Dark Matter Phenomenology

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In this Talk

- ☑ UV-complete models vs. simplified models
- ☑ Dark Photons
- ☑ Primordial black holes as a DM candidate

UV-complete Models vs. Simplified Models

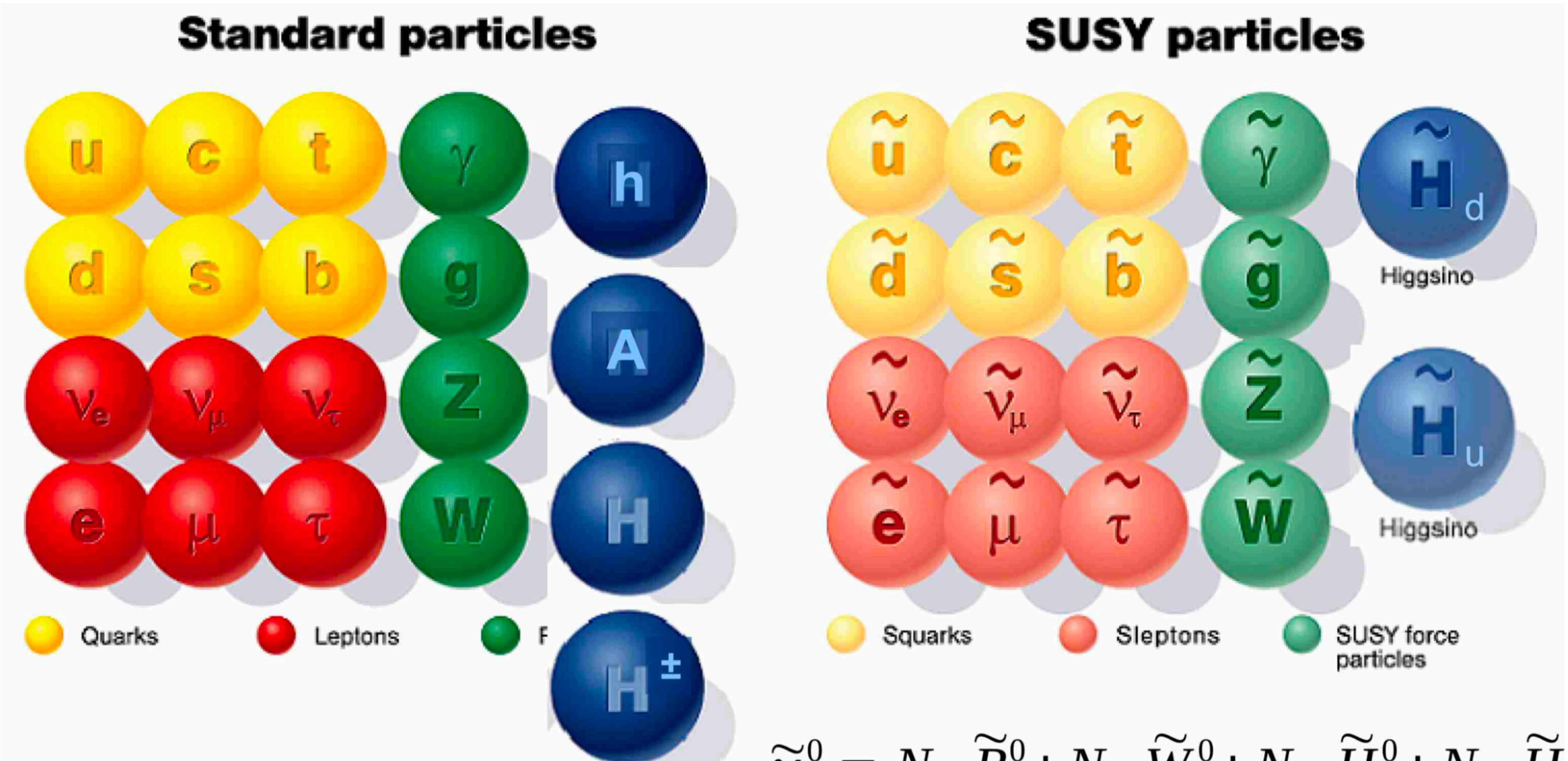
DM in UV-complete models

- Traditional approach to DM searches:
  - Work in a UV-complete scenario, guided by theoretical arguments
  - For instance MSSM (minimal supersymmetric standard model)

The MSSM

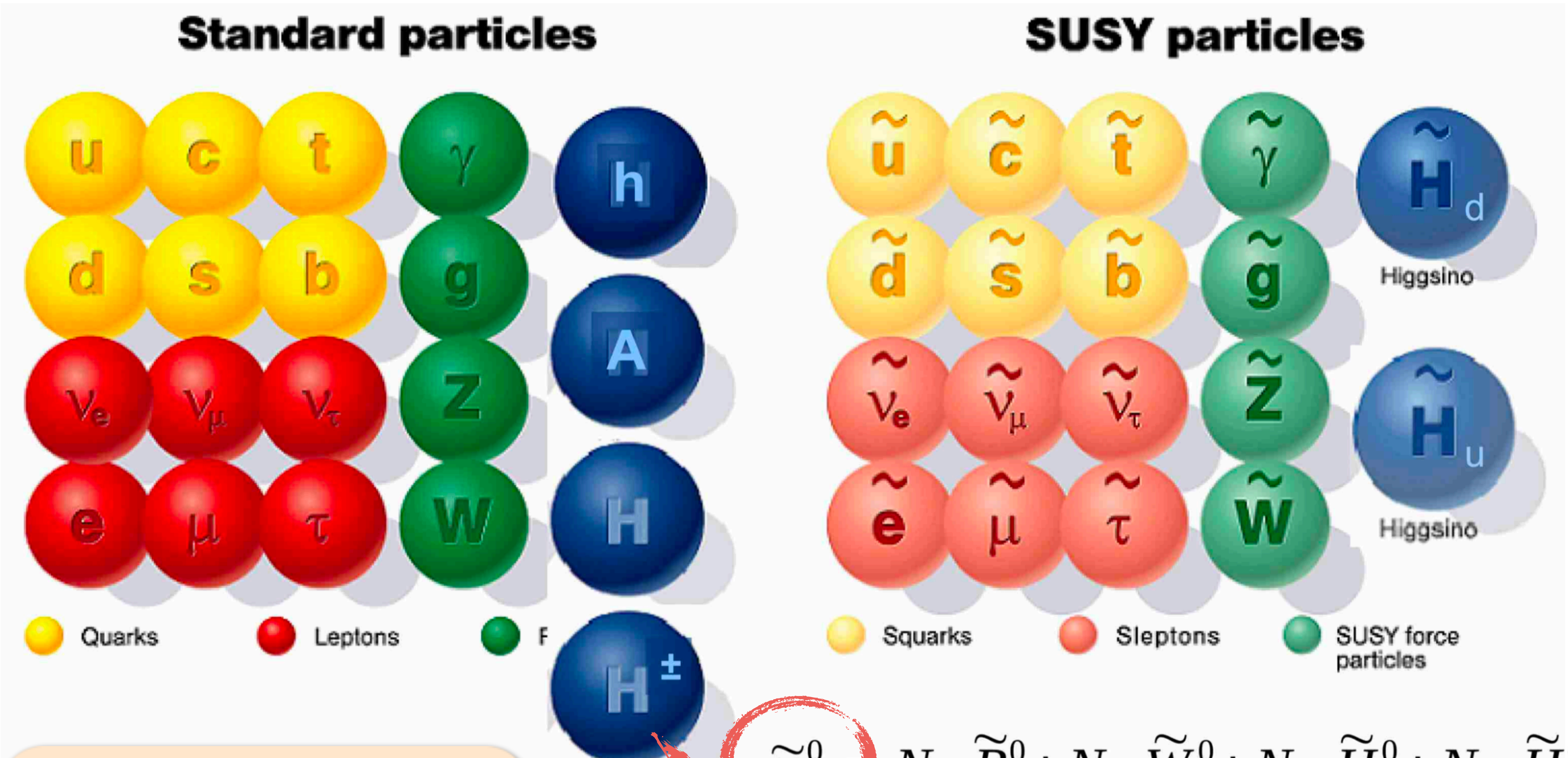

STORCK
DAS MULTIVITAMIN-BONBON · FÜR DIE GANZE FAMILIE
nimm2
Orangen- und Zitronenbonbons
mit Vitaminen
Fruchtsaft & Vitamine

The MSSM



$$\widetilde{\chi}_i^0 = N_{i1}\widetilde{B}^0 + N_{i1}\widetilde{W}^0 + N_{i3}\widetilde{H}_u^0 + N_{i4}\widetilde{H}_d^0$$

taken from a slide by Are Raklev



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Neutralino:
neutral fermion with
weak couplings to the SM

taken from a slide by Are Raklev

The MSSM

- ☑ Equal number of fermion and boson states
  (not equal number of particles!)
- ☑ Particles and their superpartners have equal mass
  - SUSY must be broken in nature
- ☑ Helps achieve Grand Unification of gauge couplings
- ☑ Has a DM candidate
- ☑ Solves hierarchy problem

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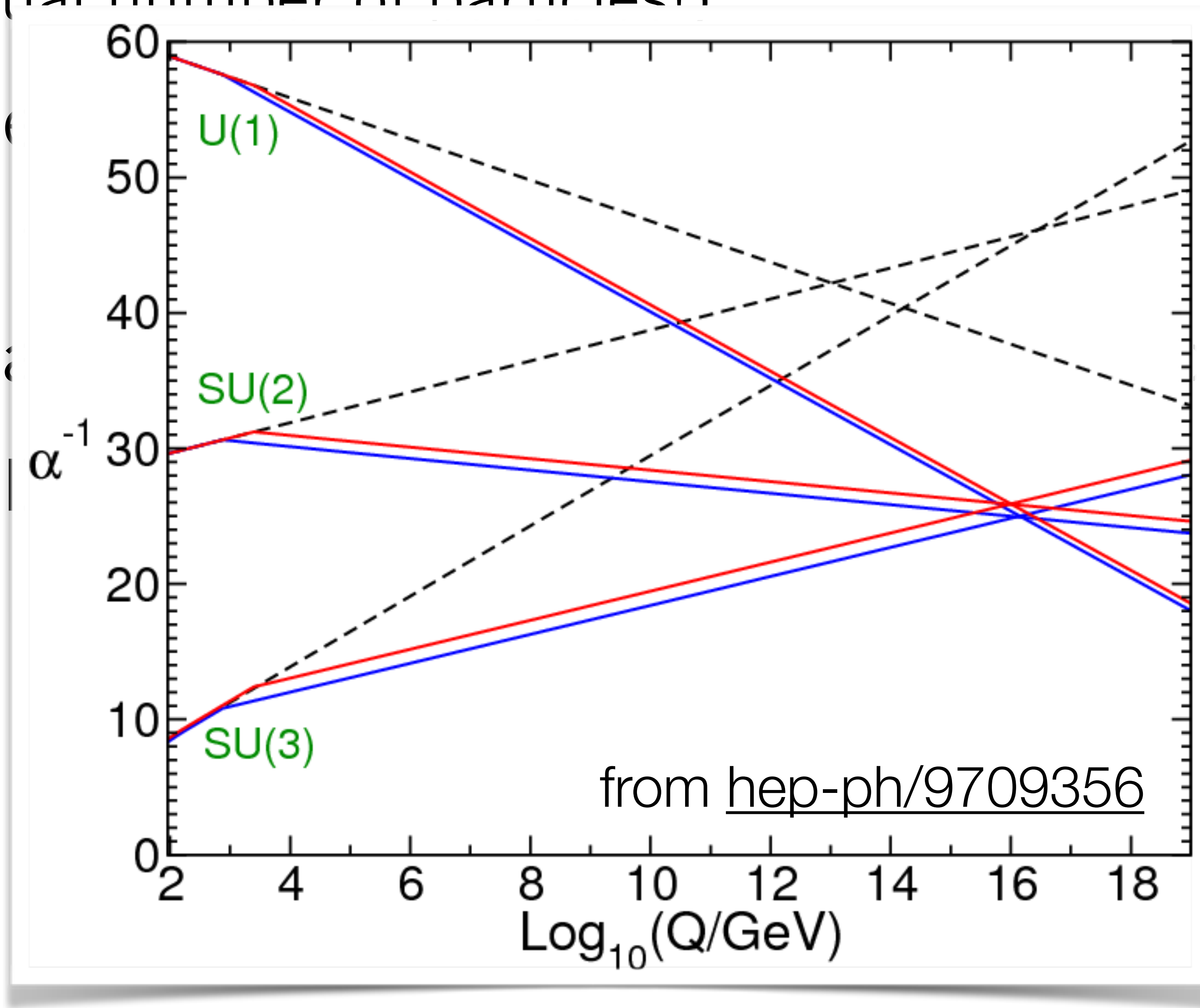

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The MSSM

- Motivation through symmetry
  - special relativity: physics invariant under Poincaré symmetry

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\mu + a^\mu$$

  - Maximal symmetry a spacetime can have (Coleman-Mandula theorem)
  - But: can be cheated by adding anti-commuting (Grassmann) coordinate θ with {θ, θ'} = θ θ' + θ' θ = 0
  - Regular fields φ(x) are replaced by superfields φ(x,θ)
  - One can always write φ(x,θ) as

$$\varphi(x,\theta) = A(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x)$$

The MSSM

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Rotations & Boosts

Translations

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Scalar field (e.g. squark)

Fermion field (e.g. quark)

Auxiliary field
(no kinetic term, can be removed using equations of motion)

Supersymmetric Dark Matter

Supersymmetric Dark Matter

- ☑ Successful thermal freeze-out
  - yields correct DM abundance automatically

Supersymmetric Dark Matter

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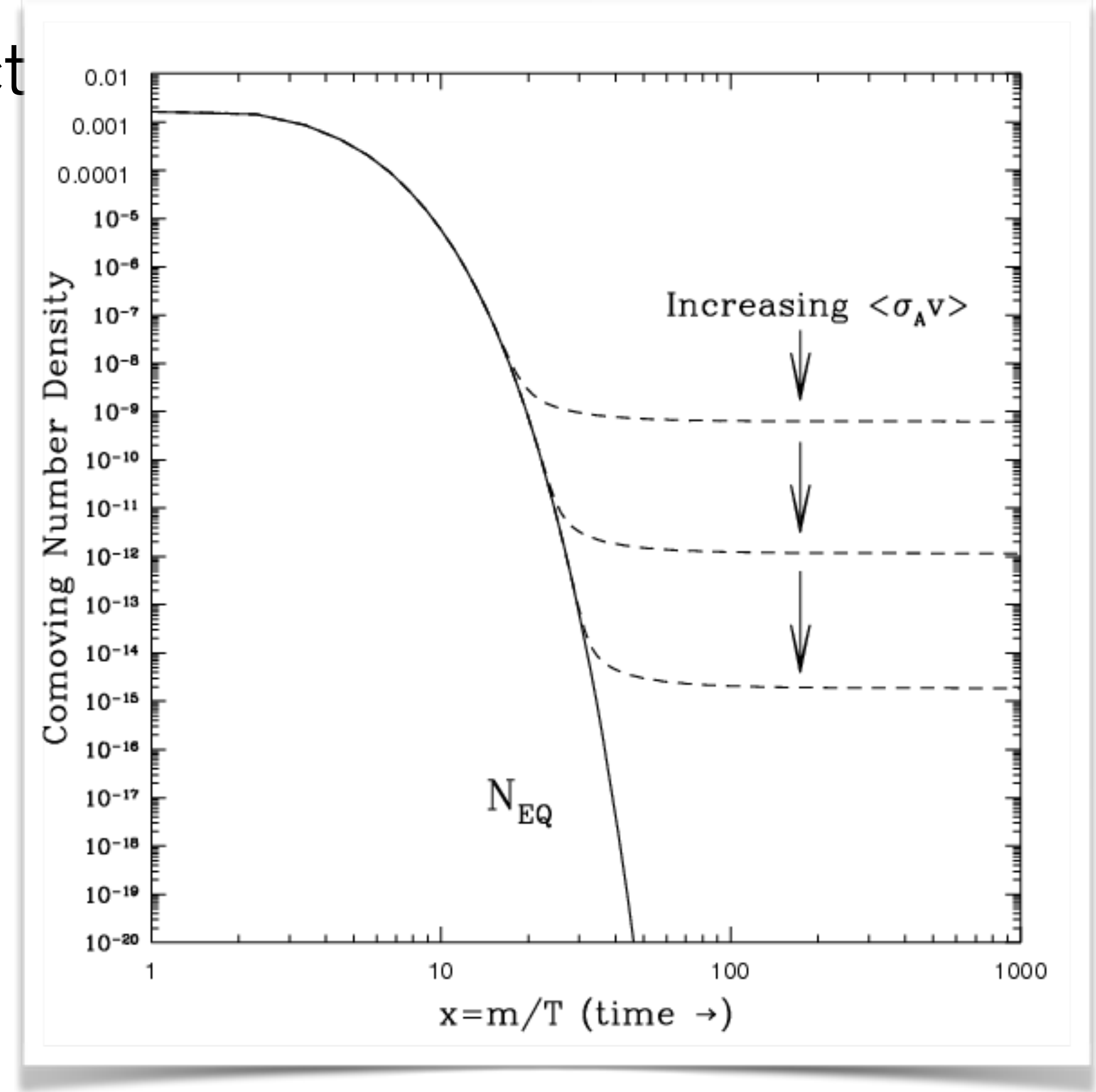
  - yields correct



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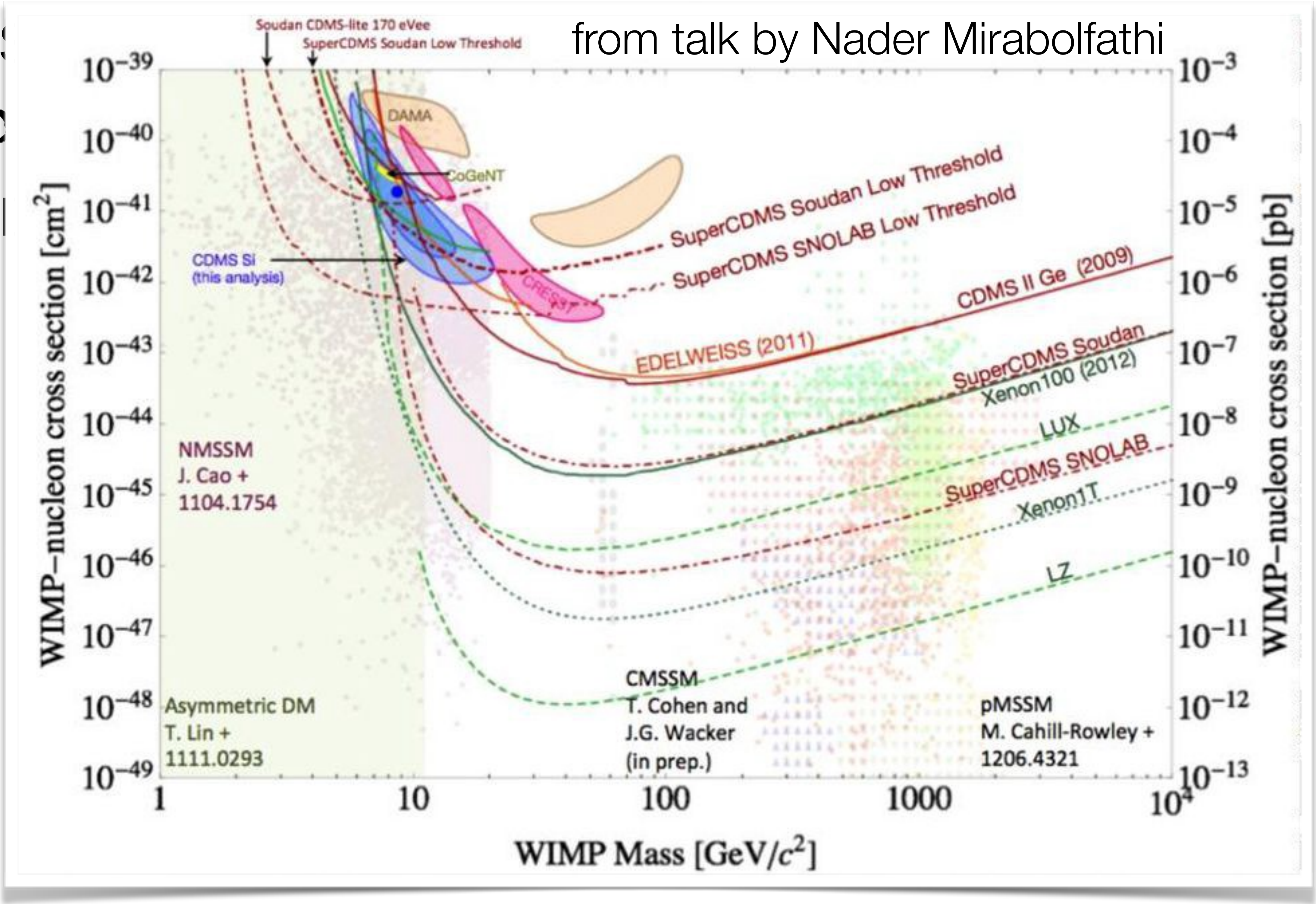

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Supersymmetric Dark Matter

- ☑ Succe
  - yield
- ☑ Detec
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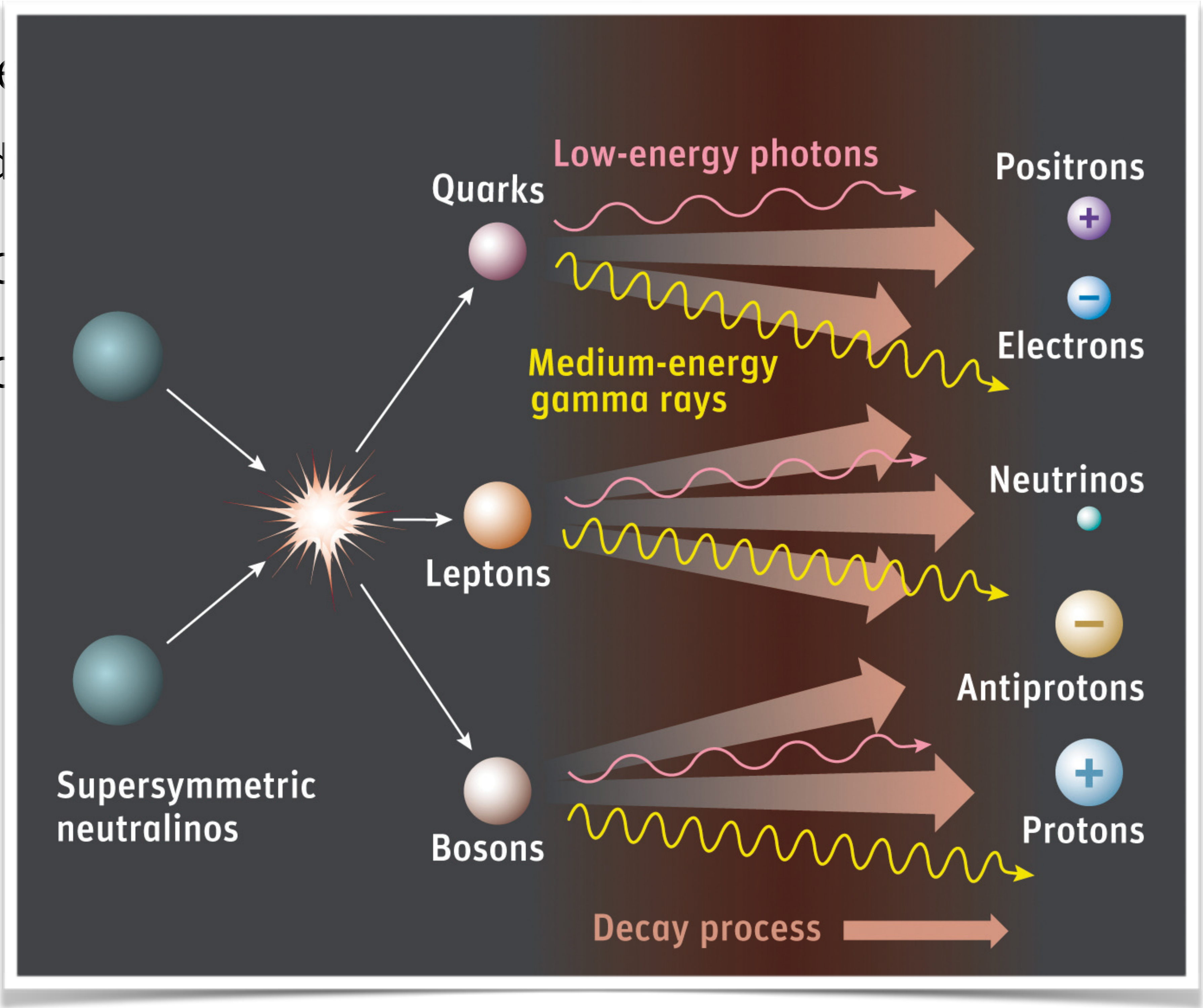

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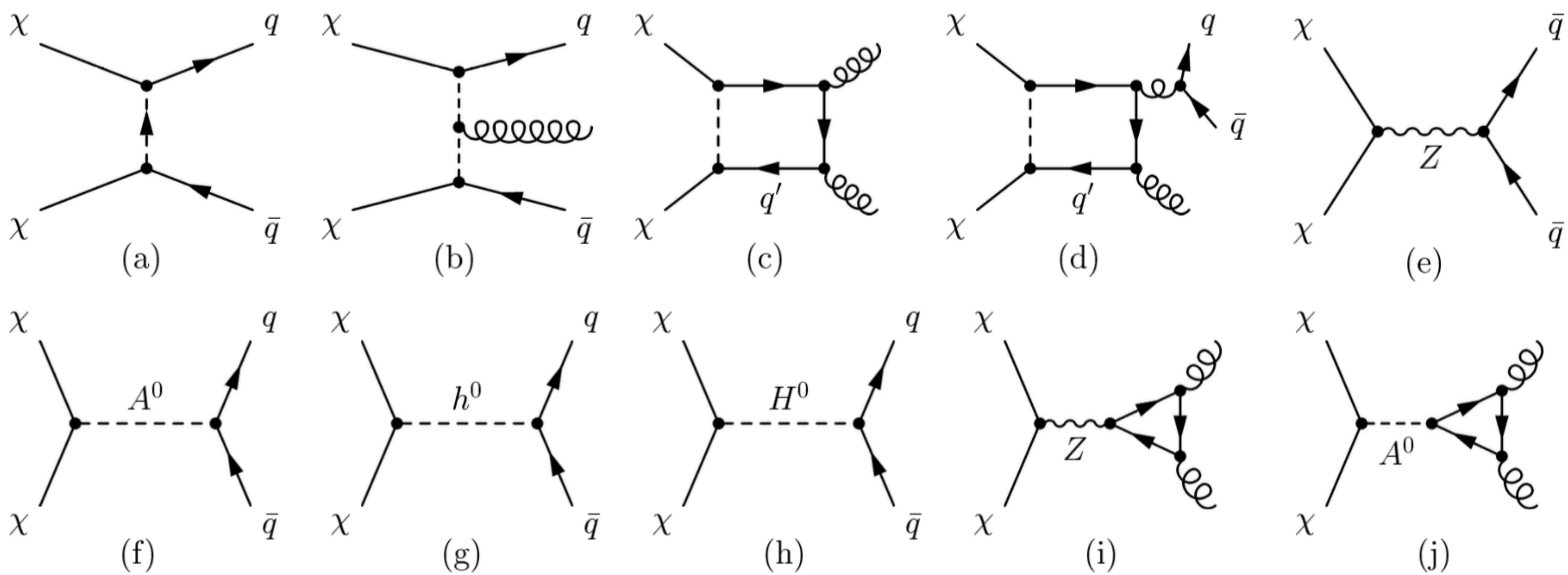
Supersymmetric DM Annihilation

- ... but each process has *many* diagrams
- e.g. annihilation:



hep-ph/0510257

- calculations have very limited applicability

Enter: the simplified model

- ☑ Extend the SM by only a minimal set of new particles
- ☑ for instance: fermionic DM ψ, new vector boson *Z'*.

$$\mathcal{L} = -\sum_{f=q,l,\nu} Z'^{\mu}\,\bar{f}\left[g_f^V \gamma_\mu + g_f^A \gamma_\mu \gamma^5\right] f - Z'^{\mu}\,\bar{\psi}\left[g_{\rm DM}^V \gamma_\mu + g_{\rm DM}^A \gamma_\mu \gamma^5\right]\psi$$

arXiv:1510.02110

- ☑ relatively simple calculations
- ☑ great for comparing experiments
- ☑ could be the low-E limit of many UV-complete models
  - but in practice, recasting simplified model constraints into constraints on UV-complete models is often difficult

Simplified DM model with a vector mediator

- Let's study fermionic DM ψ with mediator *Z'* in detail:

$$\mathcal{L} = -\sum_{f=q,l,\nu} Z'^{\mu}\,\bar{f}\left[g_f^V\gamma_\mu + g_f^A\gamma_\mu\gamma^5\right]f - Z'^{\mu}\,\bar{\psi}\left[g_{\mathrm{DM}}^V\gamma_\mu + g_{\mathrm{DM}}^A\gamma_\mu\gamma^5\right]\psi$$

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- First for the special case $g_f^A = g_{DM}^A = 0$ (pure vector couplings)
  - Direct detection (heavy *Z'* limit): $\sigma(\chi N \to \chi N) = \frac{9m_q^2}{\pi m_{Z'}^4}$
  - Relic density (heavy *Z'* limit): $\sigma(\chi\chi \to \bar{q}q) = \frac{3n_f m_\chi^2}{\pi m_{Z'}^4}$

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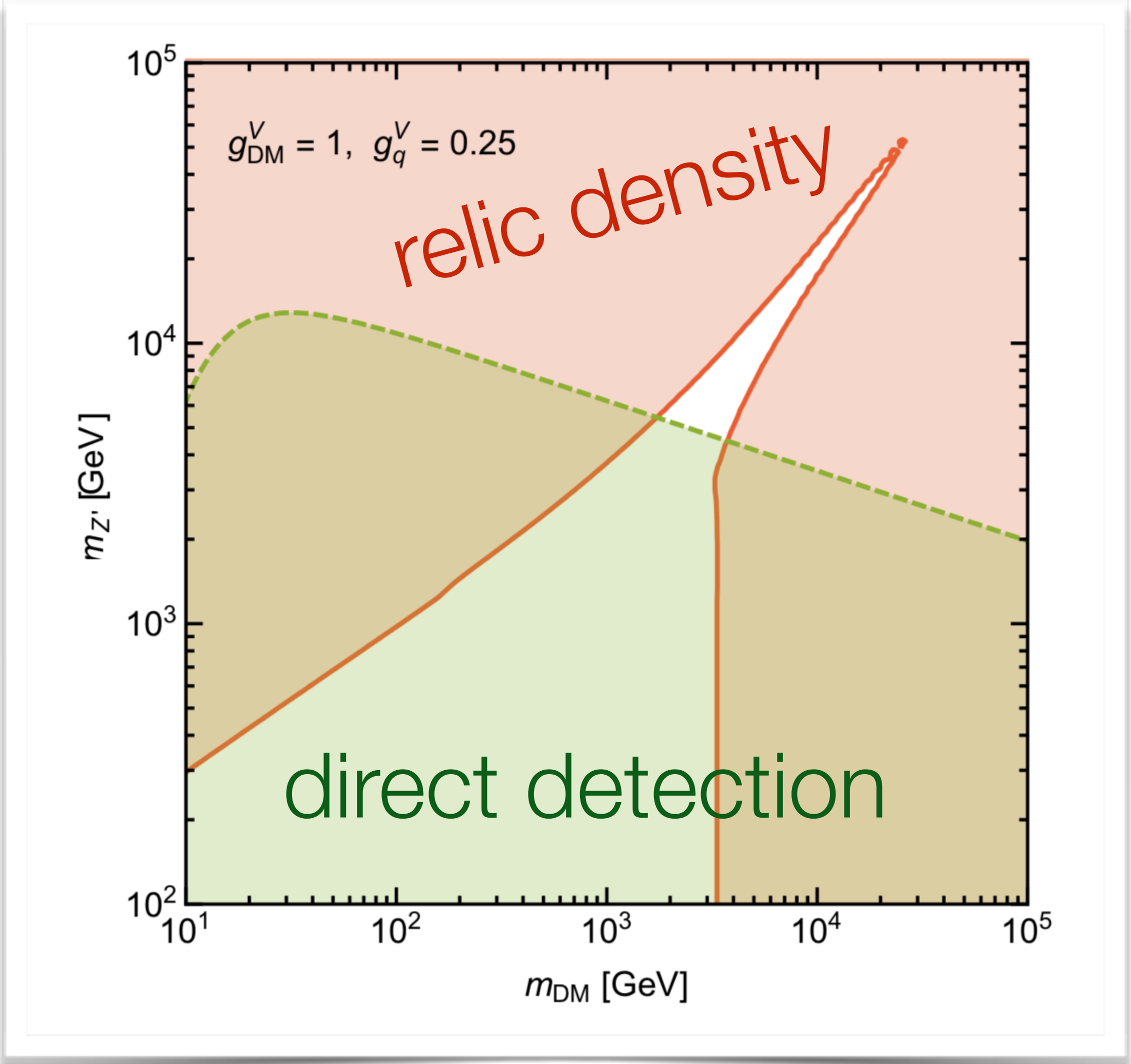

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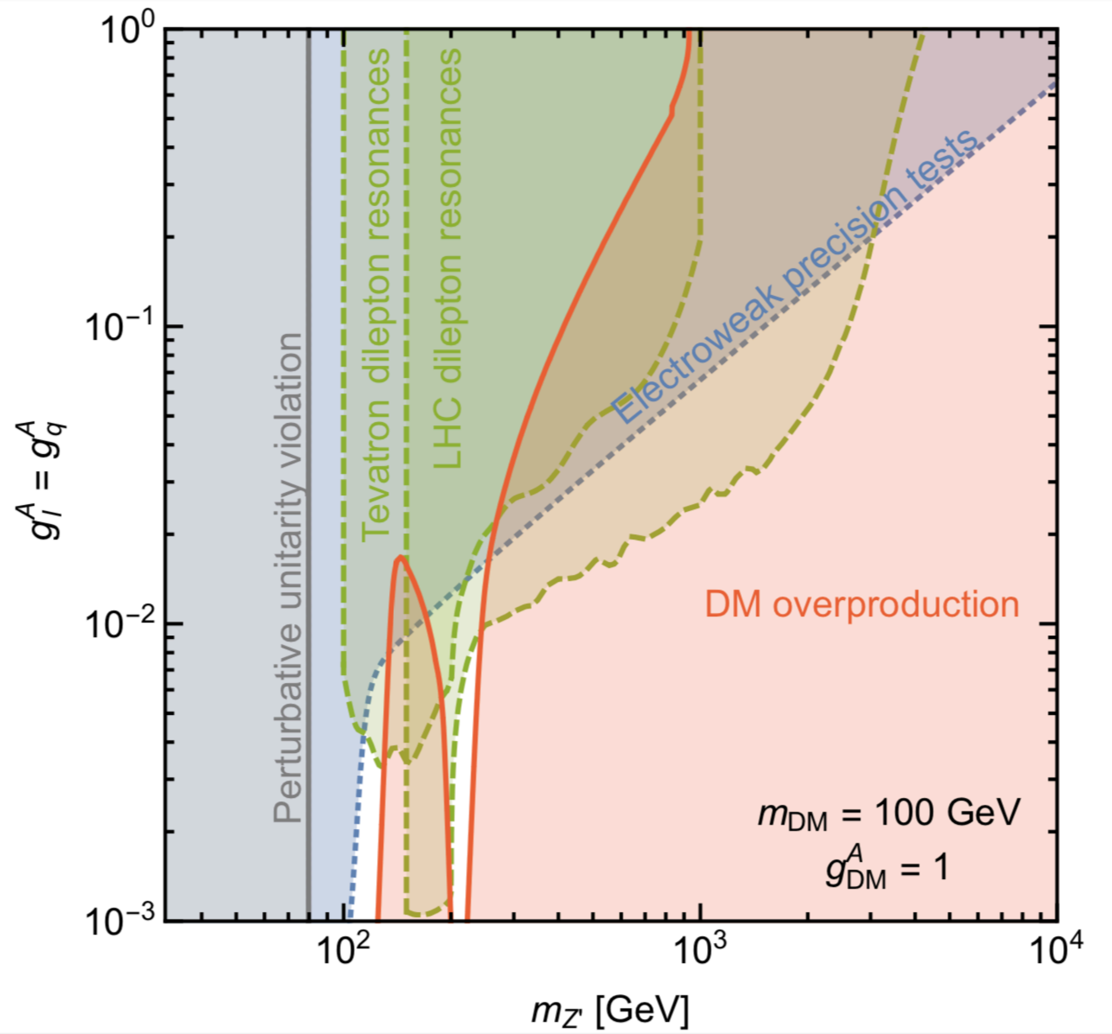
☑ Let's now include also axial vector couplings: $g_f^V = g_f^A$, but $g_{DM}^V = 0$.



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Z' resonance
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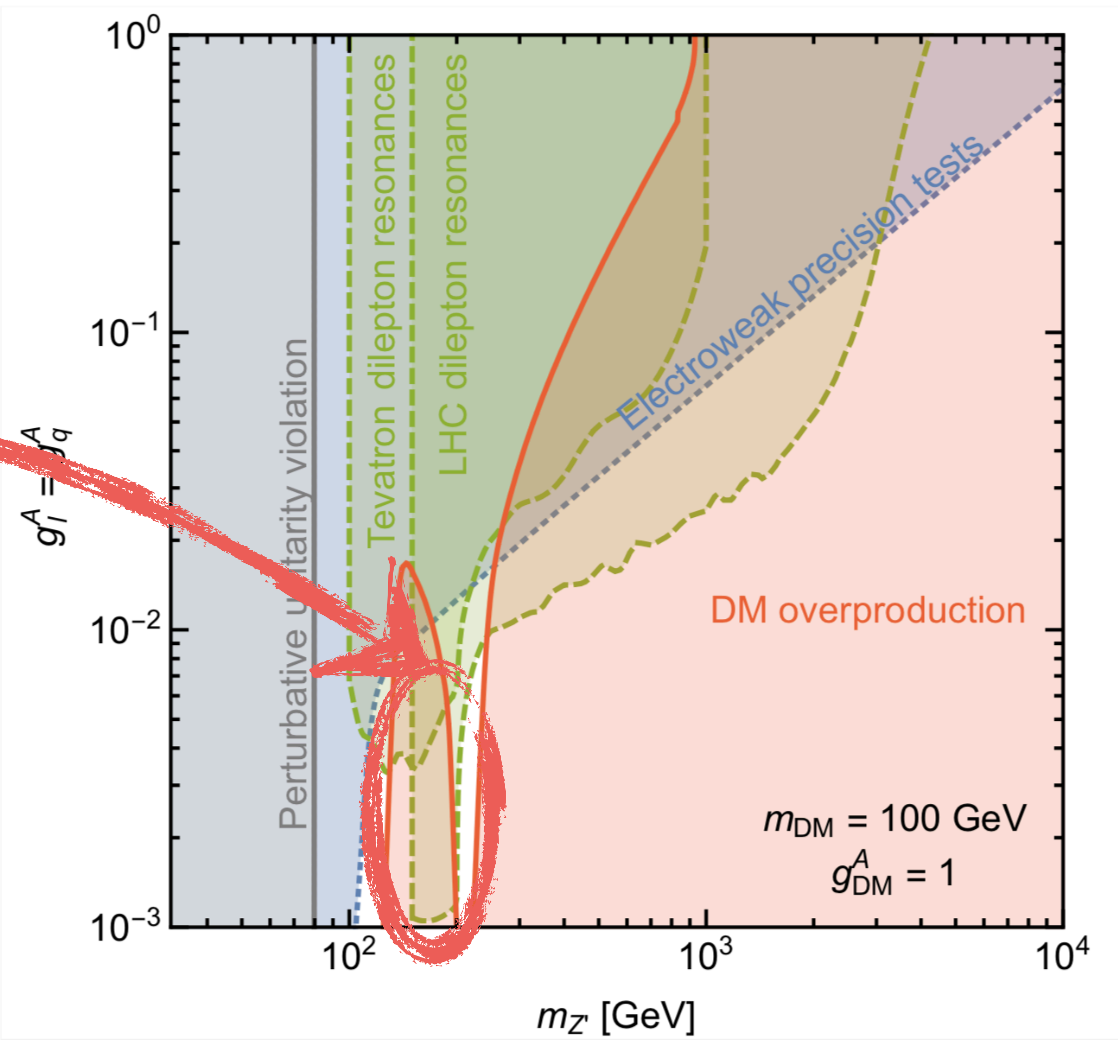

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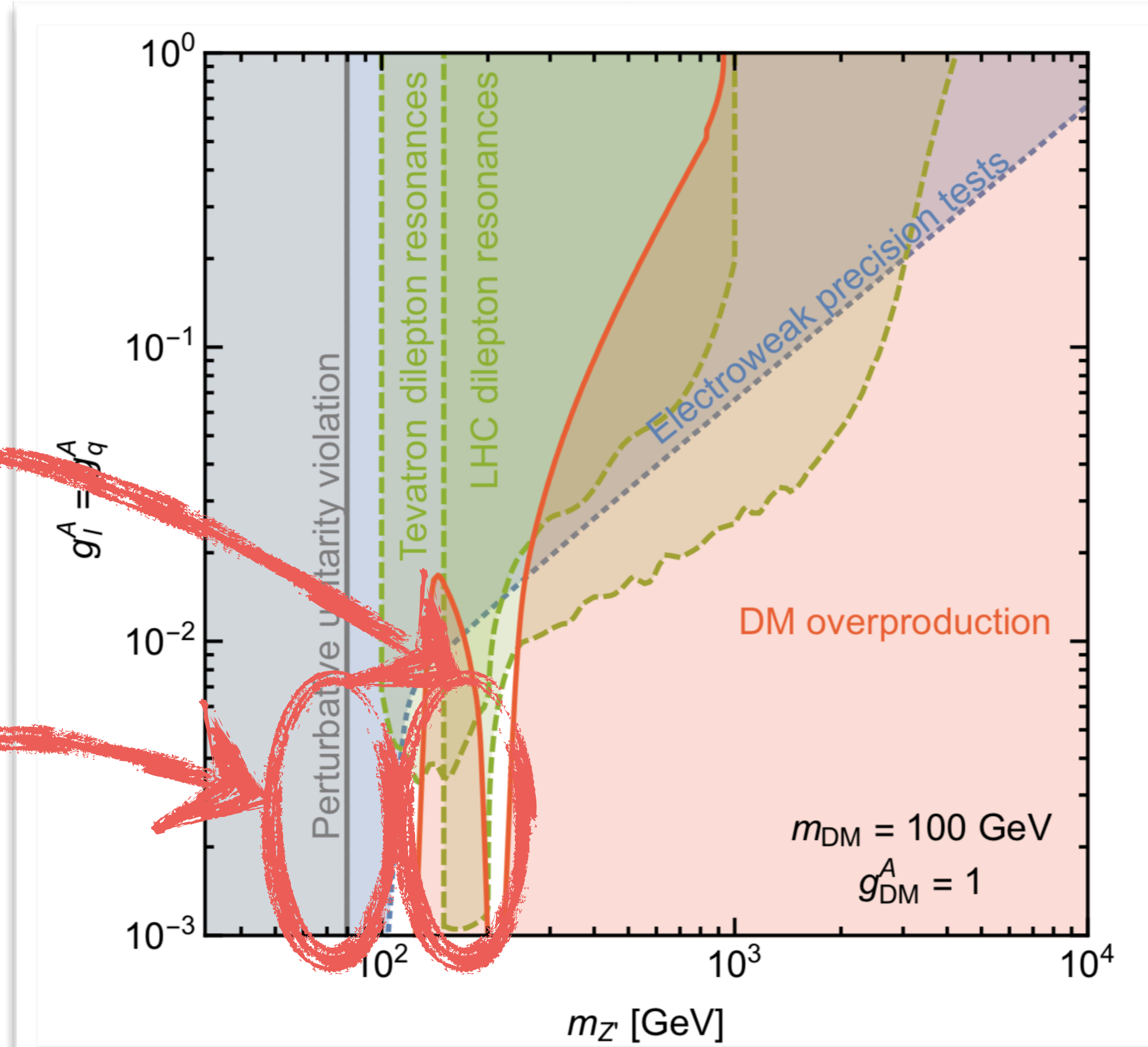
Z' resonance in annihilation

Annihilation channel **χχ → Z'Z'** becomes accessible



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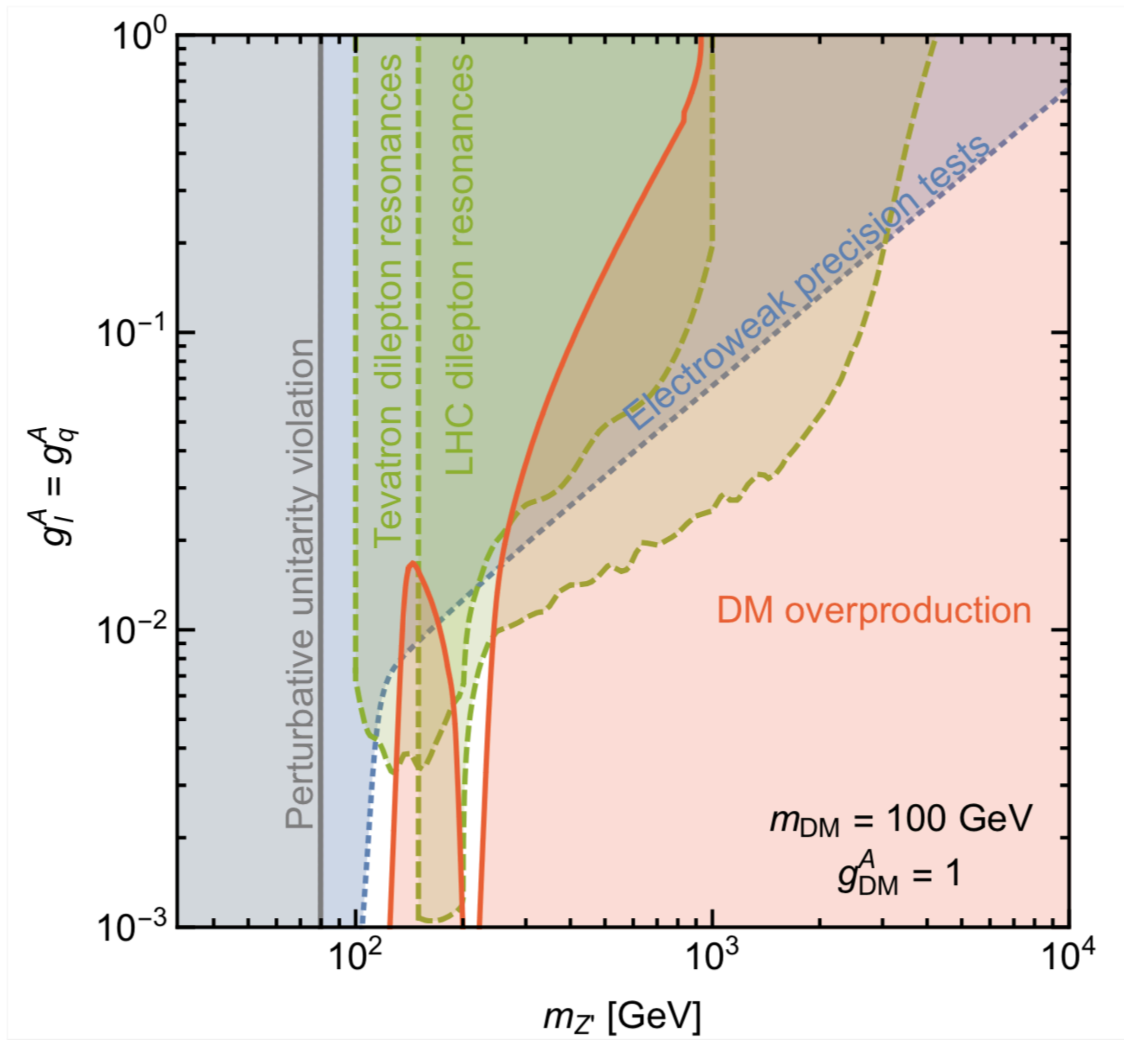

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Direct Searches for the Z' Mediator

$$\mathcal{L} = -\sum_{f=q,l,\nu} Z'^{\mu}\, \bar{f}\left[g_f^V \gamma_\mu + g_f^A \gamma_\mu \gamma^5\right] f - Z'^{\mu}\, \bar{\psi}\left[g_{\rm DM}^V \gamma_\mu + g_{\rm DM}^A \gamma_\mu \gamma^5\right]\psi$$

- ☑ Consider $\bar{q}q \to Z' \to \ell\ell$, χχ

$$\Gamma(Z' \to f\bar{f}) = \frac{m_{Z'} N_c}{12\pi}\sqrt{1-\frac{4m_f^2}{m_{Z'}^2}}\left[(g_f^V)^2 + (g_f^A)^2 + \frac{m_f^2}{m_{Z'}^2}\left(2(g_f^V)^2 - 4(g_f^A)^2\right)\right]$$

- ☑ LHC signature: dilepton resonance at the Z' mass

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Electroweak Precision Tests

$$\mathcal{L} = -\sum_{f=q,l,\nu} Z'^{\mu}\,\bar{f}\left[g_f^V\gamma_\mu + g_f^A\gamma_\mu\gamma^5\right]f - Z'^{\mu}\,\bar{\psi}\left[g_{\rm DM}^V\gamma_\mu + g_{\rm DM}^A\gamma_\mu\gamma^5\right]\psi$$

- Non-zero g_f^A ➟ LH and RH SM fermions carry opposite Z' charge
- To make the Yukawa coupling $y\, f_L\, H\, f_R$ invariant, the SM Higgs H must carry Z' charge q' as well.
- Its vev then contributes to the Z' mass
- ... and leads to mixing between Z and Z': with the covariant derivate $D^\mu \equiv \partial^\mu - ig_1 Y B^\mu - ig_2 t^a W^{a\mu} - ig' q' Z'^\mu$, and with

$$B^\mu = -s_W Z^\mu + c_W A^\mu$$
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we find

$$(D^\mu H)^\dagger (D_\mu H) \supset g' q' v^2 (g_1 Y s_W + \tfrac{1}{2} g_2 c_W) Z'^\mu Z^\mu = \tfrac{1}{2} g' q' v^2 \sqrt{g_1^2 + g_2^2}\, Z'^\mu Z^\mu$$

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SM Higgs hypercharge: Y= ½

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sine/cosine of Weinberg angle:

$$s_W = g_1/\sqrt{g_1^2 + g_2^2}$$

$$c_W = g_2/\sqrt{g_1^2 + g_2^2}$$

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Constrained by LEP measurements of **electroweak precision observables** (*S* and *T* parameters)

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Unitarity

arXiv:
1510.02110

Unitarity

- Matrix element for scattering process $i \to f$ can be decomposed into *partial waves*:

$$\mathcal{M}^J_{if}(s) = \frac{1}{32\pi} \beta_{if} \int_{-1}^{1} \mathrm{d}\cos\theta \, d^J_{\mu\mu'}(\theta) \, \mathcal{M}_{if}(s, \cos\theta)$$

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Kinematic factor

Wigner d-function
(generalized spherical harmonic)

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scattering angle

Kinematic factor

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- cf. partial wave decomposition of wave function in QM:

$$\psi(r, \theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} u_{\ell m}(r) Y_{\ell m}(\theta, \phi)$$

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- Optical theorem:

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- Optical theorem:

$$\mathrm{Im}(\mathcal{M}^J_{ii}) = \sum_f |\mathcal{M}^J_{if}|^2 = |\mathcal{M}^J_{ii}|^2 + \sum_{f\neq i}|\mathcal{M}^J_{if}|^2 \geq |\mathcal{M}^J_{ii}|^2$$

- This implies

$$0 \leq \mathrm{Im}(\mathcal{M}^J_{ii}) \leq 1\,, \quad |\mathrm{Re}(\mathcal{M}^J_{ii})| \leq \frac{1}{2}$$

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Unitarity

- Matrix element for scattering process $i \to f$ can be decomposed into *partial waves*:

$$\mathcal{M}^J_{if}(s) = \frac{1}{32\pi}\beta_{if}\int_{-1}^{1} \mathrm{d}\cos\theta \, d^J_{\mu\mu'}(\theta)\, \mathcal{M}_{if}(s,\cos\theta)$$

- cf. partial wave decomposition of wave function in QM:

$$\psi(r,\theta,\phi) \quad \ldots \quad \phi)$$

With $S = 1 + iT$
unitarity ($S^\dagger S = 1$) implies:
$-i(T^\dagger - T) = T^\dagger T$

- Optical theorem:

$$\mathrm{Im}(\mathcal{M}^J_{ii}) = \sum_f |\mathcal{M}^J_{if}|^2 = |\mathcal{M}^J_{ii}|^2 + \sum_{f \neq i} |\mathcal{M}^J_{if}|^2 \geq |\mathcal{M}^J_{ii}|^2$$

- This implies

$$0 \leq \mathrm{Im}(\mathcal{M}^J_{ii}) \leq 1\,, \qquad |\mathrm{Re}(\mathcal{M}^J_{ii})| \leq \frac{1}{2}$$

arXiv: 1510.02110

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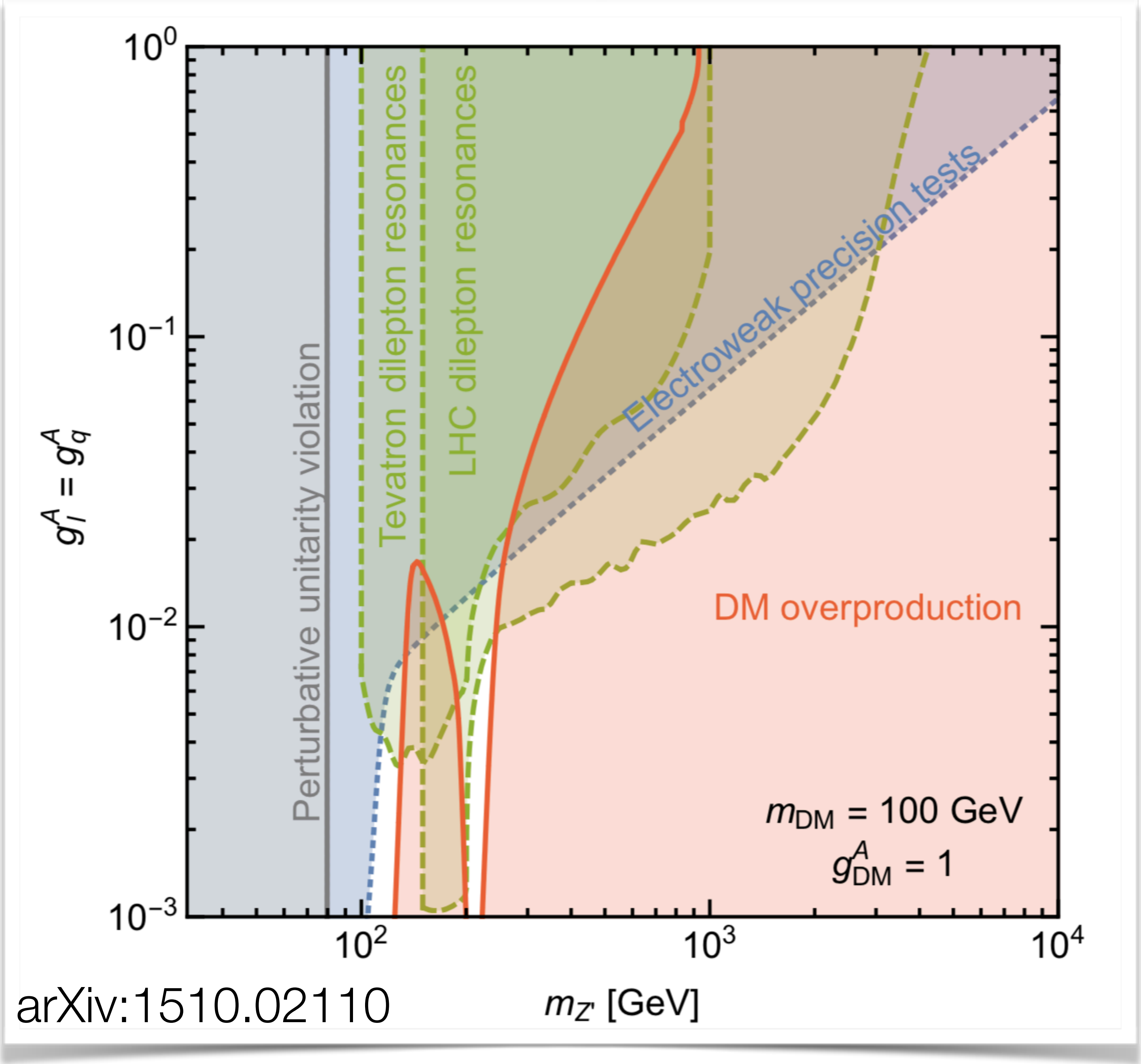
arXiv: 1510.02110

Z' Simplified Model: Axial Couplings Summary



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Dark Photons

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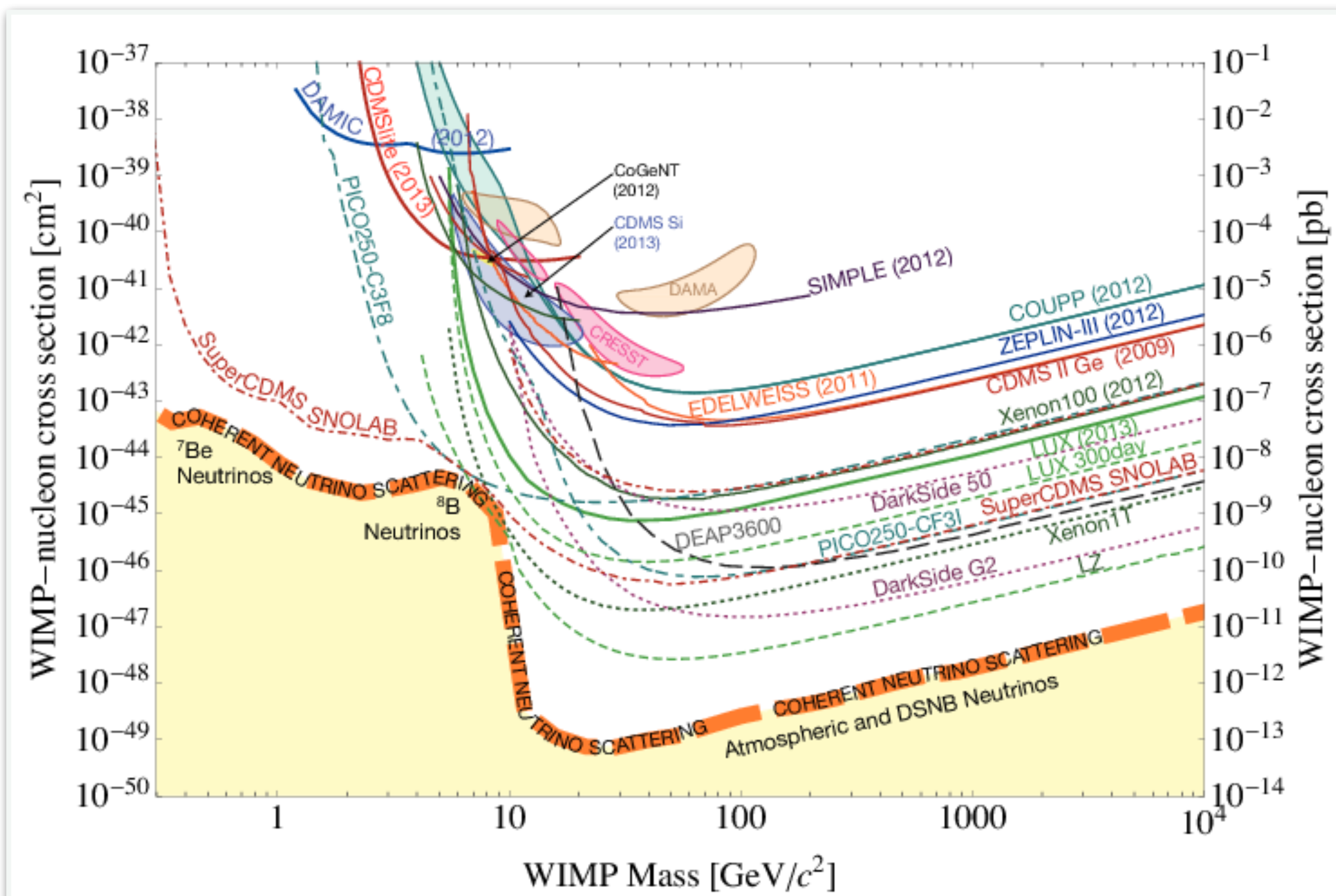

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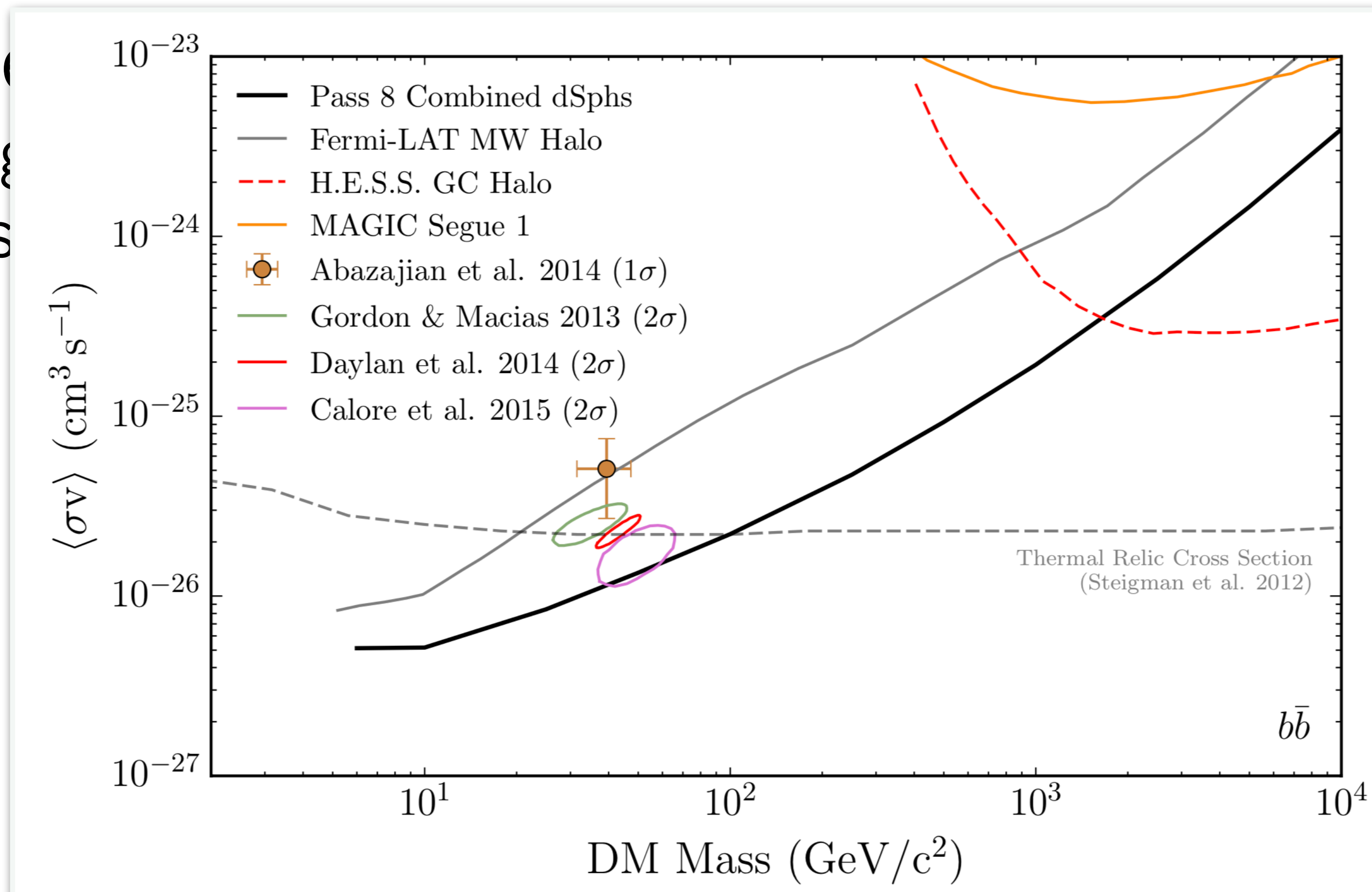

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  below threshold for annihilation into γ-rich final states ($\bar{b}b$, $\tau^+\tau^-$, ...)
- ☑ For light mediator particles, colliders are at relative disadvantage (cross section $\sigma \sim 1/E_{cm}^2$)

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Motivation

- Only three possibilities for coupling a total gauge singlet to SM particles through a renormalizable interaction
  - Singlet scalar *S*: Higgs portal $\mathcal{L} \supset \lambda(H^\dagger H)S^\dagger S$
    (typically implies $m_S \sim m_H$ ➟ back at the electroweak scale)
  - Singlet fermion *N*: Neutrino portal $\mathcal{L} \supset y\bar{L}(i\sigma^2 H^*)N$
    (relevant for instance for sterile neutrino DM ➟ Christoph Weniger's lectures)
  - Singlet gauge boson *B'*: kinetic mixing $\mathcal{L} \supset -\frac{1}{2}\sin\chi F_Y^{\mu\nu} F'_{\mu\nu}$

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Hypercharge (B_μ) field strength tensor

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Hypercharge (B_μ) field strength tensor

B'_μ field strength tensor

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Dark Photons and Dark Matter

- ☑ Dark Photons could either make up the dark matter ...
- ☑ ... or act as mediator of DM—SM couplings

Dark Photons: Formalism

$$\mathcal{L} \supset -\frac{1}{2}\sin\chi F_Y^{\mu\nu} F'_{\mu\nu}$$

- Remove kinetic mixing term by transformation

$$\begin{pmatrix} B_\mu \\ B'_\mu \end{pmatrix} = \begin{pmatrix} 1 & -\tan\chi \\ 0 & \sec\chi \end{pmatrix} \begin{pmatrix} \tilde{B}_\mu \\ \tilde{B}'_\mu \end{pmatrix}$$

to ensure *B* and *B'* have standard kinetic terms
(necessary for proper definition and normalization of 1-particle states)
Note: this trafo does not change the SM hypercharge couplings.

- Electroweak symmetry breaking mixes *B* and *W*:

$$\begin{pmatrix} \tilde{A}_\mu \\ \tilde{Z}_\mu \\ \tilde{Z}'_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w & 0 \\ -\sin\theta_w & \cos\theta_w & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{B}_\mu \\ W^3_\mu \\ \tilde{B}'_\mu \end{pmatrix}$$

see for instance arXiv:0903.1118

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Dark Photons: Formalism

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- ☑ θ_W is defined such that Ã is massless.
- ☑ Z̃ and Z̃' have mass term of the form

$$\frac{1}{2}\begin{pmatrix} \tilde{Z}_\mu & \tilde{Z}'_\mu \end{pmatrix} \begin{pmatrix} m^2 & -\Delta \\ -\Delta & M^2 \end{pmatrix} \begin{pmatrix} \tilde{Z}^\mu \\ \tilde{Z}'^\mu \end{pmatrix}$$

- ☑ Diagonalized by rotation

$$\begin{pmatrix} Z_- \\ Z_+ \end{pmatrix} = \begin{pmatrix} \cos\zeta & -\sin\zeta \\ \sin\zeta & \cos\zeta \end{pmatrix} \begin{pmatrix} \tilde{Z}^\mu \\ \tilde{Z}'^\mu \end{pmatrix}$$

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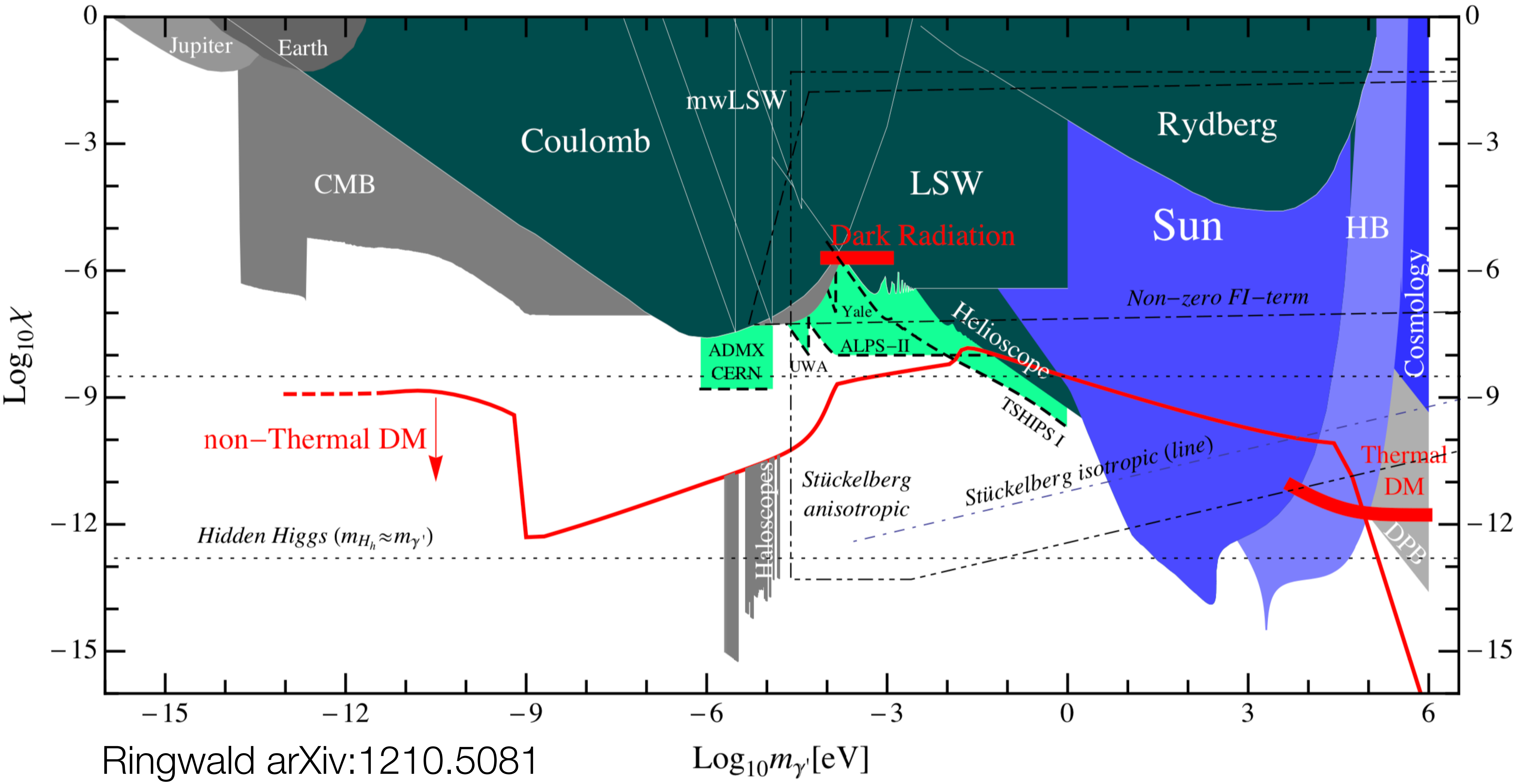
- ☑ Couplings to SM currents in the new basis:

$$\begin{pmatrix} J_A \\ J_Z \\ J_{Z'} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -\cos\theta_w \tan\chi \sin\zeta & \sin\theta_w \tan\chi \sin\zeta + \cos\zeta & \sec\chi \sin\zeta \\ -\cos\theta_w \tan\chi \cos\zeta & \sin\theta_w \tan\chi \cos\zeta - \sin\zeta & \sec\chi \cos\zeta \end{pmatrix} \begin{pmatrix} J^{\rm SM}_{\rm EM} \\ J^{\rm SM}_Z \\ J' \end{pmatrix}$$

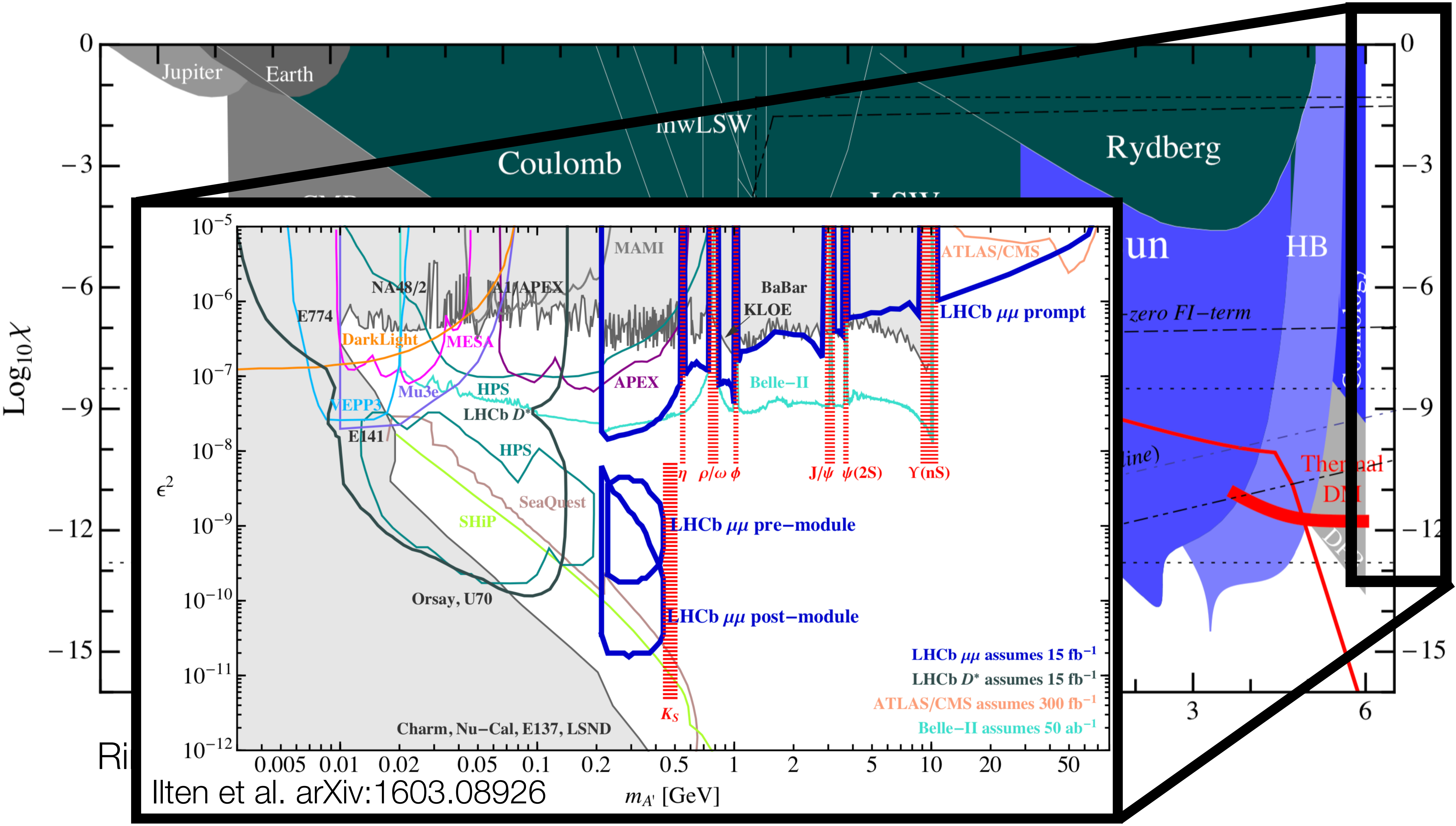
- ☑ Note: photon couplings unchanged (related to unbroken $U(1)_{em}$)

Dark Photon Constraints



Ringwald arXiv:1210.5081

Dark Photon Constraints



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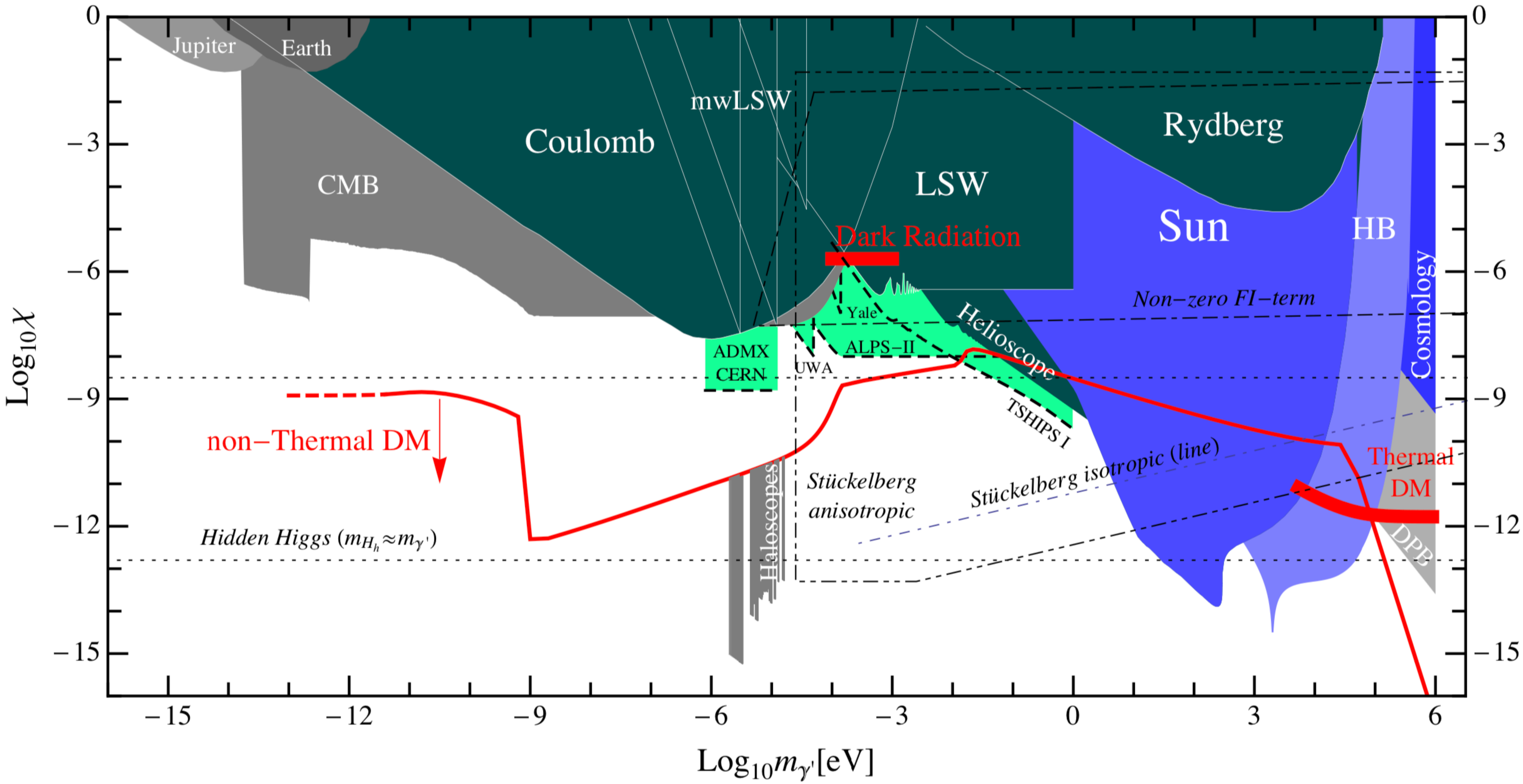
Dark Photon Constraints



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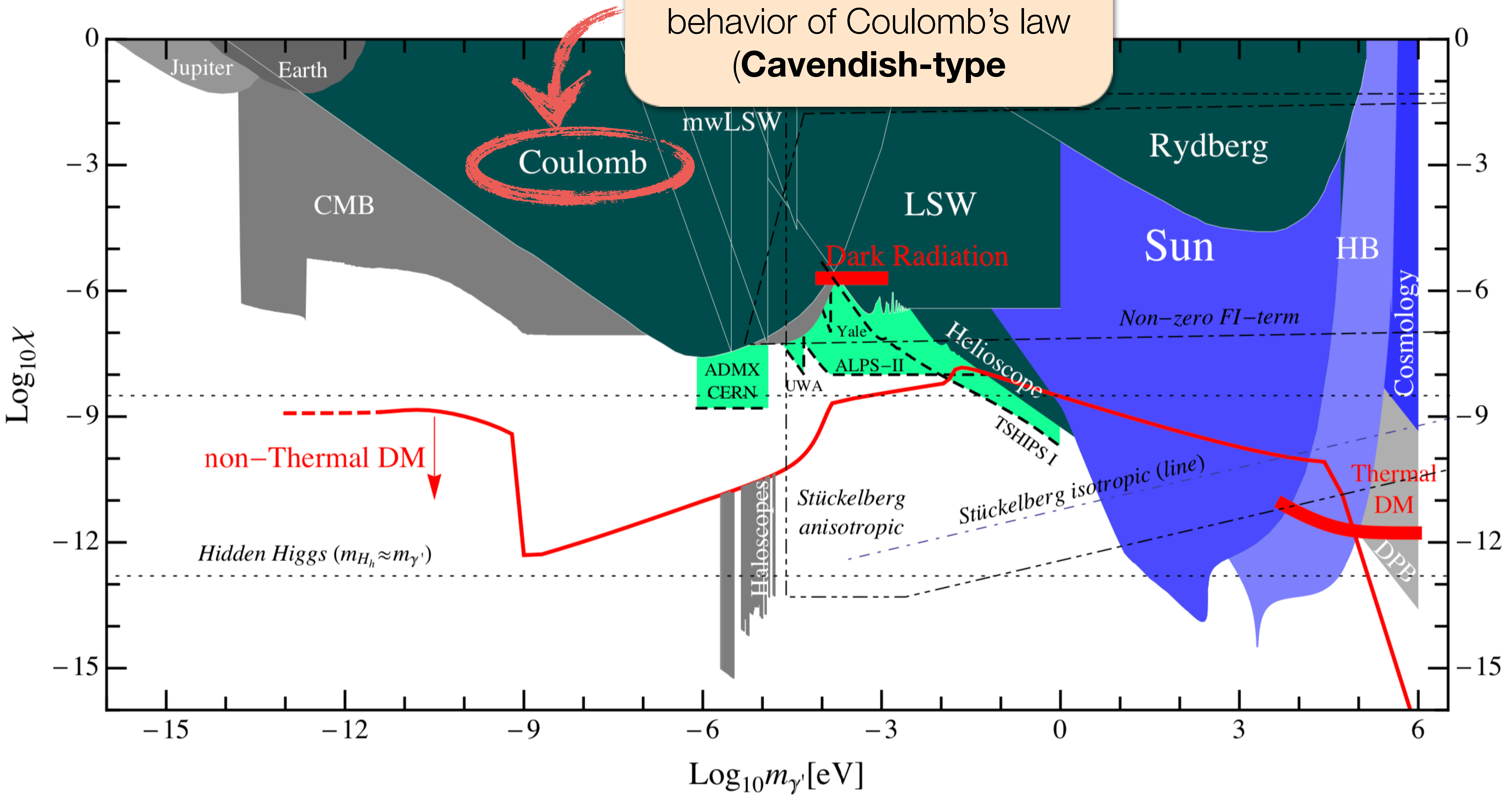
Dark Photon Constraints

Modification to the 1/r behavior of Coulomb's law (**Cavendish-type**



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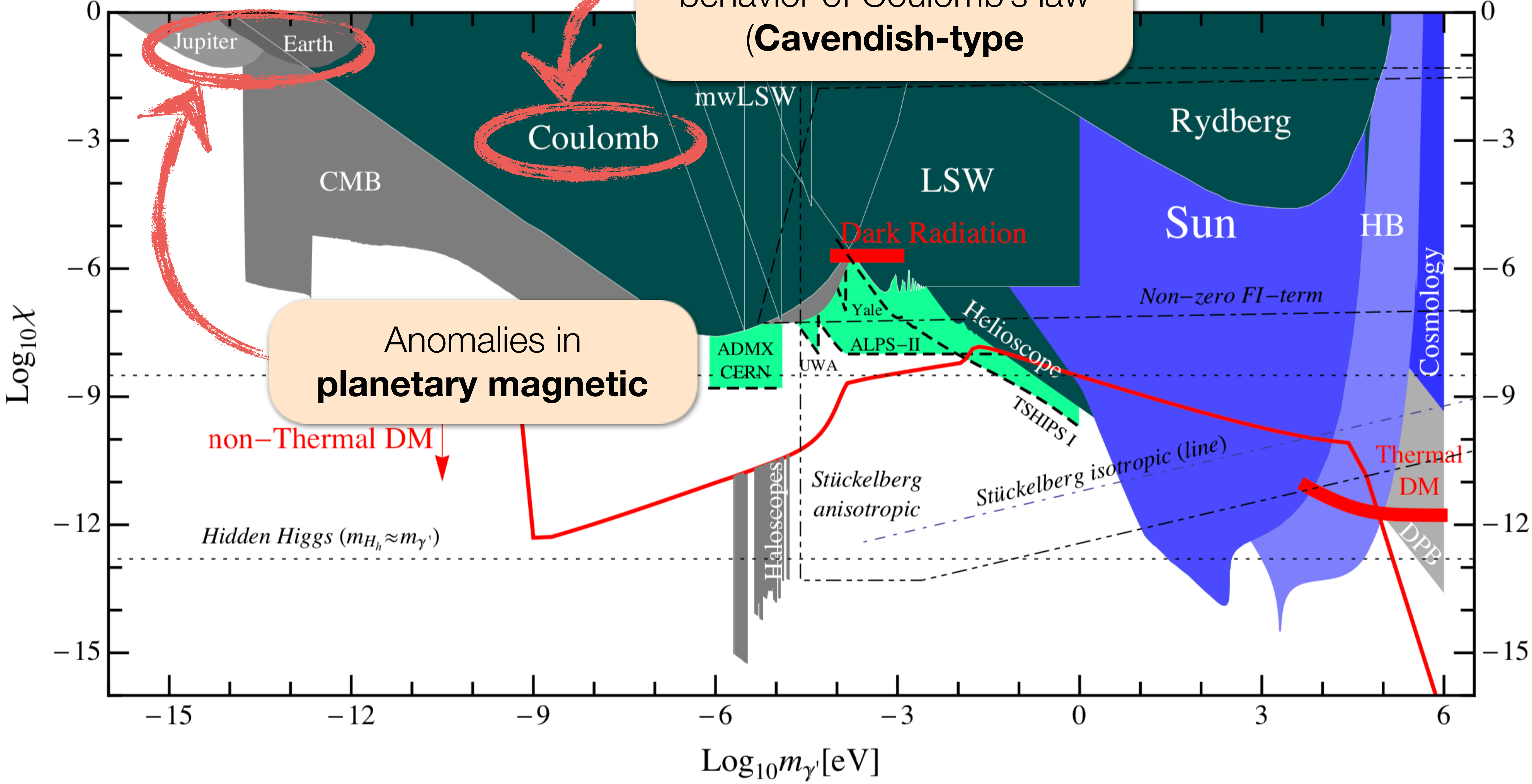
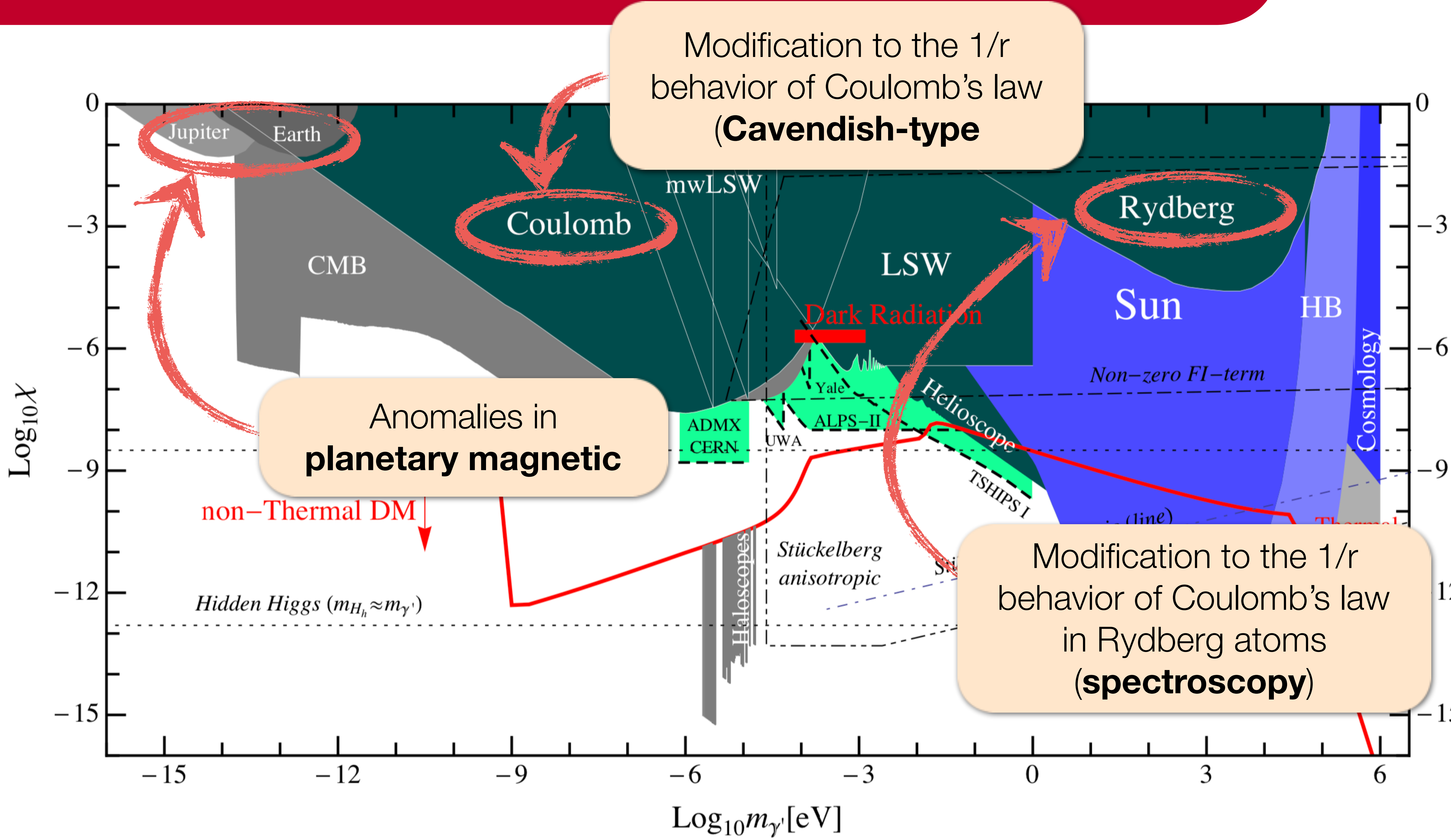
Dark Photon Constraints



Modification to the 1/r behavior of Coulomb's law (**Cavendish-type**

Anomalies in **planetary magnetic**

Dark Photon Constraints



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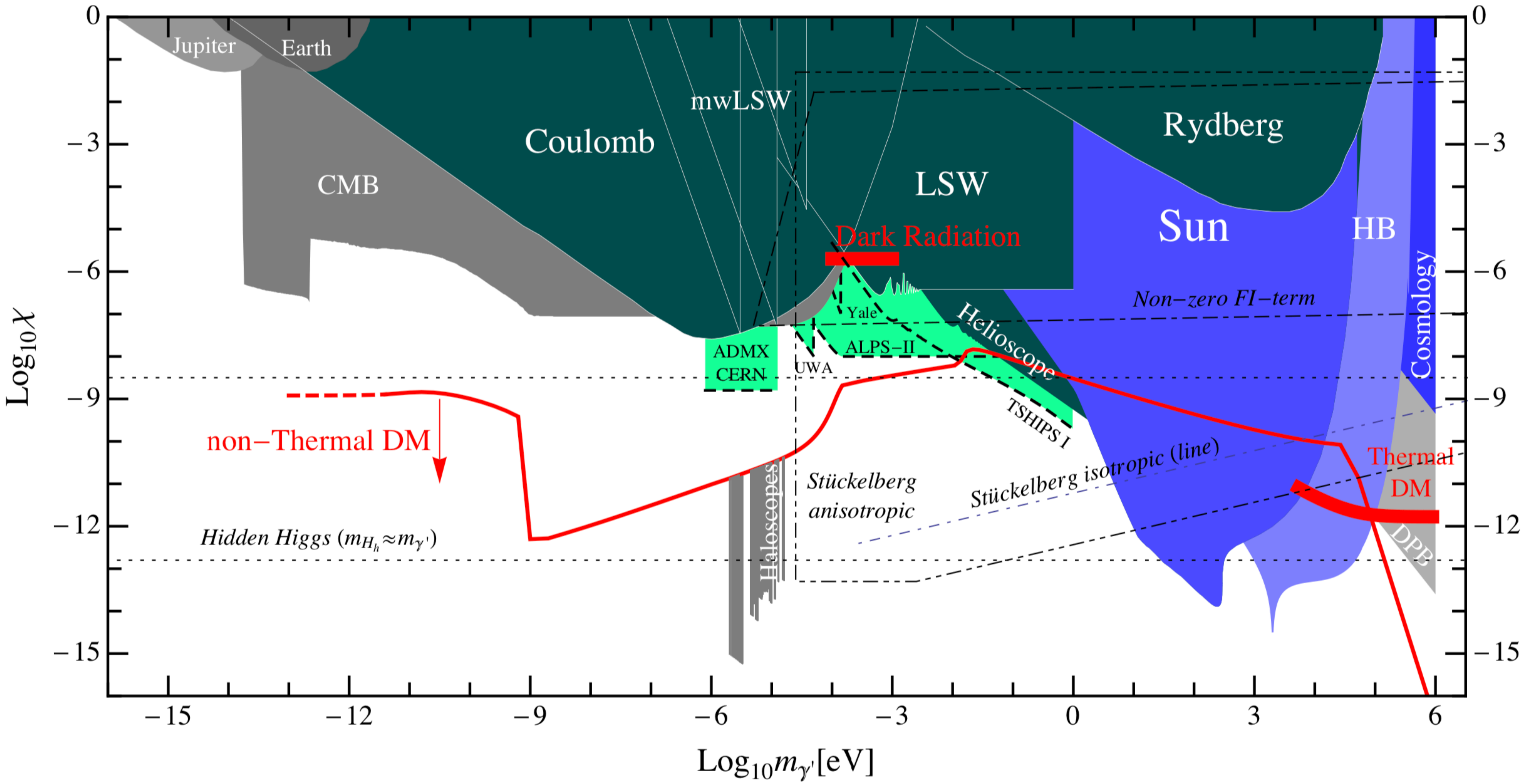

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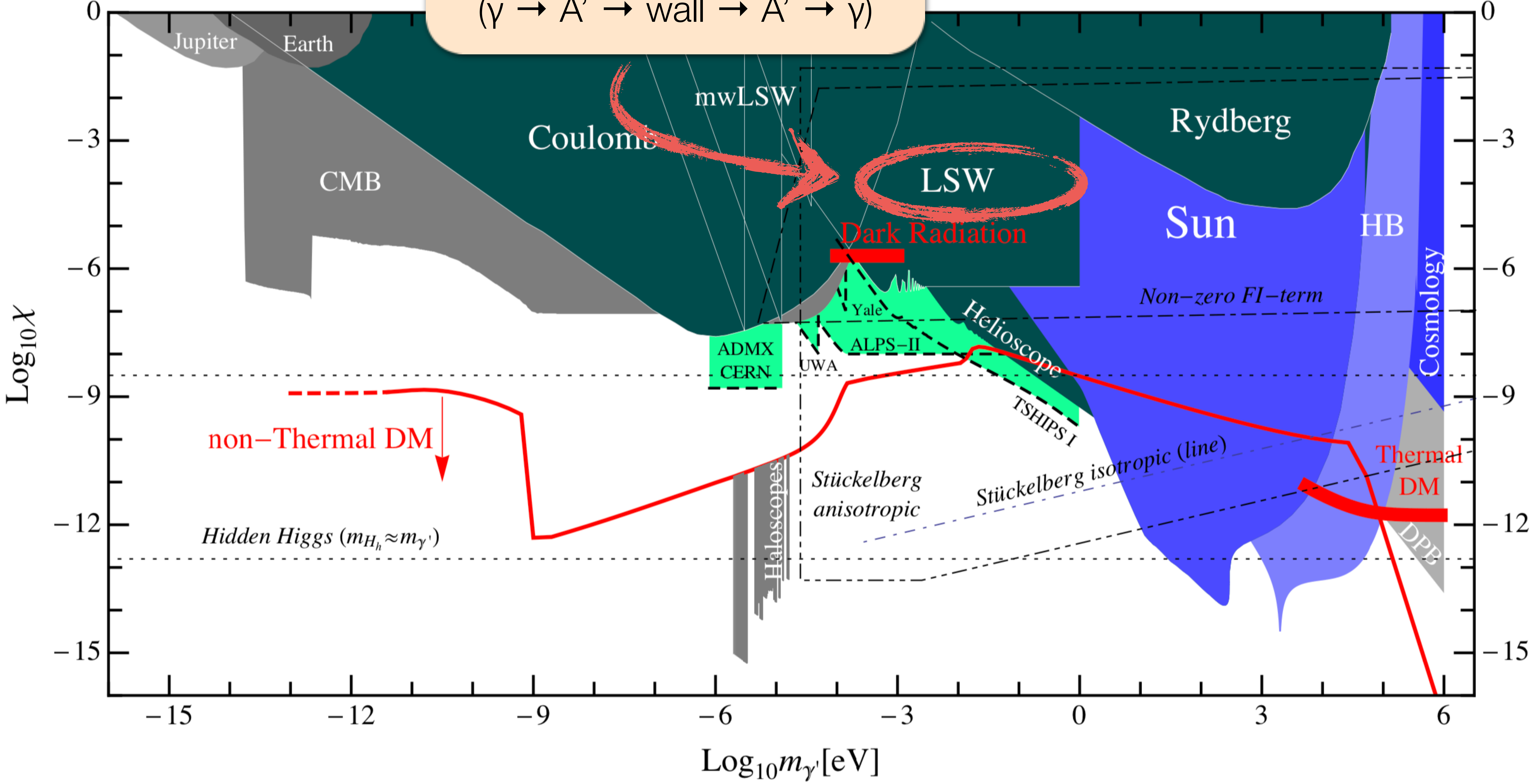
Dark Photon Constraints

Light shining through wall experiments
(γ → A' → wall → A' → γ)



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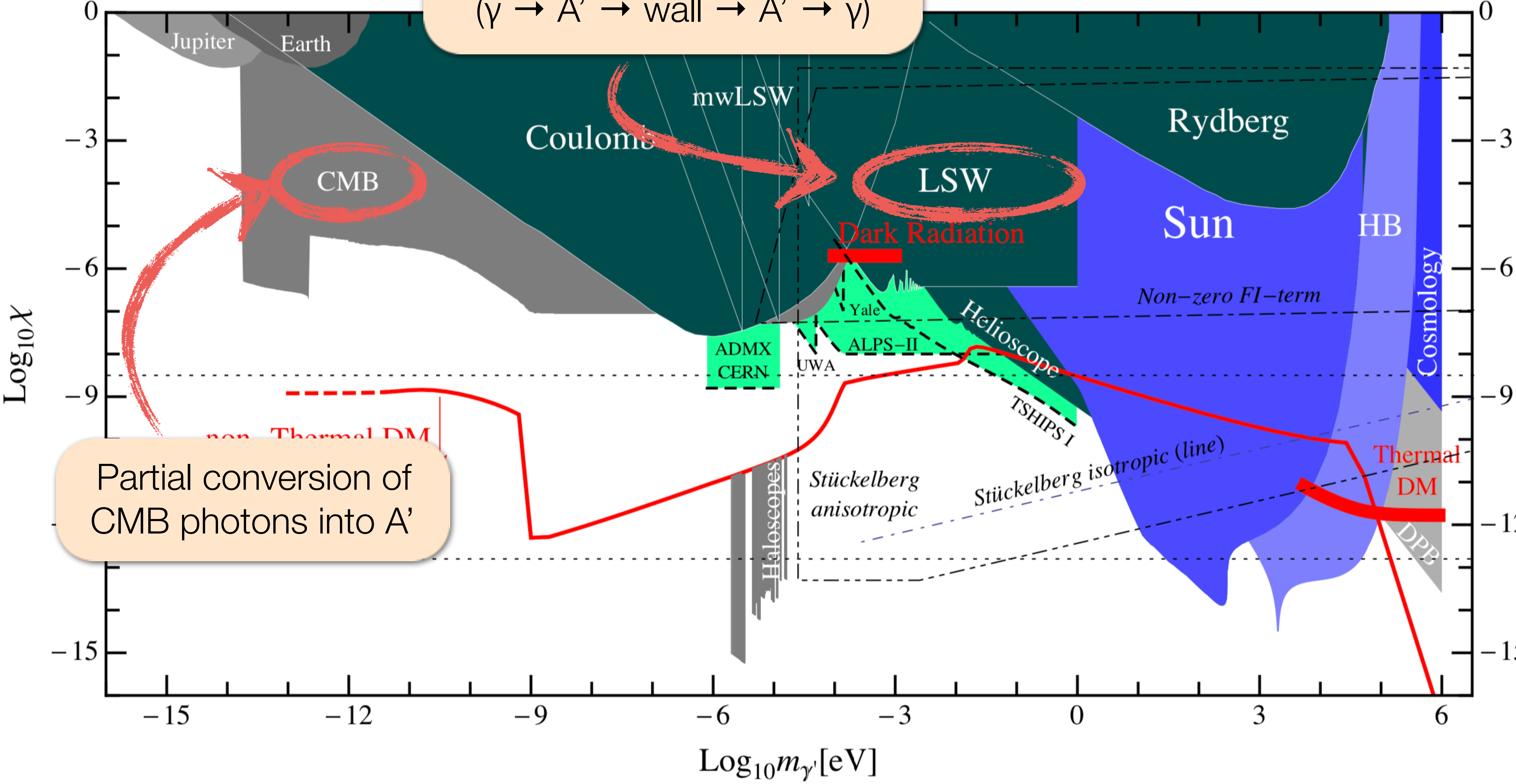

Partial conversion of CMB photons into A’

Dark Photon Constraints

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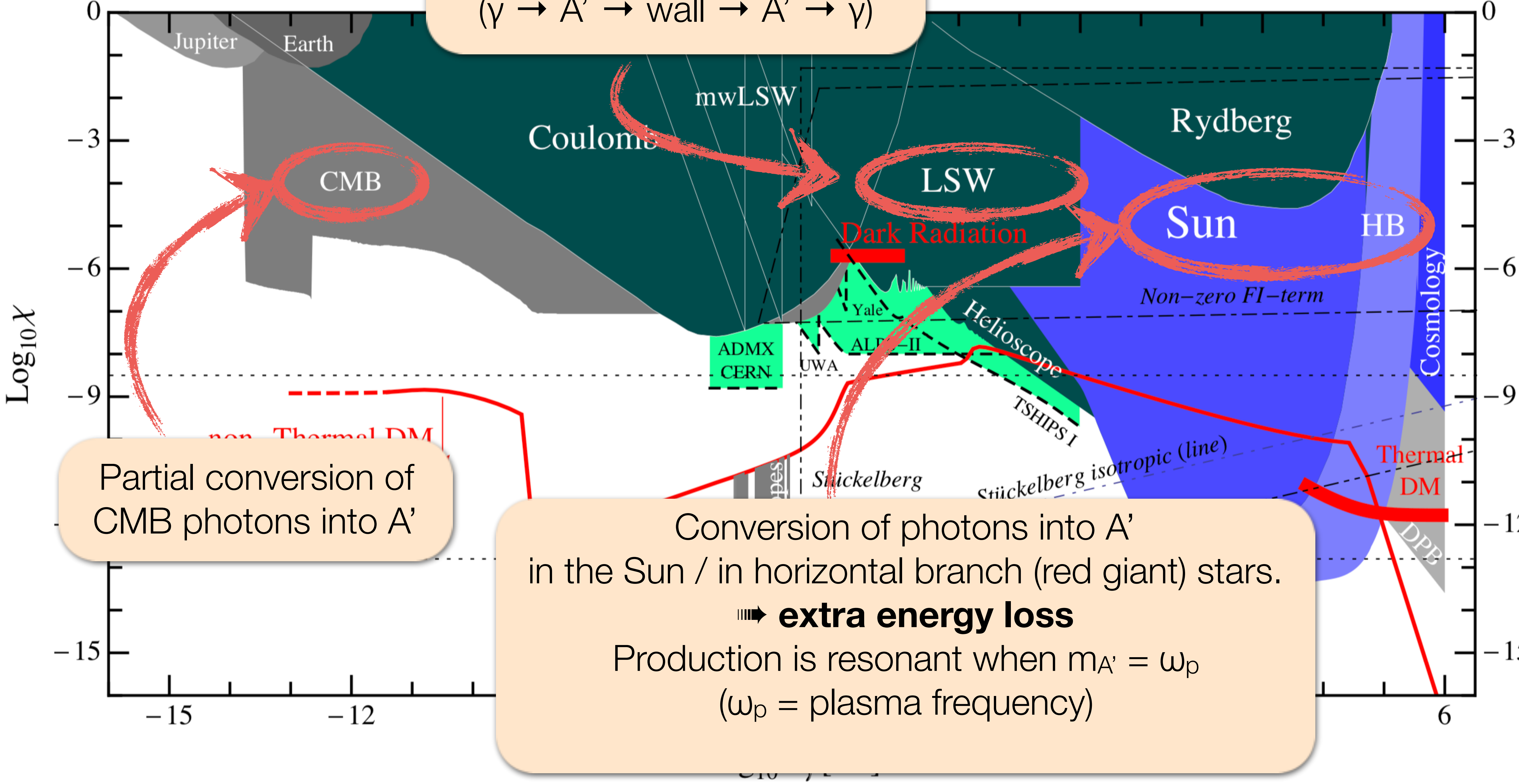
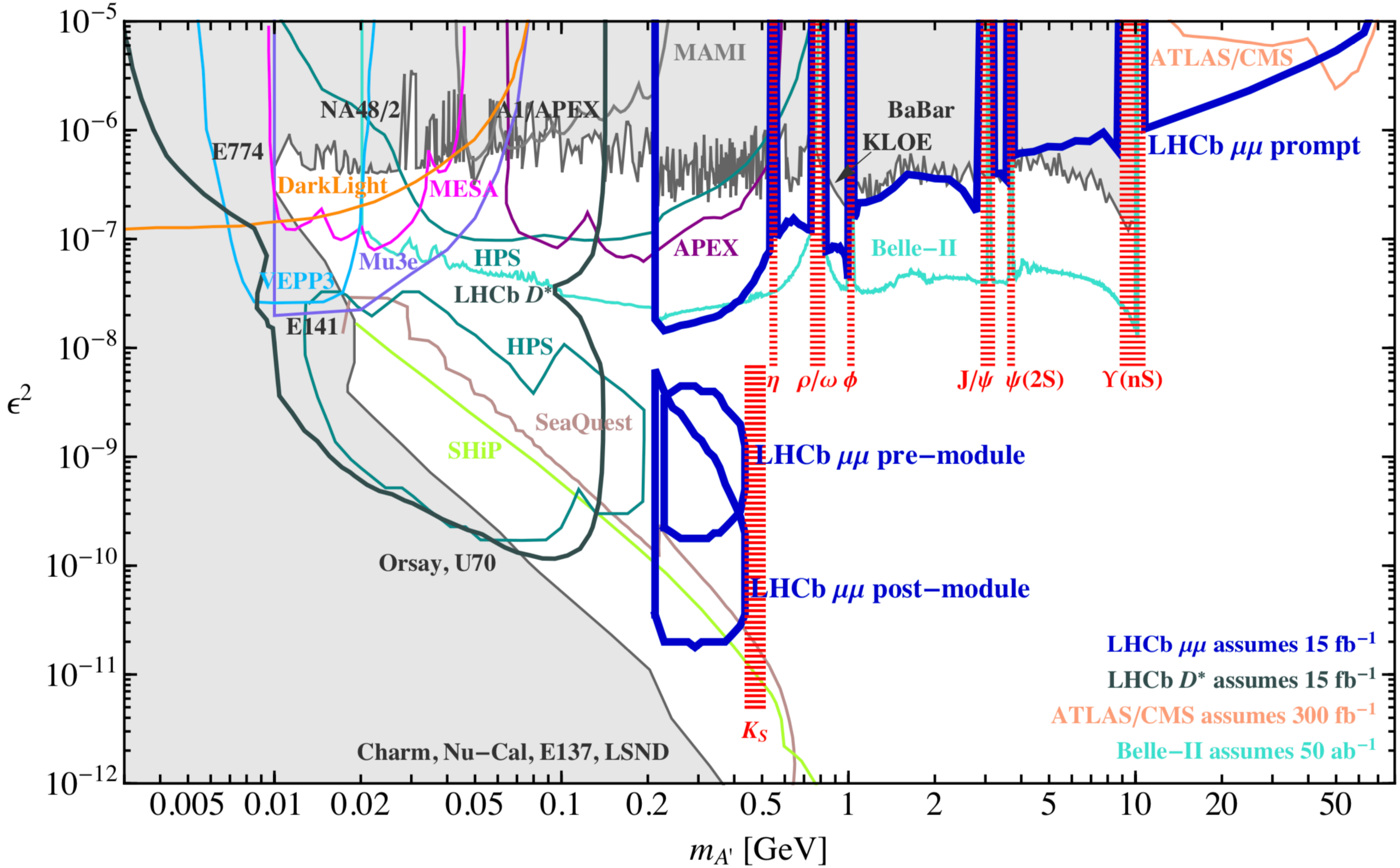


Partial conversion of CMB photons into A’

Conversion of photons into A’
in the Sun / in horizontal branch (red giant) stars.
➟ **extra energy loss**
Production is resonant when $m_{A'} = \omega_p$
(ω_p = plasma frequency)

Dark Photon Constraints



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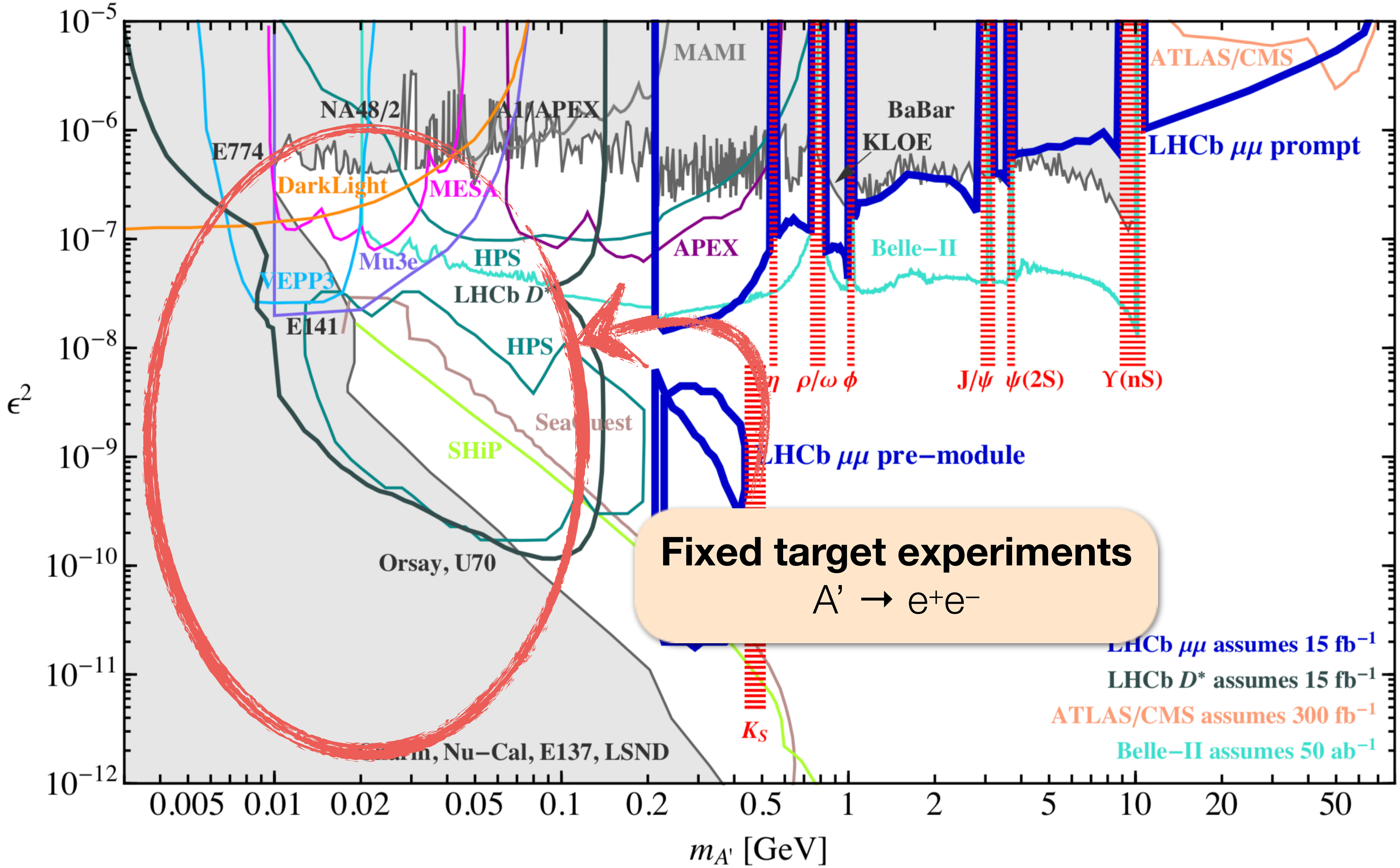

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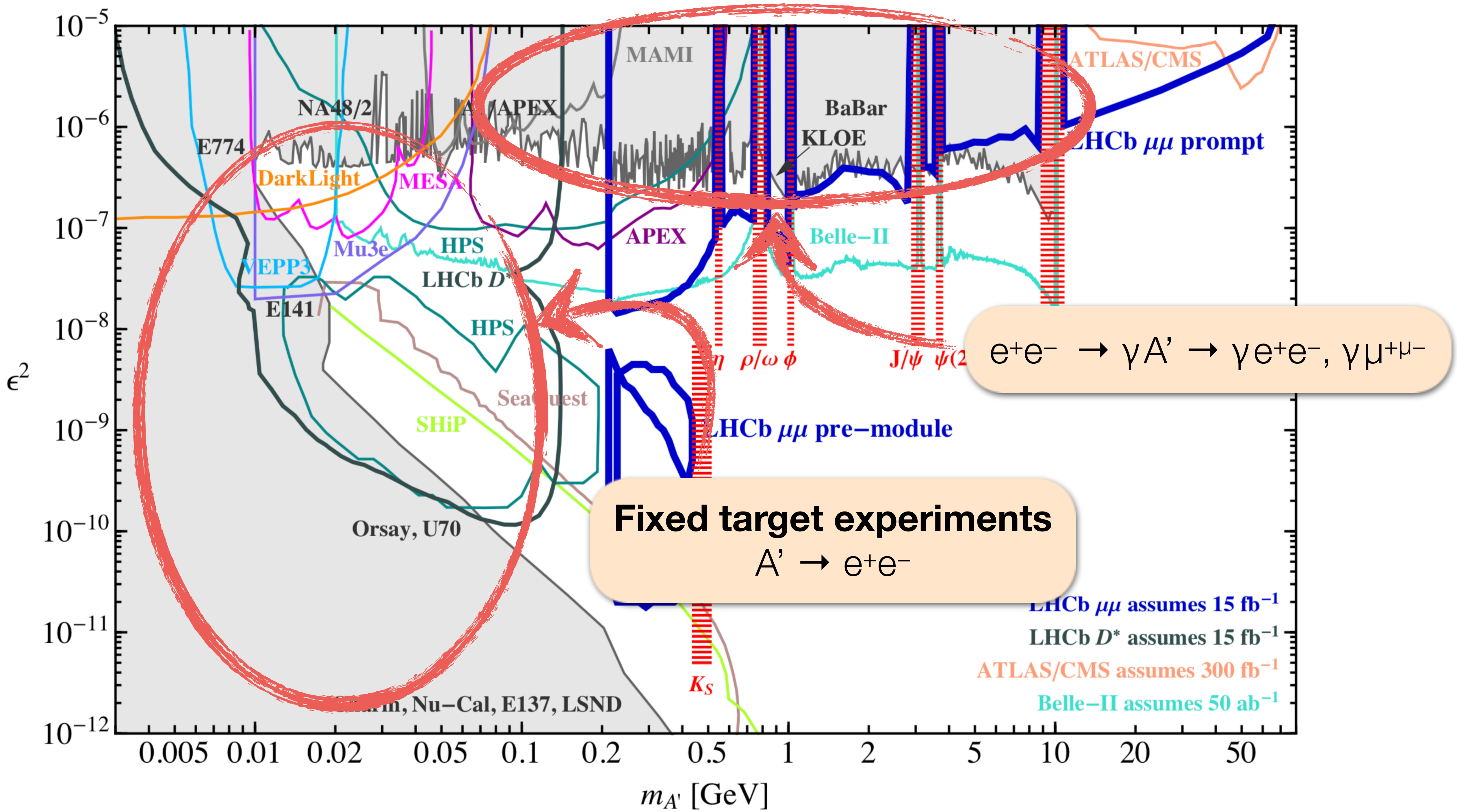

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Primordial Black Holes as Dark Matter

Basic Idea

- Upward fluctuations of the plasma density in the early Universe may gravitationally collapse into black holes.
- Criterion:
  "collapse should happen faster than rebound"
  - Collapse timescale: $1/(G\,\delta\rho)^{1/2}$ (from $R \sim G M t^2/R^2$)
  - Rebound timescale: $R/c_{sound} = R/w^{1/2}$
  - where w is the equation of state parameter ($p = w\,\rho$)
  - ➟ $R > (w/G\,\delta\rho)^{1/2}$
  - Set $R \sim 1/H \sim M_{Pl}/T^2$ (Hubble horizon) and use $G \sim 1/M_{Pl}^2$
  - ➟ $\delta\rho/T^4 > w$

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relative overdensity

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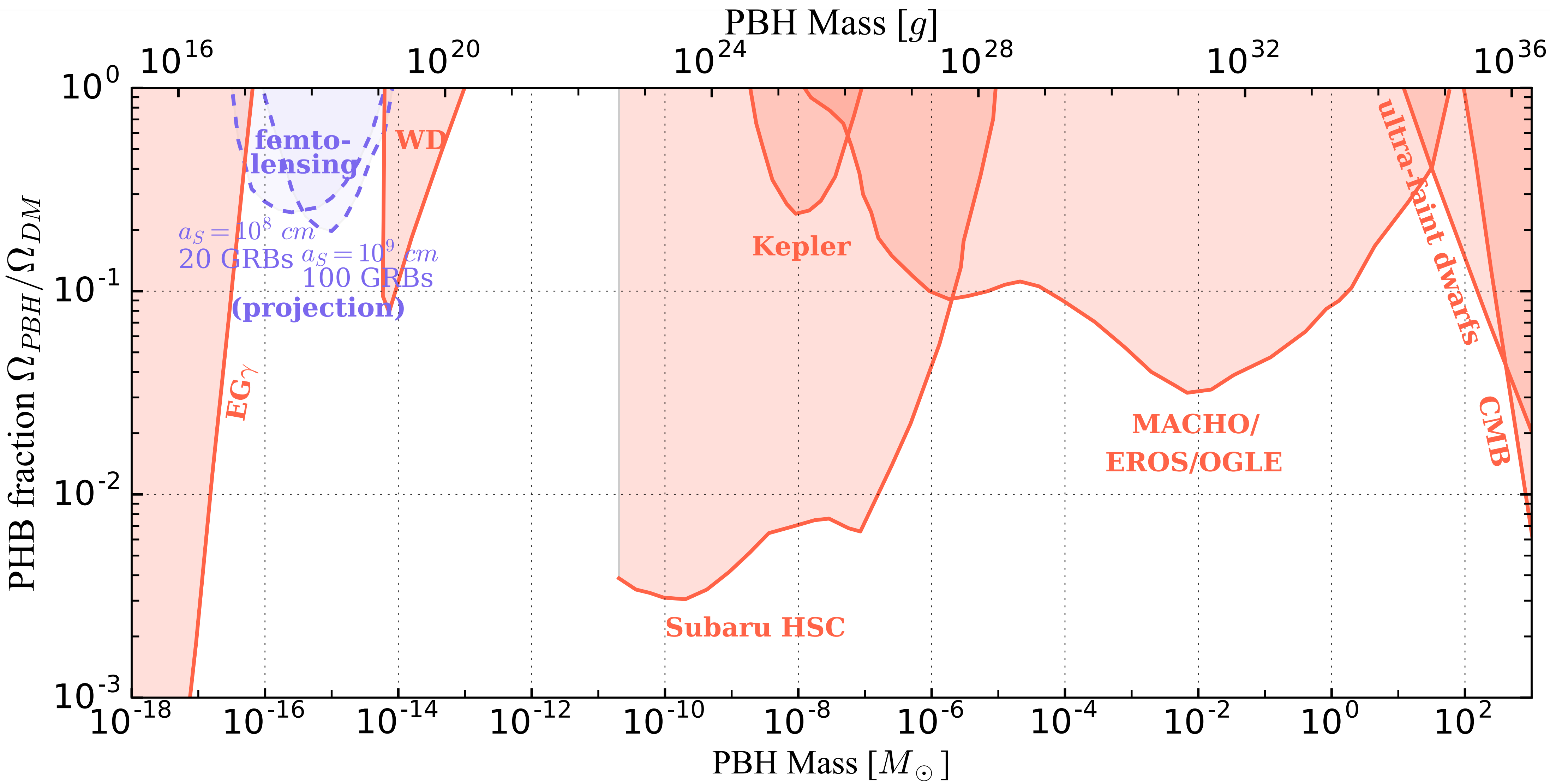
PBH Parameter Space



Katz JK Sibiryakov Xue
arXiv:1807.11495

PBH Evaporation

- ☑ Hawking 1974: black holes emit thermal radiation at temperature $T_{\rm BH} = 1/(8\pi G_N M)$ *("Hawking radiation")*

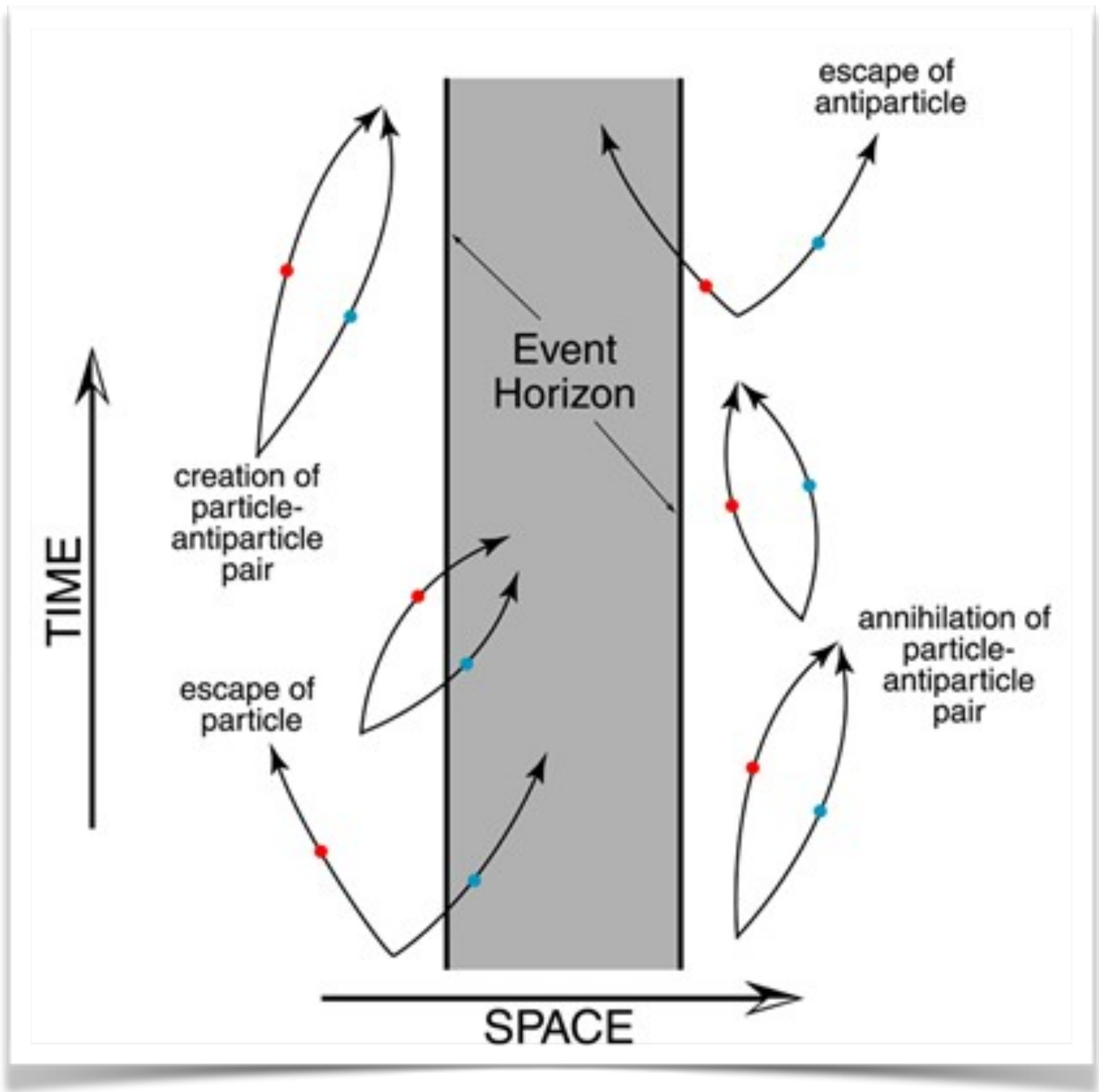


image by Stephen Dilorio

PBH Evaporation

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- Mass loss per unit area per unit time (Stefan Boltzmann law):

$$\frac{dM_{\rm BH}}{dt\,dA} = \sigma T_{\rm BH}^4$$

- Consequently, they eventually evaporate.

$$\frac{dM_{\rm BH}}{dt} = \sigma T_{\rm BH}^4 \cdot 4\pi R^2 = \frac{1}{2^{10}\pi \cdot 15}\frac{1}{G_N^2 M^2}$$

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Stefan-Boltzmann constant:
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Stefan-Boltzmann constant: $\sigma = \pi^2/60$

Schwarzschild radius $R = 2 G_N M$

PBH Evaporation

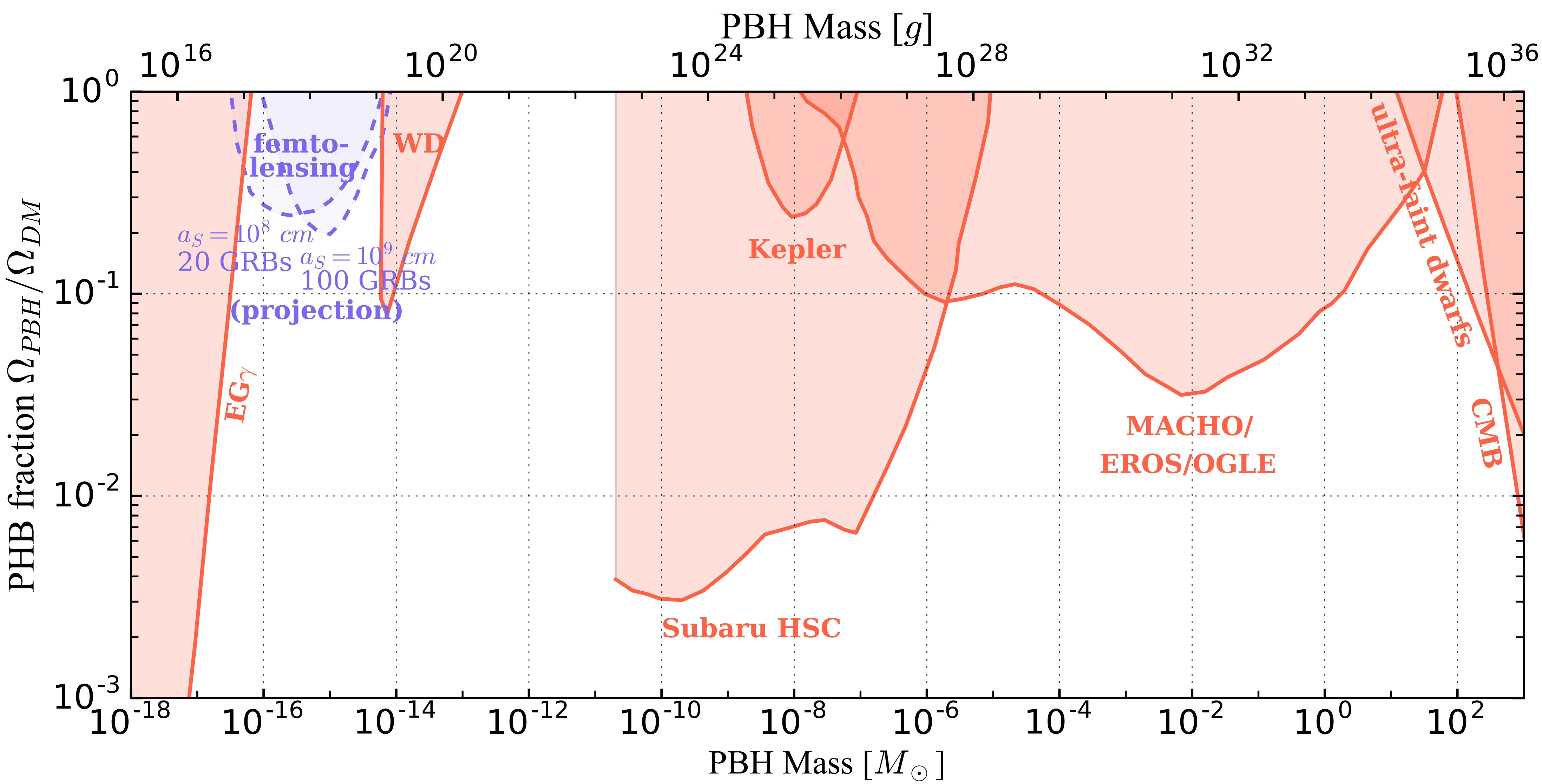
$$\frac{dM_{\rm BH}}{dt} = \sigma T_{\rm BH}^4 \cdot 4\pi R^2 = \frac{1}{2^{10}\pi \cdot 15} \frac{1}{G_N^2 M^2}$$

- Solve this differential equation by separation of variable

$$t = 5 \cdot 2^{10} \pi G_N^2 M^3 = 2 \times 10^{67}\,\text{yrs} \times \left(\frac{M}{M_\odot}\right)^3$$

- Conclusions:
  - PBH with mass ≲ $10^{-20} M_\odot$ have already evaporated
  - Even for somewhat larger masses (up to $10^{-16} M_\odot$), their Hawking radiation would contribute significantly to extragalactic background light

PBH Parameter Space



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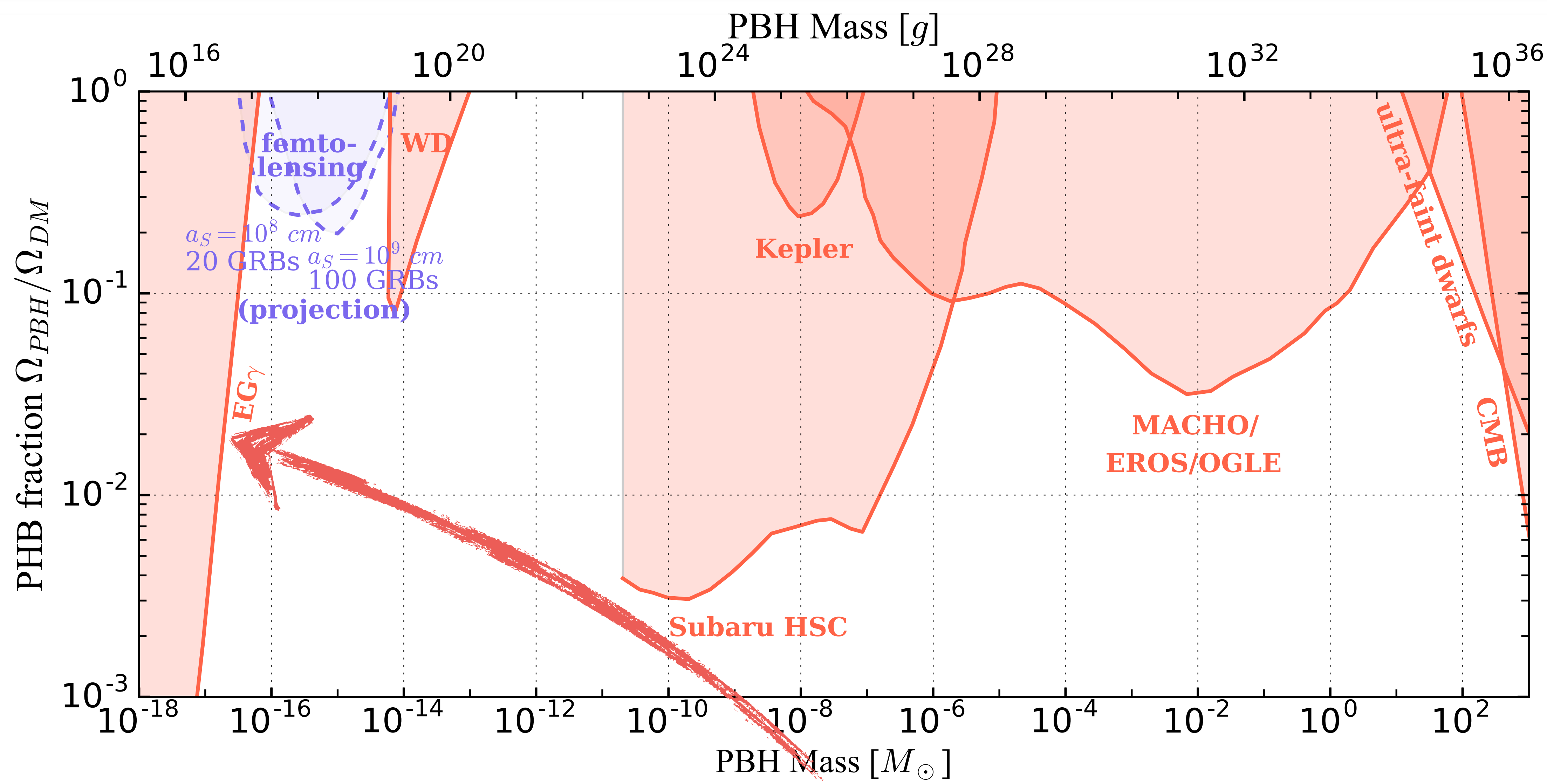

Extragalactic background light
constraint on Hawking radiation
from PBH evaporation

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Gravitational Lensing

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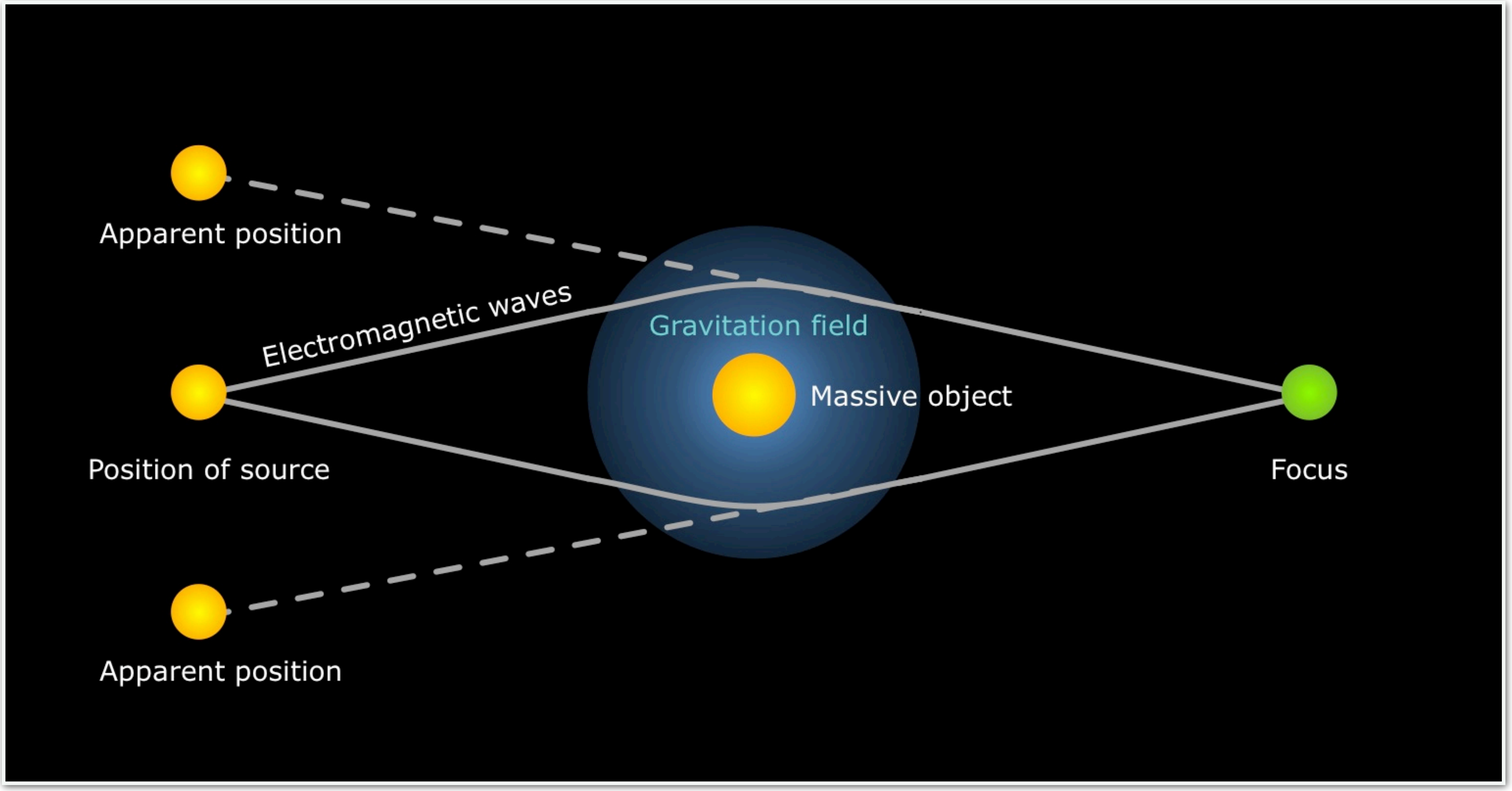
Gravitational Lensing

- Basic idea:
  PBH intersecting our line of sight to a distant source distorts the image of that source

www.spacetelescope.org

www.spacetelescope.org

Gravitational Lensing



Gravitational Lensing: Formalism

- Starting from the Minkowski metric

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

we add a weak gravitational potential

$$\eta_{\mu\nu} \rightarrow g_{\mu\nu} = \begin{pmatrix} 1+\frac{2\Phi}{c^2} & 0 & 0 & 0 \\ 0 & -(1-\frac{2\Phi}{c^2}) & 0 & 0 \\ 0 & 0 & -(1-\frac{2\Phi}{c^2}) & \\ 0 & 0 & 0 & -(1-\frac{2\Phi}{c^2}) \end{pmatrix}$$

- Corresponding line element:

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \left(1+\frac{2\Phi}{c^2}\right)c^2\mathrm{d}t^2 - \left(1-\frac{2\Phi}{c^2}\right)(\mathrm{d}\vec{x})^2$$

- Light travels along null geodesic (*ds* = 0):

$$\left(1+\frac{2\Phi}{c^2}\right)c^2\mathrm{d}t^2 = \left(1-\frac{2\Phi}{c^2}\right)(\mathrm{d}\vec{x})^2$$

based on lecture notes by Massimo Meneghetti

Gravitational Lensing: Formalism

$$\left(1+\frac{2\Phi}{c^2}\right)c^2\mathrm{d}t^2 = \left(1-\frac{2\Phi}{c^2}\right)(\mathrm{d}\vec{x})^2$$

- ☑ Speed of light in gravitational field

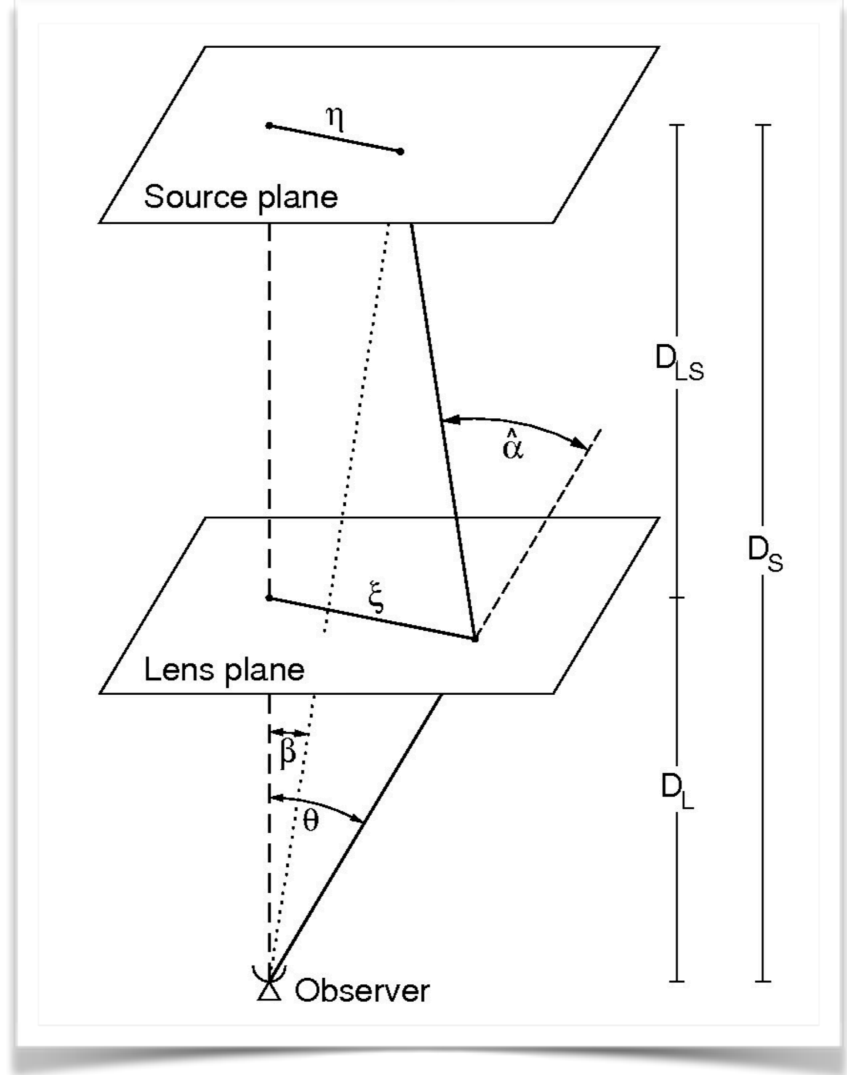
$$c' = \frac{|\mathrm{d}\vec{x}|}{\mathrm{d}t} = c\sqrt{\frac{1+\frac{2\Phi}{c^2}}{1-\frac{2\Phi}{c^2}}} \approx c\left(1+\frac{2\Phi}{c^2}\right)$$

- ☑ Corresponding index of refraction

$$n = c/c' = \frac{1}{1+\frac{2\Phi}{c^2}} \approx 1-\frac{2\Phi}{c^2}$$

- ☑ Light travel time is increased by

$$\Delta t_{\mathrm{grav}} = \int_S^O \frac{dl}{c}\, n[\vec{x}(l)] = \int_S^O \frac{dl}{c}\, \frac{2G_N M}{c^2\sqrt{l^2+\xi^2}} \simeq -\frac{4G_N M}{c^2}\log\theta$$



based on lecture notes by Massimo Meneghetti

Gravitational Lensing: Formalism

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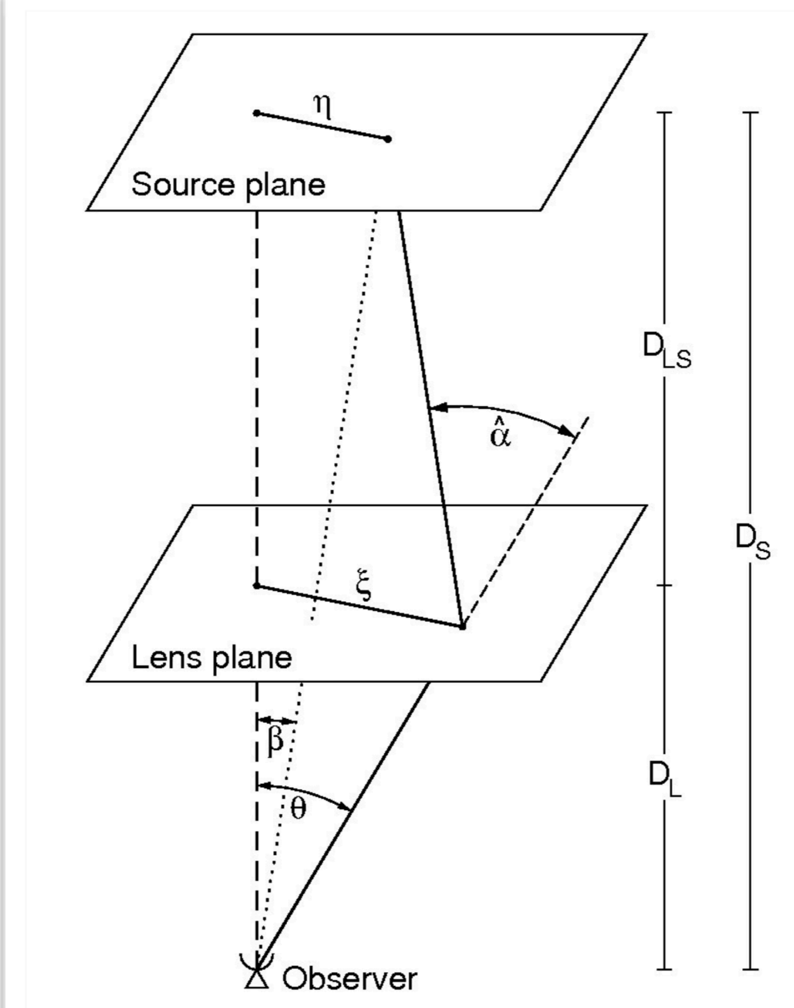


Integral from source to observer

based on lecture notes by Massimo Meneghetti

Gravitational Lensing: Formalism

$$\left(1+\frac{2\Phi}{c^2}\right)c^2\mathrm{d}t^2=\left(1-\frac{2\Phi}{c^2}\right)(\mathrm{d}\vec{x})^2$$

- ☑ Speed of light in gravitational field

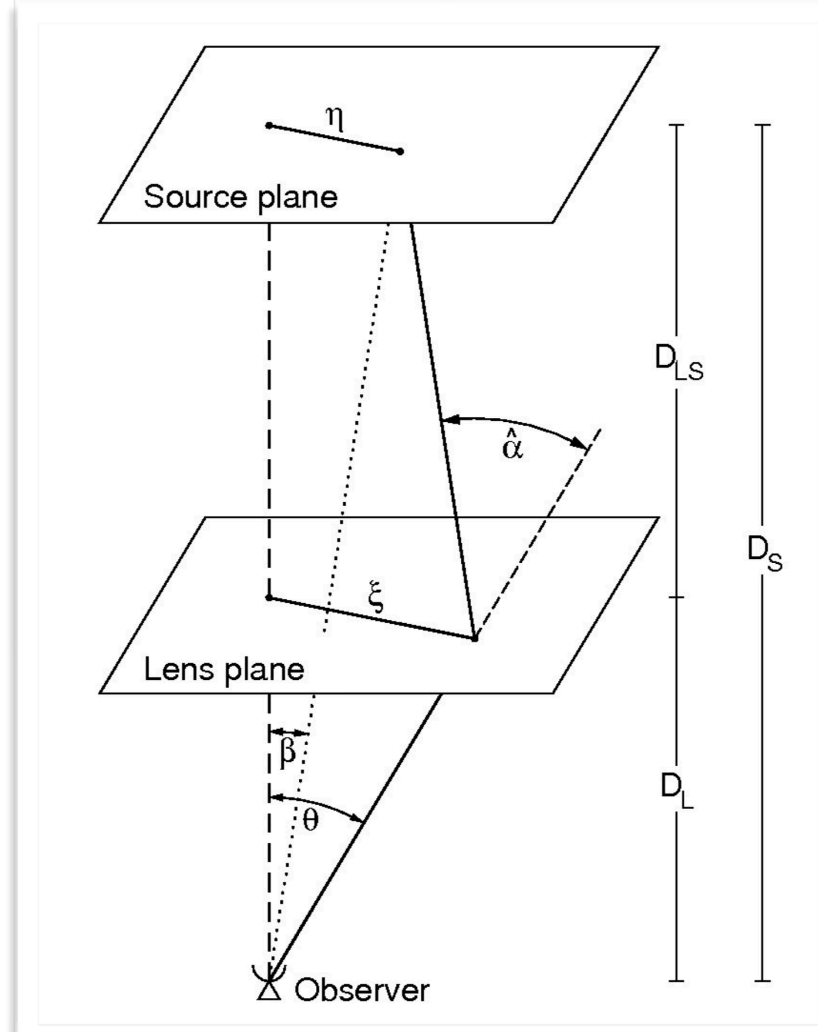
$$c'=\frac{|\mathrm{d}\vec{x}|}{\mathrm{d}t}=c\sqrt{\frac{1+\frac{2\Phi}{c^2}}{1-\frac{2\Phi}{c^2}}}\approx c\left(1+\frac{2\Phi}{c^2}\right)$$

- ☑ Corresponding index of refraction

$$n=c/c'=\frac{1}{1+\frac{2\Phi}{c^2}}\approx 1-\frac{2\Phi}{c^2}$$

- ☑ Light travel time is increased by

$$\Delta t_{\mathrm{grav}}=\int_S^O\frac{dl}{c}\,n[\vec{x}(l)]=\int_S^O\frac{dl}{c}\,\frac{2G_NM}{c^2\sqrt{l^2+\xi^2}}\simeq-\frac{4G_NM}{c^2}\log\theta$$



Integral from source to observer

Impact parameter (min. distance to lens)

based on lecture notes by Massimo Meneghetti

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Gravitational Lensing: Formalism

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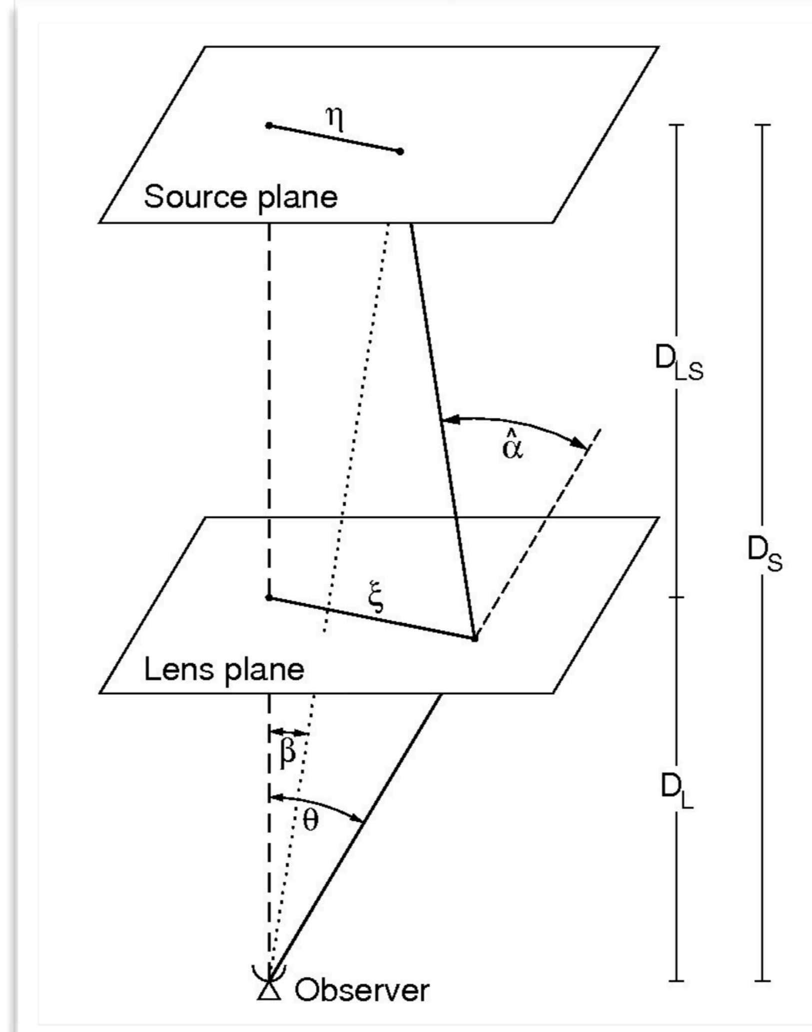


Integral from source to observer

Impact parameter (min. distance to lens)

lensing angle
$\theta = \xi/D_S$

based on lecture notes by Massimo Meneghetti

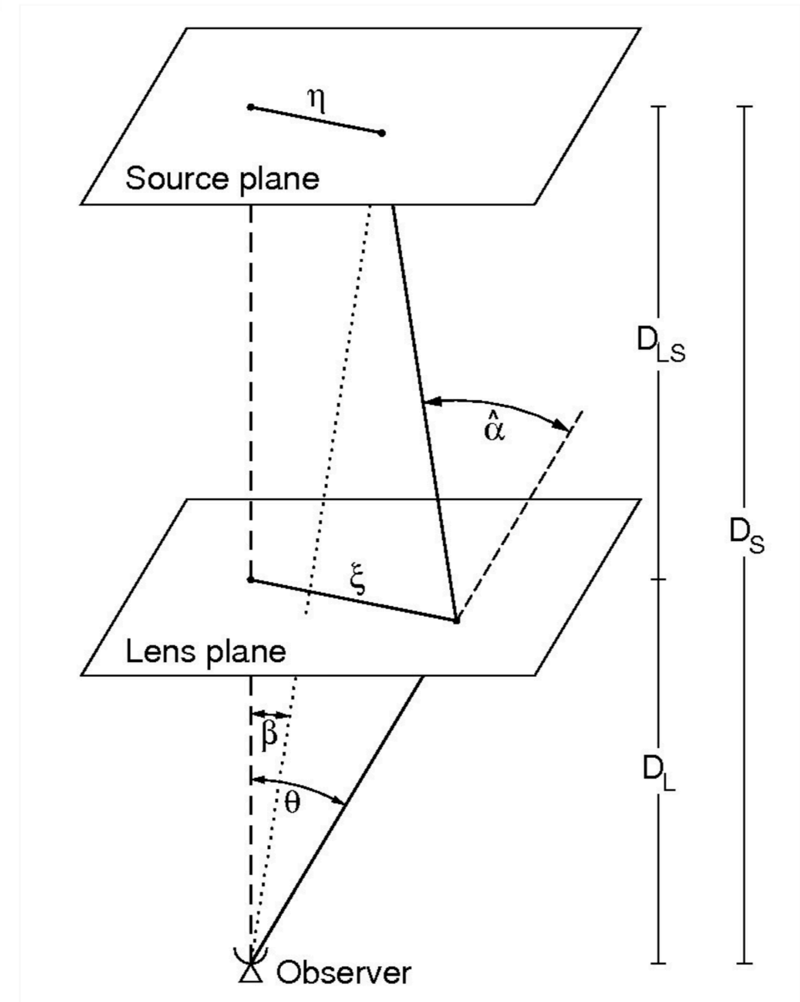
Gravitational Lensing: Formalism

- In addition: geometric time delay

$$\Delta t_{\rm geom} = \left[\frac{D_L}{c\,\cos(\theta-\beta)} - D_L\right] + \left[\frac{D_{LS}}{c\,\cos[(\theta-\beta)D_L/D_{LS}]} - D_{LS}\right]$$
$$\simeq \frac{D_L}{2c}(\theta-\beta)^2 + \frac{D_{LS}}{2c}\frac{(\theta-\beta)^2 D_L^2}{D_{LS}^2}$$
$$= \frac{D_L D_S}{2cD_{LS}}(\theta-\beta)^2$$

- Overall:

$$\Delta t = \frac{D_L D_S}{cD_{LS}}\left[\frac{(\theta-\beta)^2}{2} - \frac{4G_N M D_{LS}}{c^2 D_L D_S}\log\theta\right]$$



Gravitational Lensing: Formalism

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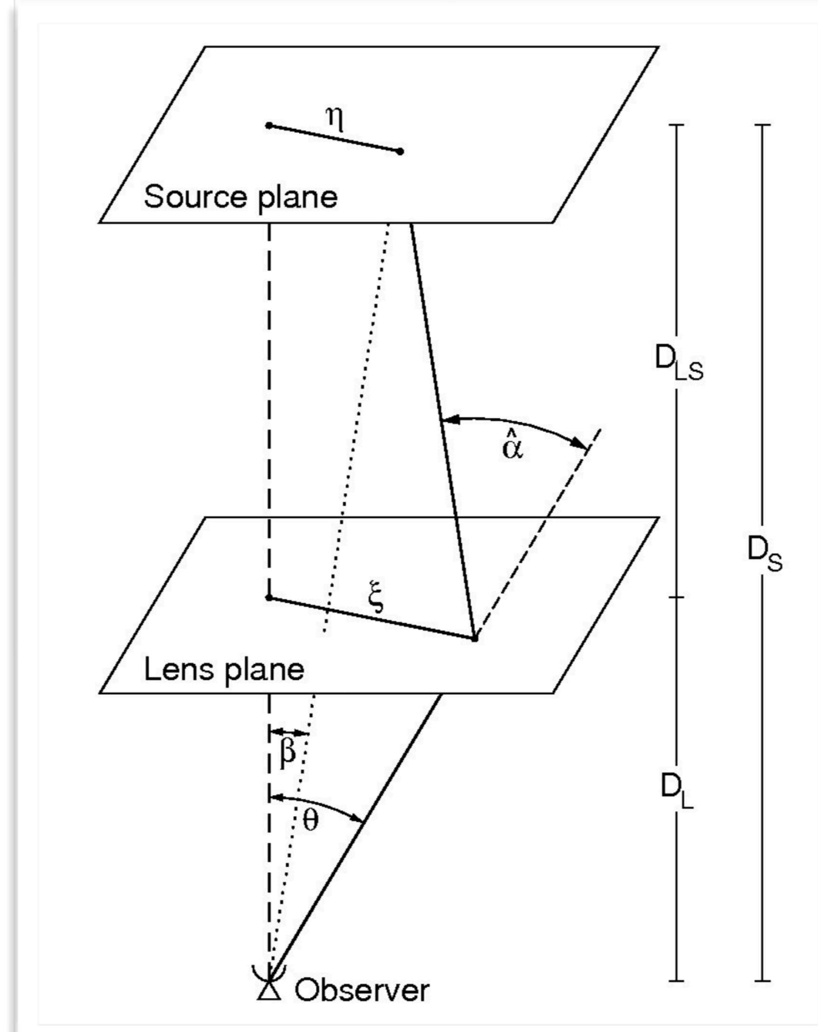
- ☑ Overall:

$$\Delta t = \frac{D_L D_S}{cD_{LS}}\left[\frac{(\theta-\beta)^2}{2} - \frac{4G_N M D_{LS}}{c^2 D_L D_S}\log\theta\right]$$

Square of the **Einstein angle**:

$$\theta_E^2 \equiv \frac{4G_N M D_{LS}}{c^2 D_L D_S}$$



Gravitational Lensing: Formalism

$$\Delta t = \frac{D_L D_S}{c D_{LS}} \left[\frac{(\theta - \beta)^2}{2} - \frac{4 G_N M D_{LS}}{c^2 D_L D_S} \log \theta \right]$$

- Light waves travelling from the source to the observer along different paths (different θ) acquire different phase: $e^{i\omega\Delta t}$.
- Fermat's principle: if $\omega\Delta t \gg 1$, contributions with different θ will interfere destructively, except at stationary points of Δt.

$$\frac{d\,\Delta t}{d\theta} = \frac{D_L D_S}{c\, D_{LS}} \left[(\theta - \beta) - \frac{\theta_E^2}{\theta} \right] \stackrel{!}{=} 0$$

- Leads to the lens equation:

$$\theta - \beta = \frac{\theta_E^2}{\theta}$$

Gravitational Lensing: Formalism

$$\theta - \beta = \frac{\theta_E^2}{\theta}$$

- The solutions are the angular positions of the lensed images

$$\theta_\pm = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right)$$

- We see that the Einstein angle is a measure for the angular deviation between the lensed and (hypothetical) unlensed images. This interpretation is exact for β = 0 (lens along the line of sight).

- One can also compute the magnification (intensity relative to the unperturbed source) of the two images:

$$\mu_\pm = \frac{y^2 + 2}{2y\sqrt{y^2 + 4}} \pm \frac{1}{2} \quad \text{with} \quad y \equiv \beta/\theta_E$$

Microlensing

- For a 1 $M_\odot$ lens at $\mathcal{O}$(kpc) distance (typical scale within the Milky Way):
  $\theta_E \sim 0.003$ arcsec
- For comparison:
  angular resolution of the Hubble telescope: 0.05 arcsec
- However: can still observe overall brightening of the source

$$\mu_\pm = \frac{y^2+2}{2y\sqrt{y^2+4}} \pm \frac{1}{2}$$ ➟ total magnification: $$\mu = \frac{y^2+2}{y\sqrt{y^2+4}}$$

- This effect is called microlensing.
- Observable because of time dependence: a PBH passing in front of a background star leads to transient magnification of that star.

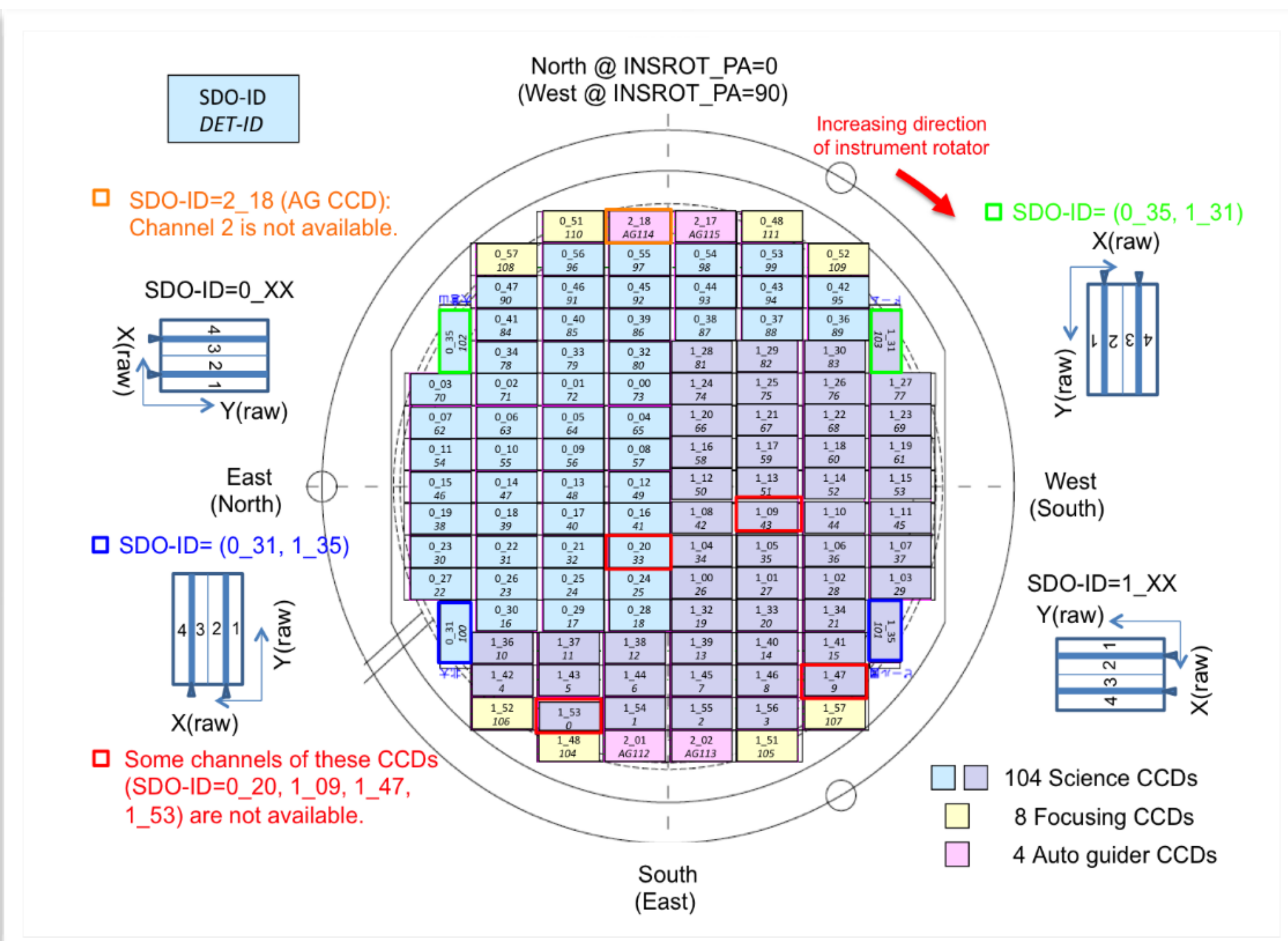
Microlensing

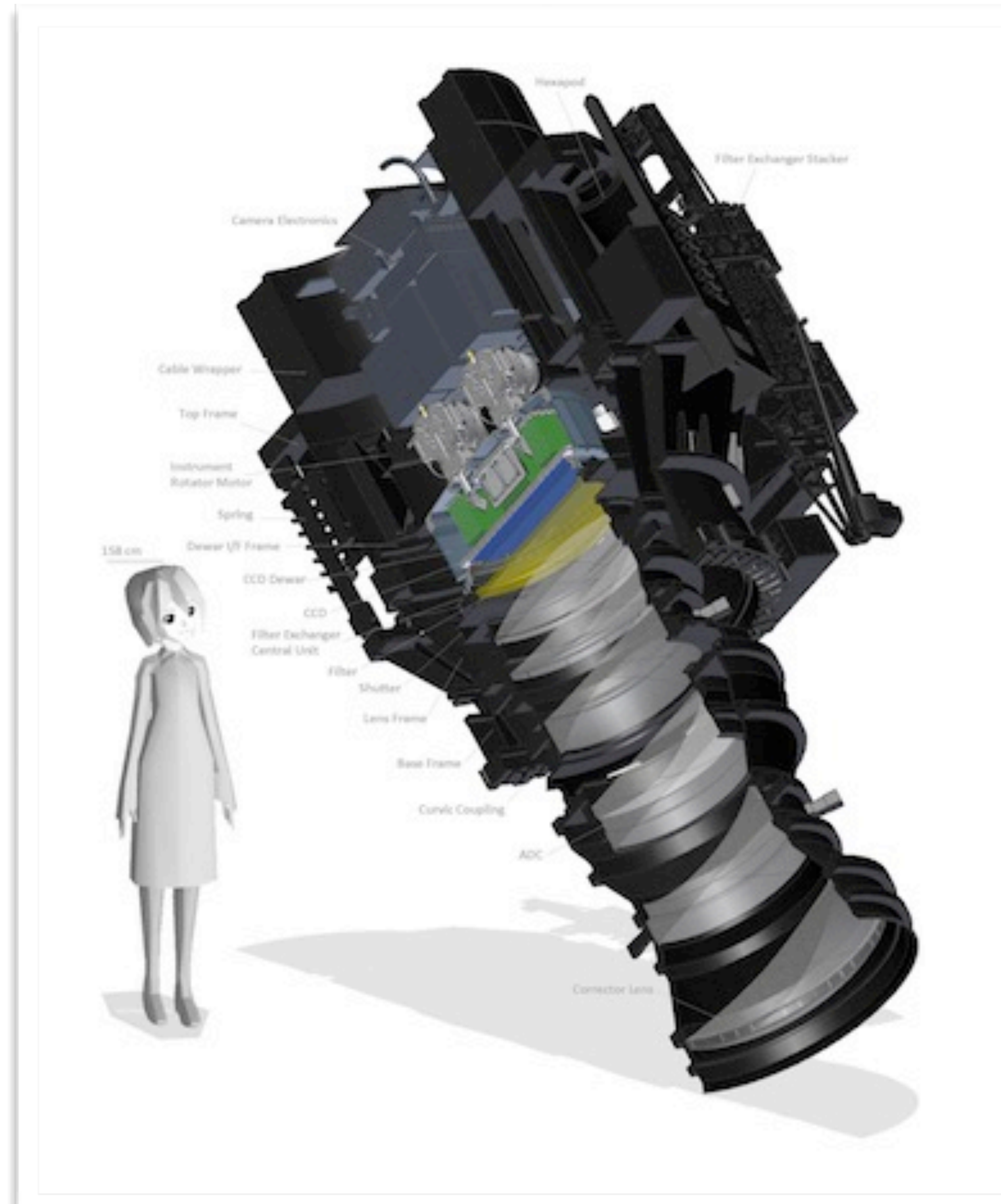
☑ Observations at the 8.2 m Subaru Telescope (Hawaii)

Niikura et al. arXiv:1701.02151

Microlensing

- In particular: Hyper Suprime-Cam
- 1.5 degree field of view (huge!)
- 900 Megapixels





Niikura et al. arXiv:1701.02151

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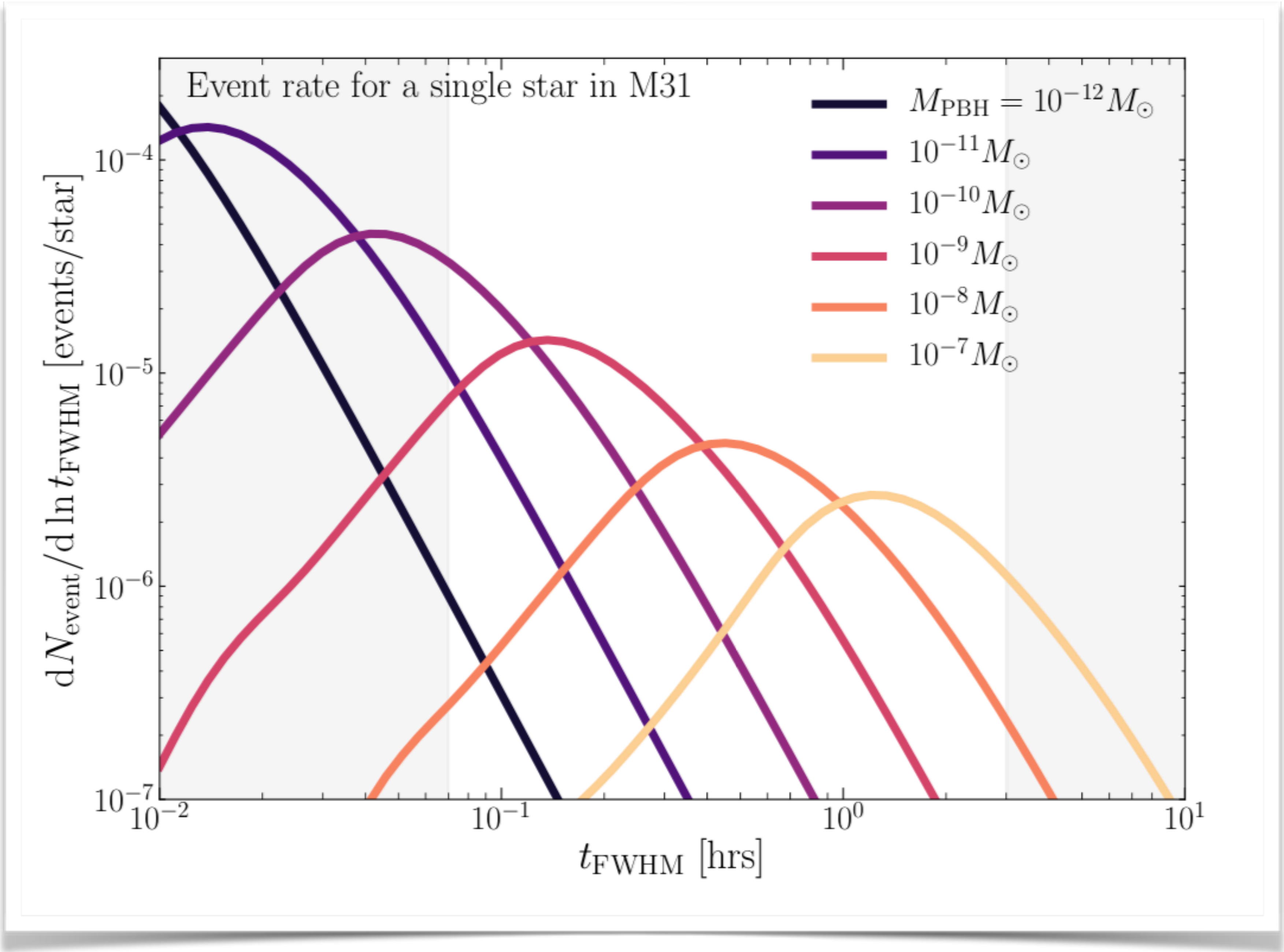
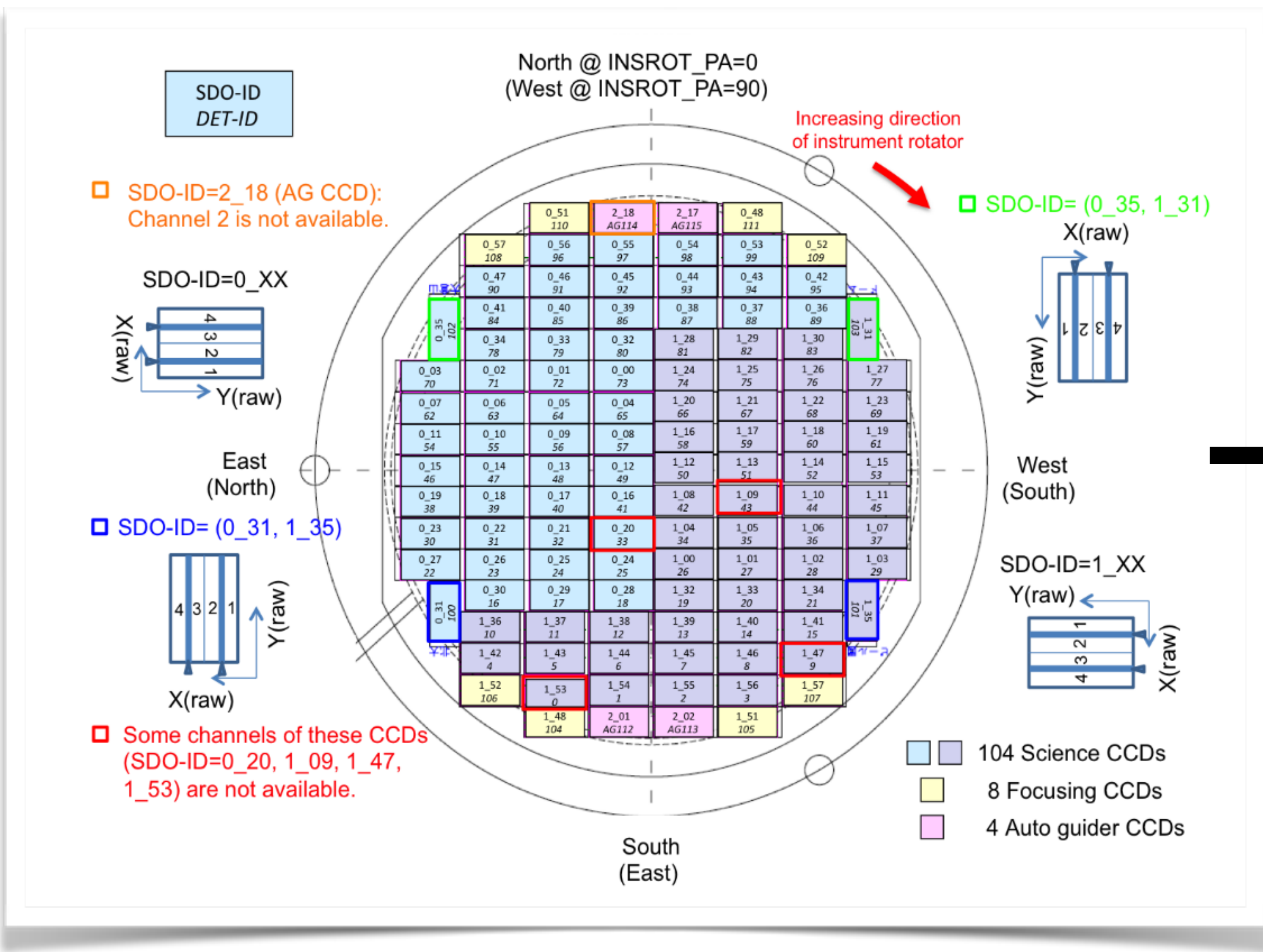
Lensing Probability

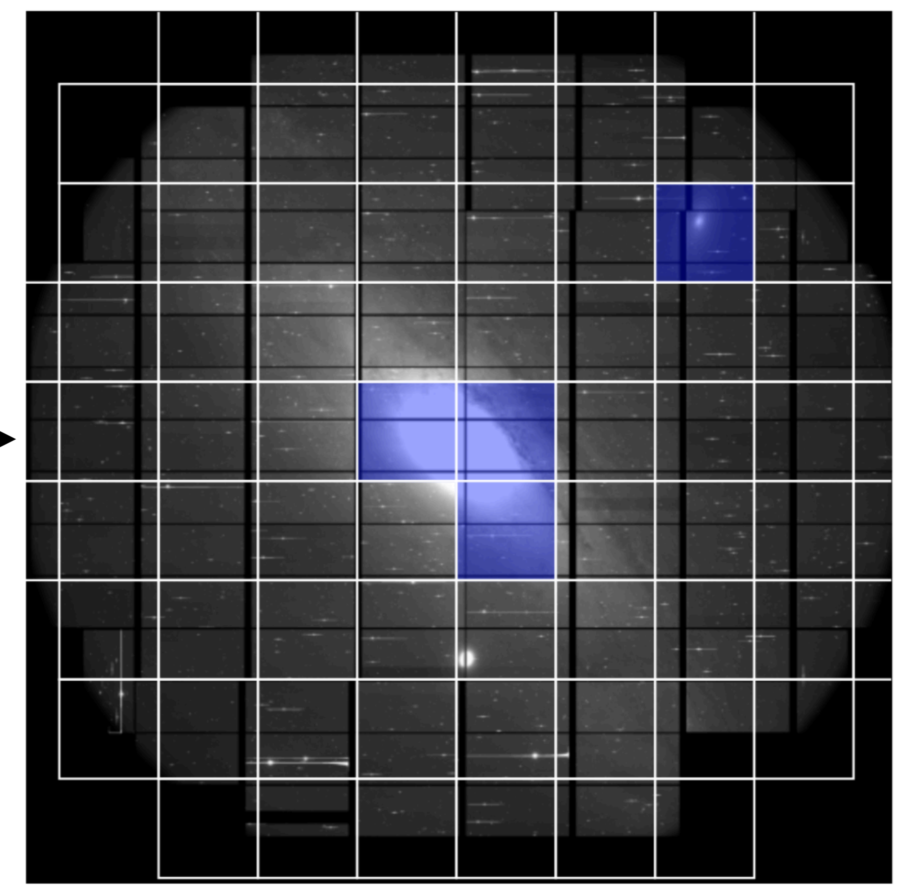


Niikura et al. arXiv:1701.02151

Observation Strategy

- ☑ Single night (7 hours of observations) sufficient
- ☑ Large field of view
  ➠ observe the whole M31 (Andromeda) galaxy at once

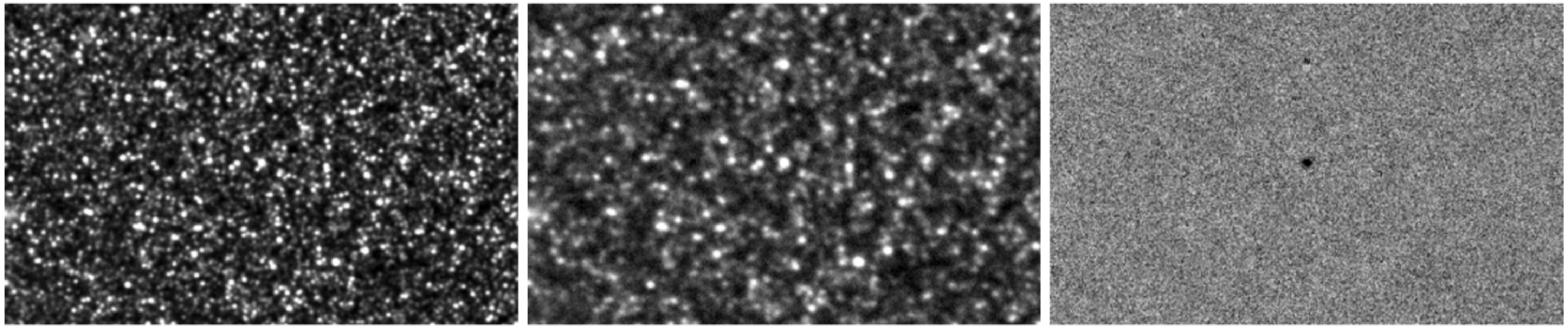




Niikura et al. arXiv:1701.02151

Observation Strategy

- ☑ Single night (7 hours of observations) sufficient
- ☑ Repeated observations of the same patch on the sky (90 sec observation time, 35 sec readout time)
- ☑ Subtract reference image to detect transients



Observation #1 | Observation #2 | Difference (including transient)

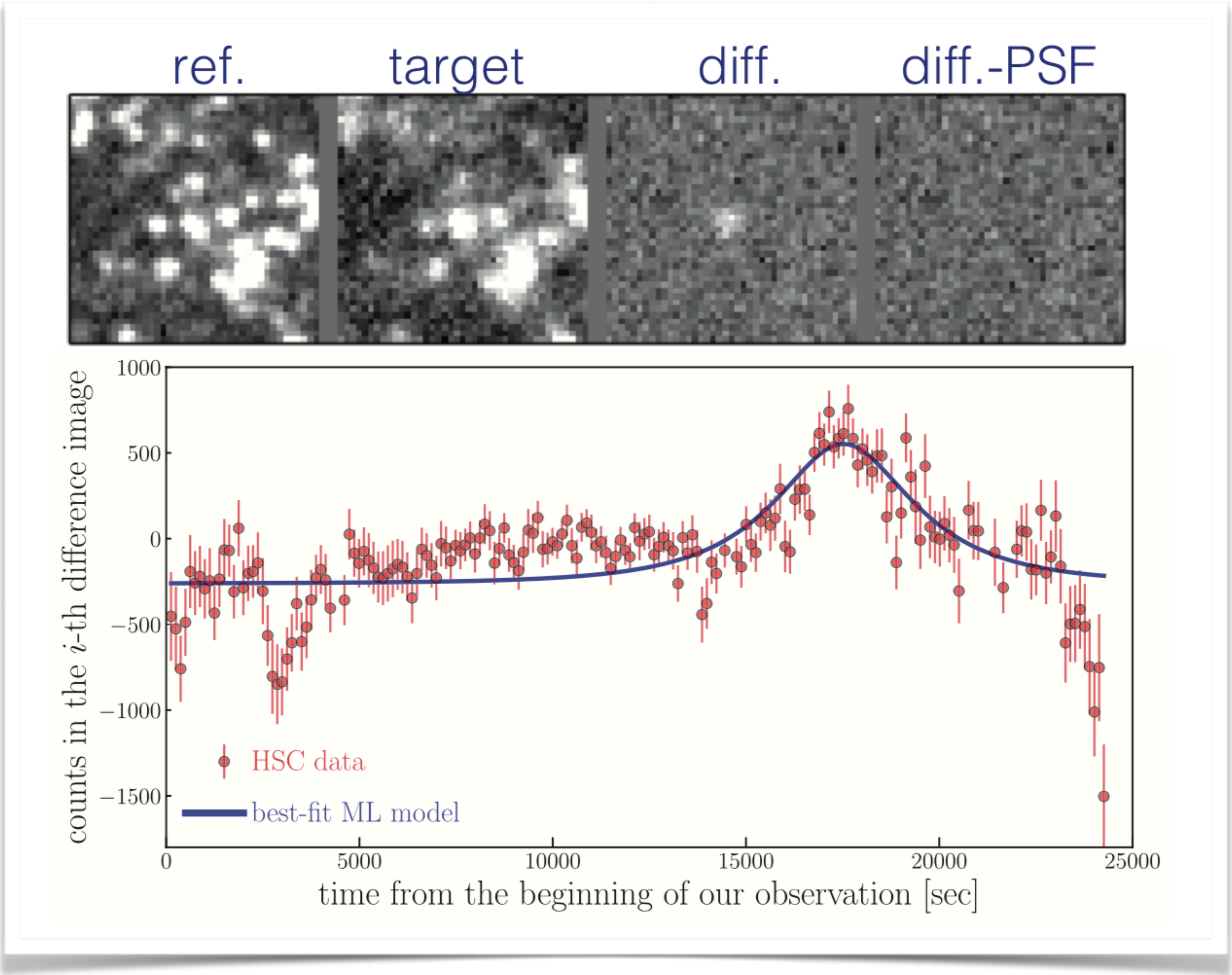
Niikura et al. arXiv:1701.02151

Data Analysis

- Analysis challenges
  - each CCD pixel contains many stars
  - central region of M31 too bright (CCDs saturated ➠ discard)
- Selection criteria for microlensing candidates
  - At least 5σ detection in any of the 188 difference images
  - difference image consistent with point spread function
- Result: 15 571 candidates
- Construct light curve for each of them

Niikura et al. arXiv:1701.02151

Data Analysis



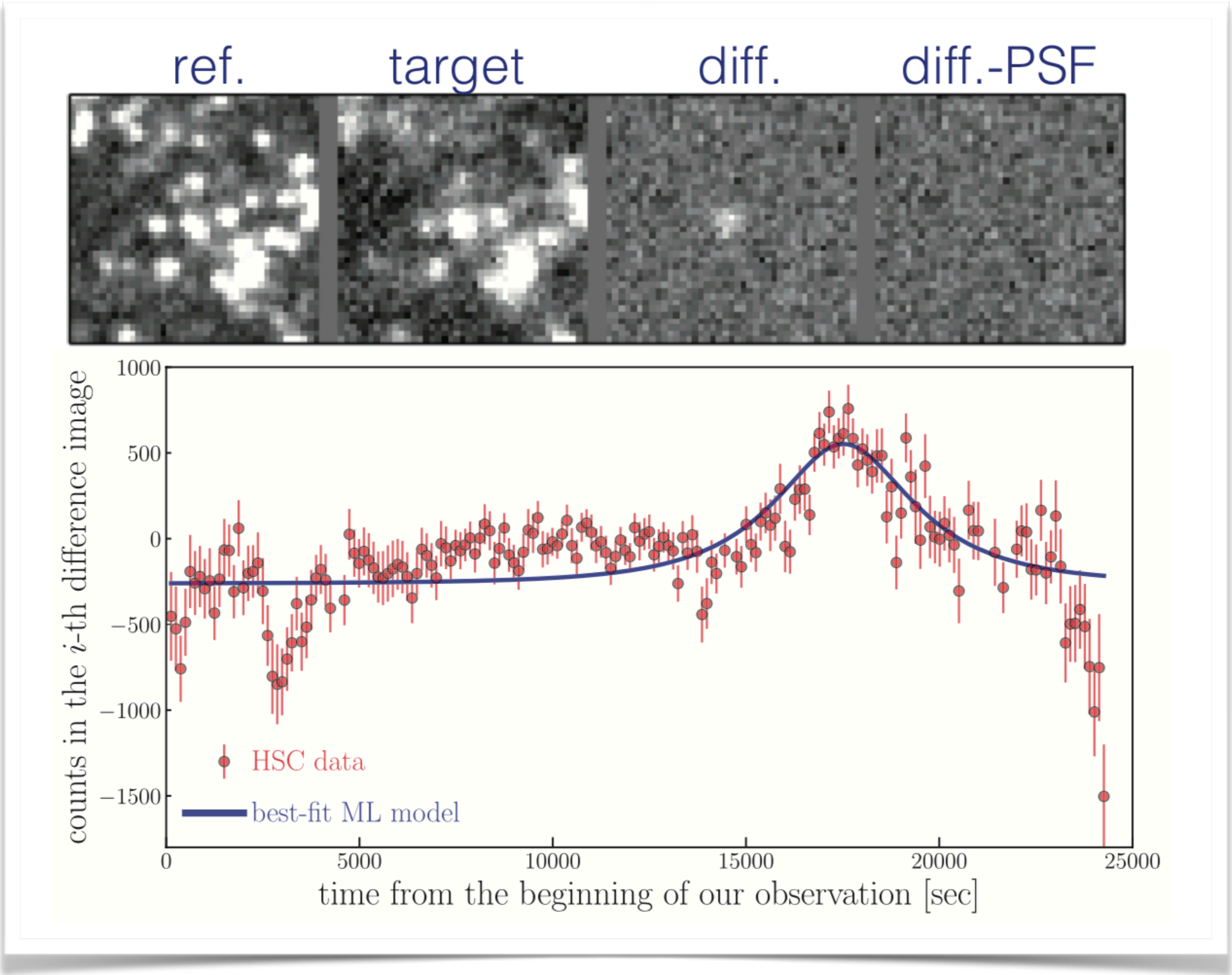
Niikura et al. arXiv:1701.02151

Further Selection Criteria

- ☑ Subject the 15 571 candidates to the following cuts
  - Require single bump to exclude periodic stars (➠ 11 703 candidates left)
  - Fit predicted microlensing light curve, require decent goodness-of-fit (➠ 66 candidates left)
- ☑ Visual inspection
  - reject 44 candidates due to cross-talk from nearby bright star
  - reject 20 additional candidates at the edges of CCDs
  - reject 1 candidate due to passing asteroid
- ☑ 1 candidate left

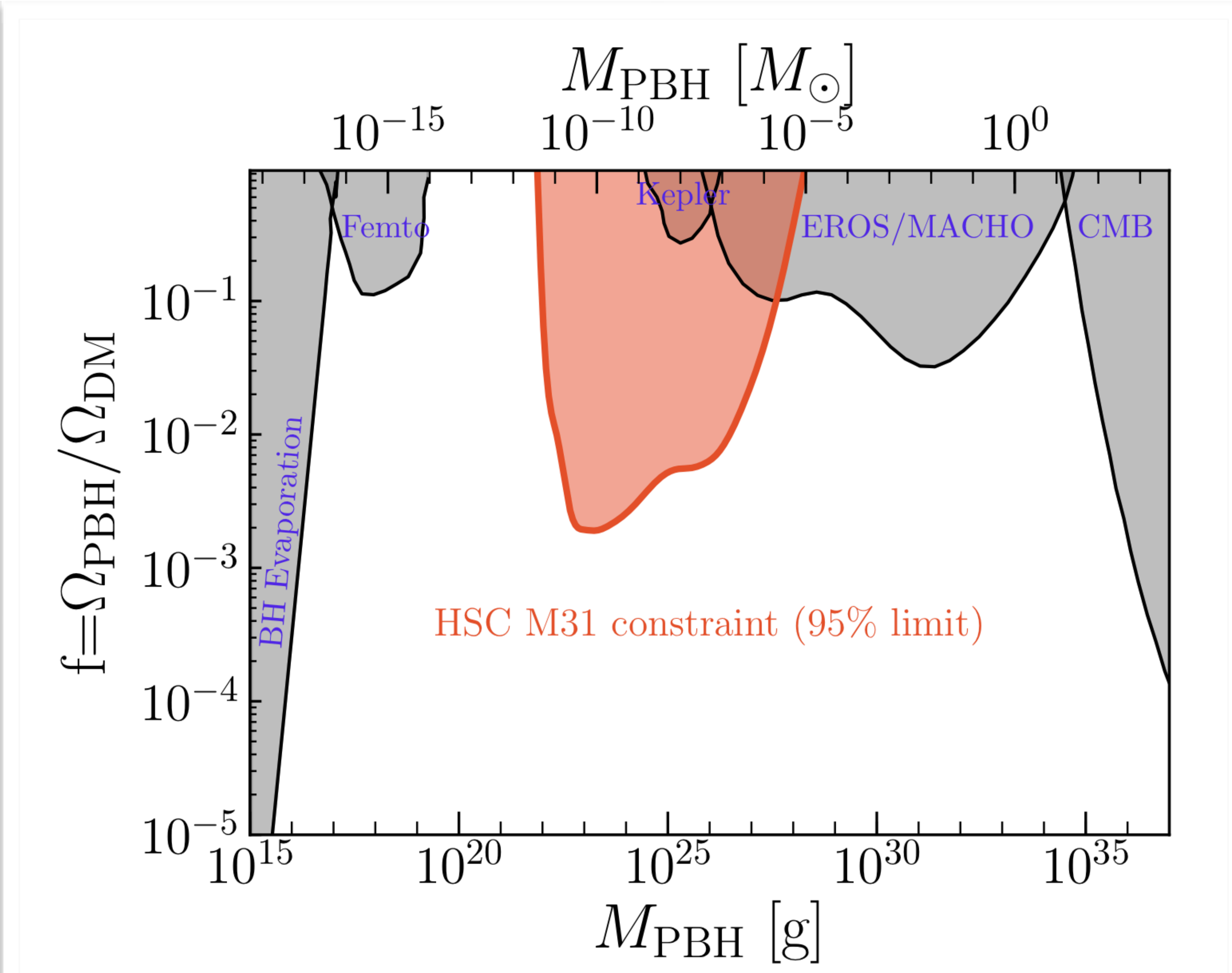
Niikura et al. arXiv:1701.02151

Data Analysis



Niikura et al. arXiv:1701.02151

Resulting Limits



Niikura et al. arXiv:1701.02151

Caveat 1: Wave Optics

- Our calculations so far relied on Fermat's principle: if $\omega\,\Delta t \gg 1$, contributions with different θ will interfere destructively, except at stationary points of Δt.
- Leads to the lens equation

$$\theta - \beta = \frac{\theta_E^2}{\theta}$$

- What if $\omega\,\Delta t \lesssim 1$?
- Need to evaluate full Fresnel integral

$$\mu \propto \left| \int d^2\vec{\theta}\, e^{i\omega\Delta t(\vec{\theta},\vec{\beta})} \right|^2$$

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Caveat 1: Wave Optics

$$\mu \propto \left| \int d^2\vec{\theta}\, e^{i\omega \Delta t(\vec{\theta},\vec{\beta})} \right|^2$$

- ☑ Can be evaluated analytically for point-like lens

$$F(y,\Omega)_{\rm BH} = e^{i\Omega|\vec{y}|^2/2} \left(-\frac{i\Omega}{2}\right)^{i\Omega/2} \Gamma\left(1-\frac{i\Omega}{2}\right) L_{-1+\frac{i\Omega}{2}} \left(-\frac{i|\vec{y}|^2\Omega}{2}\right)$$

with

$$\Omega \equiv \frac{4GM(1+z_L)}{c^3}\,\omega \qquad\qquad y \equiv \beta/\theta_E$$

- ☑ Tends to reduce magnification
(more destructive interference)

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Laguerre polynomial

with

$$\Omega \equiv \frac{4GM(1+z_L)}{c^3}\,\omega \qquad y \equiv \beta/\theta_E$$

- ☑ Tends to reduce magnification (more destructive interference)

Caveat 2: Finite Size of the Source

- Different points on the source are magnified differently
- Remember: total magnification in geometric optics:

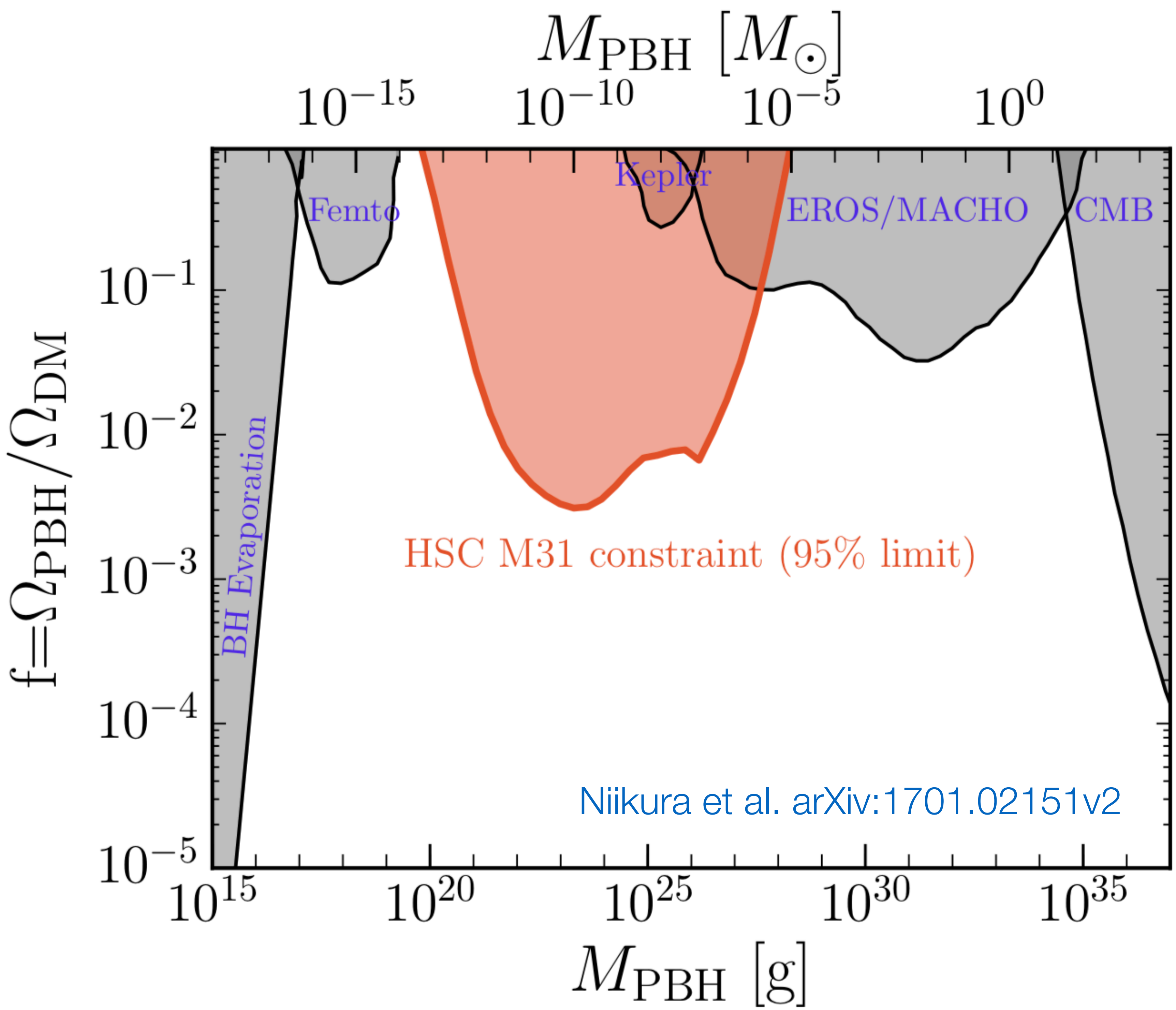
$$\mu = \frac{y^2 + 2}{y\sqrt{y^2 + 4}} \,.$$

- Now need to evaluate

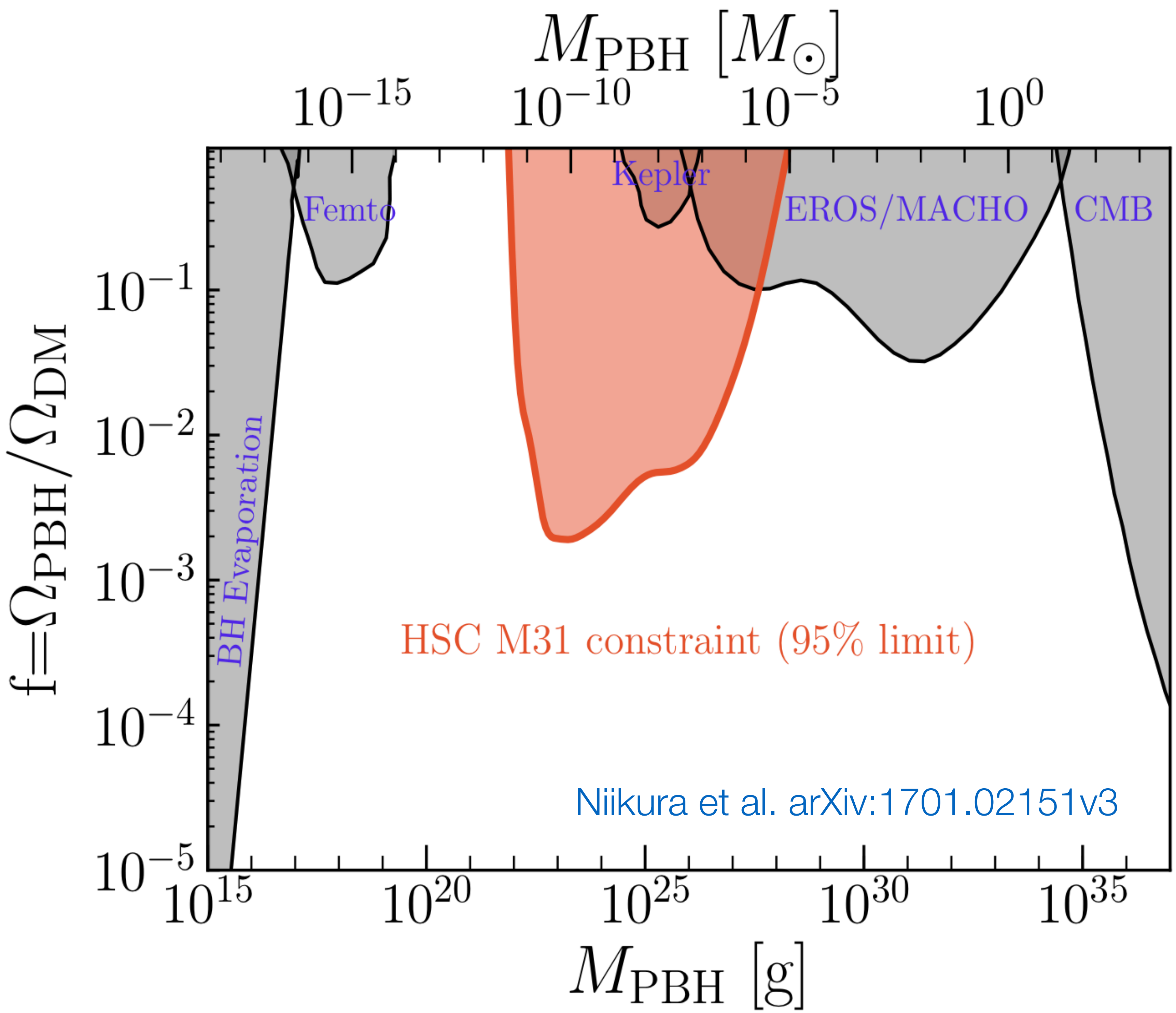
$$\int d\vec{y}\, \frac{\vec{y}^2 + 1}{|\vec{y}|\sqrt{\vec{y}^2 + 4}}$$

- Tends to reduce the magnification

Effect of Wave Optics + Finite Source Size

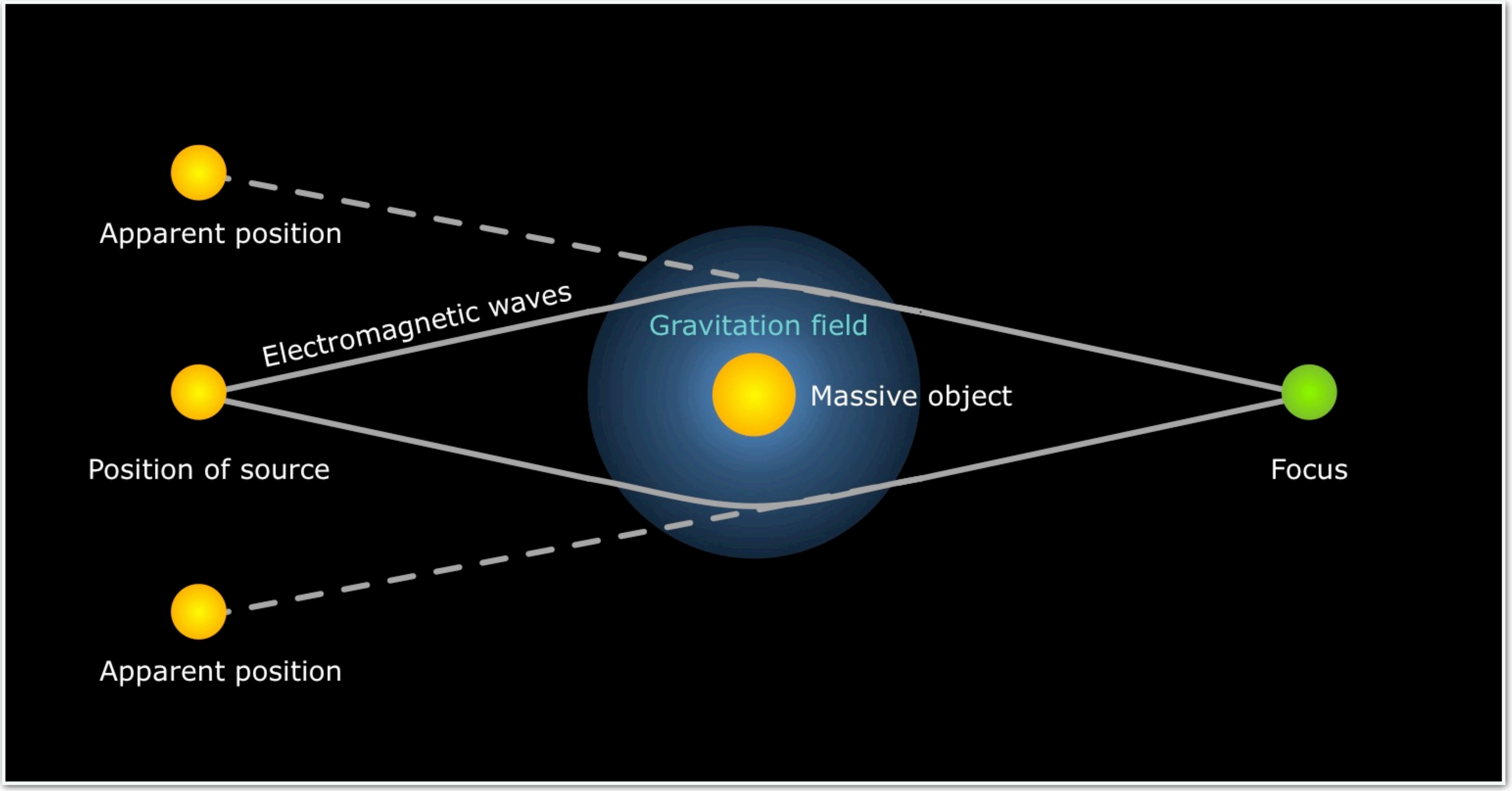


Effect of Wave Optics + Finite Source Size

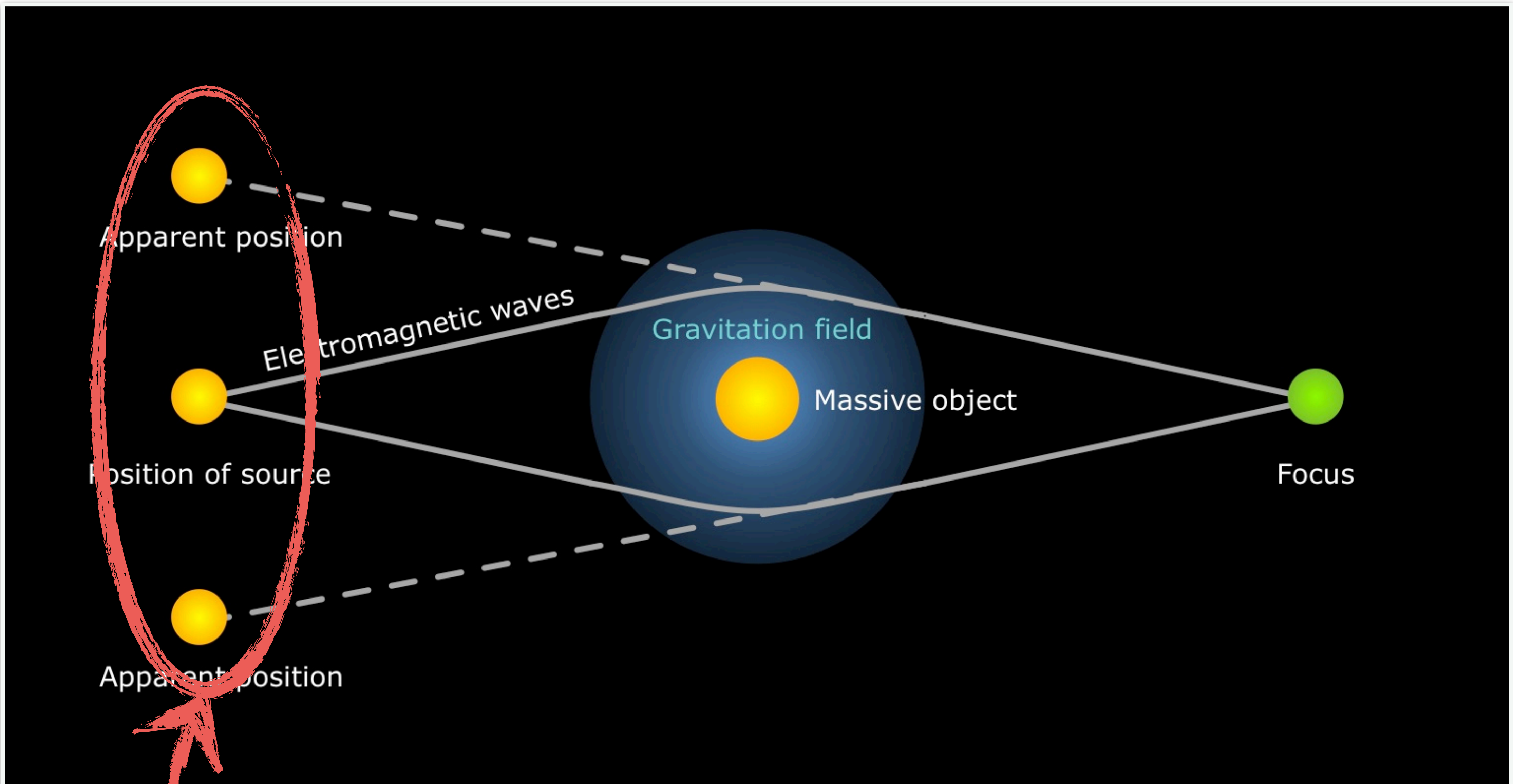


Femtolensing

Femtolensing

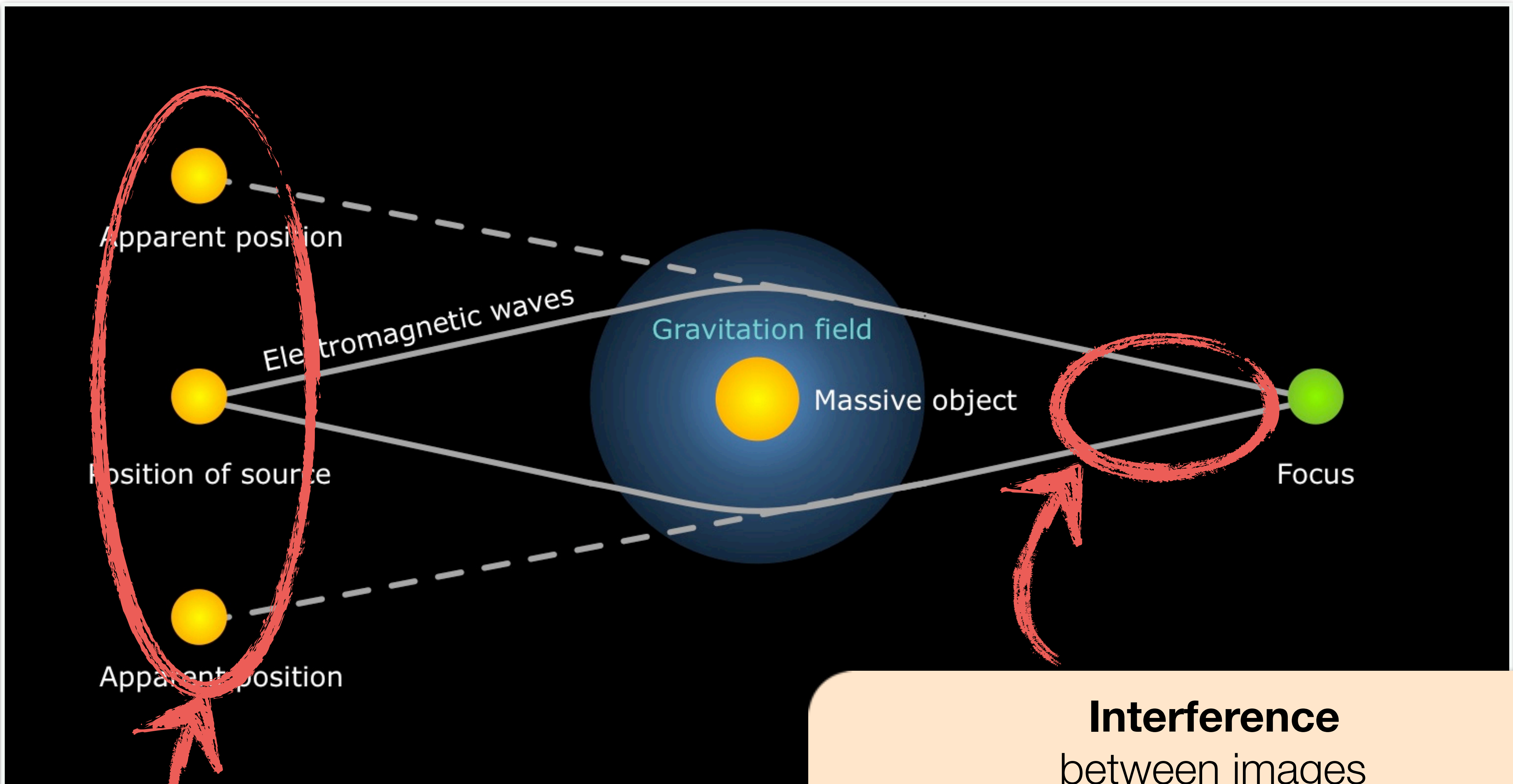


Femtolensing



Images not resolved

Femtolensing



Images not resolved

Interference
between images

$$A = A_1 e^{iEt_1} + A_2 e^{iEt_2}$$

expect wiggles in energy spectrum

Time Delay (Geometric Optics)

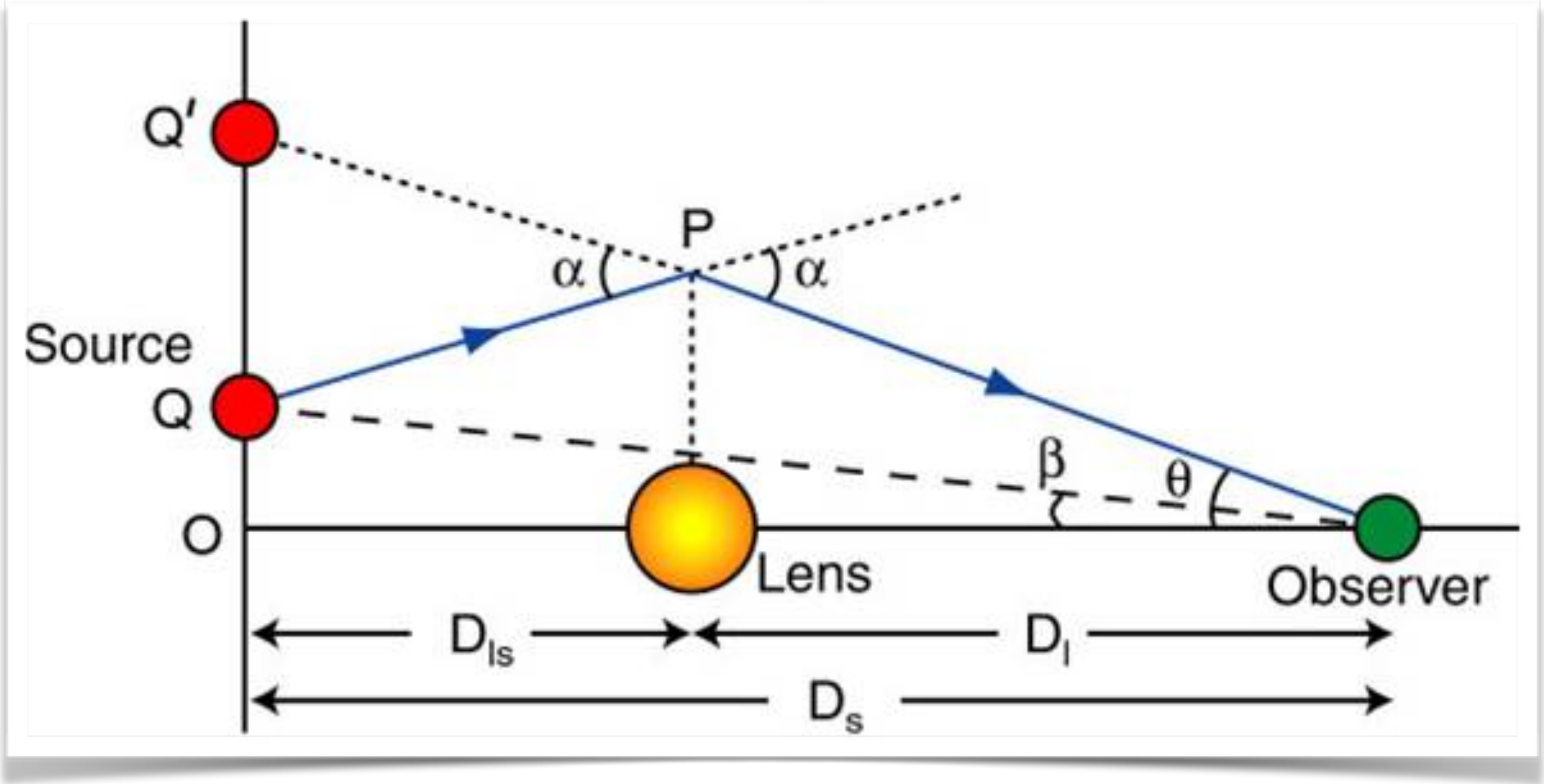
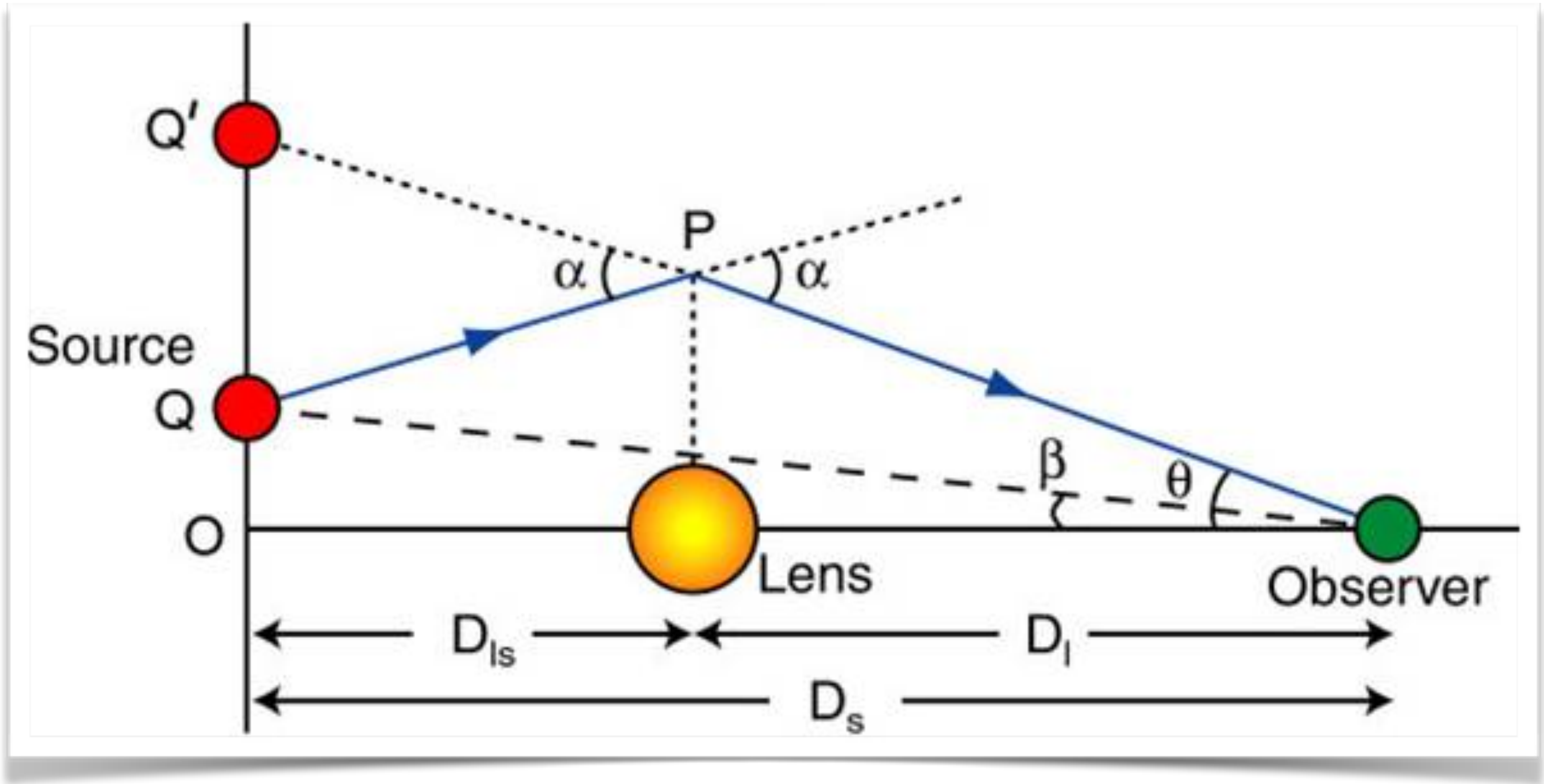


Image: University of Manchester

Time Delay: $$\Delta t = \frac{1}{c}\frac{D_L D_S}{D_{LS}}(1+z_L)\left(\frac{|\vec{\theta}-\vec{\beta}|^2}{2} - \psi(\vec{\theta})\right)$$

Time Delay (Geometric Optics)



niversity of Manchester

Geometric Time Delay

Time Delay:

$$\Delta t = \frac{1}{c}\frac{D_L D_S}{D_{LS}}(1+z_L)\left(\frac{|\vec{\theta}-\vec{\beta}|^2}{2}-\psi(\vec{\theta})\right)$$

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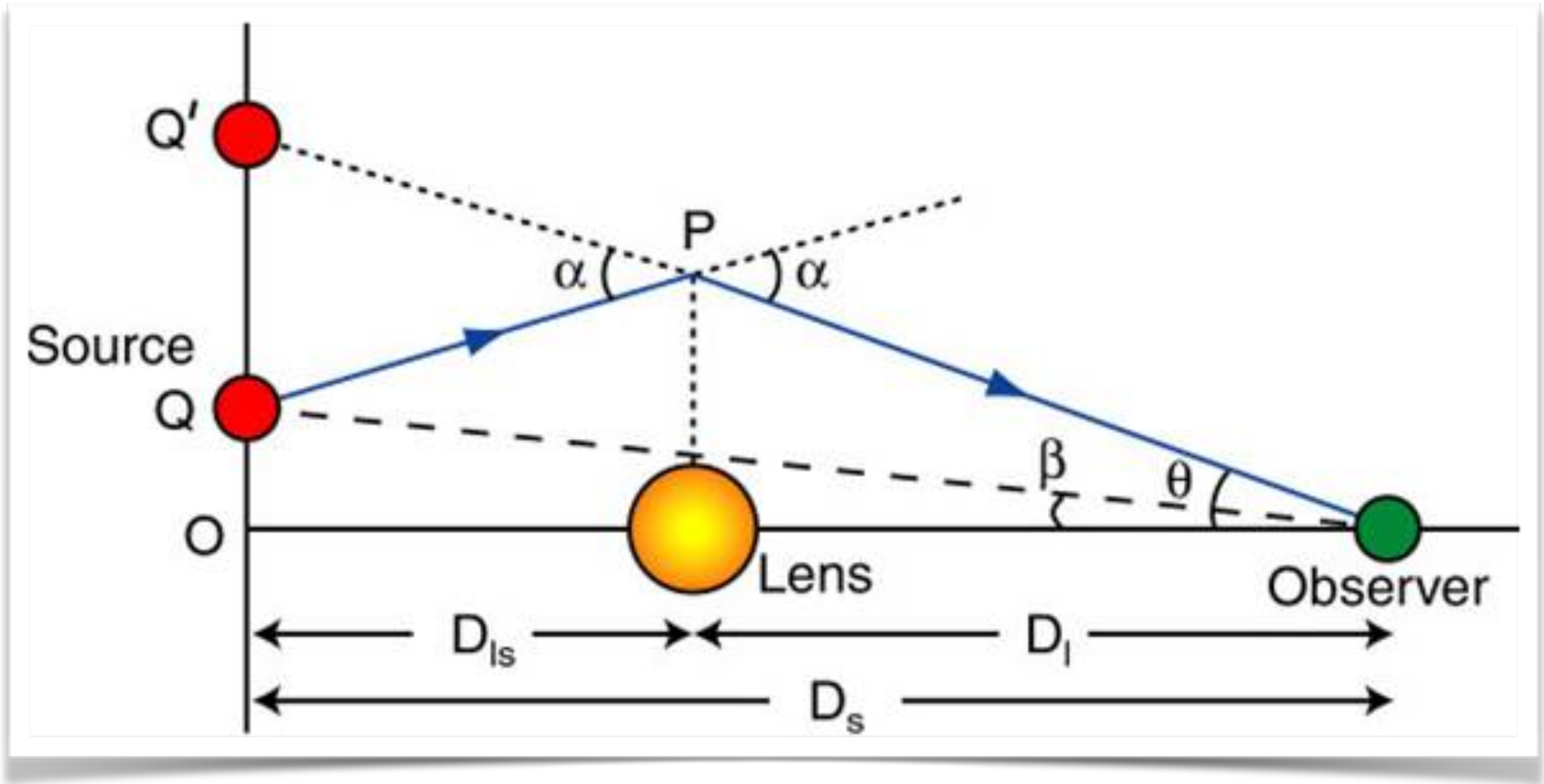
Time Delay (Geometric Optics)



University of Manchester

Geometric Time Delay

Time Delay: $$\Delta t = \frac{1}{c}\frac{D_L D_S}{D_{LS}}(1+z_L)\left(\frac{|\vec{\theta}-\vec{\beta}|^2}{2} - \psi(\vec{\theta})\right)$$

Lensing Potential

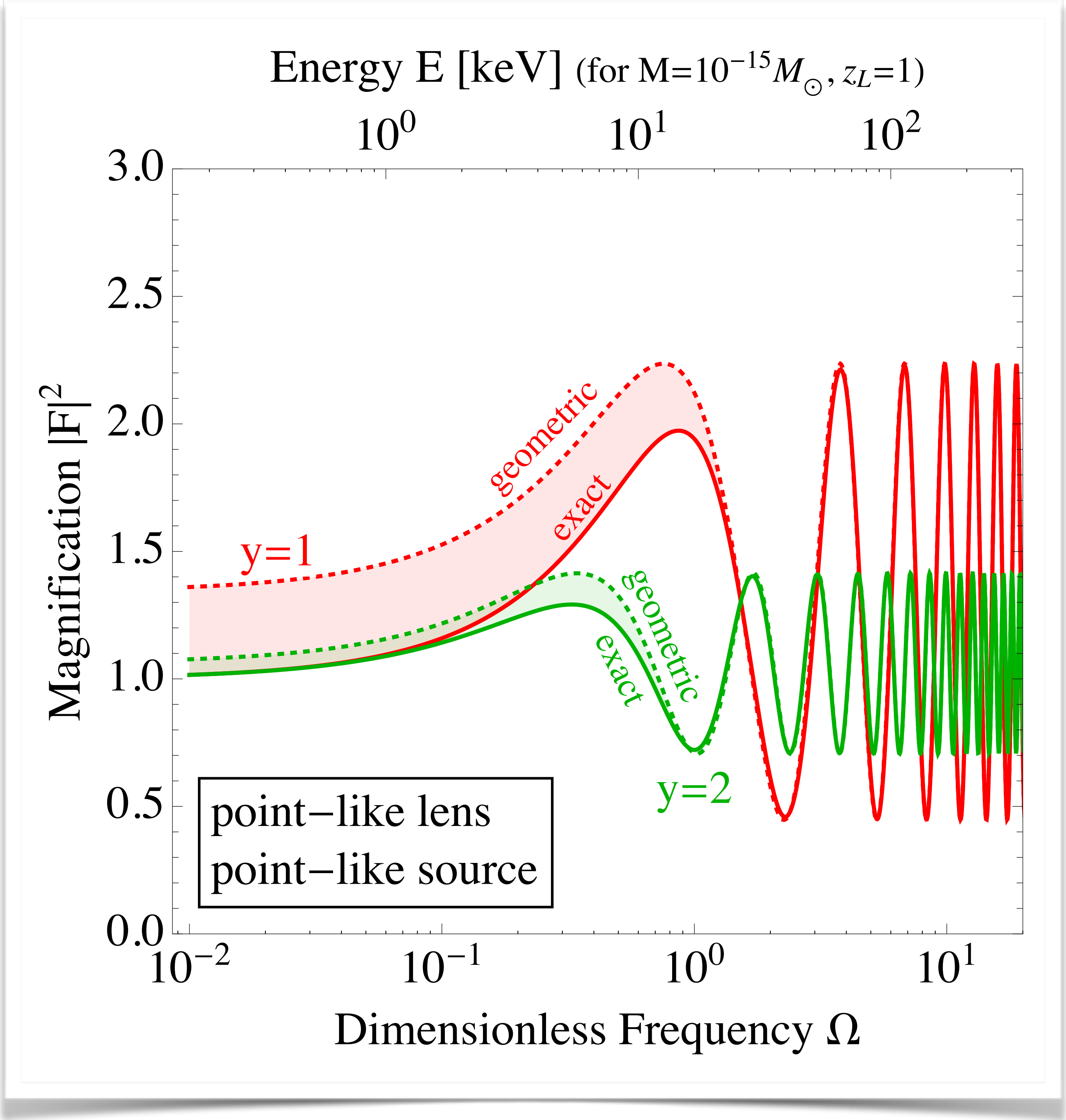
for point-like lens: $\psi(\theta) = \theta_E^2 \log\theta$

Time Delay (Geometric Optics)

$$\Delta t = \frac{1}{c}\frac{D_L D_S}{D_{LS}}(1+z_L)\left(\frac{|\vec{\theta}-\vec{\beta}|^2}{2} - \psi(\vec{\theta})\right)$$

- ☑ If $\omega\,\Delta t \lesssim 1$, expect interference between the two images
- ☑ Oscillatory features in magnification function

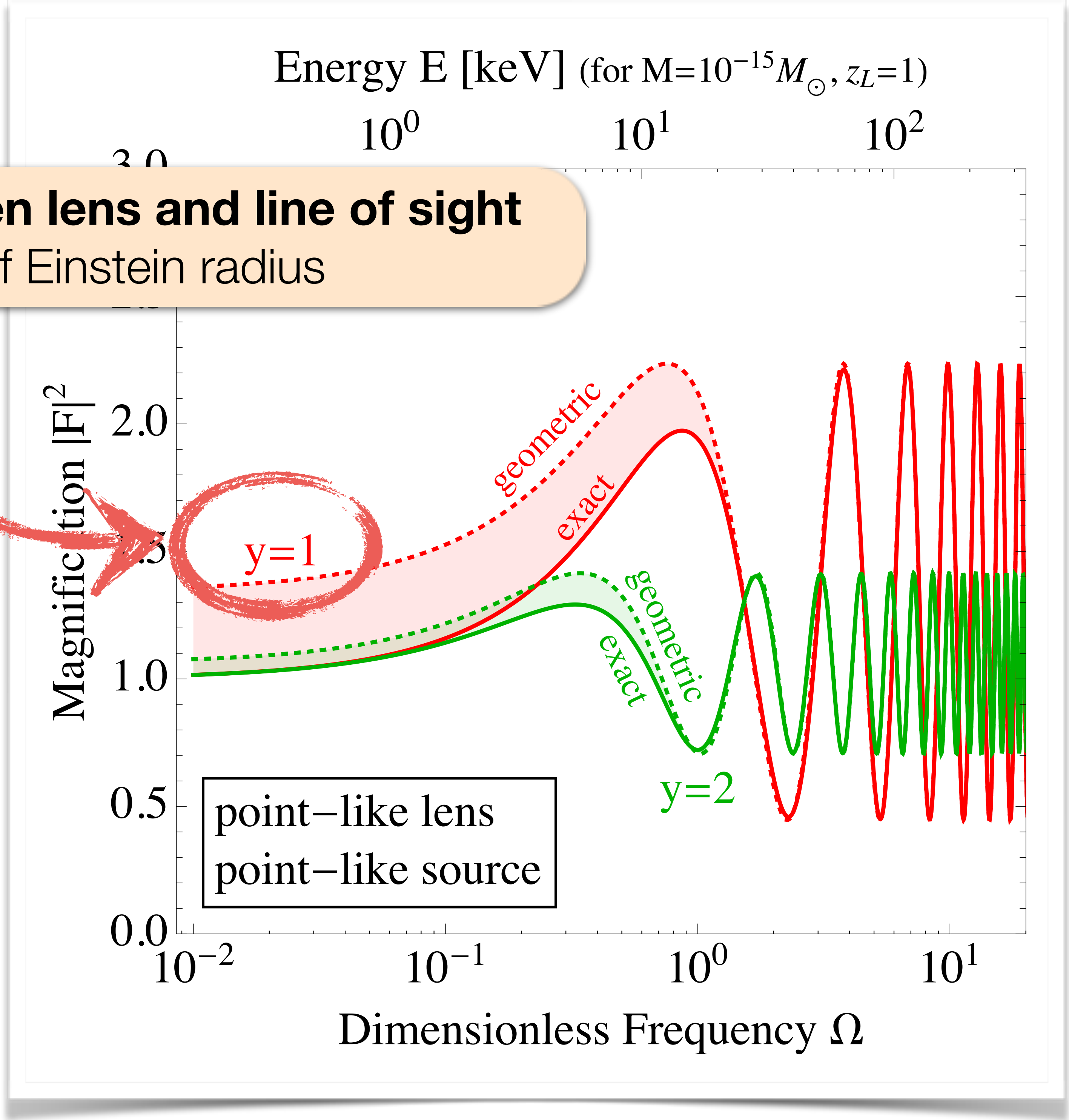
Magnification Function



Katz JK Sibiryakov Xue
arXiv:1807.11495

Magnification Function

Distance between lens and line of sight
in units of Einstein radius



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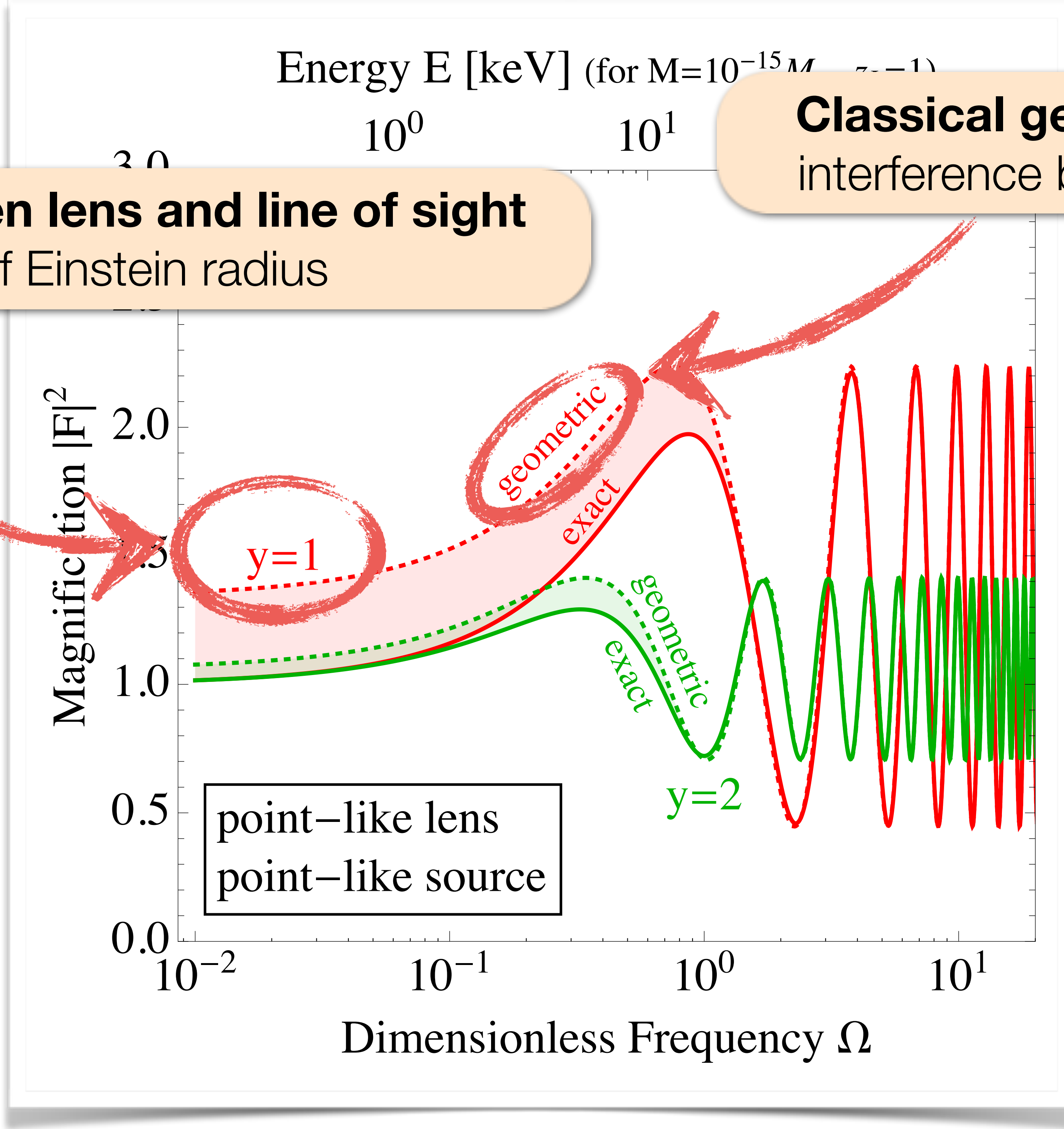
Magnification Function

Distance between lens and line of sight
in units of Einstein radius

Classical geometric picture
interference between two rays



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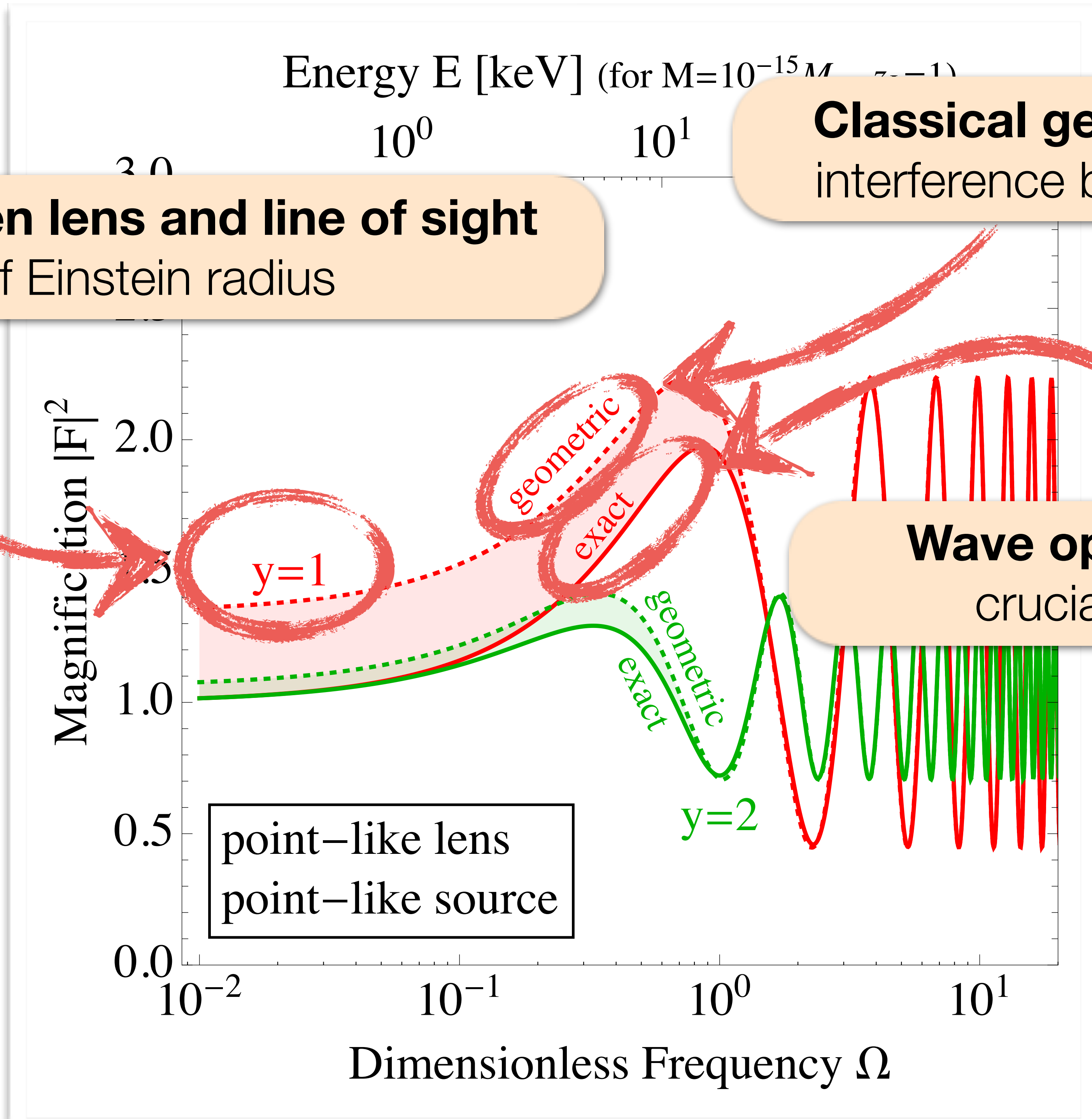
Magnification Function



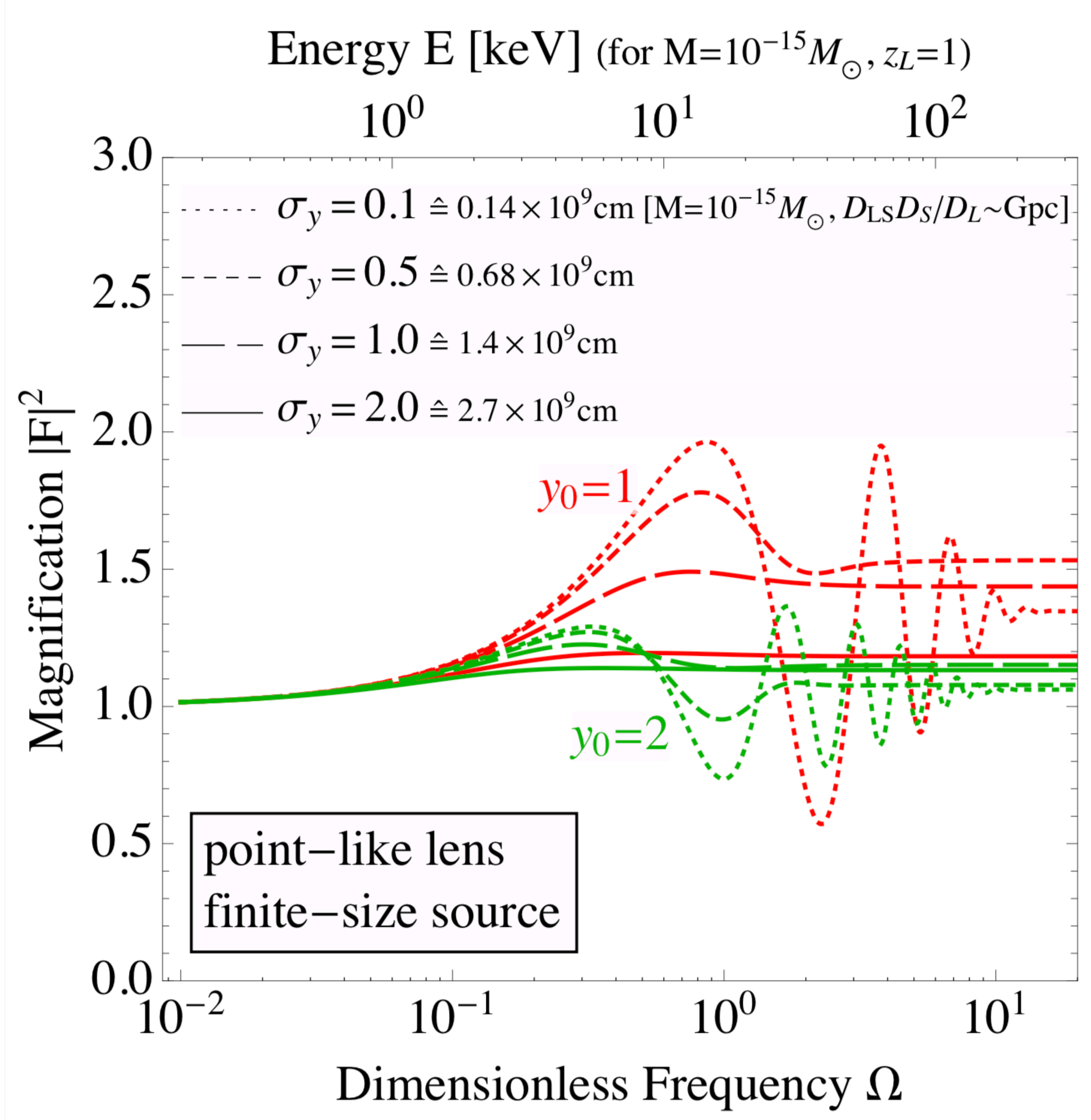
Distance between lens and line of sight
in units of Einstein radius

Classical geometric picture
interference between two rays

Wave optics corrections
crucial if $\lambda \gtrsim G_N M_{lens}$

Katz JK Sibiryakov Xue
arXiv:1807.11495

Including Finite Source Size



Katz JK Sibiryakov Xue
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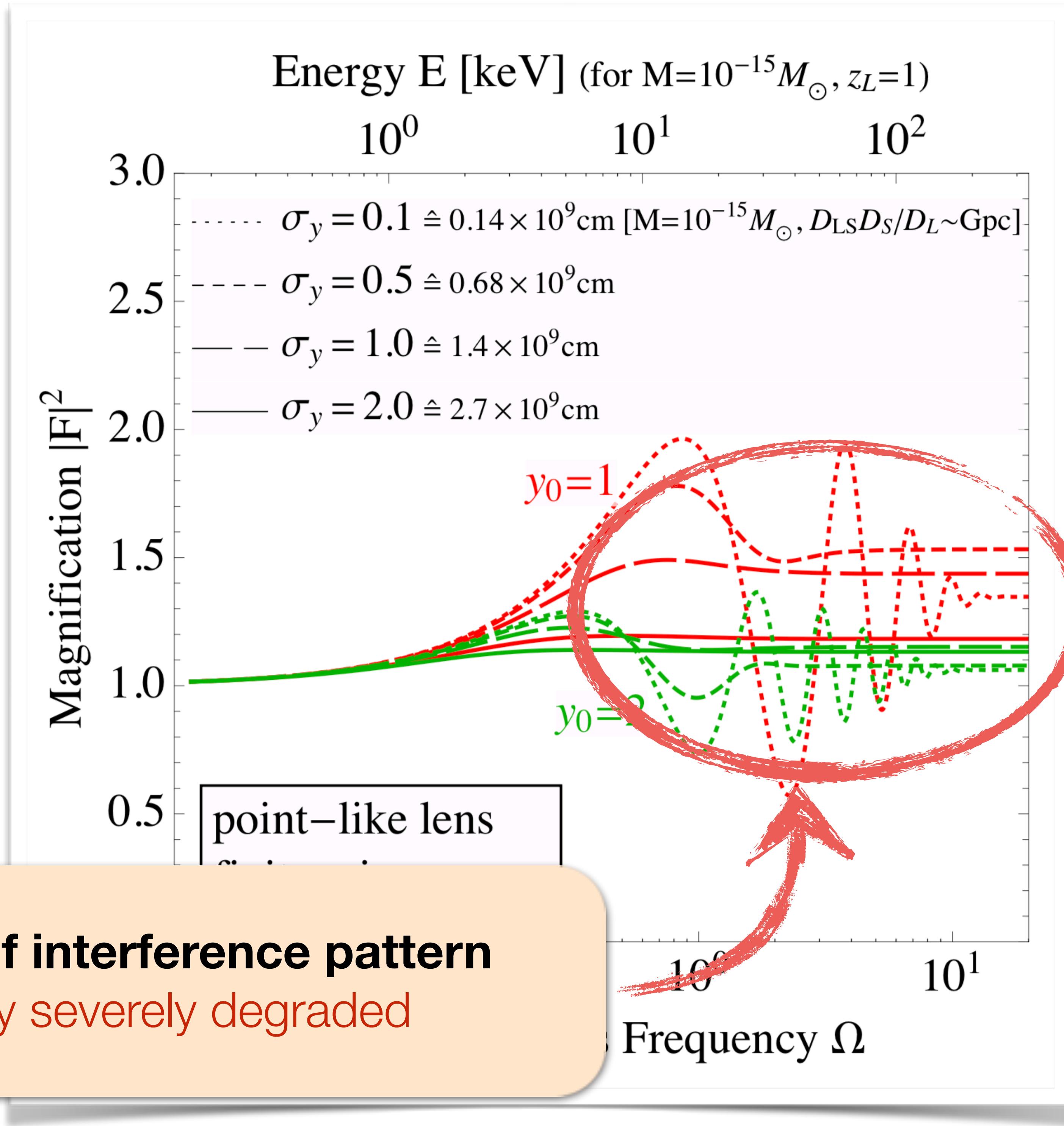
Including Finite Source Size



Katz
arXiv

Wash-out of interference pattern
sensitivity severely degraded

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Requires Source Properties

- How to realize $\omega\,\Delta t \lesssim 1$?

$$\Delta t = \frac{D_L D_S}{c\,D_{LS}}\left[\frac{(\theta-\beta)^2}{2} - \frac{4G_N M D_{LS}}{c^2 D_L D_S}\log\theta\right]$$

$$\sim \frac{4G_N M}{c^2} = 2\times 10^{-5}\,\mathrm{sec}\left(\frac{M}{M_\odot}\right)$$

or, equivalently

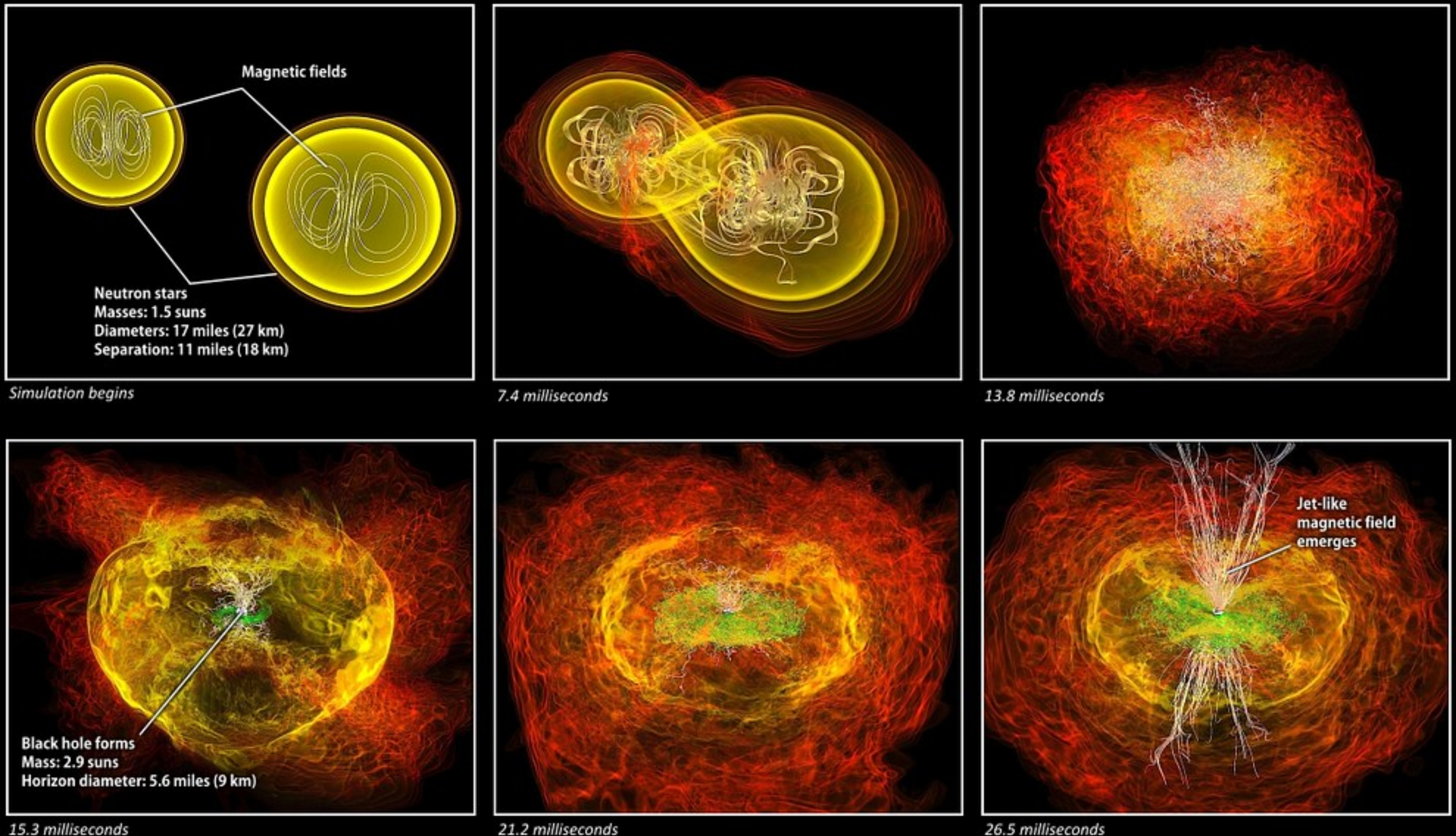
$$\frac{1}{\Delta t} \sim 0.3\,\mathrm{MeV}\left(\frac{10^{-16}M_\odot}{M}\right)$$

- Satisfied for instance for *gamma rays*

Possible Source: Gamma Ray Bursts (GRBs)

- Brightest electromagnetic events in the Universe
  - Can be observed far, far away (~ Gpc, z ~ few)
  - large probability of finding a lens in between
- Duration: ~100 ms to tens of seconds
- Proposed mechanisms
  - Supernova explosion of massive star (long GRB, duration ≳ 2 sec)
  - Binary neutron star merger (short GRB, duration ≲ 2 sec)

Crashing neutron stars can make gamma-ray burst jets

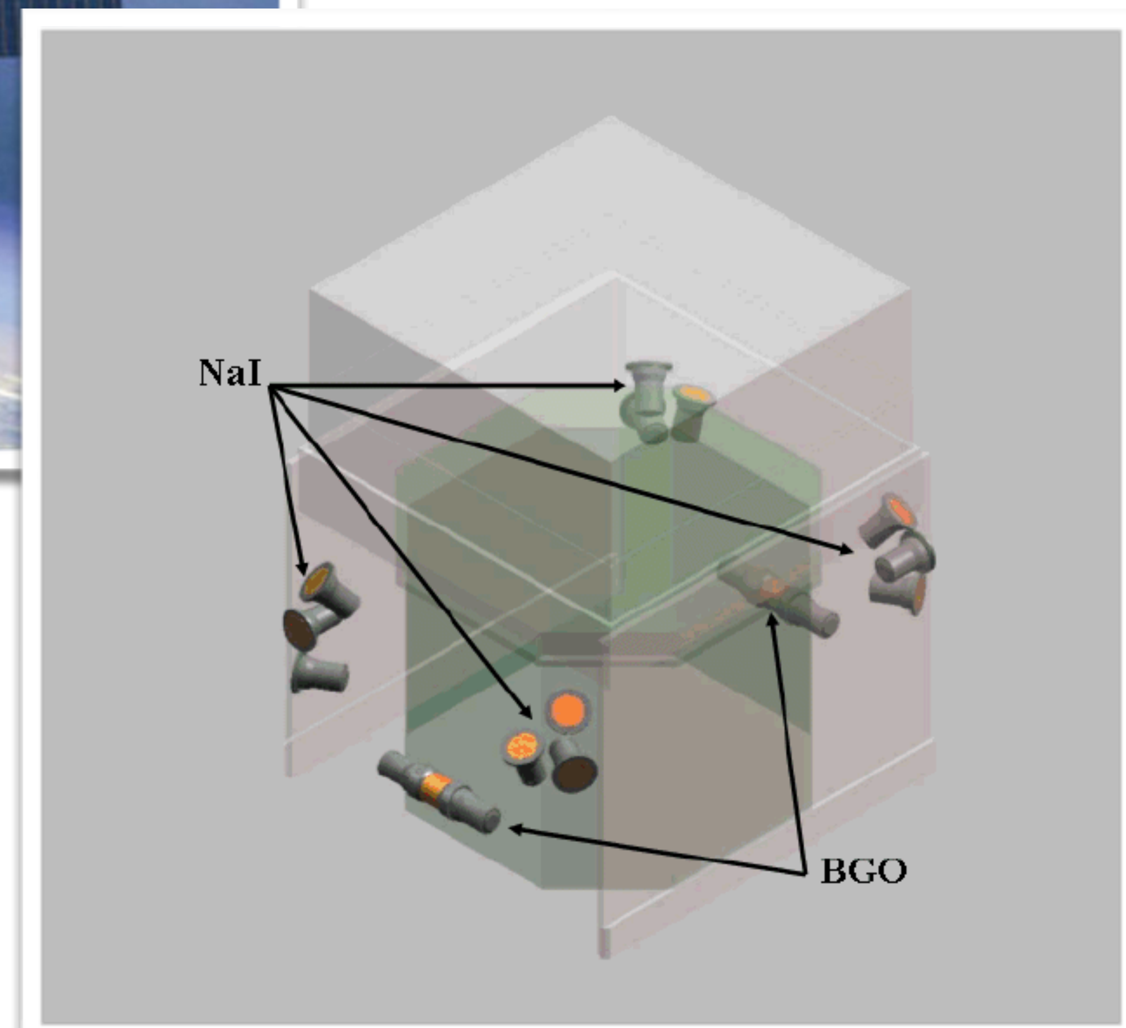


Credit: NASA/AEI/ZIB/M. Koppitz and L. Rezzolla

GRB Observations

Fermi Satellite

Fermi Gamma Ray Burst Monitor



GRB Observations

GBM Specifications & Performance

Quantity	GBM (Minimum Spec.)
Energy Range	< 10 keV - > 25 MeV
Field of View	all sky not occulted by the Earth
Energy Resolution [1]	< 10%
Deadtime per Event	< 15 μs
Burst Sensitivity [2]	< 0.5 cm^{-2} s^{-1}
Alert GRB Location [3]	~ 15°
Final GRB Location [4]	~ 3°

[1] 1-σ, 0.1 - 1 MeV

[2] 50 - 300 keV

[3] Calculated on-board; 1 second burst of 10 photons cm^{-2} s^{-1}, 50 - 300 keV

[4] Final ground computed locations; 1 second burst of 10 photons cm^{-2} s^{-1}, 50 - 300 keV

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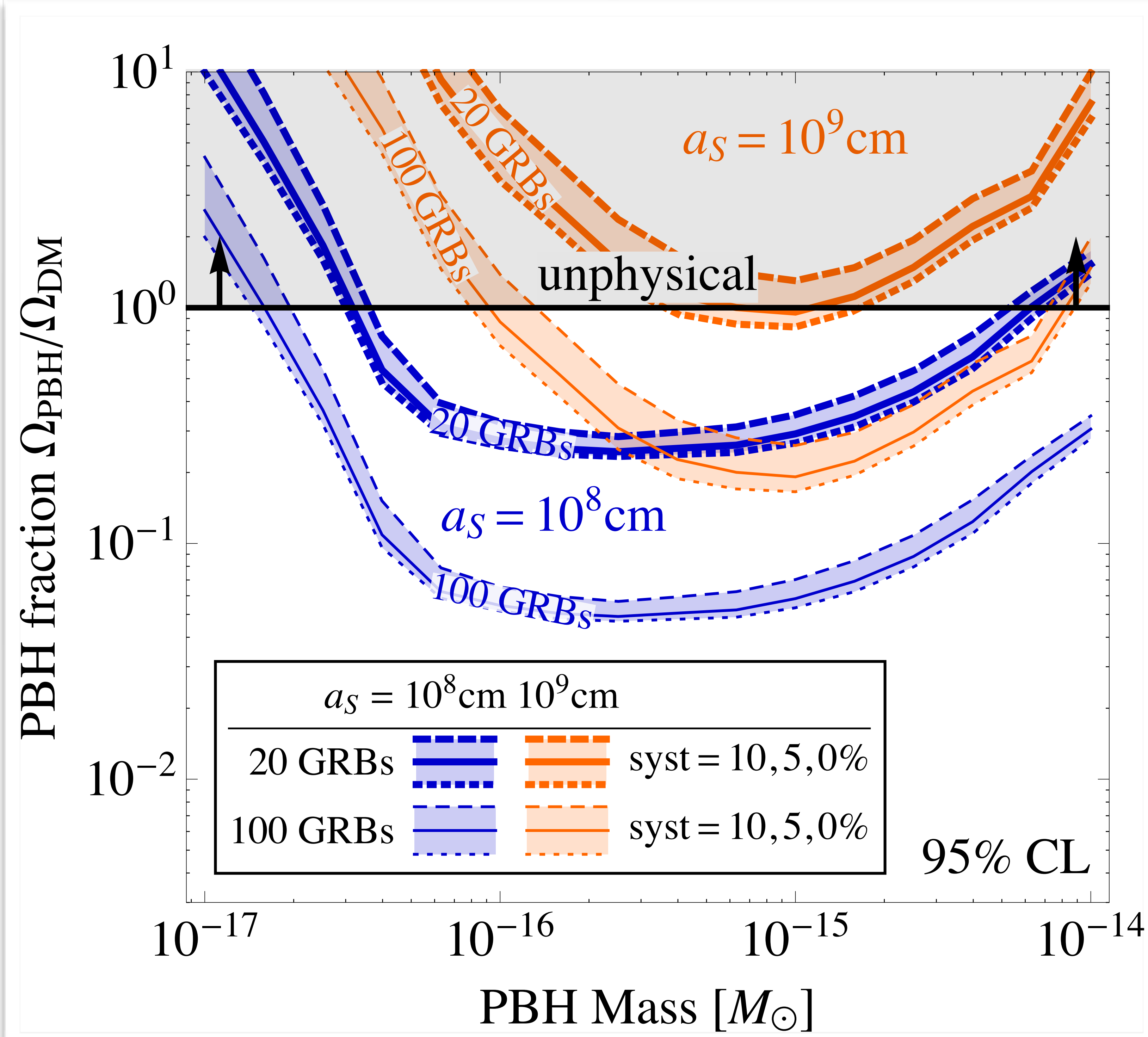
GRB Caveats

- To constrain the PBH density using (non-)observation of femtolensing, we need to know the distance to the GRB
  - Requires optical counterpart
  - Only ~20 GRBs with known distance so far
- Wave optics effects
- Finite size of GRB source

Sensitivity Estimates



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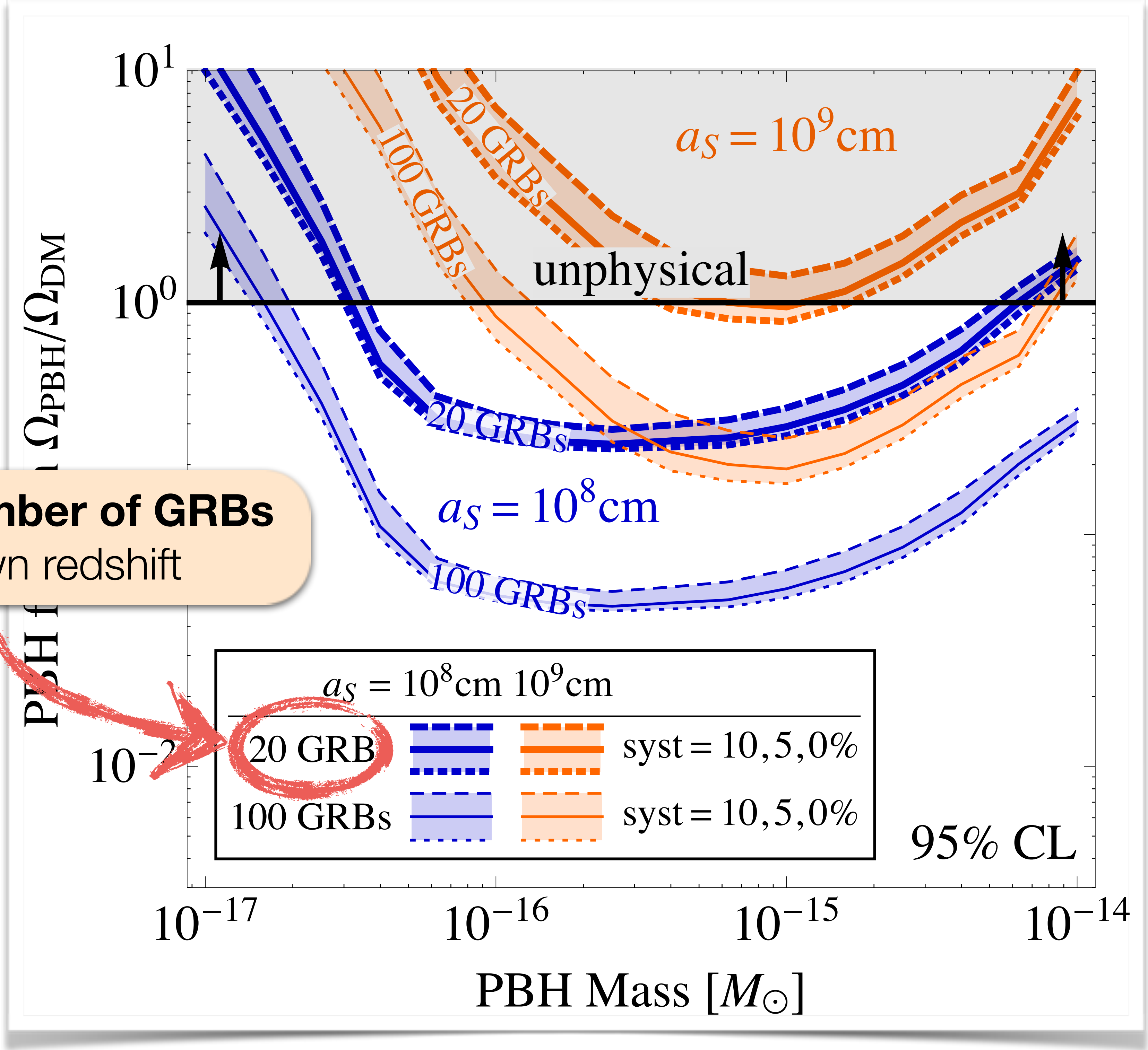
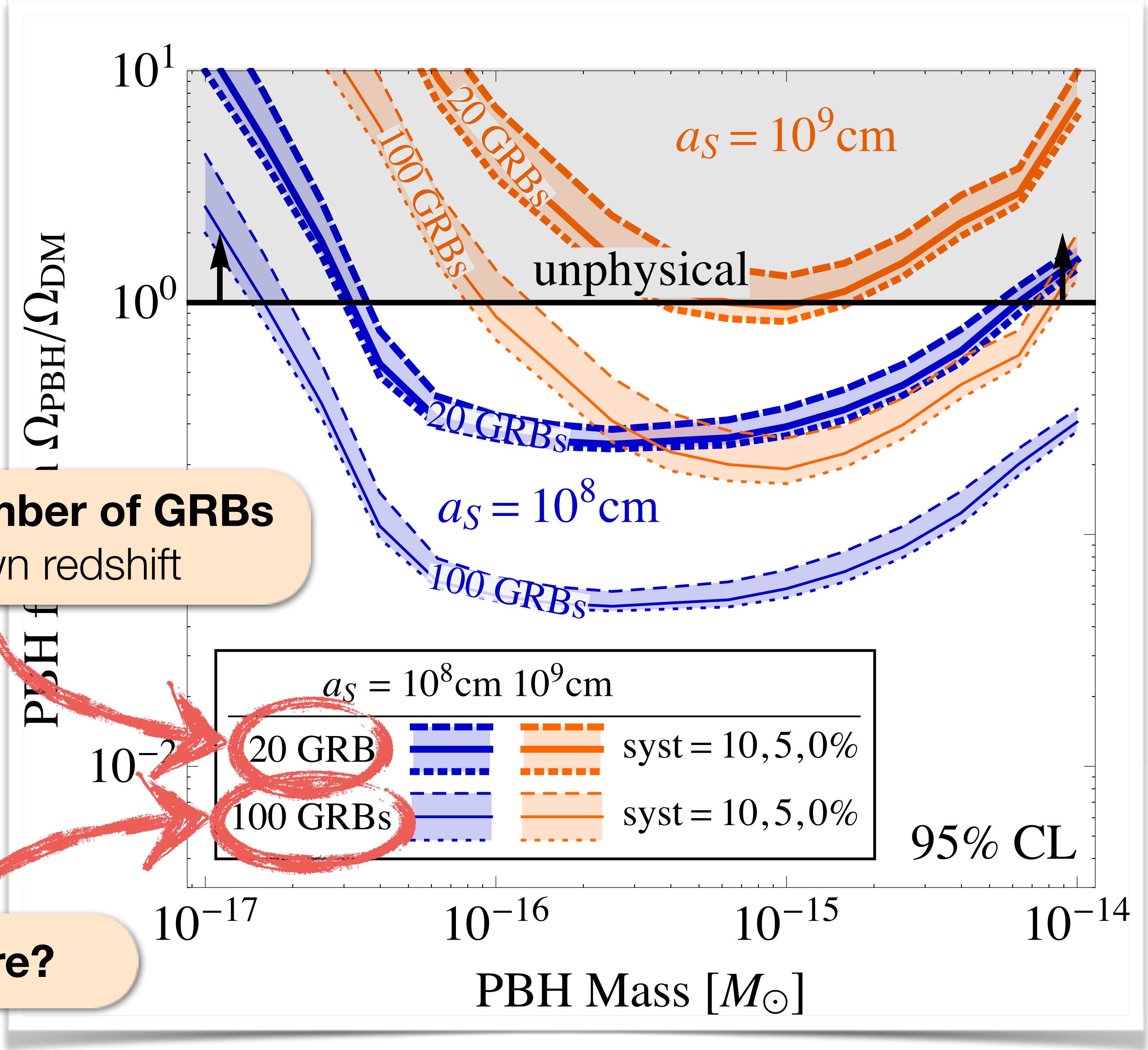
Sensitivity Estimates



~ current number of GRBs
with known redshift

Katz JK Sibiryakov Xue
arXiv:1807.11495

Sensitivity Estimates



~ current number of GRBs
with known redshift

Future?

Katz JK Sibir...
arXiv:1807.11495

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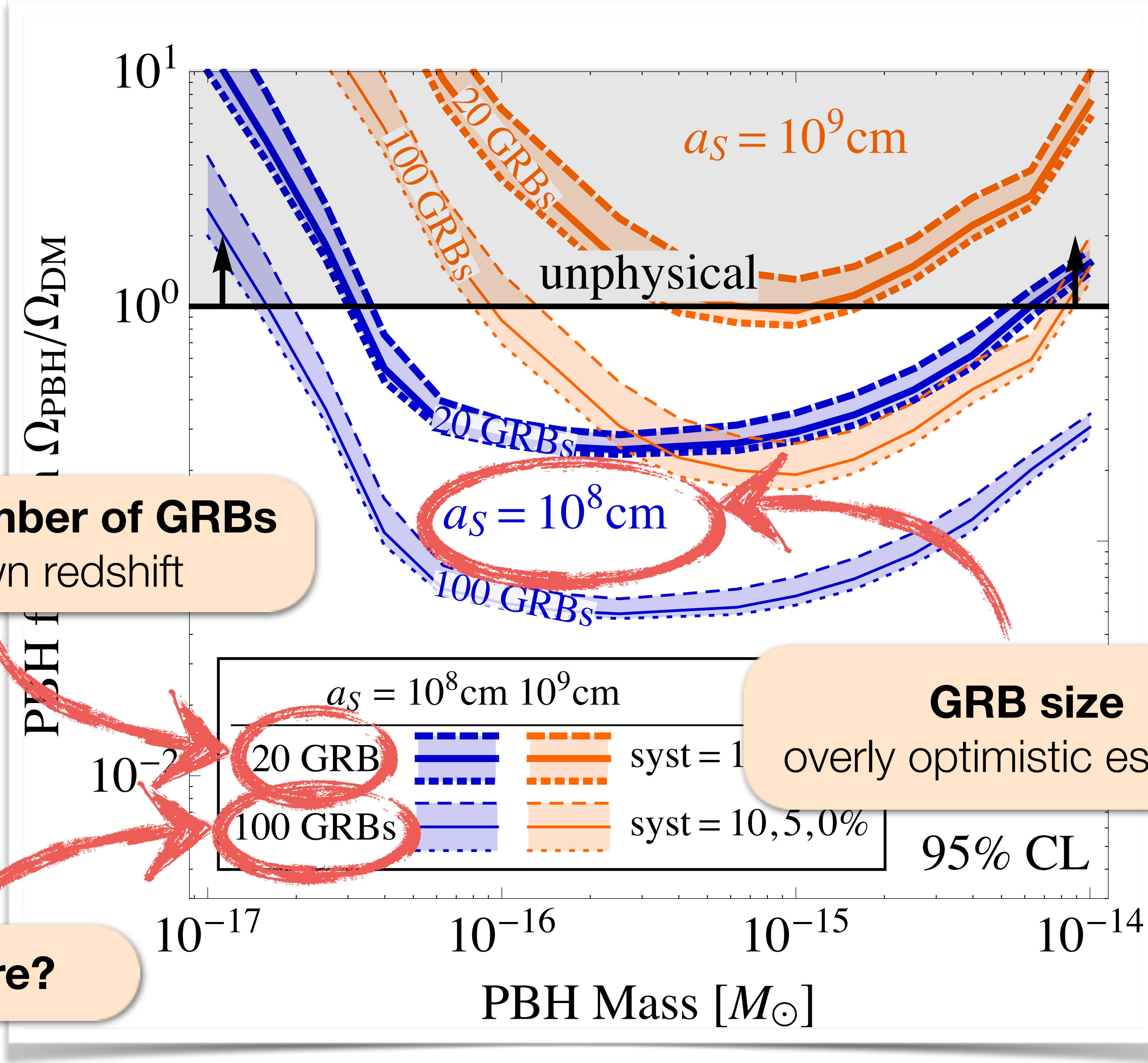
Sensitivity Estimates



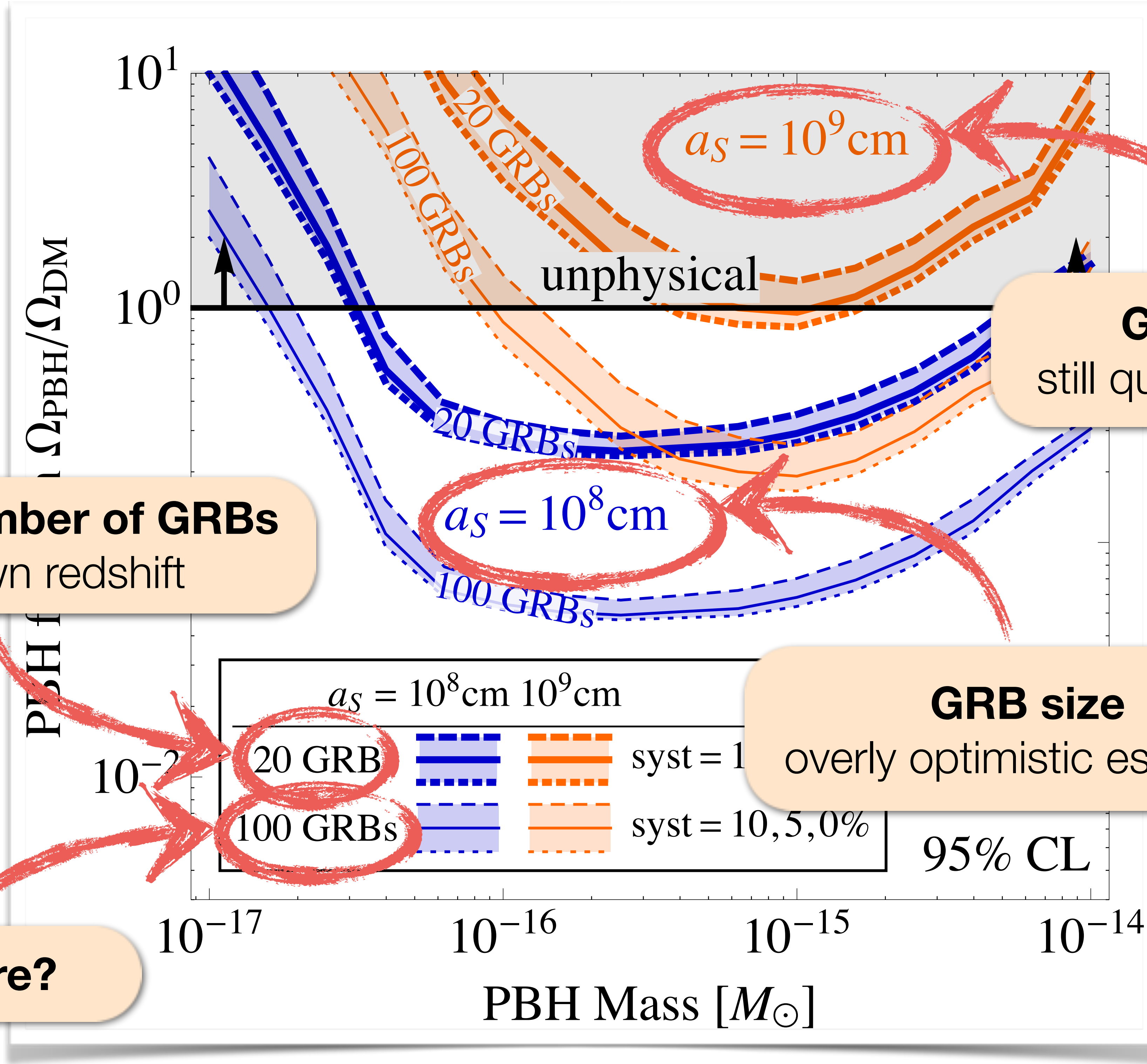
~ current number of GRBs
with known redshift

GRB size
overly optimistic estimate

Future?

Katz JK Sibir...
arXiv:1807.11495

Sensitivity Estimates



Katz JK Sibir...
arXiv:1807.11495

Finite Size of GRB Sources

- γ production in GRBs: Katz JK Sibiryakov Xue, arXiv:1807.11495
  - e^+, e^- acceleration in relativistic shock waves
- Variability time scale in rest frame for source size a_S:

$$t_{\rm var} \sim a_S/c$$

- Relativistic boost γ:

$$t_{\rm var} \sim (1+z_S)\left(1 - \frac{v}{c}\cos\theta_{\rm obs}\right)\gamma\, a_S/c$$

- Observation angle $\theta_{\rm obs} \sim 1/\gamma$
- Observed $t_{\rm var} \gtrsim 0.01$ sec (short GRB); $\gtrsim 0.1$ sec (long GRB)

$$a_S \simeq \frac{10^{11}\,{\rm cm}}{1+z_S} \times \left(\frac{t_{\rm var}}{0.03\,{\rm sec}}\right)\left(\frac{\gamma}{100}\right)$$

Finite Size of GRB Sources: Caveats

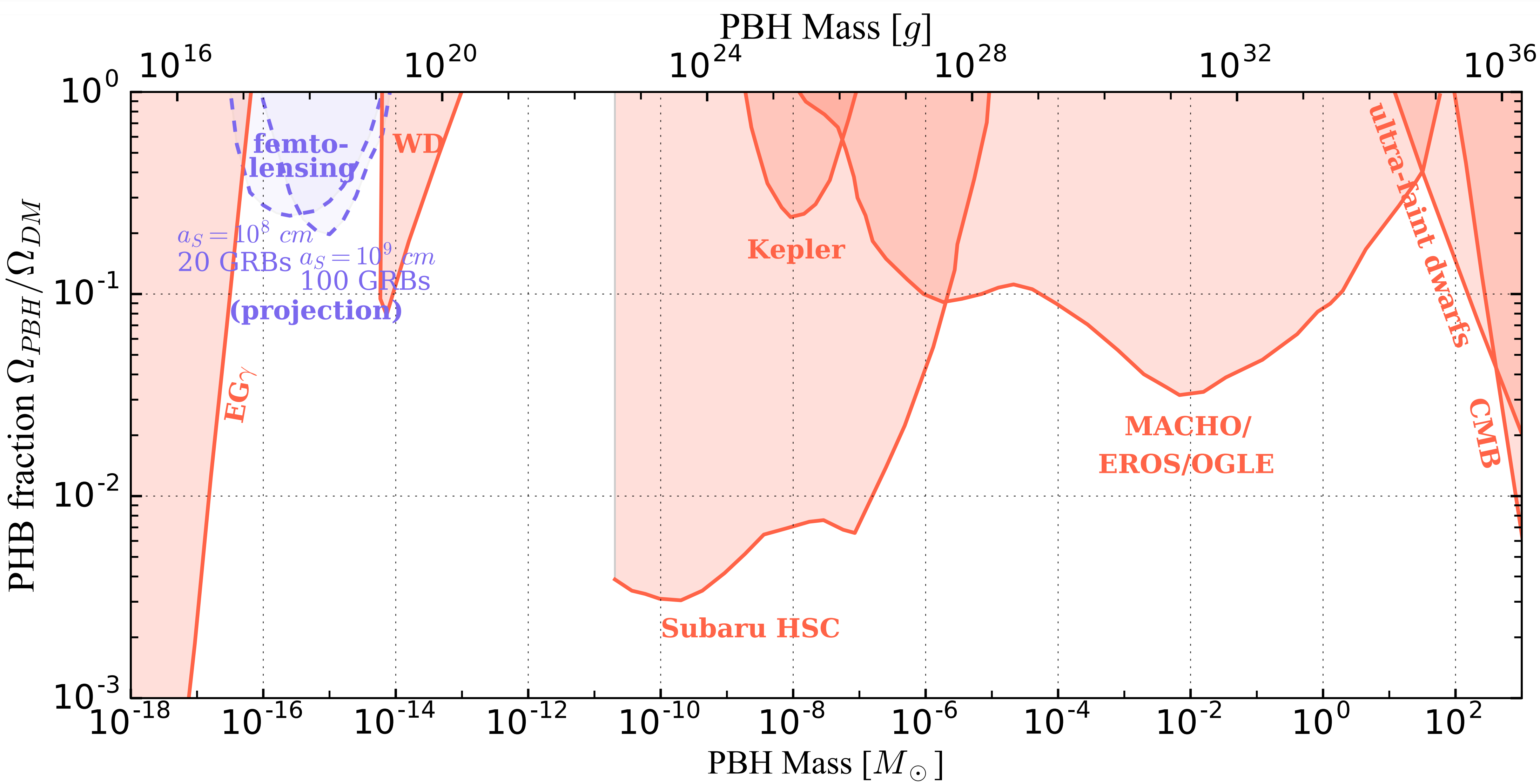
- Some GRBs with shorter variability time scale $t_{var} \lesssim 10^{-3}$ sec
  - t_{var} distribution could have a long tail → use tail for femtolensing
- Intrinsic variability might be too fast to be resolved
- Conservative estimate: require optical depth $\tau < 1$:

$$a_S > 1.8 \times 10^9 \left(\frac{d_S}{7\text{Gpc}}\right)^2 \left(\frac{f_{500}}{10^{-3}\text{sec}^{-1}\text{cm}^{-2}\,\text{keV}^{-1}}\right) \left(\frac{\gamma}{1000}\right)^{-4} \text{cm}\,.$$

- Assumptions:
  - Power law spectrum with $\alpha = -2$
  - Thomson scattering (non-relativistic in rest frame of ejecta)
  - Target e^+, e^- from pair production by γ rays
  - ...

Katz JK Sibiryakov Xue, arXiv:1807.11495

PBH Parameter Space



Katz JK Sibiryakov Xue
arXiv:1807.11495

Assuming δ-like PBH mass distribution

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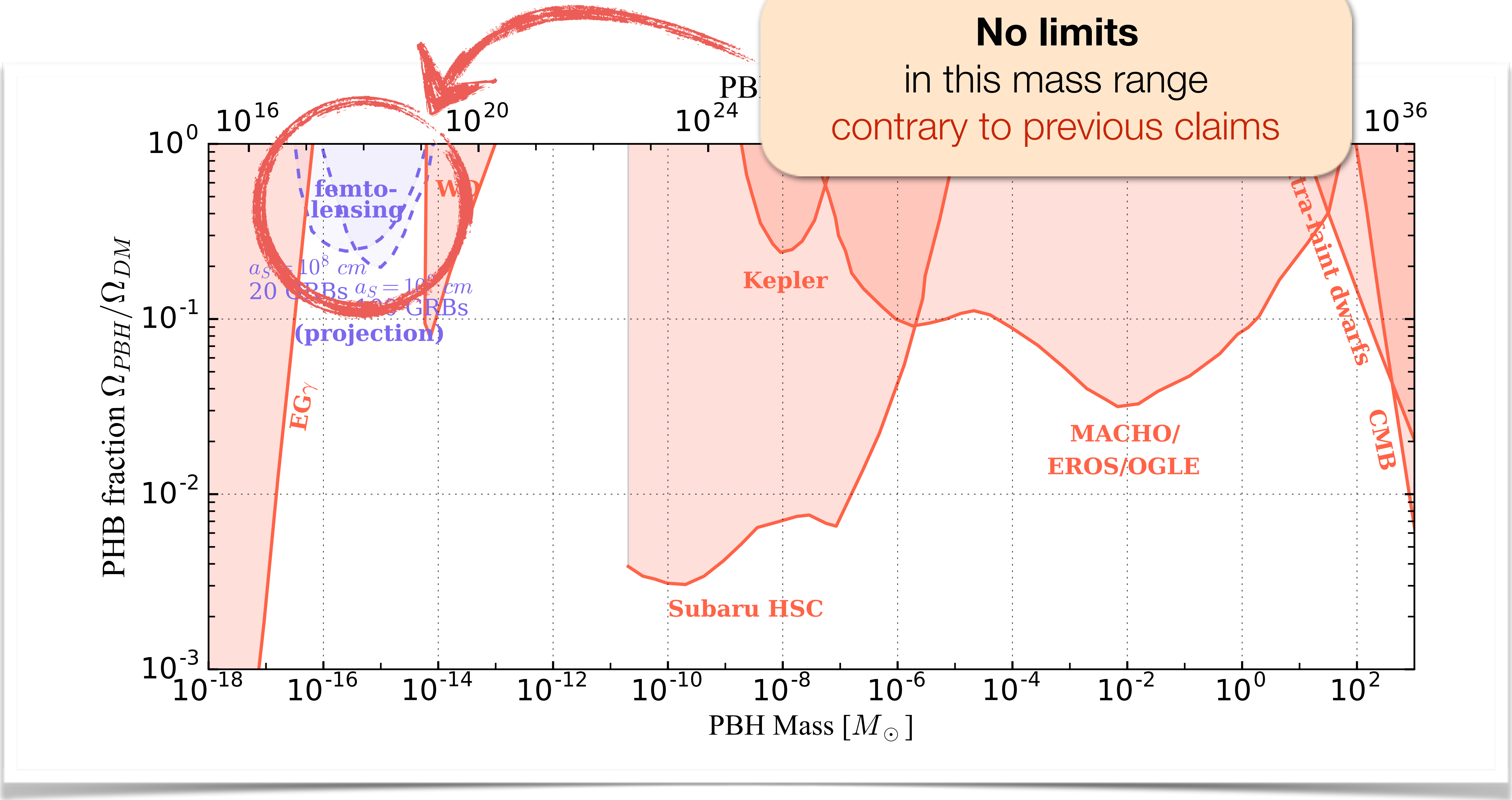
PBH Parameter Space

No limits
in this mass range
contrary to previous claims

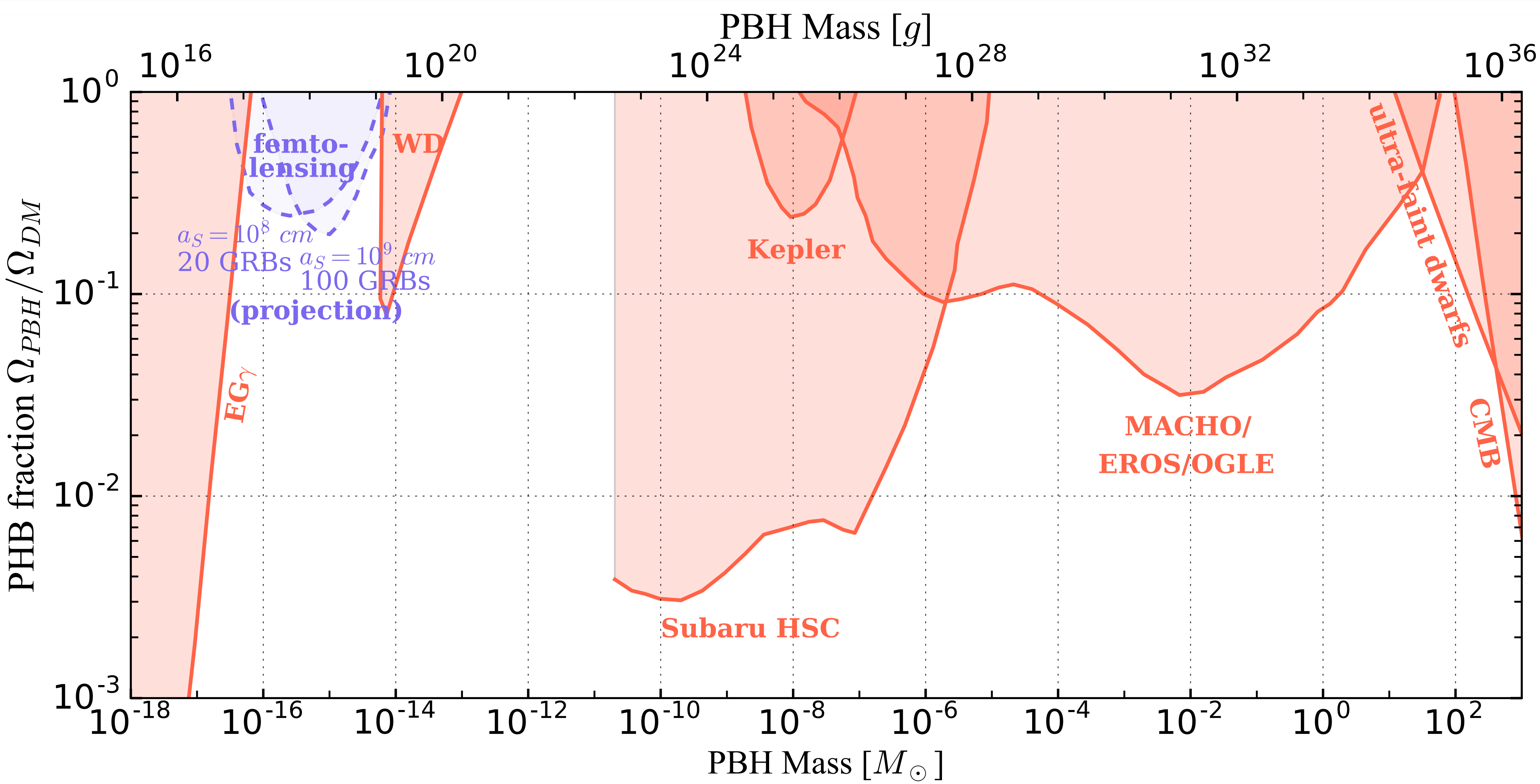


Katz JK Sibiryakov Xue
arXiv:1807.11495

Assuming δ-like PBH mass distribution

Other PBH Limits

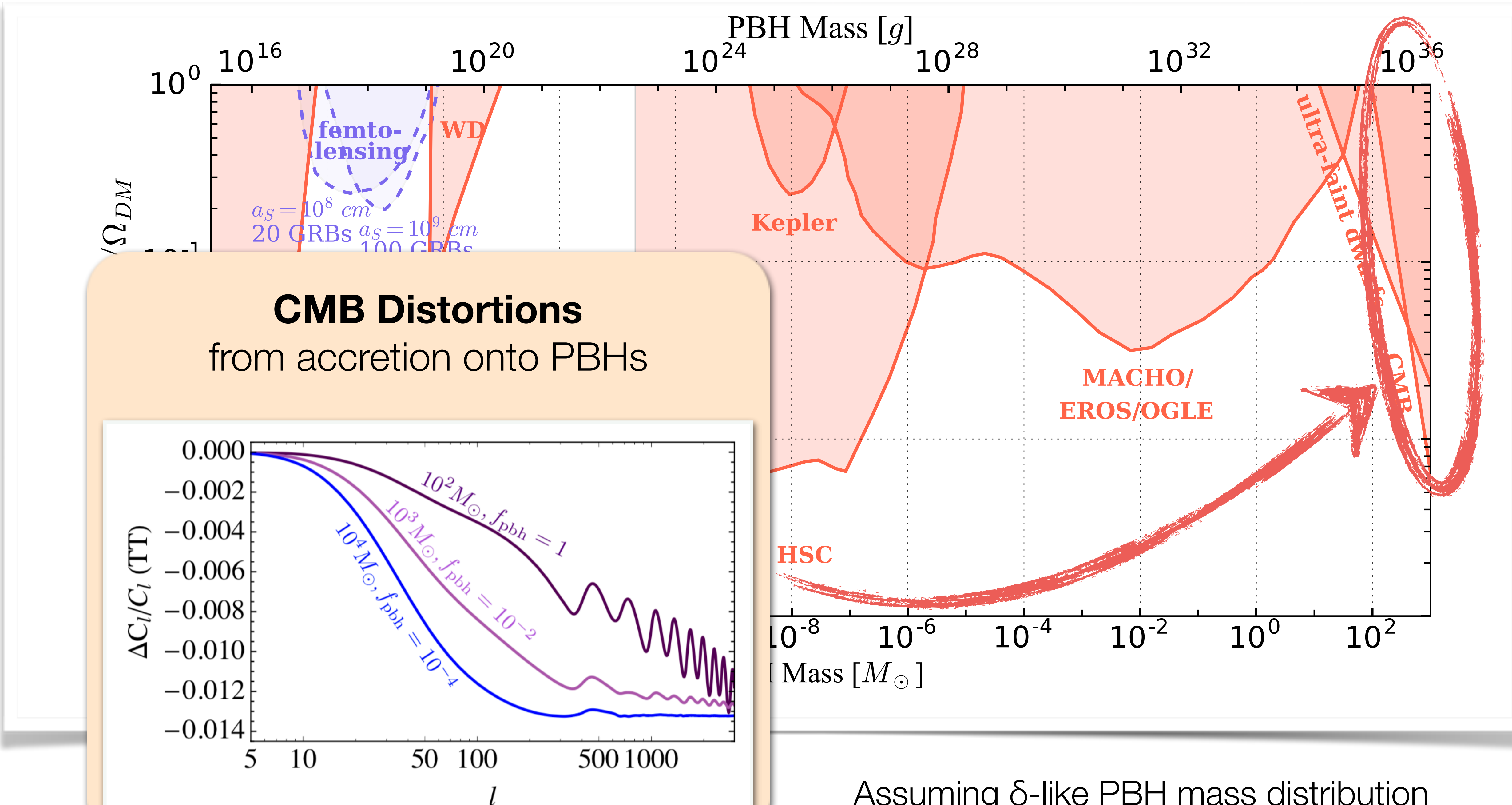
PBH Parameter Space



Assuming δ-like PBH mass distribution

PBH Parameter Space

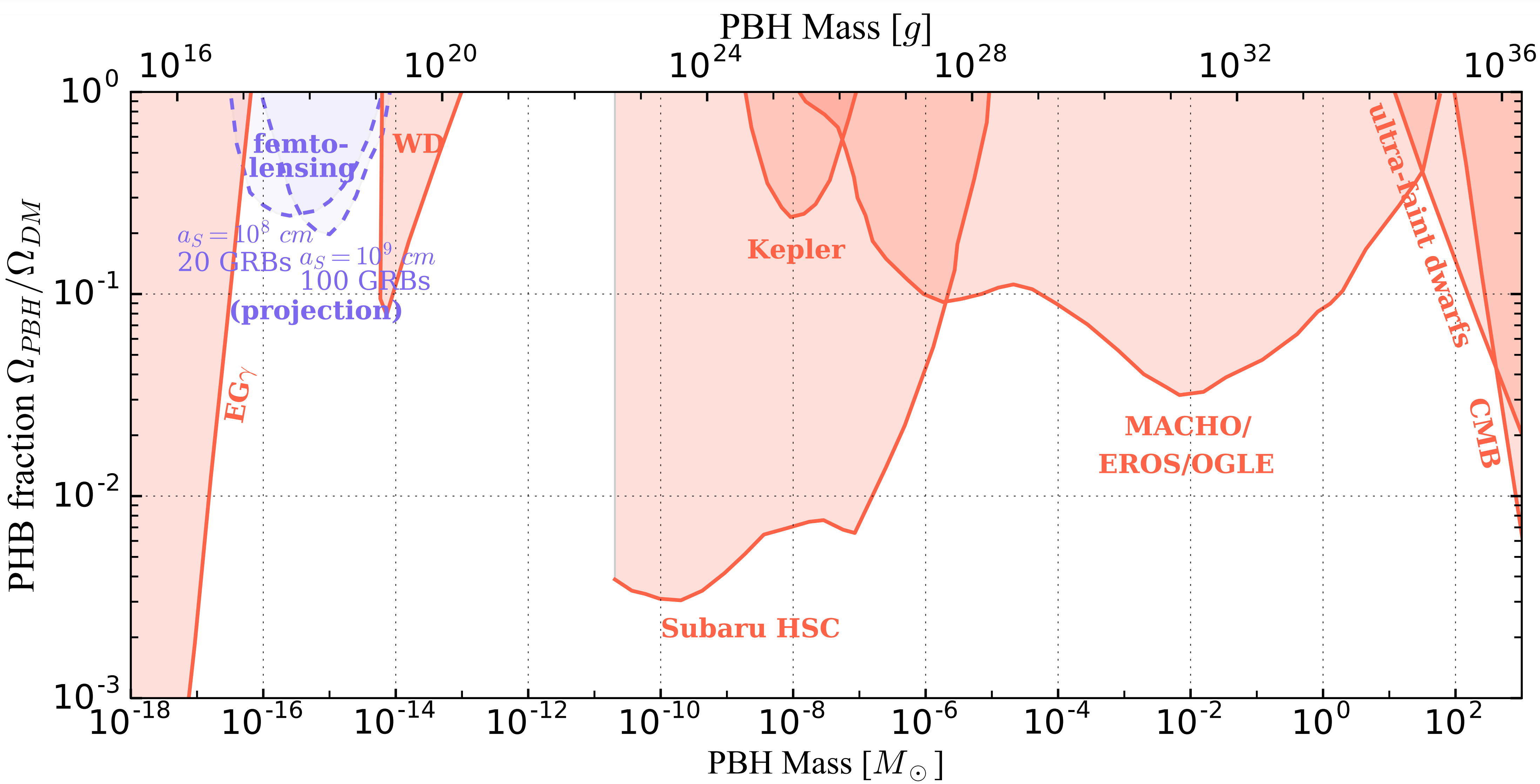
Ali-Haïmoud Kamionkowski arXiv:1612.05644



CMB Distortions
from accretion onto PBHs

Assuming δ-like PBH mass distribution

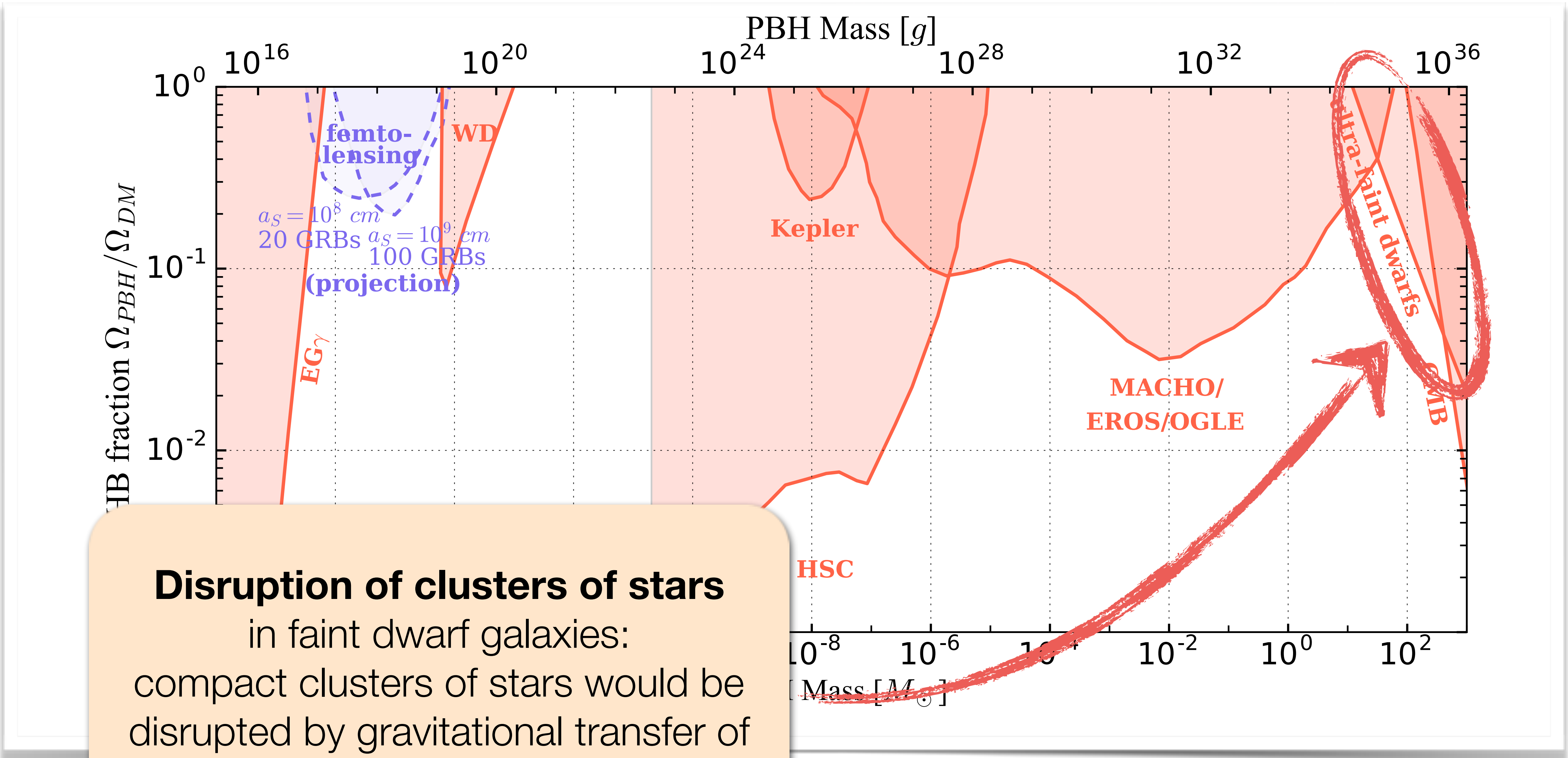
PBH Parameter Space



Assuming δ-like PBH mass distribution

PBH Parameter Space

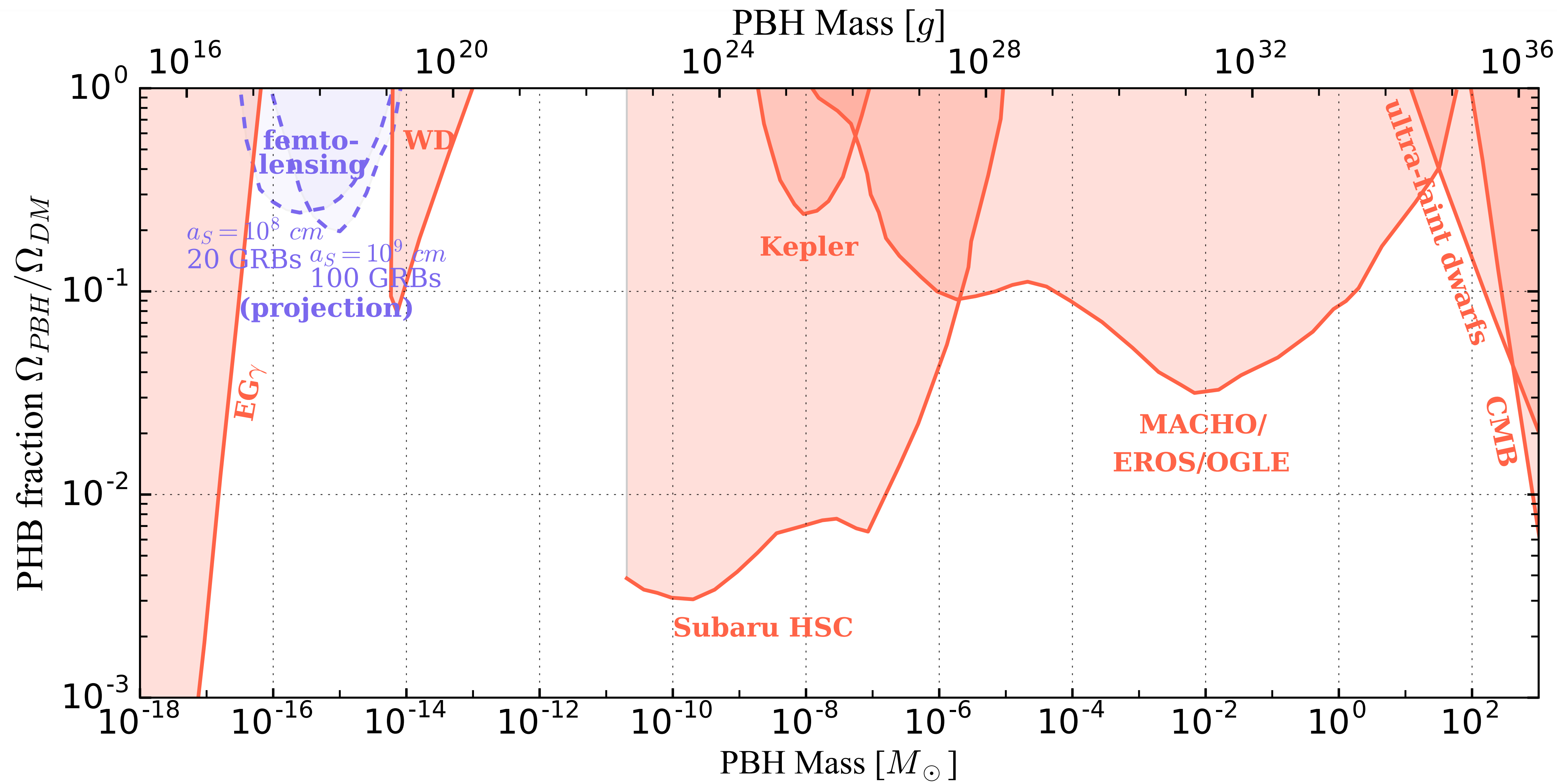
Brandt arXiv:1605.03665



Disruption of clusters of stars in faint dwarf galaxies: compact clusters of stars would be disrupted by gravitational transfer of kinetic energy from massive PBHs.

Assuming δ-like PBH mass distribution

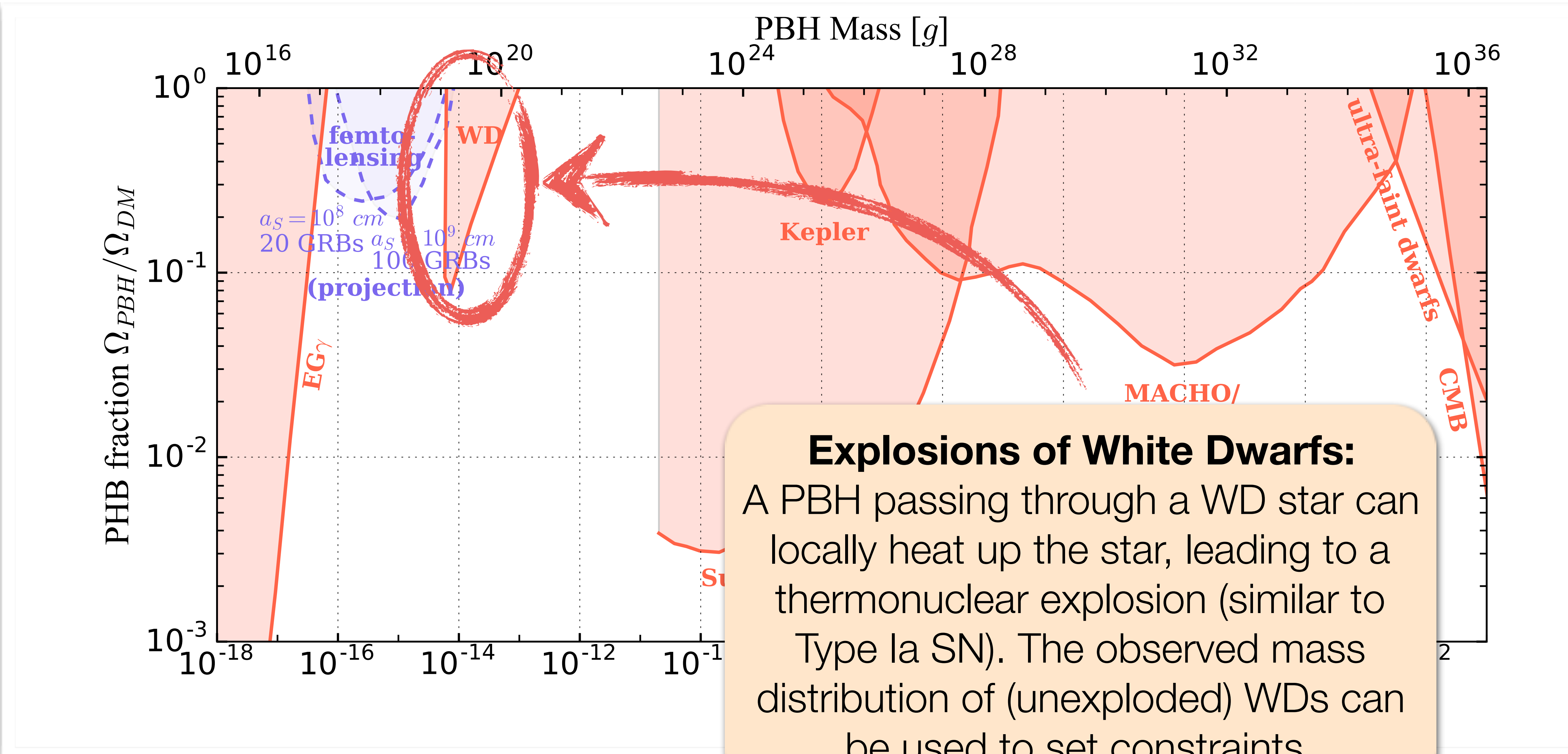
PBH Parameter Space



Assuming δ-like PBH mass distribution

PBH Parameter Space

Graham Rajendran Veral arXiv:1505.04444



Explosions of White Dwarfs:
A PBH passing through a WD star can locally heat up the star, leading to a thermonuclear explosion (similar to Type Ia SN). The observed mass distribution of (unexploded) WDs can be used to set constraints.

Assuming δ-like PBH mass distribution

Summary

Summary

- Electroweak Scale Dark Matter
  - The field has moved from UV-complete models to simplified models and EFT.
- Dark Photons
  - generically appears in low-scale ($\lesssim$ GeV) DM models
  - potpourri of constraints
- Primordial Black Holes
  - interesting DM candidate that doesn't require new particles
  - interesting astrophysical constraints
  - but lots of open parameter space

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Thank You !