modern cosmology

cosmic microwave background and gravitational lensing

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outline

1. gravitational light deflection
2. cosmic shear
3. lensing
4. int. Sachs-Wolfe effect
5. Rees-Sciama effect
6. summary

gravitational lensing: overview

- gravitational light deflection: test of general relativity (1919)
- strong lensing: giant luminous arcs in clusters of galaxies
- weak lensing: correlated distortion of background galaxy images
- multiply imaged quasars and time delays
- lensed light curves of bulge stars and search of MACHOs
- lensing of the microwave background (2007)

weak perturbations of the metric

- consider Minkowski-line element, weakly perturbed by static gravitational potential Φ

$$(ds)^2 = \left(1 + \frac{2}{c^2}\Phi\right)c^2dt^2 - \left(1 - \frac{2}{c^2}\Phi\right)d\vec{x}^2 \qquad (1)$$

- on a geodesic, the line element vanishes: derive effective index of refraction n

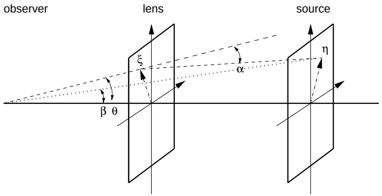
$$\frac{d|\vec{x}|}{dt} = c' = \frac{c}{n} \text{ with } n = 1 - \frac{2}{c^2}\Phi \qquad (2)$$

- Fermat's principle: photon minimises run time $\int \left|d\vec{x}\right| n$

$$\delta \int_{x_i}^{x_f} ds \sqrt{\frac{d\vec{x}^2}{ds^2}} n(\vec{x}(s)) = 0, \qquad (3)$$

for parameterisation $x(s)$ of trajectory with $\left|d\vec{x}/ds\right| = 1$

lens equation



- carry out the variation yields ($\nabla_\perp = \nabla - \vec{e}(\vec{e}\nabla)$):

$$\nabla n - \vec{e}(\vec{e}\nabla n) - n\frac{d\vec{e}}{ds} = 0 \rightarrow \frac{d\vec{e}}{ds} = \nabla_\perp \ln n \simeq -\frac{2}{c^2}\nabla_\perp\Phi \qquad (4)$$

- deflection $\hat{\alpha} = \vec{e}_f - \vec{e}_i = -\frac{2}{c^2}\int ds\nabla_\perp\Phi$
- read off lens equation, use deflection angle $\hat{\alpha}$:

$$\vec{\eta} = \frac{D_s}{D_l}\vec{\xi} - D_{ls}\hat{\alpha} \rightarrow \vec{\beta} = \vec{\theta} - \frac{D_{ls}}{D_s}\hat{\alpha}(\vec{\theta}) = \vec{\theta} - \vec{\alpha} \qquad (5)$$

approximations

- formally: $\hat{\alpha} = \vec{e}_f - \vec{e}_i = -\frac{2}{c^2}\int ds \nabla_\perp \Phi$
- nonlinear integral: the deflection determines the path on which one needs to carry out the integration
- **Born-approximation**: integration along a fiducial straight ray instead of actual photon geodesic
- if the travel path (of order c/H_0)) is large compared to the size of the lens, then the gravitational interaction can be taken to be instantaneous → **thin-lens approximation**
- in this case: project the surface mass density Σ

$$\Sigma(\vec{b}) = \int dz\, \rho(\vec{b}, z) \tag{6}$$

- deflection is the superposition of all surface density elements

$$\hat{\alpha}(\vec{b}) = \frac{4G}{c^2}\int d^2b'\, \Sigma(\vec{b}')\frac{\vec{b} - \vec{b}'}{|\vec{b} - \vec{b}'|^2} \tag{7}$$

lens mapping and the mapping Jacobian

- lens equation $\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$ relates true position $\vec{\theta}$ to observed position β with mapping field α
- if mapping $\alpha = \nabla_\perp\psi$ is not constant across galaxy image $\rightarrow$ distorsion of observed shape
- describe with Jacobian-matrix J

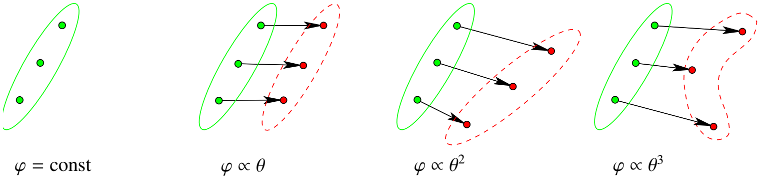
$$J = \frac{\partial\vec{\beta}}{\partial\vec{\theta}} = \left(\delta_{ij} - \frac{\partial^2\psi(\vec{\theta})}{\partial\theta_i\partial\theta_j}\right) \tag{8}$$

- decompose $A = \mathrm{id} - J$ in terms of Pauli-matrices:

$$A = \sum_a a_a\sigma_a = \kappa\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \gamma_+\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \gamma_\times\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{9}$$

- coefficients: κ (convergence), γ_+ and $\gamma_\times$ (shear)
- combine shear coefficients to complex shear $\gamma = \gamma_+ + i\gamma_\times$ (spin 2)

image distortions



- deflection not observable, actual position of a galaxy is unknown
- with assumptions on galaxy ellipticity, the shearing is observable
- bending of an image (flexion) is a new lensing method

question

why is there no rotation of a galaxy image in lensing?

mass reconstructions

- convergence $\propto$ local surface mass density Σ of a lens
- but: it is **not directly observable** $\rightarrow$ is it possible to infer κ and the mass map from the observation of gravitational shear?
- write down derivative relations in Fourier space

$$\kappa = -\frac{1}{2}(k_x^2 + k_y^2)\psi \quad \gamma_+ = -\frac{1}{2}(k_x^2 - k_y^2)\psi \quad \gamma_\times = -k_x k_y \psi \tag{10}$$

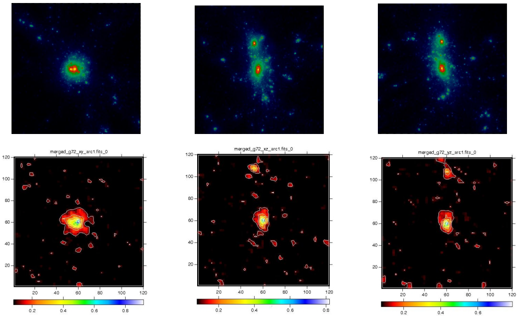
- combine into single equation

$$\begin{pmatrix} \gamma_+ \\ \gamma_\times \end{pmatrix} = \frac{1}{k^2}\begin{pmatrix} k_x^2 - x_y^2 \\ 2k_x k_y \end{pmatrix}\kappa \tag{11}$$

- operator is **orthogonal**: $A^2 = \mathrm{id}$

$$\left[\frac{1}{k^2}\begin{pmatrix} k_x^2 - k_y^2 \\ 2k_x k_y \end{pmatrix}\right]^2 = 1 \tag{12}$$

example: cluster profiles



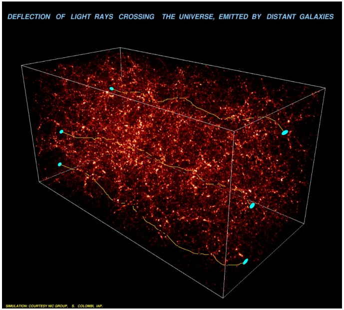
numerical cluster reconstructions, source: J. Merten

- inversion $\kappa = \frac{1}{k^2}\left[(k_x^2 - k_y^2)\gamma_+ + 2k_xk_y\gamma_\times\right]$ yields estimate of map Σ

question

derive the reconstruction operator in real space and formulate the inversion as an integration, identify the Green-function

weak cosmic shear



source: S. Colombi

- lensing on the large-scale structure: fluctuation statistics of the lensing signal reflects the fluctuation statistics of the density field
- neighboring galaxies have correlated deformations because the light rays cross similar, correlated tidal fields

tidal fields and their effect on light rays

- distance x of a gravitationally deflected light ray relative to a fiducial straight line is

$$\frac{d^2x}{d\chi^2} = -\frac{2}{c^2}\nabla_\perp\Phi \tag{13}$$

- solution (flat universes)

$$x = \chi\theta - \frac{2}{c^2}\int d\chi'\,(\chi-\chi')\nabla_\perp\Phi(\chi'\theta) \tag{14}$$

- deflection angle

$$\alpha = \frac{\chi\theta - x}{\chi} = \frac{2}{c^2}\int d\chi'\,\frac{\chi-\chi'}{\chi}\nabla_\perp\Phi(\chi'\theta) \tag{15}$$

- convergence, with $\nabla_\theta = \chi\nabla_x$

$$\kappa = \frac{1}{2}\mathrm{div}\alpha = \frac{1}{c^2}\int d\chi'\,(\chi-\chi')\frac{\chi'}{\chi}\Delta\Phi(\chi'\theta) \tag{16}$$

tidal fields and their effect on light rays

- relate to density field with (comoving) Poisson-equation

$$\Delta\Phi = \frac{3H_0^2\Omega_m}{2a}\delta \qquad (17)$$

- final result:

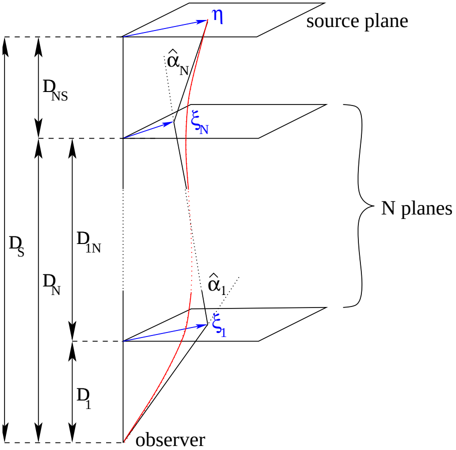
$$\kappa = \int d\chi'\, W(\chi,\chi')\delta \quad \text{with} \quad W(\chi,\chi') = \frac{3}{2}\left(\frac{H_0}{c}\right)^2 \frac{\Omega_m}{a}(\chi-\chi')\frac{\chi'}{\chi} \qquad (18)$$

- fluctuations in κ reflect fluctuations in δ **in a linear way**

cosmic shear

gravitational shear of a galaxy measures the integrated matter density along the line of sight, weighted by $W(\chi)$

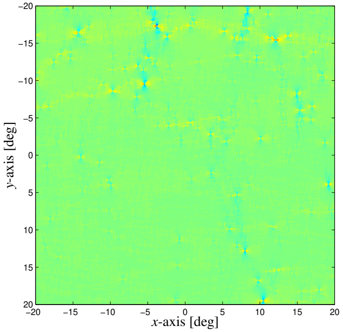
ray-tracing simulations of weak lensing



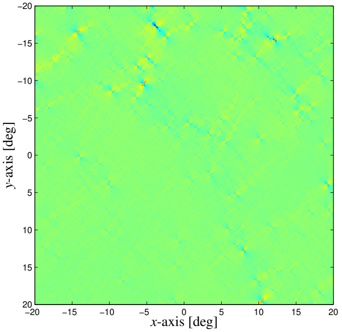
source: C. Pfrommer

- solve transport $\frac{d^2}{dw^2}x = -\frac{2}{c^2}\nabla_\perp\Phi$ by discretisation

simulated shear field on an n-body simulation



shear γ_+



shear $\gamma_\times$

- Gadget-simulated, side length 100 Mpc/h, 40 planes
- clusters of galaxies produce characteristic pattern in shear field

Limber-equation

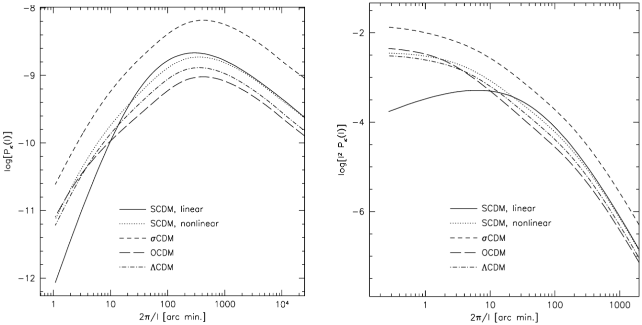
- original title: Limber (1953), The Analysis of Counts of the Extragalactic Nebulae in Terms of a Fluctuating Density Field
- relate 3d-power spectrum $P(k)$ to observed 2d-power spectrum $C(\ell)$
- define correlation function $C(\theta) = \langle g(\vec{\theta}_1)g(\vec{\theta}_2)\rangle$ of quantity g, which measures fluctuations in density field $g(\vec{\theta}) = \int d\chi W(\chi)\delta(\chi\vec{\theta}, \chi)$
- assume that weighting function $q(\chi)$ does not vary much compared to fluctuation scale:

$$C(\theta) = \int d\chi\, W(\chi)^2 \int d(\Delta\chi)\, \xi\left(\sqrt{(\chi\theta)^2 + \Delta^2\chi}, \chi\right) \qquad (19)$$

- correlation function $C(\theta)$ can be Fourier-transformed to yield angular power spectrum $C(\ell)$:

$$C(\ell) = \int d\chi \frac{W(\chi)^2}{\chi^2} P\left(k = \frac{\ell}{\chi}, \chi\right) \qquad (20)$$

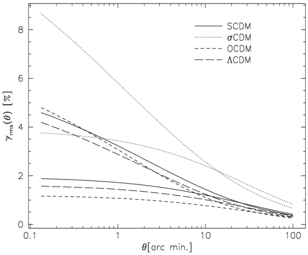
shear power spectra



source: Bartelmann & Schneider, physics reports 340 (2001)

- use Limber's equation to link the shear power spectrum to the dark matter power spectrum
- cosmology: redshift weightings $W(\chi)$, growth $D_+(a(\chi))$, normalisation reflects σ_8

shear in apertures

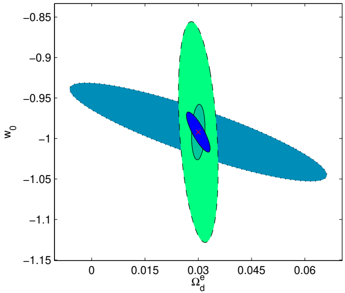


source: Bartelmann & Schneider, physics reports 340 (2001)

- improve constraint on σ_8: $C(\ell)$ should be determined by a small range of k-modes
- average γ in an aperture of size θ: $\langle|\gamma|^2\rangle(\theta)$: product in ℓ-space

$$\langle|\gamma|^2\rangle(\theta) = 2\pi \int_0^\infty \mathrm{d}\ell\,\ell C_\gamma(\ell)\left[\frac{J_1(\theta\ell)}{\pi\theta\ell}\right]^2 \quad (21)$$

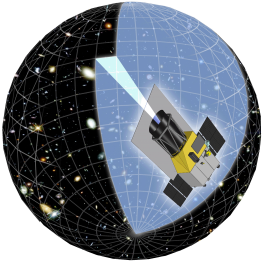
parameter estimates from weak cosmic shear



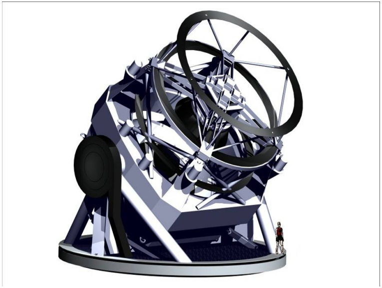
joint constraint on Ω_{EDE} and w_0, source: L. Hollenstein

- lensing is a powerful method for determining parameters
- even complicated dark energy models can be investigated

future lensing surveys



EUCLID



LSST

- coverage $\sim$ half of the sky, going to unit redshift
- precision determination of cosmological parameters, statistical errors $\sim 10^{-3\ldots-4}$
- challenge: **systematics control**

measurements of galaxy shapes

- observe distorsion in the shape of lensed galaxies
- measure second moments of brightness distribution

$$Q_{ij} = \frac{\int d^2\theta I(\vec{\theta})(\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta I(\vec{\theta})} \qquad (22)$$

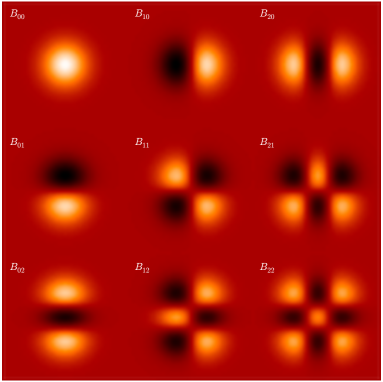
- define complex ellipticity (spin 2):

$$\epsilon = \frac{Q_{xx} - Q_{yy} + 2iQ_{xy}}{Q_{xx} + Q_{yy} + 2\sqrt{Q_{xx}Q_{yy} - Q_{xy}^2}} \qquad (23)$$

- mapping of complex ellipticity by a Jacobian with **reduced shear** $g(\vec{\theta}) = \gamma(\vec{\theta})/[1 - \kappa(\vec{\theta})]$:

$$\epsilon = \frac{\epsilon' + g}{1 + g^*\epsilon'} \text{ for } |g| \leq 1, \;\; \epsilon = \frac{1 + (\epsilon')^* g}{(\epsilon')^* - g'} \text{ for } |g| > 1 \qquad (24)$$

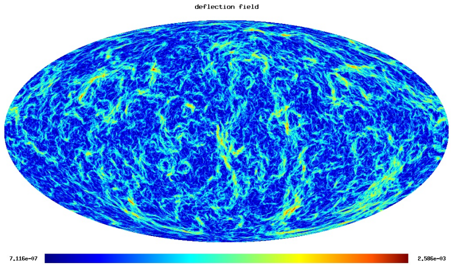
galaxy shapes with shapelets



shapelet base functions B_{ij}, source: P. Melchior

- decomposition into a set of basis functions based on the quantum mechanical harmonic oscillator: Hermite polynomials

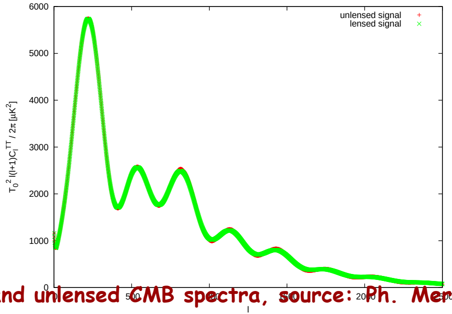
lensing of the cosmic microwave background



sky-map of the deflection angle, source: C. Carbone

- weird (non-Gaussian) patterns in the deflection field
- measurement of lensing at high redshift, in temperature and polarisation

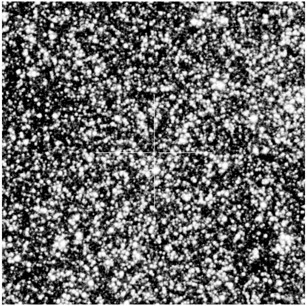
parameter estimates from CMB lensing

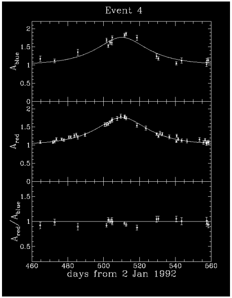


lensed and unlensed CMB spectra, source: Ph. Merkel

- lensing wipes out structures in the CMB (compare to frosted glass)
- amplitudes of the CMB spectrum decreases, non-Gaussianitites in the CMB are generated
- polarisation correlations more strongly affected, B-modes

microlensing and MACHOs

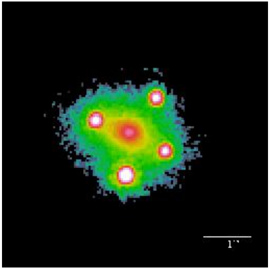


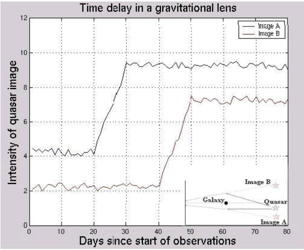


source: C. Alcock

- compact massive objects (historical dark matter candidates) orbit the Milky Way
- observe a large number of bulge stars or stars in the LMC
- find lensed light curves, very typical signature

time delay measurements with quasars

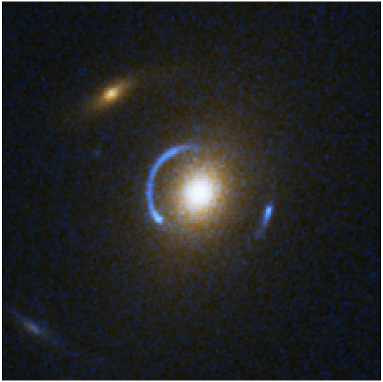




source: universe review

- image appears if the variation of the gravitational time delay is zero
- time delays between different images differ by days
- geometry of the lens can be determined, including the distance

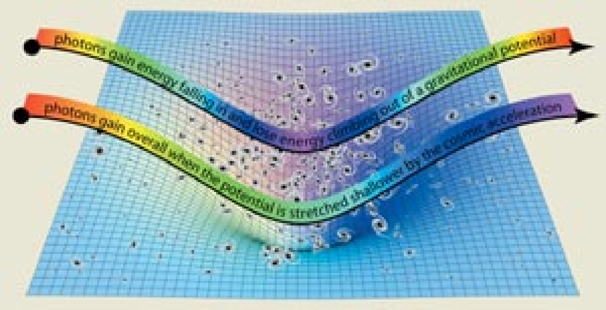
strong lensing and Einstein-rings



Einstein ring around an elliptical galaxy, source: SLACS survey

- perfect alignment of source and lens give rise to **Einstein rings**

integrated Sachs-Wolfe effect



- gravitational interaction of CMB photons with time-varying potentials
- sensitive to the growth of structures
- secondary anisotropy in the CMB, large angular scales

iSW-derivation

- grav. interaction of CMB photons with time-evolving potentials
- temperature perturbation τ, conformal time η

$$\tau = \frac{\Delta T}{T_{CMB}} = -\frac{2}{c^2}\int d\eta\,\frac{\partial\Phi}{\partial\eta} = \frac{2}{c^3}\int d\chi\, a^2 H(a)\frac{\partial\Phi}{\partial a}$$

- reformulation:
  - use comoving distance χ as a distance measure: $d\chi = -c d\eta$
  - scale factor a as a time variable:

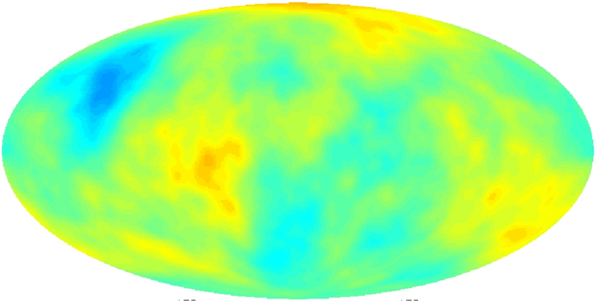
$$\frac{d}{d\eta} = a^2 H(a)\frac{d}{da}$$

- generate potential from density field with comoving Poisson equation

$$\Delta\Phi = \frac{3H^2\Omega_m}{2a}\delta \rightarrow \frac{\Phi}{c^2} = \frac{3\Omega_m}{2a}\frac{\Delta^{-1}\delta}{d_H^2}$$

- iSW-effect measures $d/da(D_+/a)$

iSW sky map



- iSW-induced temperature fluctuations on large scales
- need to be separated from the primary CMB fluctuations

cross correlation techique

- iSW-perturbation have the same spectrum as the CMB
- use a tracer (i.e. galaxy density) which marks the potential wells
- cross-correlation between the CMB and the tracer

$$\langle(\tau_{iSW} + \tau_{CMB})\gamma_{tracer}\rangle = \langle\tau_{iSW}\,\gamma_{tracer}\rangle$$

  - tracer is uncorrelated with primary CMB
  - tracer picks out iSW-perturbations
- tracer density: redshift distribution $p(z)$, bias b

$$\gamma = \int d\chi\, p(z)\frac{dz}{d\chi} b\, D_+\, \delta$$

- careful: iSW-effect measures φ, but tracers follow $\delta \rightarrow$ different scales

iSW-spectra

- line of sight expressions, $\varphi = \Delta^{-1}\delta/d_H^2$, $d_H = c/H_0$

$$\tau = \frac{3\Omega_m}{c}\int dx\, a^2 H(a)\frac{d}{da}\frac{D_+}{a}\varphi = \int dx\, W_\tau(x)\varphi$$

$$\gamma = \int dx\, p(z)\frac{dz}{dx}D_+ b\,\delta = \int dx\, W_\gamma(x)\delta$$

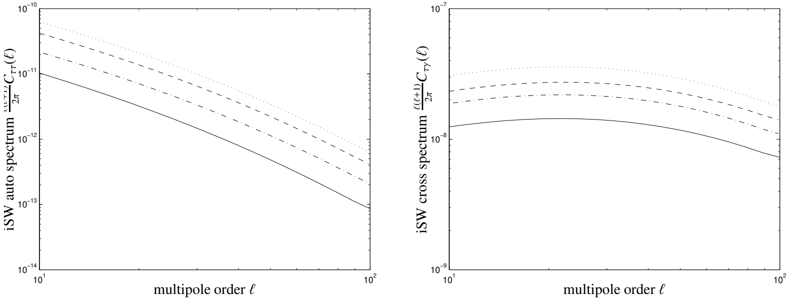
- Limber-equation: project 3d spectrum to 2d spectrum, flat-sky approximation
- angular power spectra: fluctuation on angular scale $\ell = \pi/\Delta\theta$

$$C_{\tau\tau}(\ell) = \int dx\, \frac{W_\tau^2(x)}{x^2}\left.\frac{P(k)}{(d_H k)^4}\right|_{k=\ell/x}$$

$$C_{\tau\gamma}(\ell) = \int dx\, \frac{W_\tau(x)W_\gamma(x)}{x^2}\left.\frac{P(k)}{(d_H k)^2}\right|_{k=\ell/x}$$

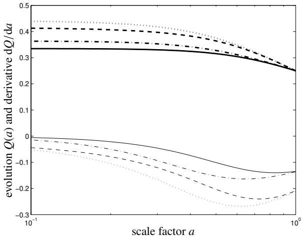
- Poisson-equation in Fourier-space: $\Delta\Phi \propto \delta \rightarrow (-k^2)\Phi \propto \delta$

iSW-spectra



- most signal at low ℓ, cosmic variance limitations
- easy to remember:
  - $C_{\tau\gamma}(\ell) \propto \ell^{-2}$
  - $C_{\tau\tau}(\ell) \propto \ell^{-4}$

which redshift contributes most to iSW?



- rewrite line of sight integral τ

$$\tau = \frac{3H_0^2}{c^3}\int_0^{\chi_H} d\chi\, a^2 H(a)\,\frac{dQ}{da}\,\Delta^{-1}\delta,$$

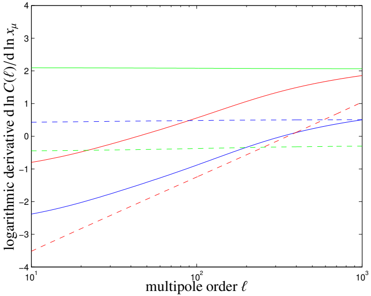
- special redshift: $\Omega_m(z) = \Omega_{DE}(z)$
- less negative eos-parameter $w \rightarrow$ signal from higher redshift

parameter sensitivity

$$C_{\tau\gamma}(\ell) = \frac{3\Omega_m}{c}\int\frac{d\chi}{\chi^2}\left[D_+ b p(z)\frac{dz}{d\chi}\right]\left[a^2H(a)\frac{d}{da}\frac{D_+}{a}\right]\frac{P(k)}{(d_H k)^2}\bigg|_{k=\ell/\chi}$$

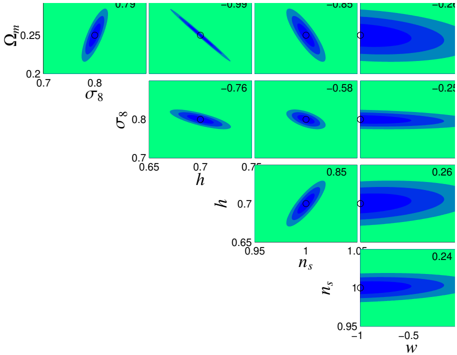
- prefers intermediate values for Ω_m
- signature for dark energy:
  - SCDM: $D_+(a) = a$, $\rightarrow d/da(D_+/a)$ vanishes
- σ_8 is completely degenerate with bias b
  - external prior on σ_8
  - combination of $C_{\tau\gamma}(\ell)$ with $C_{\gamma\gamma}(\ell)$
- minor dependency on n_s and h (via shape parameter)
- sensitivity to w, from growth and cosmology
- compare to lensing: very similar, $\propto D_+/a$

parameter sensitivity



- logarithmic derivatives of the spectrum
- parallel curves → degeneracies

parameter constraints



- ideal measurment
- CMB priors on Ω_m, σ_8, n_s and h
- 10% accuracy on Ω_{DE}, 20% accuracy on w

constraints: covariance

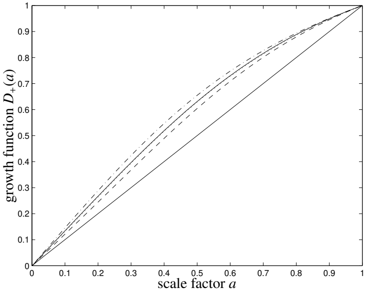
- cross correlation technique: $C_{\mathsf{TY}}(\ell)$ does not CMB-fluctuations
    - tracer is uncorrelated with CMB
- primary CMB fluctuations enter as correlation noise!

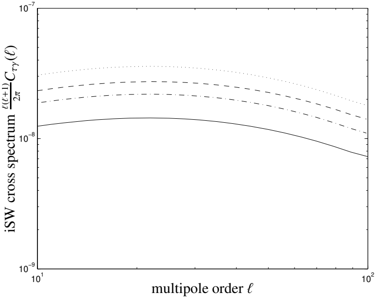
$$\mathrm{cov}[C_{\mathsf{TY}}] = \frac{2}{2\ell+1}\frac{1}{f_{\mathrm{sky}}}\left[\tilde{C}^2_{\mathsf{TY}}(\ell) + \tilde{C}_{\mathsf{YY}}(\ell)\tilde{C}_{\mathsf{TT}}(\ell)\right]$$

- $\tilde{C}_{\mathsf{TY}}(\ell) = C_{\mathsf{TY}}(\ell)$, cross correlation!
- $\tilde{C}_{\mathsf{YY}}(\ell) = C_{\mathsf{YY}}(\ell) + C_{\mathrm{Poisson}}(\ell)$, Poissonian error
- $\tilde{C}_{\mathsf{TT}}(\ell) = C_{\mathsf{TT}}(\ell) + C_{\mathrm{CMB}}(\ell) + C_{\mathrm{noise}}(\ell)$, primary CMB

- cosmic variance: important, highest amplitudes at low ℓ
- iSW-effect much weaker (10σ) than gravitational lensing ($> 100\sigma$)
- weaker constraints, but
    - useful for degeneracy breaking
    - weird models can be investigated

application: coupled fluids





- recent flurry in the literature: coupled DM/DE
- construct cosmologies with very similar growth functions
- iSW-effect can still distinguish them!
- other field: modified gravity theories, DGP-gravity

iSW-effect: pros and cons

- maps structure growth, compares D_+ to a
- signature of dark energy, vanishes in SCDM
- sensitivity for non-standard Poisson equation
  $\rightarrow$ DE/CDM coupling or modified gravity
- weak constraints (CMB noise), total significance $\simeq 10\sigma$
- can access information hidden to geometrical probes, gravitational
- analogy to lensing:
  - lensing $\kappa \propto \int d\chi\, D_+/a\delta$
  - iSW-effect $\tau \propto \int d\chi\, d(D_+/a)/da\varphi$
- strongest for intermediate Ω_m: coupling vs. growth
- uncertainties related to bias
  - bias decreases with time: $db/da < 0$, different for every tracer
  - scale dependence $b(k)$, different for every tracer

Rees-Sciama effect

- RS-effect: iSW-effect from nonlinear structures
- distinction a bit artificial (similar to kin. Sunyaev-Zeldovich-effect vs. Ostriker-Vishniac-effect)
- two different approaches in perturbation theory
  - perturbed density field, solve for potential

$$\delta(a) = D_+(a)\delta^{(1)} + D_+^2(a)\delta^{(2)} + \ldots$$

  - continuity equation: velocity-density products, get $\dot{\Phi}$

$$\dot{\delta} = -\mathrm{div}(\delta\vec{v}) \rightarrow \text{Poisson-equation}$$

- first approach: 2^{nd} order, second approach: 1^{st} order
- both involve computation of 4-point correlation functions

iSW-effect vs. RS-effect

- perturbation: $\delta(a) = D_+\delta^{(1)} + D_+^2\delta^{(2)} + \ldots$
  - first order

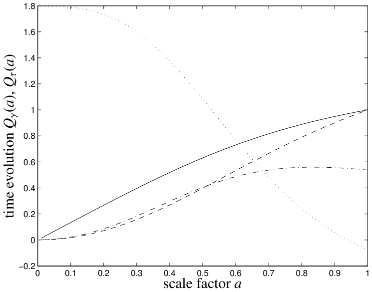
$$\tau^{(1)} = \frac{3\Omega_m}{c}\int_0^{\chi_H} d\chi\, a^2 H(a)\, \frac{d}{da}\left(\frac{D_+}{a}\right)\frac{\Delta^{-1}}{d_H^2}\delta^{(1)}$$

  - second order

$$\tau^{(2)} = \frac{3\Omega_m}{c}\int_0^{\chi_H} d\chi\, a^2 H(a)\, \frac{d}{da}\left(\frac{D_+^2}{a}\right)\frac{\Delta^{-1}}{d_H^2}\delta^{(2)}$$

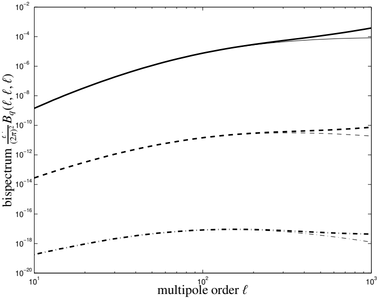
- dark energy sensitivity:
  - linear iSW-effect:
    vanishes in SCDM, $D_+ = a$, nonzero in DE cosmologies
  - nonlinear iSW-effect: largest in SCDM, smaller in DE

time evolution of source terms



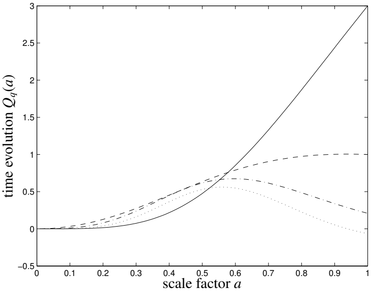
- individual time-evolution for first- and second order fields
- plotted for ΛCDM with $\Omega_m = 0.25$

iSW bispectrum - non-Gaussian signature



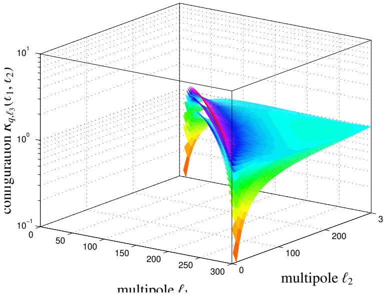
- angular equilateral bispectra between galaxy overdensity and iSW-effect, $\langle \tau^q \gamma^{3-q} \rangle$, $q = 0, 1, 2$
- perturbation theory for galaxy density and iSW-effect

time evolution of bispectra



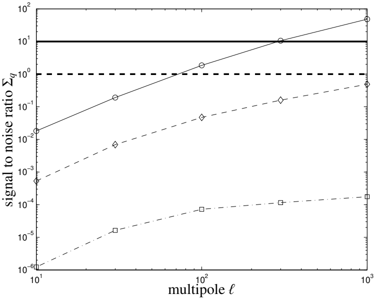
- time evolution of mixed spectra $\langle \tau^n \gamma^{3-n} \rangle$
- non-Gaussianity of late-time structure formation

configuration dependence



- configuration dependence $R_{\ell_3}(\ell_1, \ell_2) \propto \sqrt{\left|\frac{B(\ell_1,\ell_2,\ell_3)}{B(\ell_3,\ell_3,\ell_3)}\right|}$
- φ peaks on larger scales than δ

measurability of the iSW-bispectrum



- bispectrum covariance, with Gaussian approximation
- max. signal to noise: 0.6, for PLANCK vs. DUNE
- push to $\ell \simeq 10^4$ for 3σ significance

gravitomagnetic potentials

- change of photon energy in time-variable potential wells:
  $\tau = \frac{\Delta T}{T} = -\frac{2}{c^3}\int \mathrm{d}\chi\, \frac{\partial\Phi}{\partial\eta}$ with conformal time η
- connection to gravitomagnetic potentials: continuity
  $\frac{\partial}{\partial\eta}\Phi = -G\int \mathrm{d}^3r'\, \frac{\dot\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} = +G\int \mathrm{d}^3r'\, \frac{\nabla'\vec{\jmath}(\vec{r}')}{|\vec{r}-\vec{r}'|}$
- integration by parts (ignore boundary terms)
  $\ldots = -G\int \mathrm{d}^3r'\, \vec{\jmath}(\vec{r}')\cdot\nabla'\frac{1}{|\vec{r}-\vec{r}'|}$
- use identity $\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = -\nabla\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)$, pull out ∇:
  $\ldots = -\nabla\left(-\int \mathrm{d}^3r'\, \frac{\vec{\jmath}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right) \rightarrow \tau = \frac{2}{c^3}\int \mathrm{d}\chi\, \mathrm{div}\vec{A}$
- interpretation: iSW-effect is due to
  1. formation of objects: $\dot\rho > 0 \rightarrow \dot\Phi > 0$, or equivalently
  2. converging matter streams: $\mathrm{div}\vec{\jmath} < 0 \rightarrow \mathrm{div}\vec{A} < 0$
- $\vec{A}$ is called gravitomagnetic potential, sourced by $\vec{\jmath}$

analogies: RS-effect and lensing

$P_\delta(k) \leftarrow$ density $\delta(\vec{r})$ | flux $\vec{j}(\vec{r}) \rightarrow P_j(k)$

$\Phi(\vec{r}) = -G\int d^3r' \frac{\delta(\vec{r}')}{|\vec{r}-\vec{r}'|}$ ↓ | ↓ $\vec{A}(\vec{r}) = -G\int d^3r' \frac{\vec{j}(\vec{r}')}{|\vec{r}-\vec{r}'|}$

potential $\Phi(\vec{r})$ | potential $\vec{A}(\vec{r})$

$\vec{\alpha} = -\nabla_\perp \Phi$ ↓ | ↓ $\tau = \mathrm{div}_\perp \vec{A}$

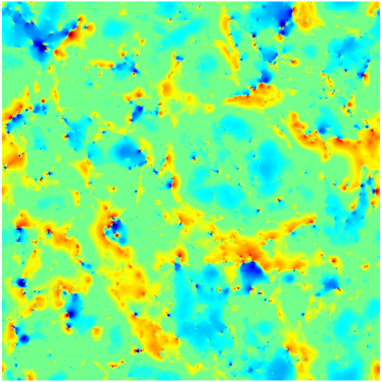
$C_\alpha(\ell) \leftarrow$ deflection $\vec{\alpha}(\vec{\theta}) \Longleftrightarrow$ iSW – effect $\tau(\vec{\theta}) \rightarrow C_\tau(\ell)$

$\kappa = \frac{1}{2}\mathrm{div}\vec{\alpha}$ ↓ | ↓ $\vec{\chi} = -\nabla\tau$

$C_\kappa(\ell) \leftarrow$ convergence $\kappa(\vec{\theta})$ | gradient $\vec{\chi}(\vec{\theta}) \rightarrow C_\chi(\ell)$

- Φ conserves energy $|\vec{k}|$, rotates direction $\vec{k}/k$
- $A_\perp$ does not influence $\vec{k}/k$, but stretches wave length

RS-effect: visual impression



credit: V.Springel (MPA), Millenium simulation

- non-Gaussian fluctuations, sharp features in the temperature field
- fluctuatios on small scales, structure formation activity

summary: Friedmann-Lemaître cosmologies

- dynamic world models based on general relativity
- Robertson-Walker line element as a solution to the field equation
- Copernican principle: homogeneous and isotropic metric
- homogeneous fluids, with a certain pressure density relation, parameterised by $w = p/\rho$
  - radiation ($w = +1/3$)
  - (dark) matter ($w = 0$)
  - curvature ($w = -1/3$)
  - cosmological constant ($w = -1$)
- Hubble parameter H_0 defines the critical density $\rho_{crit} = 3H_0^2/(8\pi G)$
- distance definitions become ambiguous
- geometrical probes constrain the model parameters to a few percent, in particular $\Omega_k < 0.01$

summary: random fields and spectra

- inflation: epoch of rapid **accelerated** expansion of the early universe
- Hubble expansion dominated by a fluid with very negative w
  - drives curvature towards zero → flatness problem
  - grows observable universe from a small volume → horizon problem
- fluctuations in the energy density of the inflaton field couple gravitationally to the other fluids
- fluctuations are Gaussian and have a finite correlation length
  - characterisation with a correlation function $\xi(r)$
  - homogeneous fluctuations: spectrum $P(k)$
- inflationary fluctuations can be observed as temperature anisotropies in the CMB
- shape of the spectrum: inflation gives $P(k) \propto k^{n_s}$, changed by transfer function $T(k)$ in the Meszaros effect, normalised by σ_8

summary: structure formation

- cosmic structures and the large-scale distribution of galaxies form by **gravitational instability** of inflationary perturbation
  - continuity equation
  - Euler equation
  - Poisson equation
- linearisation for small amplitudes: homogeneous growth, described by $D_+(a)$, conservation of Gaussianity of initial conditions
- nonlinear growth is inhomogeneous and destroys Gaussianity by mode coupling
- three basic difficulties
  - nonlinearities in the continuity and Euler-equation
  - collisionlessness of dark matter
  - non-extensivity of gravity
- galaxy formation: gravitational collapse, Jeans argument
- halo density: predicted from $P(k)$ with Press-Schechter

summary: standard model ΛCDM

- ΛCDM is a flat, accelerating Friedmann-Lemaître cosmology with dark matter and a cosmological constant
- ΛCDM has 7 parameters, and is in remarkable agreement with observations, both of geometrical and growth probes
  1. $\Omega_m = 0.25$, low density, required by supernova observations
  2. $\Omega_b = 0.04$, small value, good measurement from CMB
  3. $\Omega_\Lambda = 0.75$, flatness from CMB, $\Omega_m + \Omega_\Lambda = 1$
  4. $w = -1$, cosmological constant, no dynamic dark energy
  5. $\sigma_8 = 0.8$, low value (compared to history), largest uncertainty
  6. $n_s = 0.96$, predicted by inflation to be $\lesssim 1$
  7. $h = 0.72$, sets expansion time scale, or age/size of the universe
- up to now, there is no theoretical understanding of Λ

summary: open questions in cosmology

- precision determination of cosmological parameters and verification of the standard model
- Gaussianity of initial conditions and constraints on the inflationary model, tensor excitations and gravitational waves
- quantification of the nonlinearly evolved cosmic density field
- substructure of dark matter haloes and an explanation of their kinematical structure
- biasing of galaxies and relations between host halo properties and member galaxies
- distinguishing between cosmological constant, dark energy or modified gravity
- tidal interactions of haloes with the large-scale structure