Swampland Notes

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1 Introduction and Recap

These are some notes for the Workshop Seminar.

Let us first recall the main message of last lecture: what is the Swampland Program? We have seen that the number of string vacua is huge, and we have called this "String Landscape". Since the landscape is so huge, in the past 10 years people tried to make model building with a bottom-up philosophy. Namely just start with a a 4d EFT which we like for phenomenological reasons (like some SUSY extension of SM for example) and then couple it gravity, and study it. One could then argue that since the number of String Theory vacua is so huge, then there must be among all these possibilities one which correspond to the low energy EFT we picked.

This line of thought is completely wrong.

Indeed if such line of thought would be correct, then *every* low energy EFT can be realized from some String Theory setup, then String Theory is completely useless for real world physics, as it would really make no predictions at all. One could get everything from it.

Of course, we all know that this is false. String Theory is a sensible and predictive theory of quantum gravity. The Swampland Program is all about quantifying *how much* String Theory is predictive. Namely, there are some EFTs which can never be obtained from String Theory. We will call the set of such theories *The Swampland*.

The Swampland copnjectures are a set of (so far 10) conjectures about which properties a gravitational EFT needs to have, in order for admitting some UV completion in String Theory. Some of these conjectures are more speculative than others, some can be proven at least in some setups, and almost all of them have phenomenological implications.

In the last lecture we saw two of these conjectures, namely

1. There are no global symmetries in a theory of quantum gravity. Every symmetry which looks global in the EFT must be either broken or gauged in the full quantum gravitational theory. If this is impossible, the EFT is in the Swampland.

2. The weak gravity conjecture. In every EFT with a $U(1)$ gauge group, there must exist at least one state with $q/m > 1$ in Plank units.

The plan of this talk is to briefly introduce three other Swampland Conjectures, namely

1. The Swampland Distance Conjecture

2. The No-susy AdS Conjecture

3. The No dS Conjecture

For a schematic review of all the different Swampland Conjectures see [1].

2 The Swampland Distance Conjecture

In order to discuss this conjecture we will first need to review some basic aspects about Kaluza-Klein compactifications. This is useful because it will provide us the easiest application of this conjecture. The discussion that follows is standard textbook meterial, in particular it is taken from [2].

2.1 Kaluza-Klein compactification

Let us consider a five-dimensional spacetime of the form $\mathbb{R}^4 \times \mathbb{S}^1$. Let us denote with x^M $(M = 0, \ldots 4)$ the coordinates of such spacetime, and let us further split them in x^μ for the Minkowski factor and y for the circle. Obviously y is periodic of period $2\pi R$ where R is the radius of the circle $\mathbb{S}^1$.

Consider a real scalar field in this background

$$S = \int d^5x \left(-\frac{1}{2} \partial_M \phi \partial^M \phi \right) \tag{1}$$

The scalar field needs to be periodic in y in order to be well-defined. Namely

$$\phi(x^\mu, y) = \phi(x^\mu, y + 2\pi R) \tag{2}$$

One can then expand in Fourier modes

$$\phi(x^\mu, y) = \sum_{k\in\mathbb{Z}} \phi_k(x^\mu) e^{iky/R} \tag{3}$$

By plugging this into 1 and integrating over y we obtain

$$S = (2\pi R) \int d^4x \left(-\frac{1}{2} \partial_\mu \phi_0 \partial^\mu \phi_0 \right) - 2\pi R \sum_{k=1}^{\infty} \int d^4x \left(\partial_\mu \phi_k \partial^\mu \phi_k^* + \frac{k^2}{R^2} \phi_k^2 \right) \tag{4}$$

where $\phi^* = \phi_{-k}$. This describes a 4d theory with a massless scalar ϕ_0 and an infinite tower of massive scalars, known as KK modes, labeled by the KK momentum k, and with masses

$$m_k^2 = \frac{k^2}{R^2} \tag{5}$$

A crucial point is that the masses of the Kaluza-Klein modes are inversely proportional to the radius. At energies $\Lambda < 1/R$, only the zero mode ϕ_0 would be observable, so the effective theory is a 4d field theory with one massless scalar. The extra dimensions are unobservable at low energies, as their detection requires energies of order $1/R$ to access the tower of Kaluza-Klein modes.

2.1.1 KK reduction of a gravitational theory.

One can do a Kaluza-Klein reduction also starting for 5d gravity. Now the metric G_{MN} needs to be split in three parts, namely $G_{\mu\nu}$, $G_{\mu 4}$ and G_{44}. Schematically

$$G_{MN} = \left(\begin{array}{c|c} g_{\mu\nu} & g_{\mu 4} \\ \hline g_{\mu 4} & g_{44} \end{array} \right) \tag{6}$$

so we see this will give rise to a 4d metric, a vector, and a scalar. It is useful to parametrize the metric in the following way

$$G_{MN} = e^{\sigma/3} \left(\begin{array}{c|c} g_{\mu\nu} + e^{-\sigma} A_\mu A_\nu & e^{-\sigma} A_\mu \\ \hline e^{-\sigma} A_\mu & e^{-\sigma} \end{array} \right) \tag{7}$$

Now the 5d metric can be expanded in Fourier modes and both the 4d scalar, the 4d vector and the 4d metric can will have their own KK tower. The effective action in 4d will be given by

$$S = M_P^2 \int d^4x \sqrt{-g} \left(R_{4d} - \frac{1}{6} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{4e^\sigma} F_{\mu\nu}^2 \right) \tag{8}$$

The scalar field σ has no scalar potential, so it is a *modulus field.* Its vacuum expectation value can be set at wish. Furthermore, it comes from the 44 componment of the 5d metric, so its vaacum expectation value is related to the size of the circle. The physical radius will be given by

$$R_{phys} = R_0 e^{-2\sigma/3} \tag{9}$$

Therefore we say that the moduli space of the 4d EFT is given by a semi-line parametrized by the expectation value $\langle R_{phys} \rangle$.

It is possible to consider the action in terms of this physical radius modulus, and we get

$$\sigma = -\frac{3}{2}\log\left(\frac{R_{phys}}{R_0}\right) \tag{10}$$

Therferore the physical radius is not canonically normalized, but has kinetic term given by

$$\int d^4x \frac{1}{R_{phys}^2}\partial_\mu R_{phys}\partial^\mu R_{phys} \tag{11}$$

The metric on the moduli space is given by $h = \frac{1}{R_{phys}^2}$.

2.1.2 String Winding modes.

Suppose that in the 5d theory there are strings, and they have tension γ. The worldsheet of those strings is 2-dimensional, one spatial and one temporal. So one thing they can do is wrapping the S^1 circle. From the point of view of the 4d EFT, such strings are perceived like particles. Their mass is given by

$$m \sim \gamma \cdot R_{phys} \cdot w \tag{12}$$

This is very intuitive, as the tension is by definition mass per unit length, and therefore the mass is given by tension times total length. The total length depends both on the size of R_{phys} and the number of times the string is wrapping the circle. Such number is w, called *winding number.* We call such a tower *winding mode tower.*

2.1.3 Properties of the moduli space

Now we need to make some very important comments.

1. The moduli space is non-compact.

2. Let us compute distances in the Moduli Space of the circle compactificaion. Let us fix a radius R_0 and compute the distance from R_0 to $\tilde{R}$ in the Moduli Space. This is given by

$$\begin{aligned} T := d(R_0, \tilde{R})) = \int_{R_0}^{\tilde{R}} \sqrt{h(R_{phys})} dR_{phys} = \int_{R_0}^{\tilde{R}} \frac{1}{R_{phys}} dR_{phys} = \\ = \log(\tilde{R}) - \log(R_0) = \log\left(\frac{\tilde{R}}{R_0}\right) \end{aligned} \tag{13}$$

   Therefore we see that starting from R_0 and going either to very large radius or very small radius means going at infinite distance in the Moduli Space.

3. If we go towards $R \to \infty$, the Kaluza Klein tower becomes more and more light. Remember we fixed a cutoff Λ at the very beginning. If we fix the cutoff and go too far into moduli space, this tower will become so light that it will invalidate my EFT description.

4. If we go towards $R \to 0$, the Winding Mode tower becomes more and more light. Remember we fixed a cutoff Λ at the very beginning. If we fix the cutoff and go too far into moduli space, this tower will become so light that it will invalidate my EFT description.

5. Let us compute how fast the KK tower becomes lighter, in terms of the distance in the moduli space. Let us consider the masses at the reference value R_0

$$m_k^2(R_0) = \frac{k^2}{R_0^2} \tag{14}$$

At the value $\tilde{R}$ we have

$$m_k^2(\tilde{R}) = \frac{k^2}{\tilde{R}^2} = \frac{k^2}{R_0^2 e^T} = m_k^2(R_0)e^{-T} \tag{15}$$

So we have learned that the tower is becoming light exponentially fast in the distance. It will eventually become massless at infinite distance. In this limit, this implies that does not matter how low my cutoff is, by going infinitely far in moduli space, some extra massless states will appear, and invalidate my EFT.

2.2 The general statement

We have seen a very nice explicit example of a Moduli space, and some of its properties. Let us now generalize these properties to the complete statement of the Swampland distance conjecture. This conjecture is supposed to hold true *for any* Moduli Space.

2.2.1 Moduli space is non-compact

The moduli space $\mathcal{M}$ of vacua (if non-trivial) is non-compact. In more detail, fix a point $p_0 \in \mathcal{M}$. Then $\forall\ T > 0\,, \exists\ p \in \mathcal{M}$ such that

$$d(p_0, p) > T\ . \tag{16}$$

where $d(p_0, p)$ is the distance between p_0 and p, computed by using the moduli space metric as the length of the geodesic passing through p and p_0.

To elucidate this criterion, we first need to discuss what we mean by the moduli space metric. Consider an EFT coupled to gravity, with N massless scalar fields Φ_i, $i = 1, ..., N$ with no potential. Such scalar fields arise generically in string compactification, and their vacuum expectation values $\langle \Phi_i \rangle$ is related to geometrical quantities in the compactification manifolds such as for example the volumes of some cycles or their shapes. We will call the algebraic variety parametrized by the various $\langle \Phi_i \rangle$ the *moduli space* $\mathcal{M}$. In the EFT, the kinetic term for those scalar fields typically takes the form

$$\mathcal{L}_{eff} = g_{ij}(\Phi)\partial_\mu \Phi^i \partial_\mu \Phi^j + \dots\ , \tag{17}$$

where g_{ij} is the metric on $\mathcal{M}$. We can use this metric to compute distances in the moduli space, and ask if $\mathcal{M}$ is compact or not. As it turns out in all known examples from compactifying string theory, the moduli space is non-compact [3].

2.2.2 Tower of states becoming lighter

Fix a point $p_0 \in \mathcal{M}$. In the limit of infinite distance from p_0, that is as $d(p_0, p) = T \to \infty$, there will be a tower of states in the EFT whose mass decreases exponentially with T,

$$m \sim e^{-\alpha T} \; . \tag{18}$$

We saw that for any choice of a starting point $p_0 \in \mathcal{M}$ and any real number $T > 0$, we will always be able to find a (in general not unique) point $p \in \mathcal{M}$ such that the distance between p and p_0 is larger than T. So in general $\mathcal{M}$ will have some non-compact directions. We want to ask now what happens when we go extremely far away in moduli space along one of those directions, or equivalently we take T to be extremely large [3].

Heuristically, we can understand this by considering the point compactification of moduli space, $\hat{\mathcal{M}}$, so that $\hat{\mathcal{M}}$ is a finite manifold where the infinities of $\mathcal{M}$ correspond to singular points of $\bar{\mathcal{M}}$. Now going to infinity corresponds to going to a singularity where generically extra massless degrees of freedom appear.

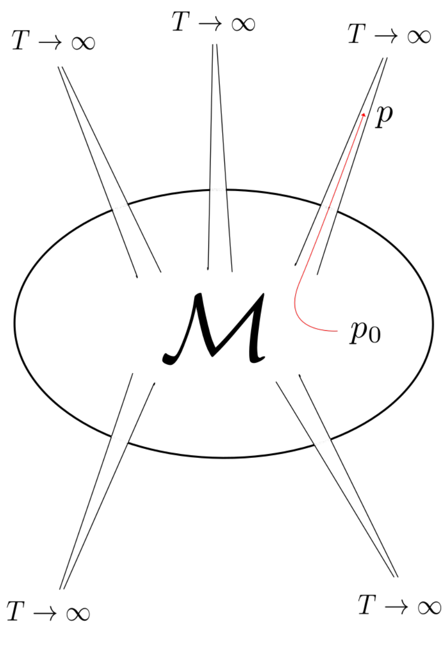

Figure 1: A schematic picture of the moduli space.

One question one can ask is how small does the α needs to be, according to the conjecture.

3 Non-Susy AdS conjecture

Non-supersymmetric AdS belongs to the Swampland. This in particular implies that non-supersymmetric AdS/CFT holography belongs to the Swampland as well.[4].

There are two ways to motivate this conjecture. Let us review one. Let us consider the Weak Gravity Conjecture, as discussed in the precious lecture. Recall that such conjecture, in its weakest form, demanded the existence in the spectrum of at least one particle with mass m and electric charge q such that

$$\left(\frac{m}{M_P}\right) \leq q \tag{19}$$

A natural question to ask now is for which states the WGC bound is saturated. The answer is given by a more sharpened version of the WGC, which is the following [4]:

The equality sign in the Weak Gravity conjecture holds if and only if

1. *The underlying theory is supersymmetric*
2. *The states saturating the WGC bound are BPS states.*

Let us assume that this refined version of the weak gravity conjecture is true. Then the motivation for the AdS criterion is very intuitive and follows from this assumption.

In case the underlying theory is supersymmetric, there will be BPS branes and therefore they saturate the weak gravity bound. This means that their "electric" repulsion is equal to their gravitational attraction, and we can pile many branes on top of each other. We typically get holography in string theory by piling branes in this way and then taking the near horizon limit. We could naively think to get non-supersymmetyric AdS/CFT in the same way: namely first we break supersymmetry in some manner, and then we pile branes and take the near-horizon geometry.

However, the problem is that if the branes are not supersymmetric, then they cannot saturate the WGC bound. Then the repulsion between branes wins over attraction due to WGC. In this case there is no way to keep the branes close to each other. The refined WGC is simply saying that in the non-supersymmetric setup, those branes will repel and fly apart!

4 dS conjecture

This last Swampland Conjecture is a congecture about the universal behaviour of scalar potentials.

Let us consider any QFT coupled to gravity and suppose that in the EFT there are scalar fields ϕ_i with a potential $V(\phi)$. Then,

$$|\nabla V| \geq c \cdot V \tag{20}$$

for every possible value of the scalar fields ϕ_i, or otherwise the EFT is in the Swampland. Here c is a positive constant of order 1 *in Planck units, but still undermined precisely.*[5].

4.1 Some examples

Let us try to have a intuitive feeling of what this condition means. Let us consider the easier case of a single scalar field ϕ. Then

$$|V'(\phi)| \geq c \cdot V(\phi) \tag{21}$$

and for simplicity let us set $c = 1$. Let us test this conjecture with some possible scalar potential one can think of. Let us also suppose that ϕ can range from -1 to $+1$ in order to be safe with the Swampland distance conjecture.

1. Quadratic potential.
   Let us suppose to take $V(\phi) = \phi^2$. Then the conjecture reads

   $$2|\phi| \geq \phi^2 \tag{22}$$

   It is easy to check that this inequality is satisfied for all possible values of $\phi \in [-1, 1]$.

2. Upwards shifted quadratic potential.
   Let us suppose to take $V(\phi) = \phi^2 + 1$.Then the conjecture reads

   $$2|\phi| \geq \phi^2 + 1 \tag{23}$$

   And we see we immediately run into a problem, for example the inequality does not hold for $\phi = \frac{1}{2}$ as in this case we would have

   $$1 \geq \frac{5}{4} \tag{24}$$

   which is clearly false.

3. Downwards shifted quadratic potential.
   Let us suppose to take $V(\phi) = \phi^2 - 1$.Then the conjecture reads

   $$2|\phi| \geq \phi^2 - 1 \tag{25}$$

   It is easy to check that this inequality is satisfied for all possible values of $\phi \in [-1, 1]$.

4. Always decreasing positive potential.
   Let us suppost to take $V(\phi) = e^{-\phi}$. Then the conjecture reads

   $$|-e^{-\phi}| \geq e^{-\phi} \tag{26}$$

   Which is satisfied for all possible values of ϕ.

5. Always decreasing potential, but slower.
   Let us suppost to take $V(\phi) = \frac{1}{\phi}$ and also suppose to take $\phi \in]0, 1]$. Then the conjecture reads
$$\left| -\frac{1}{\phi^2} \right| \geq \frac{1}{\phi} \tag{27}$$
   which is
$$\frac{1}{\phi} \geq 1 \tag{28}$$
   Which is satisfied for all possible values of ϕ.

What have we learned from these examples? In conclusion, this conjecture means that *the slope of V cannot be too small when V is positive*. Now, the key point is that local extrema of $V(\phi)$ occur when the derivative of ϕ vanishes. Therefore this conjecture generically rules out scalar potentials if their minima (or maxima, or inflection point) occur at a positive value of $V(\phi)$.

This implies that there is no way in String Theory to get a (meta)stable dS solution. Indeed in all such cases the scalar potential has the form given by figure 2, where the minimum lies at positive value of $V(\phi)$. This has tremendous consequences as we measure positive cosmological constant. The way out that is proposed is quintessence.

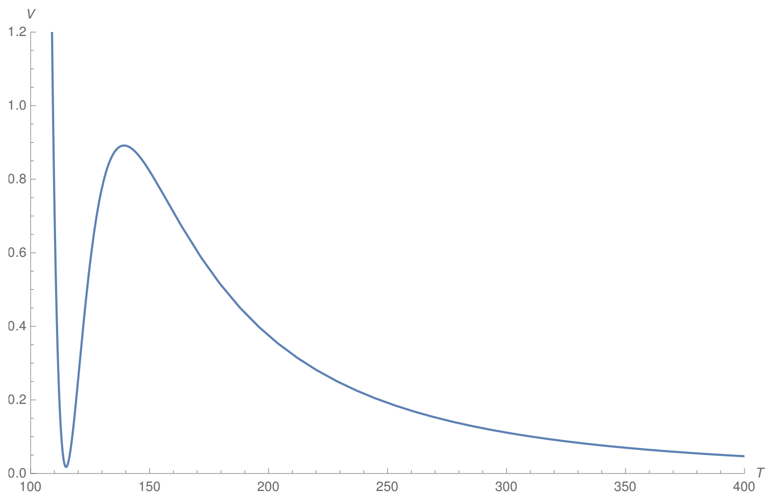

Figure 2: The scalar potential for a metastable dS. As there are both a minimum and a maximum at positive $V(\phi)$, this is ruled out by the dS conjecture.

4.2 Motivation

Motivation for this conjecture arises from direct experience in String Theory. Building a metastable dS vacuum *is hard*. For a review about how hard it is, look at [6]. Up to date, there is no known example of a construction that gets dS in String Theory parametrically under control. Mild evidence for this conjecture is given by some classical no-go theorems like [7], altought it is very easy to evade such no-go theorems as soon as non-perturbative effects or quantum effects (like euclidean branes, orientifolds, gaugino condensations etc) are taken into account. Constructions certainly worth to mention are the famous KKLT and LVS, which claim to get a metastable dS vacuum and indeed use quantum effects to evade the no-go theorem. However, it is also fair to say that they both come with their own set of difficulties.

4.3 A counterexample

As pointed out in [8], the Higgs potential is clearly a counterexample, as the maximum of the Higgs potential occurs as positive $V(H)$. Recall that

$$V(H) = \lambda_H(|H|^2 - v^2)^2 \tag{29}$$

has the famous Mexican-hat shape with a local maximum at $H = 0$. However this might not be a problem, as we do not think that the Higgs field is the only scalar field in the full theory, then the scalar potential will not be the $V(H)$ anymore. However, suppose we add to the theory a quintessence scalar with $V_Q(\phi) = V_0 e^{-\lambda\phi}$, completely decoupled from the Higgs field, namely

$$V = V(H) + V_Q(\phi) \tag{30}$$

then it was argued that this violates the dS conjecture by 55 orders of magnitudes, namely by taking $|\nabla V_Q| \sim V_Q \sim 10^{-120}$ and $V_H(0) = 10^{-65} M_P^4$ one gets

$$\frac{\nabla V}{V} \simeq \frac{10^{-120}}{10^{-65}} \simeq 10^{-55} \tag{31}$$

in Plank units.

In order to avoid this problem a new version of the dS conjecture was developed. [9] The new conjecture says that the EFT is in the Swampland if BOTH of the two following conditions are violated.

1. $|\nabla V| \geq c \cdot V$
2. $\min \partial_i \partial_j V(\phi) \leq -V(\phi)$

where min means the smallest eigenvalue of the Hessian matrix. If at least one holds true, then it is OK. The effect of this second condition is that now maxima are generically admitted.

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