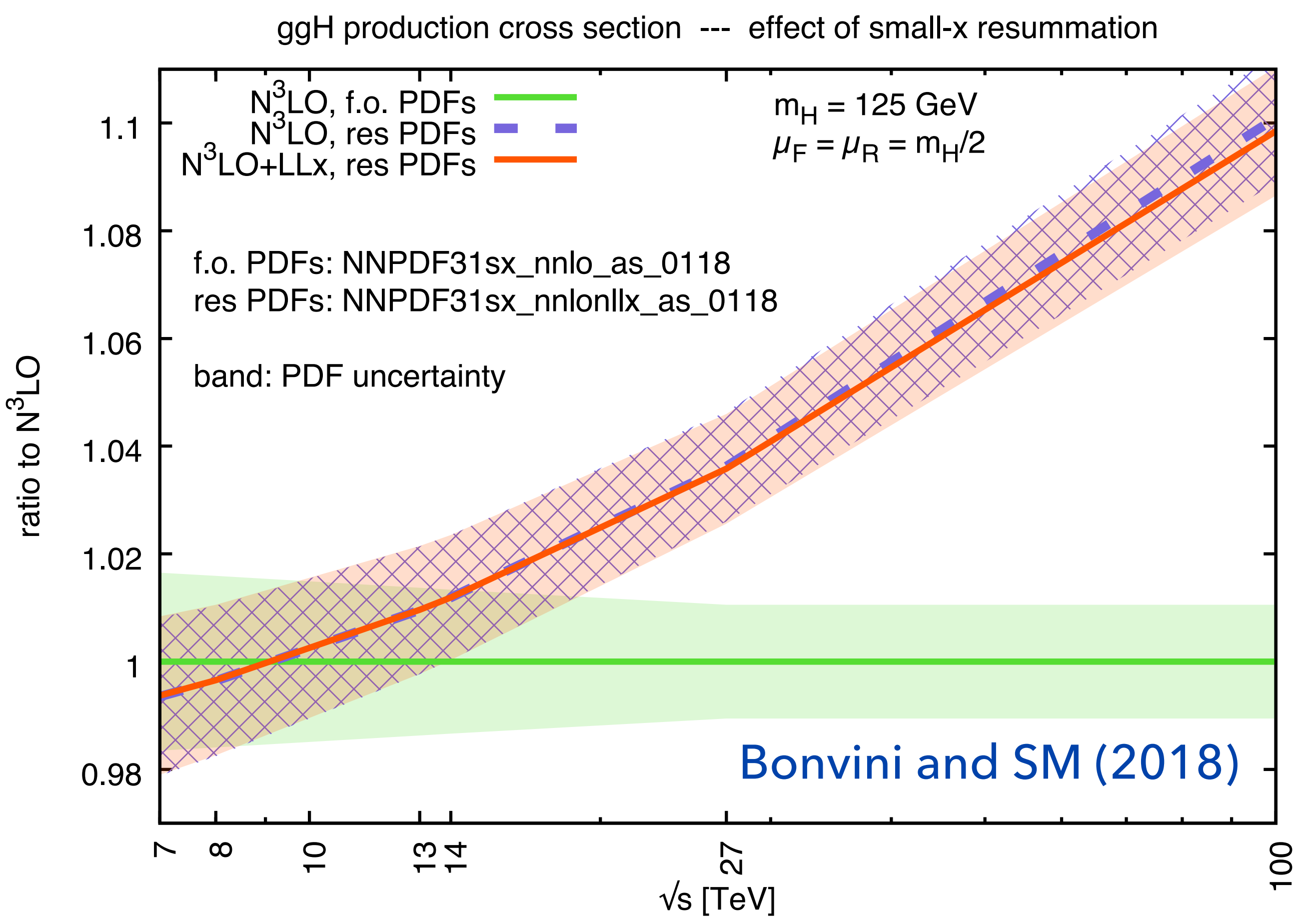
BFKL: THE GHOST OF CHRISTMAS YET-TO-COME



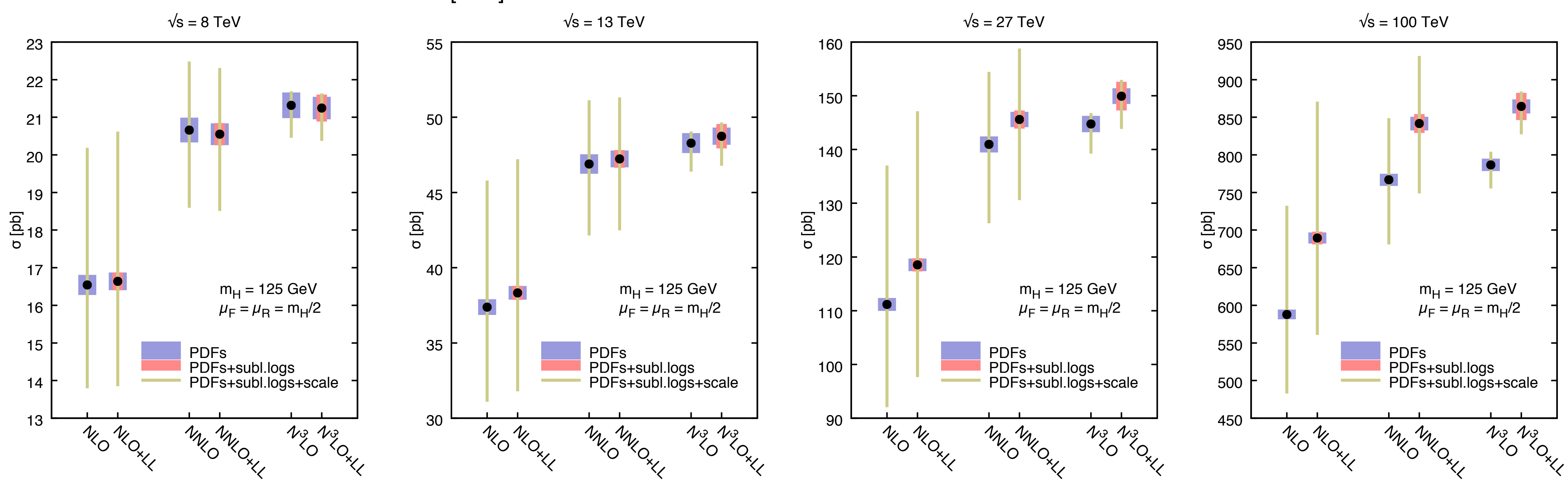
- small-x physics will be crucial at future circular colliders
- *e* (60 GeV) - *p* (7 TeV or 50 TeV) collisions
- to gauge the impact: fits including (resummed) pseudo-data

	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$ NNLO	$\chi^2/N_{\rm dat}$ NNLO+NLLx	$\Delta\chi^2$
HERA I+II incl. NC	922	1.22	1.07	-138
LHeC incl. NC	148	1.71	1.22	-73
FCC-eh incl. NC	98	2.72	1.34	-135
Total	**1168**	**1.407**	**1.110**	**-346**

THE HIGGS CROSS-SECTION



- resummation has little effect on the coefficient functions (and also on evolution)
- significant effects driven by the resummed gluon luminosity
- 10% correction at FCC

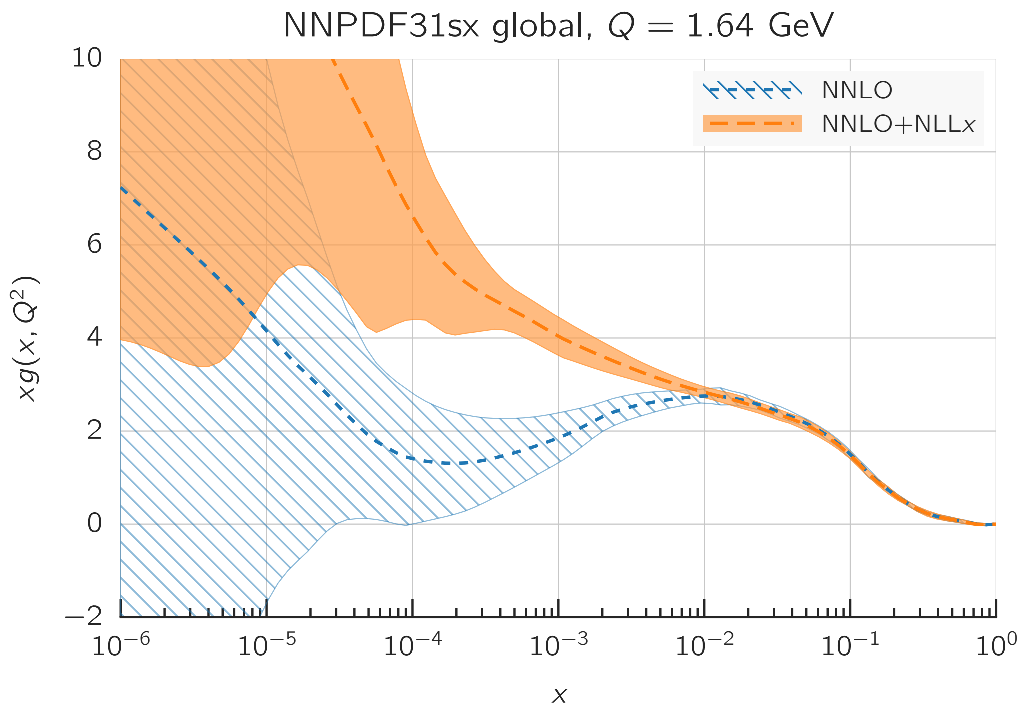


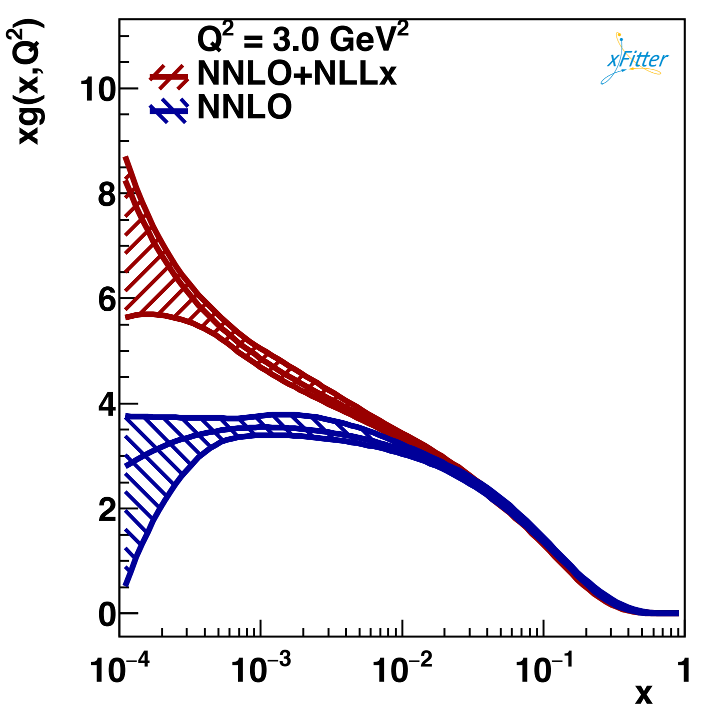
Bonvini (2018)

FINAL REMARKS AND CONCLUSIONS

COMPARISON TO XFITTER ANALYSIS

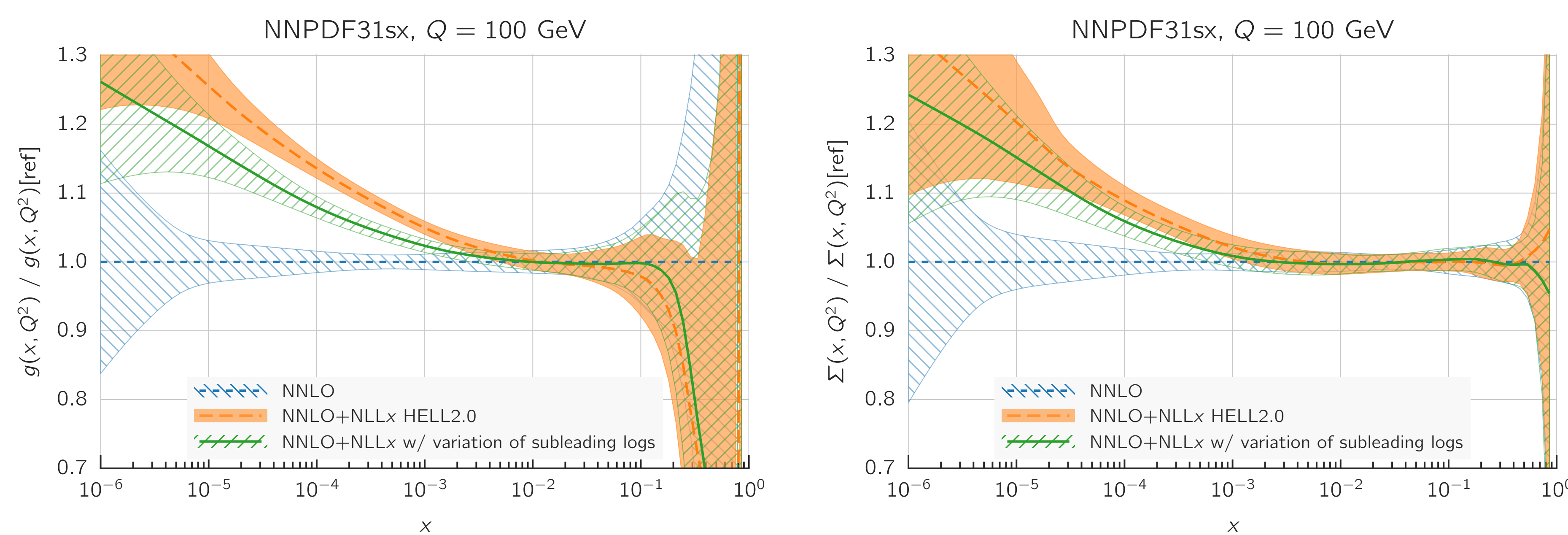
- PDF analysis presented so far was done within the NNPDF framework
- recently small-x resummation effects also studied by xFitter collaboration
- results in good agreement





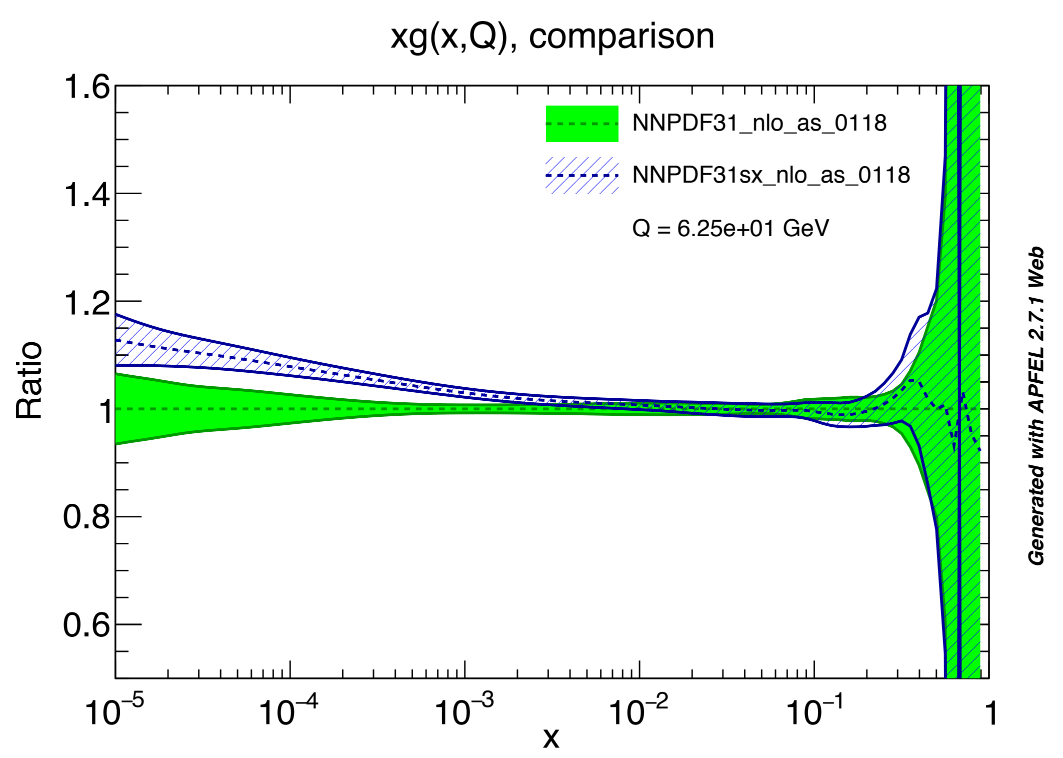
OPEN QUESTION I: IMPACT OF THEORETICAL UNCERTAINTIES

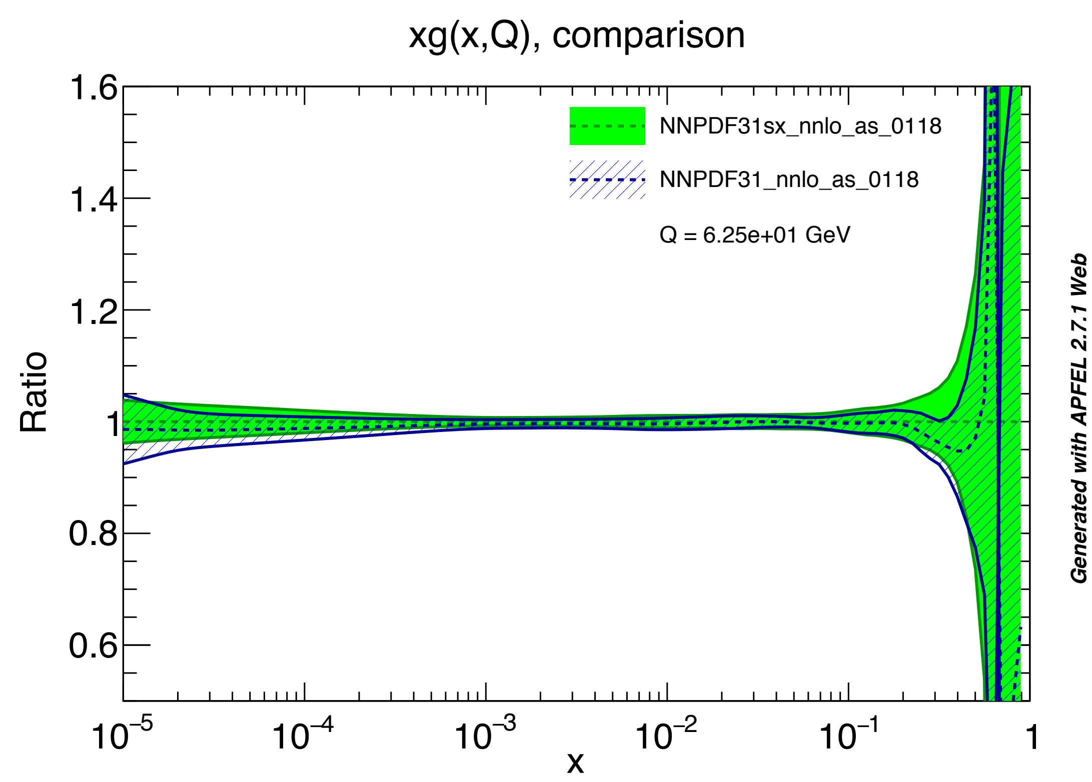
- quark splitting functions and coefficient functions start at NLLx (so we effectively include only their leading behaviour)
- the inclusion of theory errors in PDF fit is currently an active area of research
- we can use a setting that varies from the standard one beyond NLL*x*
- a DIS-only study shows that the fit quality is unchanged
- qualitative behaviour on solid grounds, however quantitative results do change



this shows the need for NNLL*x* resummation (at least in the quark sector)

OPEN QUESTION II: DEPENDENCE ON THE DATASET





- perturbative instability seems less pronounced in standard NNPDF3.1
- differences in methodology (truncated vs full solution of DGLAP) can affect NLO
- differences in datasets (less likely because NNLO are similar)
  - NNPDF31sx has an extra bin at low Q^2
  - NNPDF31sx does not include low-*x* hadronic data

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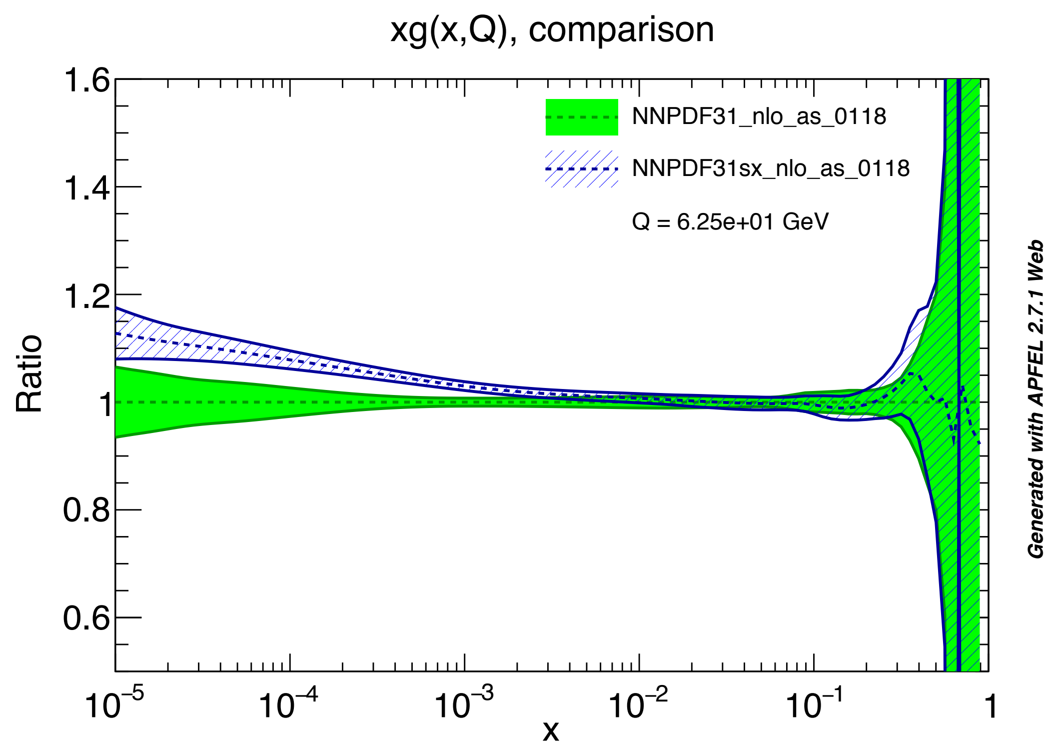

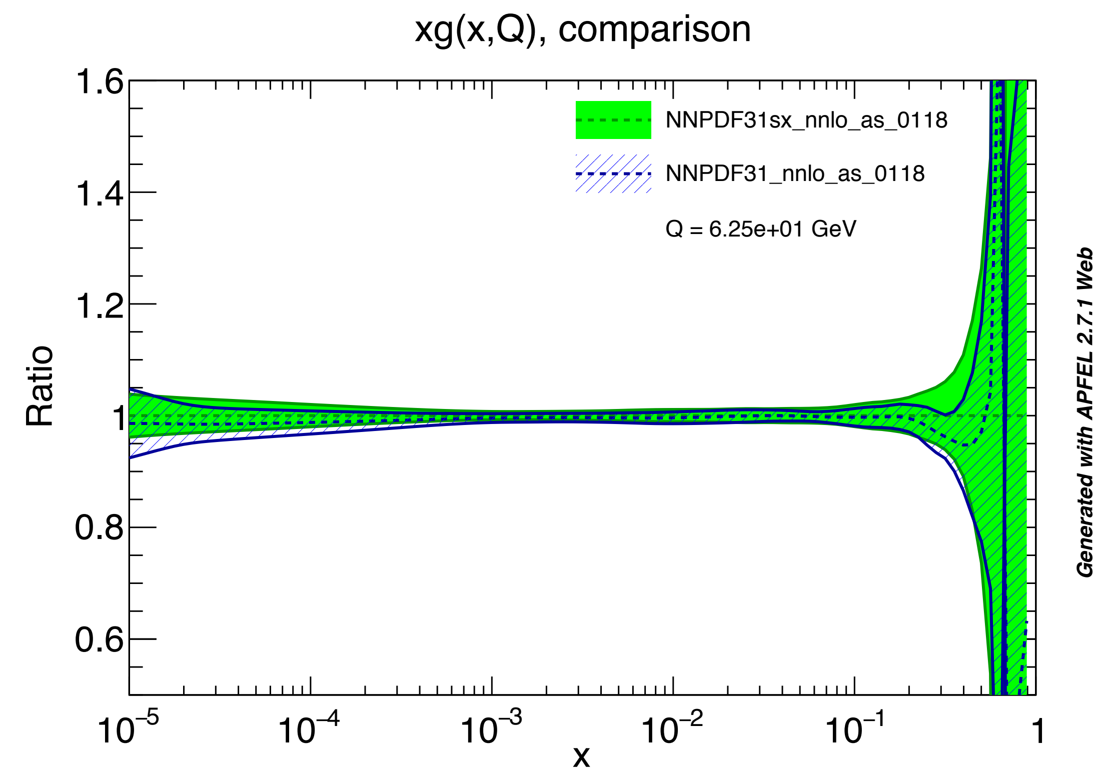

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  - NNPDF31sx has an extra bin at low Q^2
  - NNPDF31sx does not include low-x hadronic data

we need small-x resummation for Drell-Yan

OPEN QUESTION III: WHAT NEXT?

- small-x physics often thought as a niche topic
- we are now appreciating that it is an important ingredient in precision QCD calculations
- effects become bigger and bigger as c.o.m energy increases (it is called high-energy resummation after all!)
- electroweak physics at FCC is small-x physics!
- we can (and we should!) exploit LHC data and resummation to constrain PDFs at small-x
- what are the best observables? (3-D DY, jets, Z+jet…)

CONCLUSIONS

- better determinations of PDFs require both data and theory
- resummation offers a complementary direction
- small-x resummed fit shows evidence of BFKL dynamics in HERA inclusive data
- application to Higgs production: small correction to the N^3LO at the LHC, which becomes larger as the c.o.m. energy grows
- (almost) entirely due to small-x resummation in PDFs

OUTLOOK

- Towards truly global resummed fits:
  - DY at small *x* is the next item on the agenda
  - then jets (although small-x effects should be moderate
- Differential distributions:
  - resummation of rapidity distributions has been known for a while but never implemented
    Caola, Forte, SM (2011)
  - double Q_T and small-*x* resummation
    SM (2016); Zhou *et al*. (2016)

THANK YOU!