Dark Energy Theory
ISAP Heidelberg
June 4th 2019

1. The accelerating Universe and Problems

a) Coincidence Problem

$(-+++)$ Signature
$k=0$ no spatial curvature

FLRW metric:

$$ds^2 = -N^2(t)\,dt^2 + a^2(t)\,dx^2$$
$$dx^2 = \delta_{ij}\,dx^i dx^j \quad i,j = 1,2,3$$

Hubble parameter: $H = \frac{\dot a}{Na}$
in physical time t, $H = \frac{\dot a}{a}$ with $\dot a = \frac{da}{dt}$
in conformal time τ, $H = \frac{a'}{a^2}$ with $a' = \frac{da}{d\tau} = a\dot a$

N : Lapse
a : Scale factor

we can always choose a gauge where $N=1 \to$ physical time
or $N = a \to$ conformal time
or anything else

but for now we don't fix the gauge
so we can derive the Friedmann eq
by varying w.r.t N (mini Superspace approximation)

$$S = \frac{M_{pl}^2}{2}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\,\mathcal{L}_m$$

$\mathcal{L}_m \to$ energy density ρ
$\to$ pressure $p = w\rho$
$\dot\rho = -3H(\rho + p)$

$\downarrow$ FRW, after integration by parts,

$$S = \frac{M_{pl}^2}{2}\int d^4x\left[-6\frac{a\dot a^2}{N}\right] + \int d^4x\left[Na^3(-\rho(a))\right]$$

Friedmann Eq: $\frac{\delta}{\delta N}$ $\quad 3M_{pl}^2\frac{\dot a^2}{a^2} = \rho(a) \Rightarrow H^2 = \frac{1}{3M_{pl}^2}\rho(a)$

Raychadhuri Eq: $\frac{\delta}{\delta a}$ $\quad\to$ gives $\frac{d}{dt}$ (Friedmann Eq) if we use conservation of energy $\dot\rho = -3H\rho(1+w)$

or in other words, consistency of Friedmann eq & Raychadhuri eq imposes conservation of energy $\dot\rho = -3H\rho(1+w)$

* Cosmological constant

$\rho = \text{const} = \Lambda$
$H^2 = \text{const} = \frac{\Lambda}{3M_{pl}^2}$
$\dot H = 0 = \frac{\ddot a}{a} - H^2 \Rightarrow \ddot a = aH^2 > 0$
Universe accelerating

Today, the Universe is accelerating at a rate of $H_0 \sim 70$ km/s/Mpc $\sim (10^{-33}\,\text{eV})$
$\to$ needs $\Lambda \sim M_{pl}^2 H_0^2$
$\sim (10^{18}\,\text{GeV})^2 (10^{-33}\,\text{eV})^2$
$\Lambda \sim (10^{-3}\,\text{eV})^4$

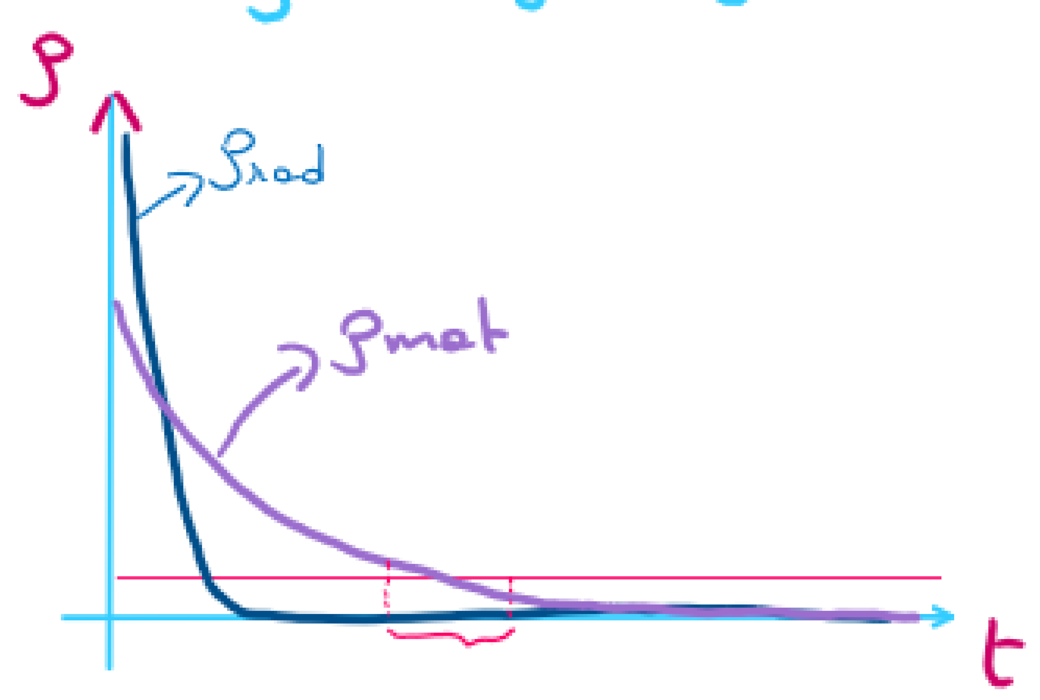
Very close to neutrino mass.
coincidence or connection ???

* other fluids

In the past, the Universe was dominated by radiation and then dust (matter)

$\rho_{rad} \sim a^{-4}$, $\omega = 1/3$ $\qquad$ $\rho_m \sim a^{-3}$, $\omega = 0$

$$\rho = \Lambda + \rho_{rad} + \rho_m$$



Today
$\Omega_{mat} \sim 30\%$
$\Omega_\Lambda \sim 70\%$

But in history of Universe, there is only a very small amount of time where Ω_{mat} and Ω_Λ are comparable...
It seems like a "coincidence" that we happen to be living precisely at that time out of the whole history of Universe

Coincidence problem

→ Reason why we may think of dark energy which evolves in time.
(maybe dark energy could be a tracker?)

b) Old & New C.C. problem

In what follows, we take a quantum effective field theory (EFT) approach.

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_{pl}} h_{\mu\nu}$$

$h_{\mu\nu}$ → Spin-2 QF

There is no issue treating gravity (GR) as a quantum EFT with Mpl cutoff.

$$S_{EFT} = \int d^4x \sqrt{-g} \Big[\Lambda + \frac{M_{pl}^2}{2} R + \underbrace{c(R^2 + \dots) + \dots}_{\text{irrelevant operators}} \Big]$$

Λ → Most Relevant operator

$$+ S_{matter}[g_{\mu\nu}, \Psi_i]$$

in QFT, we expect matter field loops to contribute to running of GR operators.

eg. for a massive scalar field ϕ of mass m,

$$S_\phi = \int d^4x \sqrt{-g} \Big\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu\phi \partial_\nu\phi - \frac{1}{2} m^2\phi^2 \Big]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_{pl}}$$

$$S_\phi = \int d^4x \Big[-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2} m^2\phi^2$$

(← everything on flat ST $(\partial\phi)^2 = \eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$)

$$+ \frac{h_{\mu\nu}}{M_{pl}} \Big[\frac{1}{2} \partial^\mu\phi\partial^\nu\phi - \frac{1}{2}\eta^{\mu\nu}\Big(-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2\Big)\Big]$$

$$+ \frac{h_{\mu\nu}h_{\alpha\beta}}{M_{pl}^2}\left(\partial\phi\partial\phi + m^2\phi^2\right) + \dots \Big]$$

⇒ interactions between ϕ and graviton $h_{\mu\nu}$

At one-loop, this leads to contributions of the form:

All loops diverge but if we just focus on log-running (eg. just do dimensional regularization) then the running of the coupling constants are of the form:

$$\delta\Lambda \sim m^4 \log\left(\frac{\mu}{m}\right)$$

$$\delta M_{Pl}^2 \sim m^2 \log\left(\frac{\mu}{m}\right) \ll M_{Pl}^2 \text{ (typically)}$$

$$\delta c \sim \log\left(\frac{\mu}{m}\right)$$

Rest just gives finite contributions.

→ the running from massive loops (below Planck Scale) to Mpl are irrelevant (Suppressed compared to bare Value).

→ Running to C.C. scales as m^4 where m is mass of any massive particle we have integrated out.

So just from known physics of the SM, we expect (roughly):

$$\Lambda_{vacuum} \sim -m_{e^-}^4 - m_{\mu}^4 - m_t^4 + m_{Higgs}^4 + m_Z^4 + m_{W^+}^4 + m_{W^-}^4 + \dots$$
$$\sim (200\ \text{GeV})^4$$

This would lead to

$$H_0 \sim \frac{(200\,\text{GeV})^2}{(10^{18}\,\text{GeV})} \sim \frac{10^4\, 10^{18}}{10^{18}\, 10^9}\text{eV} \sim 10^{-5}\text{eV} \sim (2\,\text{cm})^{-1}$$

If this was correct, the size of the "observable Universe" would roughly the same as a lychee...

Old Cosmological Constant problem

formulated in the 80's, how come $\Lambda = 0$ when matter loops contribute with such a large amount...

how can we set $\Lambda = 0$?

Since then, we know that the Universe is accelerating

New Cosmological Constant Problem

why is the observed C.C. so small?

Most models of Dark Energy ignore the old C.C. problem (ie assume that $\Lambda = 0$ for a reason or another) and try to explain the late-time acceleration of the Universe using instead a dynamical fluid.

Some models of modified gravity try to tackle the old C.C. problem directly

2. Dynamical Dark Energy

a) Quintessence

Consider gravity and a (minimally coupled) Scalar field ϕ

$$\mathcal{L} = \frac{M_{Pl}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi)$$

K.G. eq : $\Box\phi = V'(\phi)$

work on FLRW with $N=1$, $\ddot{\phi} + 3H\dot{\phi} = V'(\phi)$

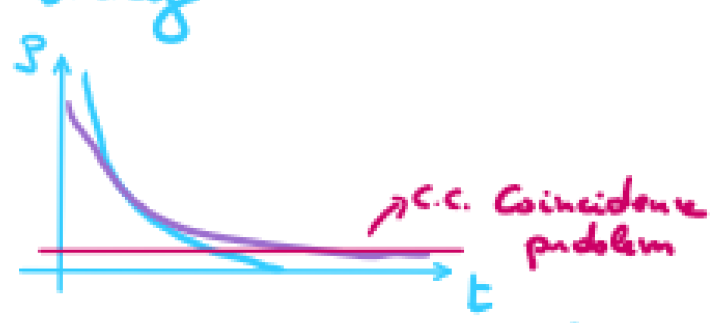
$$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}(\partial\phi)^2 + V\right)$$

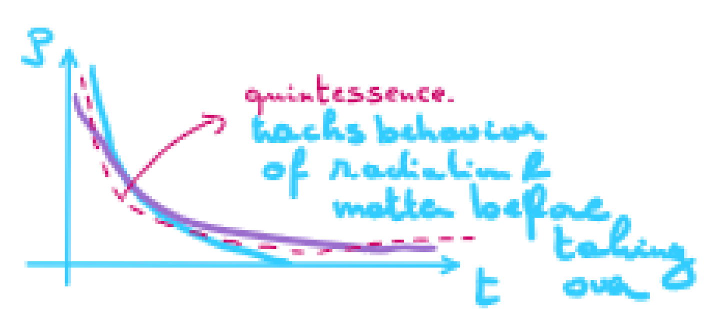
$$\left.\begin{aligned} \rho &= -T^0{}_0 = \frac{1}{2}\dot{\phi}^2 + V \\ P &= T^1{}_1 = T^2{}_2 = \frac{1}{2}\dot{\phi}^2 - V \end{aligned}\right\} \omega = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}$$

$\Rightarrow$ if $\dot{\phi} \ll \sqrt{V}$, then $\omega_\phi \sim -1 \rightarrow$ acts as an effective c.c. (but still dynamical)

if $V \gg \dot{\phi}^2$, $\omega = -1 + \underbrace{\frac{\dot{\phi}^2}{V}}_{>0} + \dots$

The idea is to have a field that mimics radiation and matter during those eras and only starts dominating today





Then the abundance of dark energy was roughly comparable to ρ_{rad} & then ρ_{mat} during whole HBB history of Universe ...

This can occur for Some types of potentials, eg. exp : $V(\phi) = e^{-\alpha\phi/M_{Pl}}$

exercise : Show that at late-time (when $\rho_{rad} = \rho_{mat} \sim 0$), we can get an accelerated expansion Solution

During the matter-dominated era we can show that we can have tracker Solutions. To end the tracking phase, we need the potential to vary a bit (not exactly of that form)

Tuning / Naturalness issues :

let's imagine $V(\phi) = \mu^4 U_0\left(\frac{\phi}{f}\right)$

with $U_0 \sim U_0' \sim U_0'' \sim \mathcal{O}(1)$

$$m_\phi^2 \sim V''(\phi) \sim \frac{\mu^4}{f^2} U_0 \quad \text{and} \quad V \sim \mu^4,\ V' \sim \frac{\mu^4}{f}$$

$$\sim \frac{V'^2}{V}$$

During the acceleration phase, we like

$$H^2 \sim \frac{V}{M_{Pl}^2} \sim \frac{\mu^4}{M_{Pl}^2} \quad ; \quad \dot{\phi} \ll \sqrt{V} \sim M_{Pl} H$$

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0 \Rightarrow V' \sim H\dot{\phi} \ll M_{Pl} H^2$$

$$m_\phi^2 \sim \frac{V'^2}{V} \ll \frac{M_{Pl}^2 H^4}{M_{Pl}^2 H^2} \sim H^2$$

So effectively, the mass of the dark energy field has to be $\sim H_0 \sim (10^{-33}\,\text{eV})$

$\rightarrow$ In addition to the old c.c. problem there is an additional fine-tuning problem on the mass...

typically if we integrate out heavier particles they would lead to corrections to the mass that go as

$$\delta m \sim \frac{M^2}{f} \rightarrow M \text{ mass of these heavier fields.}$$

b) $f(R)$ gravity

$$S = \int d^4x \sqrt{-g}\left[\underbrace{\frac{M_{pl}^2}{2} R}_{\text{could change this}} - \underbrace{\frac{1}{2}(\partial\phi)^2 - V(\phi)}_{}\right]$$

or this

or how matter couples...

or any combination

or even more complicated things...

↳ always leads to similar fine-tuning problems

In the end of the day we need to explain why at late-time $V \sim \Lambda \sim$ small

Idea behind $f(R)$ is to change GR part

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{pl}^2}{2} f(R) + \mathcal{L}_{matter}(g, \psi_i)\right]$$

Modified Eq:

$$f'(R) R_{\mu\nu} - \underbrace{\nabla_\mu \nabla_\nu f'(R) + g_{\mu\nu} \Box f'(R)}_{\text{could maybe lead to acceleration?}} - \frac{1}{2} f(R) g_{\mu\nu} = \frac{1}{M_{pl}^2} T_{\mu\nu}$$

originally, a popular choice was

$$f(R) = R - \frac{\mu^4}{R}$$

μ^4 ↳ dimension-4 scale

So that clearly we could no longer have $R = 0$ even in the vacuum

instead, in the vacuum, there is a dS solution with $R = \text{Const} = C$, $R_{\mu\nu} = +\frac{C}{4} g_{\mu\nu}$

$f'(R) = 1 + \frac{\mu^4}{C^2}$ $\quad \nabla_\mu R = 0 \quad \Box R = 0$

$$\left(1 + \frac{\mu^4}{C^2}\right) \frac{C}{4} g_{\mu\nu} - \frac{1}{2}\left(C - \frac{\mu^4}{C}\right) g_{\mu\nu} = 0$$

$$\Rightarrow \quad -\frac{1}{4} + \frac{3}{4}\frac{\mu^4}{C^2} = 0$$

→ have (A)dS solutions in the vacuum... $\quad \boxed{C = \pm\sqrt{3}\,\mu^2}$

however, we will see that

$$\boxed{f(R) \text{ is equivalent to GR + Scalar field}}$$

To see that, we consider an auxiliary scalar field φ

$$S = \int d^4x \sqrt{-g} \quad \frac{M_{pl}^2}{2}\left[f'(\varphi) R + f(\varphi) - \varphi f'(\varphi)\right] + \mathcal{L}_m(g, \psi_i)$$

EOM for φ: $\quad \cancel{f''}(\varphi) R + \cancel{f'(\varphi)} - \cancel{f'(\varphi)} = \varphi \cancel{f''}(\varphi)$

If $f'' \neq 0$ then $\varphi = R$

and $f'(R) R + f(R) - R f'(R) = f(R)$ so it is indeed equivalent to our original formulation

however with the scalar field, it is easier to see what is going on and go back to what we call Einstein frame

At the moment matter couples to $g_{\mu\nu}$ (φ does not enter that coupling) this is the Jordan frame

we can make a change of frame to go to Einstein frame

Jordan frame ⟶ Einstein frame

$g_{\mu\nu}$ $\qquad \bar{g}_{\mu\nu} = f'(\varphi) g_{\mu\nu}$

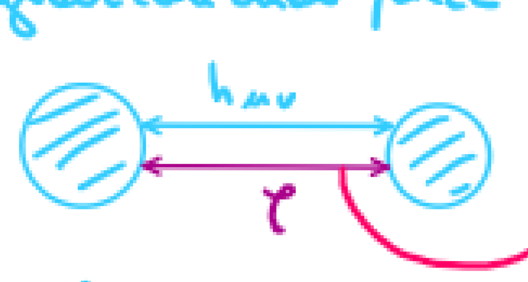
φ is the real source behind acceleration

$$S = \int d^4x \sqrt{-\bar{g}} \frac{M_{pl}^2}{2}\left[\bar{R} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2 (\bar{\partial}\varphi)^2 + f(\varphi) - \varphi f'(\varphi)\right]$$

would need to then normalize scalar field and this would describe the effective potential

In Einstein frame, the matter couples to $\bar{g}_{\mu\nu} = f'(\varphi) g_{\mu\nu}$

This implies that φ mediates a gravitational force



Violates the equivalence Principle.

Unless there is a Screening Mechanism in place the scalar field generates an additional "gravitational fifth force" that is incompatible with observations.

c) Fifth forces

To see this more specifically, let's look a gravitational exchange amplitude (gauge-indep) and compare with GR

* in GR

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{pl}^2}{2} R + \mathcal{L}_m(g, \psi_i)\right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_{pl}} h_{\mu\nu}$$

to quadratic order in $h_{\mu\nu}$,

$$S = \int d^4x \left[-\frac{1}{4} h^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - \frac{1}{2M_{pl}} h_{\mu\nu} T^{\mu\nu}\right]$$

with $T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}}\left[\sqrt{-g}\,\mathcal{L}_m\right]$ $\boxed{\partial_\mu T^\mu{}_\nu = 0}$

$$\hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} = -\frac{1}{2}\left[\Box h_{\mu\nu} - \partial_{(\mu}\partial_\alpha h^\alpha{}_{\nu)} + \partial_\mu\partial_\nu h - \eta_{\mu\nu}(\Box h - \partial_\alpha\partial_\beta h^{\alpha\beta})\right]$$

gauge invariance $\to$ de Donder gauge

$$\partial_\mu h^\mu{}_\nu = \frac{1}{2}\partial_\nu h$$

$\Rightarrow$ EOM are

$$\Box\left(h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}\right) = \frac{1}{2M_{pl}} T_{\mu\nu}$$

$$\mathcal{A} = \int d^4x \; h_{\mu\nu} T'^{\mu\nu} = \int d^4x \; T'^{\mu\nu} \frac{1}{\Box}\left(T_{\mu\nu} - \frac{1}{2} T \eta_{\mu\nu}\right)$$

* Now let's imagine we are dealing with GR + Scalar field

$$S = \int d^4x \left[\sqrt{-g}\, \frac{M_{pl}^2}{2} R - \frac{1}{2}(\partial\varphi)^2 + \mathcal{L}_m\left(\left(1 + \frac{\varphi}{M_{pl}}\right) g, \psi_i\right)\right]$$

$\downarrow$ linearized $\qquad = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_{pl}} + \frac{\varphi\eta_{\mu\nu}}{M_{pl}}$

$$\mathcal{L} = -\frac{1}{4} h \mathcal{E} h - \frac{1}{2M_{pl}} h_{\mu\nu} T^{\mu\nu} - \frac{1}{2M_{pl}} \varphi T$$

EOM for $h_{\mu\nu}$ is the same as in GR and Scalar field now satisfies

$$(\Box - V'')\varphi = \frac{1}{2M_{pl}} T$$

o Gravitational exchange amplitude has now
\- new contribution

$$\mathcal{A} = \int d^4x \; T'^{\mu\nu}(h_{\mu\nu} + \varphi\eta_{\mu\nu})$$

$$= \int d^4x \left[T'^{\mu\nu} \frac{1}{\Box}\left(T_{\mu\nu} - \frac{1}{2} T \eta_{\mu\nu}\right) + \frac{1}{6} T' \frac{1}{\Box - m^2} T\right]$$

if $m = 0$, get $T'^{\mu\nu} \frac{1}{\Box}\left(T_{\mu\nu} - \frac{1}{3} T \eta_{\mu\nu}\right)$

o and we would have very different observables
\-s compared with GR.

we see, (at least symbolically here) that one way out is to have m very large
In practise this would mean that the Scalar field is frozen in its vacuum and it would cost too much to excite it

The larger the mass the hardest it is to excite the field.

As we have seen, typically the mass needs to be very small for DE but very large for Solar System tests...

lo
iSlea behind Chameleon mechanism
e to make the mass dependent on the
nvironment.

3) Screening Mechanisms

a) Various Mechanisms

In many models of dark Energy, we need to "hide" or "Screen" the dark energy field in Solar System, Galaxy, maybe for LSS,...

Chameleon is a way to make mass dependent on environment but there is other ways...

$$\mathcal{L} = \underbrace{-\frac{1}{2}(\partial\phi)^2}_{\text{Vainshtein}} - \frac{1}{2}\underbrace{m^2}_{\text{chameleon}}\phi^2 + \underbrace{A(\phi)}_{\text{weak coupling}}T$$

many models rely on a weak coupling to matter,

effectively

$$A(\phi)T \sim \frac{\phi}{M_c}T$$

weak coupling if $M_c \ggg M_{pl}$

but naively we would expect $M_c \simeq M_{pl}$

→ leads to naturalness issues

Chameleon, Symmetron, Vainsthein are Screening mechanism to have an order $M_c \sim M_{pl}$ coupling and yet avoid 5^{th} force constraints

b) Kinetic Screening
or Vainshtein Mechanism

Most Screening Mechanisms involve Additional tuned parameters with their own naturalness issues in addition to old & New CC tuning issue problems

Vainshtein mechanism is one of the few mechanisms that:

- arises in models of modified gravity that attempt to tackle the old C.C. problem

- Enjoy a Shift Symmetry that makes the Small C.C. value technically natural (ie the value of parameters are small but don't get Spoiled by Quantum corrections)

- The mechanism itself does not require additionally tuned parameters beyond the ones required for DE and is technically Stable.

i) Example of the Cubic Galileon

can consider

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda^3}(\partial\phi)^2\Box\phi + \frac{\phi}{M_{pl}}T$$

Λ: Strong Coupling Scale (typically $\Lambda \ll M_{pl}$)

T: trace of Stress-Energy tensor for all other matter fields

Eq of motion are:

$$\Box\phi + \frac{1}{\Lambda^3}\left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right] = \frac{T}{M_{pl}}$$

There is a Shift & Galileon Symmetry, indeed eom are invariant under

$$\phi \to \phi + c + v_\mu x^\mu$$

where c and v_μ are constant.

It is a Space-dependent global Symmetry. The Symmetry protects theory from ever generating a (large) mass term.

In Vacuum ($T=0$), we can have DE-like Solutions by taking $\phi \sim \alpha t \to \partial^2\phi \sim \alpha H_0$

$$\Rightarrow \frac{\alpha H_0}{\Lambda^3} \sim 1 \to \alpha \sim \frac{\Lambda^3}{H_0}$$

ii) Tuning & Non-Renormalization theorem

$$\rho_\phi \sim (\partial\phi)^2 + \frac{1}{\Lambda^3}(\partial\phi)^2\Box\phi \Rightarrow H_0^2 \sim \frac{\alpha^2}{M_{pl}^2} \rightarrow H_0 \sim \frac{\Lambda^3}{H_0 M_{pl}}$$

$$\boxed{H_0^2 \sim \frac{\Lambda^3}{M_{pl}^2}}$$

To have dark-energy behaviour,

$$\Lambda^3 \sim H_0^2 M_{pl} \sim (1000\,km)^{-3}$$
$$\sim (10^{-13}\,eV)^3$$

$$\Lambda \lll M_{pl}$$

of course $\frac{\Lambda^3}{M_{pl}^3} \sim \frac{H_0^2}{M_{pl}^2}$ Same Tuning as original cosmological constant, but it is a tuning which remains stable against quantum corrections.

These galileon Lagrangian satisfy a non-renormalization theorem which means that within this framework, higher order operators can be generated by quantum corrections (like for all the EFT, this is entirely normal), but these Galileon operators themselves do not get renormalized

iii) Vainshtein Screening

This Galileon models also exhibit a Veinsthein Screening Mechanism

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda^3}(\partial\phi)^2\Box\phi + \frac{\phi T}{M_{pl}}$$

$$\phi = \bar{\phi} + \delta\phi$$
$$T = \bar{T} + \delta T$$

$$\mathcal{L} = -\frac{1}{2} Z^{\mu\nu}\partial_\mu\delta\phi\,\partial_\nu\delta\phi + \frac{1}{\Lambda^3}(\delta\phi)^2\Box\delta\phi + \frac{\delta\phi\,\delta T}{M_{pl}}$$

$$Z^{\mu\nu} = \eta^{\mu\nu} + \underbrace{\frac{1}{\Lambda^3}\left[\partial^\mu\partial^\nu\bar{\phi} - \eta^{\mu\nu}\Box\bar{\phi}\right]}_{\gg 1 \text{ in vicinity of matter}}$$

Canonically Normalizing the Scalar field, Symbolically,

$\sqrt{Z}\,\delta\phi \sim \chi$ then

$$\mathcal{L}_\chi \sim -\frac{1}{2}(\partial\chi)^2 + \frac{1}{(\sqrt{Z}\Lambda)^3}(\partial\chi)^2\Box\chi + \frac{1}{\sqrt{Z}\,M_{pl}}\chi\,\delta T$$

with $Z \gg 1$, So $\Lambda_* \sim \sqrt{Z}\,\Lambda \gg M_{pl}$

and the Scalar field χ couples now to external Sources with coupling constant $\sqrt{Z}\,M_{pl} \gg M_{pl}$ So the force mediated by the Scalar field is Suppressed in the Vicinity of a Source

$\Rightarrow$ force Suppressed in Solar System, galaxy, Early Universe, ...