modern cosmology

ingredient 2: fluid mechanics

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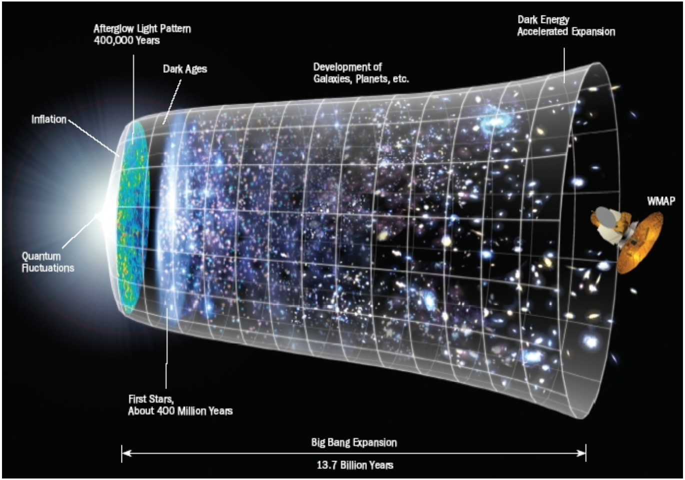
May 16, 2019

outline

1. inflation
2. random processes
3. CMB
4. secondary anisotropies
5. random processes
6. large-scale structure
7. CDM spectrum
8. structure formation

expansion history of the universe



expansion history of the universe

Planck-scale

- at $a = 0$, $z = \infty$ the metric diverges, and $H(a)$ becomes infinite
- description of general relativity breaks down, quantum effects become important
- relevant scales:
  - quantum mechanics: de Broglie-wave length: $\lambda_{QM} = \frac{2\pi\hbar}{mc}$
  - general relativity: Schwarzschild radius: $r_s = \frac{2Gm}{c^2}$
- setting $\lambda_{QM} = r_s$ defines the **Planck mass**

$$m_P = \sqrt{\frac{\hbar c}{G}} \simeq 10^{19} \mathrm{GeV}/c^2 \qquad (1)$$

question

how would you define the corresponding Planck length and the Planck time? what are their numerical values?

flatness problem

- construct a universe with matter $w = 0$ and curvature $w = -1/3$
- Hubble function

$$\frac{H^2(a)}{H_0^2} = \frac{\Omega_m}{a^3} + \frac{\Omega_K}{a^2} \tag{2}$$

- density parameter associated with curvature

$$\frac{\Omega_K(a)}{\Omega_K} = \frac{H_0^2}{a^{3(1+w)}H^2(a)} = \frac{H_0^2}{a^2 H^2(a)} \tag{3}$$

- Ω_K increases always and was smaller in the past

$$\Omega_K(a) = \left(1 + \frac{\Omega_m}{\Omega_K}\frac{1}{a}\right)^{-1} \simeq \frac{\Omega_K}{\Omega_m} a \tag{4}$$

- we know (from CMB observations) that curvature is very small today, typical limits are $\Omega_K < 0.01 \rightarrow$ **even smaller in the past**
- at recombination $\Omega_K \sim 10^{-5}$

horizon problem

- horizon size: light travel distance during the age of the universe

$$\chi_H = c \int \frac{da}{a^2 H(a)} \tag{5}$$

- assume $\Omega_m = 1$, integrate from $a_{min} = a_{rec} \dots a_{max} = 1$

$$\chi_H = 2\frac{c}{H_0}\sqrt{\Omega_m a_{rec}} = 175\sqrt{\Omega_m}\mathrm{Mpc}/h \tag{6}$$

- comoving size of a volume around a point at recombination inside which all points are in causal contact
- angular diameter distance from us to the recombination shell:

$$d_{rec} \simeq 2\frac{c}{H_0} a_{rec} \simeq 5\mathrm{Mpc}/h \tag{7}$$

- angular size of the particle horizon at recombination: $\theta_{rec} \simeq 2^\circ$
- points in the CMB separated by more than 2° have never been in causal contact $\to$ **why is the CMB so uniform?!**

inflation: phenomenology

- curvature $\Omega_K \propto$ to the **comoving Hubble radius** $c/(aH(a))$
- if by some mechanism, $c/(aH)$ could decrease, it would drive Ω_K towards 0 and solve the fine-tuning required by the flatness problem
- shrinking comoving Hubble radius:

$$\frac{d}{dt}\left(\frac{c}{aH}\right) = -c\frac{\ddot{a}}{\dot{a}^2} < 0 \rightarrow \ddot{a} > 0 \rightarrow q < 0 \qquad (8)$$

- equivalent to the notion of accelerated expansion
- accelerated expansion can be generated by a dominating fluid with sufficiently negative equation of state $w = -1/3$
- horizon problem: fast expansion in inflationary era makes the universe grow from a small, causally connected region

question

what's the relation between deceleration q and equation of state w?

inflaton-driven expansion

- analogous to dark energy, one postulates an **inflaton field** φ, with a small kinetic and a large potential energy, for having a sufficiently negative equation of state for accelerated expansion
- pressure and energy density of a homogeneous scalar field

$$p = \frac{\dot{\varphi}^2}{2} - V(\varphi), \quad \rho = \frac{\dot{\varphi}^2}{2} + V(\varphi) \tag{9}$$

- Friedmann equation

$$H^2(a) = \frac{8\pi G}{3}\left(\frac{\dot{\varphi}^2}{2} + V(\varphi)\right) \tag{10}$$

- continuity equation

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{dV}{d\varphi} \tag{11}$$

slow roll conditions

- inflation can only take place if $\dot{\varphi}^2 \ll V(\varphi)$
- inflation needs to keep going for a sufficiently long time:

$$\frac{d}{dt}\dot{\varphi}^2 \ll \frac{d}{dt}V(\varphi) \rightarrow \ddot{\varphi} \ll \frac{d}{d\varphi}V(\varphi) \tag{12}$$

- in this regime, the Friedmann and continuity equations simplify:

$$H^2 = \frac{8\pi G}{3}V(\varphi), \quad 3H\dot{\varphi} = -\frac{d}{d\varphi}V(\varphi) \tag{13}$$

- conditions are fulfilled if

$$\frac{1}{24\pi G}\left(\frac{V'}{V}\right)^2 \equiv \varepsilon \ll 1, \quad \frac{1}{8\pi G}\left(\frac{V''}{V}\right) \equiv \eta \ll 1 \tag{14}$$

- ε and η are called **slow-roll parameters**

stopping inflation

- flatness problem: shrinkage by $\simeq 10^{30} \simeq \exp(60) \rightarrow$ **60 e-folds**
- due to the slow-roll conditions, the energy density of the inflaton field is almost constant
- all other fluid densities drop by huge amounts, ρ_m by 10^{90}, ρ_γ by 10^{120}
- eventually, the slow roll conditions are not valid anymore, the effective equation of state becomes less negative, acclerated expansion stops
- but energy is stored in φ as kinetic energy $\dot{\varphi}^2$
- **reheating:** couple φ to other particle fields, and generate particles from the inflaton's kinetic energy
- how exactly reheating occurs, is largely unknown

generation of fluctuations

- fluctuations of the inflation field can perturb the distribution of all other fluids
- mean fluctuation amplitude is related to the variance of φ
- fluctuations in φ perturb the metric, and all other fluids feel a perturbed potential
- relevant quantity

$$\sqrt{\langle\delta\Phi^2\rangle} \simeq \frac{H^2}{V} \tag{15}$$

  which is approximately constant during slow-roll
- Poisson-equation in Fourier-space $k^2\Phi(k) = -\delta(k)$
- variance of density perturbations:

$$\left|\delta(k)\right|^2 \propto k^4\left|\delta\Phi\right|^2 \propto k^3 P(k) \tag{16}$$

- defines **spectrum** $P(k)$ of the initial fluctuations, $P(k) \propto k^n$ with $n \simeq 1$
- fluctuations are Gaussian, because of the central limit

random fields

- random process → probability density $p(\delta)d\delta$ of event δ
- alternatively: all moments $\langle\delta^n\rangle = \int d\delta\, \delta^n p(\delta)$
- in cosmology:
  - random events are values of the density field δ
  - outcomes for $\delta(\vec{x})$ form a statistical ensemble at fixed $\vec{x}$
  - ergodic random processes:
    one realisation is consistent with $p(\delta)d\delta$
- special case: Gaussian random field
  - only **variance** relevant

characteristic function $\varphi(t)$

- for a continuous pdf, all moments need to be known for reconstructing the pdf
- reconstruction via **characteristic function** $\varphi(t)$ (Fourier transform)

$$\varphi(t) = \int \mathrm{d}x p(x) \exp(itx) = \int \mathrm{d}x p(x) \sum_n \frac{(itx)^n}{n!} = \sum_n \langle x^n \rangle_p \frac{(it)^n}{n!} \tag{17}$$

with moments $\langle x^n \rangle = \int \mathrm{d}x x^n p(x)$

- Gaussian pdf is special:
  - all moments exist! (counter example: Cauchy pdf)
  - all odd moments vanish
  - all even moments are expressible as products of the variance
  - σ is enough to statistically reconstruct the pdf
  - pdf can be differentiated arbitrarily often (Hermite polynomials)
- funky notation: $\varphi(t) = \langle \exp(itx) \rangle$

cosmic microwave background

- inflation has generated perturbations in the distribution of matter
- the hot baryon plasma feels fluctuations in the distribution of (dark) matter by gravity
- at the point of (re)combination:
  - hydrogen atoms are formed
  - photons can propagate freely
- perturbations can be observed by two effects:
  - plasma was not at rest, but flowing towards a potential well → Doppler-shift in photon temperature, depending to direction of motion
  - plasma was residing in a potential well → gravitational redshift
- between the end of inflation and the release of the CMB, the density field was growth **homogeneously** → all statistical properties of the density field are conserved
- testing of inflationary scenarios is possible in CMB observations

formation of hydrogen: (re)combination

- temperature of the fluids drops during Hubble expansion
- eventually, the temperature is sufficiently low to allow the formation of hydrogen atoms
- but: high photon density (remember $\eta_B = 10^9$) can easily reionise hydrogen
- Hubble-expansion does not cool photons fast enough between recombination and reionisation
- neat trick: recombination takes place by a 2-photon process

question

at what temperature would the hydrogen atoms form if they could recombine directly? what redshift would that be?

CMB motion dipole

- the most important structure on the microwave sky is a dipole
- CMB dipole is interpreted as a relative motion of the earth
- CMB dispole has an amplitude of 10^{-3}K, and the peculiar velocity is $\beta = 371\mathrm{km/s} \cdot c$

$$T(\theta) = T_0\,(1 + \beta\cos\theta) \qquad (18)$$

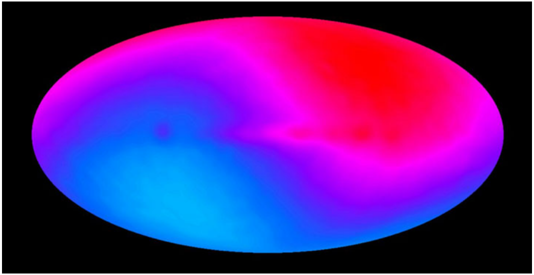
question

is the Planck-spectrum of the CMB photons conserved in a Lorentz-boost?

question

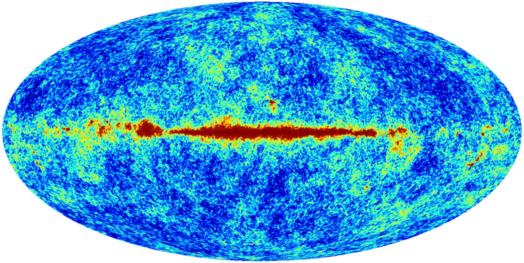
would it be possible to distinguish between a motion dipole and an intrinsic CMB dipole?

CMB dipole



source: COBE

subtraction of motion dipole: primary anisotropies



source: PLANCK simulation

CMB angular spectrum

- analysis of fluctuations on a sphere: decomposition in $Y_{\ell m}$

$$T(\theta) = \sum_\ell \sum_m t_{\ell m} Y_{\ell m}(\theta) \quad \leftrightarrow \quad t_{\ell m} = \int d\Omega\, T(\theta) Y^*_{\ell m}(\theta) \tag{19}$$

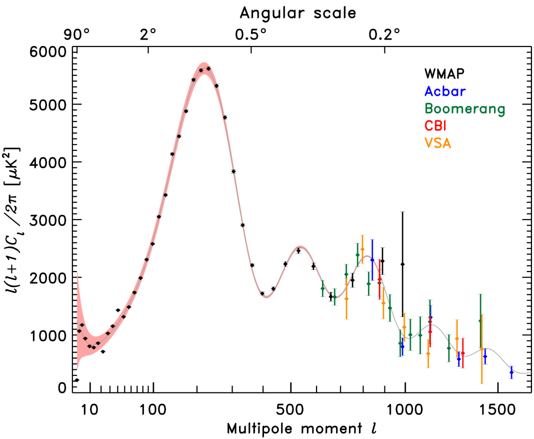
- spherical harmonics are an orthonormal basis system
- average **fluctuation variance** on a scale $\ell \simeq \pi/\theta$

$$C(\ell) = \langle |t_{\ell m}|^2 \rangle \tag{20}$$

- averaging $\langle \ldots \rangle$ is a hypothetical ensemble average. in reality, one computes an estimate of the variance,

$$C(\ell) \simeq \frac{1}{2\ell+1} \sum_{m=-\ell}^{m=+\ell} |t_{\ell m}|^2 \tag{21}$$

parameter sensitivity of the CMB spectrum



source: WMAP

features in the CMB spectrum

- predicting the spectrum $C(\ell)$ is **very complicated**
- perturbations in the CMB photons $n \propto T^3$, $u \propto T^4$, $p = u/3 \propto T^4$:

$$\frac{\delta n}{n_0} = 3\frac{\delta T}{T} \equiv \Theta, \quad \frac{\delta u}{u_0} = 4\Theta = \frac{\delta p}{p_0} \tag{22}$$

- continuity and Euler equations:

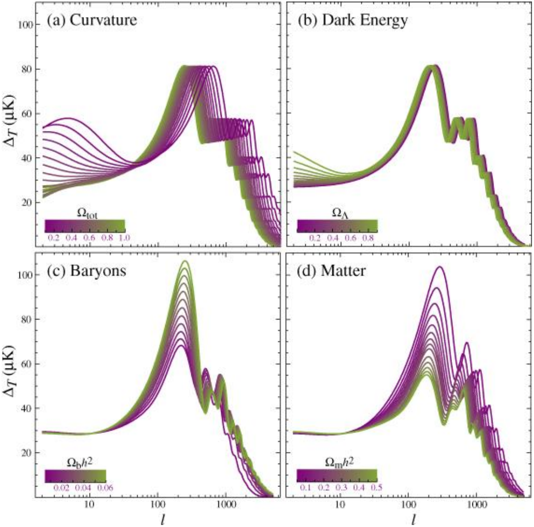
$$\dot{n} = n_0 \mathrm{div} \upsilon = 0, \quad \dot{\upsilon} = -c^2\frac{\nabla p}{u_0 + p_0} + \nabla\delta\Phi \tag{23}$$

- use $u_0 + p_0 = 4/3 u_0 = 4p_0$
- combine both equations

$$\ddot{\Theta} - \frac{c^2}{3}\Delta\Theta + \frac{1}{3}\Delta\delta\Phi = 0 \tag{24}$$

- identify two mechanisms:
  - oscillations may occur, and photons experience **Doppler shifts**
  - photons feel fluctuations in the potential: **Sachs-Wolfe effect**

parameter sensitivity of the CMB spectrum

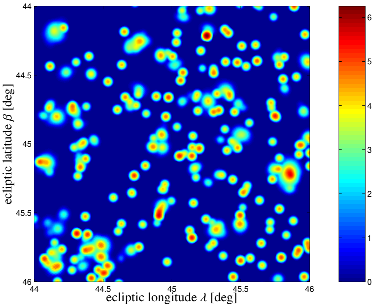


source: Wayne Hu

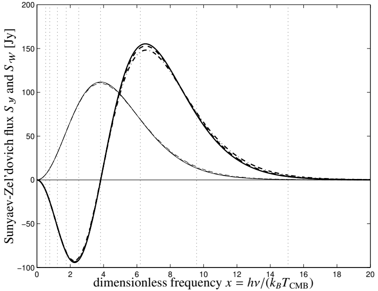
secondary CMB anisotropies

- CMB photons can do interactions in the cosmic large-scale structure on their way to us
- two types of interaction: **Compton-collisions** and **gravitational**
- consequence: secondary anisotropies
- study of secondaries is very interesting: observation of the growth of structures possible, and precision determination of cosmological parameters
- all effects are in general important on **small angular scales** below a degree

thermal Sunyaev-Zel'dovich effect



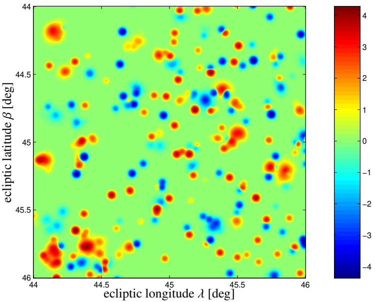
thermal SZ sky map



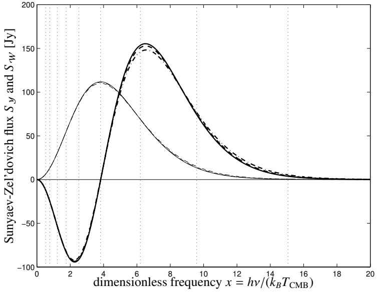
CMB spectrum distortion

- Compton-interaction of CMB photons with **thermal** electrons in clusters of galaxies
- characteristic redistribution of photons in energy spectrum

kinetic Sunyaev-Zel'dovich/Ostriker-Vishniac effect



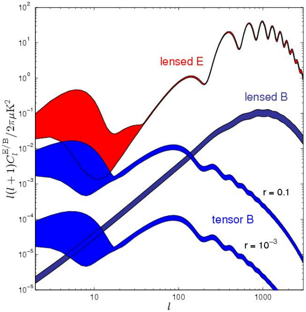
thermal SZ sky map



CMB spectrum distortion

- Compton-interaction of CMB photons with electrons in **bulk flows**
- increase/decrease in CMB temperature according to direction of motion

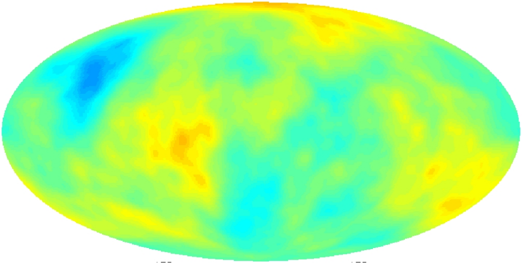
CMB lensing



source: A. Lewis, A. Challinor

- gravitational deflection of CMB photons on potentials in the cosmic large-scale structure
- CMB spots get distorted, and their fluctuation statistics is changed, in particular the polarisation

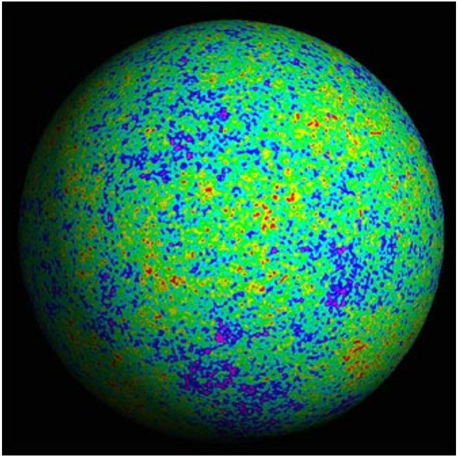
integrated Sachs-Wolfe effect



source: B. Barreiro

- gravitational interaction of photons with time-evolving potentials
- higher-order effect on photon geodesics in general relativity

inflationary fluctuations in the CMB



source: WMAP

random processes

- inflation generates fluctuations in the distribution of matter
  - fluctuations can be seen in the cosmic microwave background
  - seed fluctuations for the large-scale distribution of galaxies
  - amplified by self-gravity
- **cosmology is a statistical subject**
- fluctuations form a **Gaussian random field**
- random processes: specify
  - probability density $p(x)dx$
  - covariance, in the case of multivariate processes $p(\vec{x})d\vec{x}$
- measurement of $p(x)dx$ by determining moments $\langle x^n\rangle = \int dx\, x^n p(x)$
- cosmology: random process describes the fluctuations of the **overdensity**

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} \tag{25}$$

with the mean density $\bar{\rho} = \Omega_m \rho_{crit}$

double pendulum

- **simple example of a random process**
- double pendulum is a chaotic system, dynamics depends **very** sensitively on tiny changes in the initial condition
- random process: imagine you want to know the distribution of φ one minute after starting
  - move to initial conditions and let go
  - wait 1 minute and measure φ (one realisation)
  - repeat experiment $\rightarrow$ distribution $p(\varphi)d\varphi$ (ensemble of realisations)
- 2 more types of data
  - distributions and moments of more than one observable
  - moments of observables across different times

question

write down the Lagrangian, perform variation and derive the equation of motion! show that there is a nonlinearity

double pendulum: ergodicity and homogeneity

ergodicity

with time, the dynamics generates values for the observables with the same probability as in the statistical ensemble, $p(\varphi(t))dt \propto p(\varphi)d\varphi$

- time averaging = ensemble averaging, for measuring moments

homogeneity

statistical properties are invariant under time-shifts Δt
$p(\varphi(t))d\varphi = p(\varphi(t + \Delta t))d\varphi$

- necessary condition for ergodicity
- double pendulum: not applicable if there is dissipation

Gaussian random fields in cosmology

- fluctuations in the density field are a Gaussian random process $\rightarrow$ sufficient to measure the **variance**
  - ergodicity: postulated (theorem by Adler)
  - volume averages are equivalent to ensemble averages

$$\langle\delta^n\rangle = \frac{1}{V}\int_V d^3x\,\delta^n(\vec{x})p(\delta(\vec{x})) \tag{26}$$

  - homogeneity: statistical properties independent of position $\vec{x}$

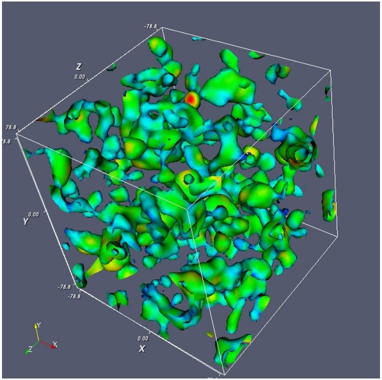
$$p(\delta(\vec{x})) \propto p(\delta(\vec{x}+\Delta\vec{x})) \tag{27}$$

- the density field is a 3d random field $\rightarrow$ **isotropy**

$$p(\delta(\vec{x})) = p(\delta(R\vec{x})), \text{ for all rotation matrices } R \tag{28}$$

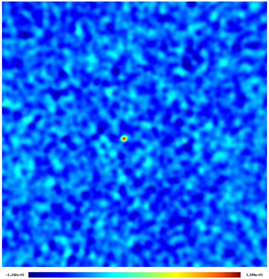
- finite correlation length: amplitudes of δ at two positions $\vec{x}_1$ and $\vec{x}_2$ are not independent:
  - covariance needed for Gaussian distribution $p(\delta(\vec{x}_1),\delta(\vec{x}_2))$
  - measurement of cross variance/covariance $\langle\delta(\vec{x}_1)\delta(\vec{x}_2)\rangle$
  - $\langle\delta(\vec{x}_1)\delta(\vec{x}_2)\rangle$ is called **correlation function** ξ

Gaussian random field

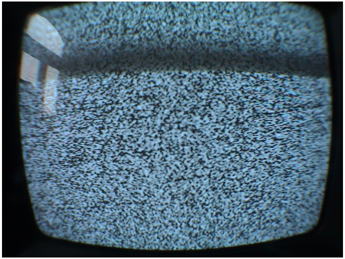


isodensity surfaces, threshold 2.5σ, shading ∼ local curvature, CDM power spectrum, smoothed on 8 Mpc/h-scales

statistics: correlation function and spectrum



finite correlation length



zero correlation length

correlation function

quantification of fluctuations: correlation function $\xi(\vec{r}) = \langle\delta(\vec{x}_1)\delta(\vec{x}_2)\rangle$, $\vec{r} = \vec{x}_2 - \vec{x}_1$ for Gaussian, homogeneous fluctuations, $\xi(\vec{r}) = \xi(r)$ for isotropic fields

statistics: correlation function and spectrum

- Fourier transform of the density field

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\,\delta(\vec{k})\exp(i\vec{k}\vec{x}) \leftrightarrow \delta(\vec{k}) = \int d^3x\,\delta(\vec{x})\exp(-i\vec{k}\vec{x}) \tag{29}$$

- variance $\langle\delta(\vec{k}_1)\delta^*(\vec{k}_2)\rangle$: use homogeneity $\vec{x}_2 = \vec{x}_1 + \vec{r}$ and $d^3x_2 = d^3r$

$$\langle\delta(\vec{k}_1)\delta^*(\vec{k}_2)\rangle = \int d^3r\,\langle\delta(\vec{x}_1)\delta(\vec{x}_1+\vec{r})\rangle\exp(-i\vec{k}_2\vec{r})(2\pi)^3\delta_D(\vec{k}_1-\vec{k}_2) \tag{30}$$

  - definition spectrum $P(\vec{k}) = \int d^3r\,\langle\delta(\vec{x}_1)\delta(\vec{x}_1+\vec{r})\rangle\exp(-i\vec{k}\vec{r})$
  - spectrum $P(\vec{k})$ is the Fourier transform of the correlation function $\xi(\vec{r})$
  - homogeneous fields: Fourier modes are mutually uncorrelated
  - isotropic fields: $P(\vec{k}) = P(k)$

Gaussianity and the characteristic function

- for a continuous pdf, all moments need to be known for reconstructing the pdf
- reconstruction via **characteristic function** $\varphi(t)$ (Fourier transform)

$$\varphi(t) = \int dx p(x) \exp(itx) = \int dx p(x) \sum_n \frac{(itx)^n}{n!} = \sum_n \langle x^n \rangle_p \frac{(it)^n}{n!}$$

with moments $\langle x^n \rangle = \int dx x^n p(x)$

- Gaussian pdf is special:
  - all moments exist! (counter example: Cauchy pdf)
  - all even moments are expressible as products of the variance
  - σ is enough to statistically reconstruct the pdf
  - pdf can be differentiated arbitrarily often (Hermite polynomials)

question

show that for a Gaussian pdf $\langle x^{2n} \rangle \propto \langle x^2 \rangle^n$, what's $\varphi(t)$?

moment generating function

- variance σ^2 characterises a Gaussian pdf completely
- $\langle x^{2n}\rangle \propto \langle x^2\rangle^n$, but what is the constant of proportionality?
- look at the **moment generating function**

$$M(t) = \int dx p(x) \exp(tx) = \langle \exp(tx)\rangle_p = \sum_n \langle x^n\rangle_p \frac{t^n}{n!}$$

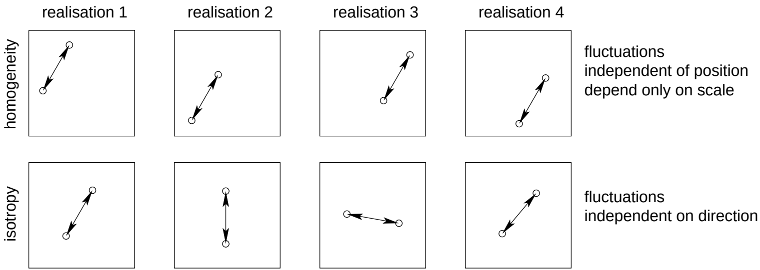
- $M(t)$ is the Laplace transform of pdf $p(x)$, and $\varphi(t)$ is the Fourier transform
- nth derivative at $t = 0$ gives moment $\langle x^n\rangle_p$:

$$M'(t) = \langle x \exp(tx)\rangle_p = \langle x\rangle_p$$

question

compute $\langle x^n\rangle$, $n = 2, 3, 4, 5, 6$ for a Gaussian directly (by induction) and with the moment generating function $M(t)$

homegeneity and isotropy in $\xi(r)$



isotropy and homogeneity in an ensemble

- homogeneity: a measurement of $\langle\delta(\vec{x})\delta(\vec{x}+\vec{r})\rangle$ is independent of $\vec{x}$, if one averages over ensembles
- isotropy: a measurement of $\langle\delta(\vec{x})\delta(\vec{x}+\vec{r})\rangle$ does not depend on the direction of $\vec{r}$, in the ensemble averaging

why correlation functions?

a proof for climate change and global warming

please be careful: we measure the correlation function because it characterises the random process generating a realisation of the density field, not because there is a badly understood mechanism relating amplitudes at different points!
(PS: don't extrapolate to 2009)

tests of Gaussianity

Gaussianity

all moments needed for reconstructing the probability density

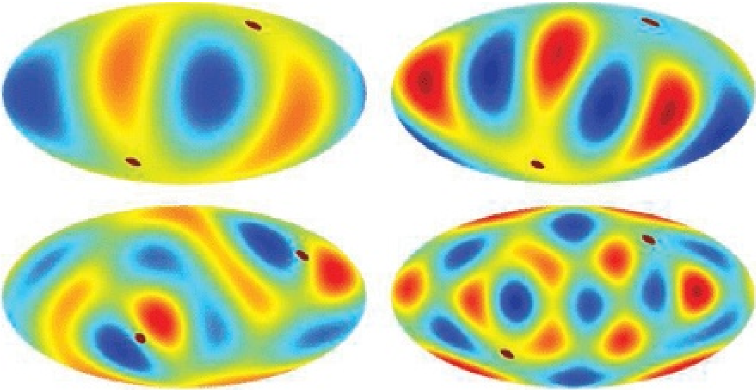
- data is finite: only a limited number of estimators are available
- classical counter example: **Cauchy-distribution**

$$p(x)dx \propto \frac{dx}{x^2 + a^2} \tag{31}$$

$\rightarrow$ all even moments are infinite

- genus statistics: peak density, length of isocontours
- independency of Fourier modes

tests of Gaussianity: axis of evil



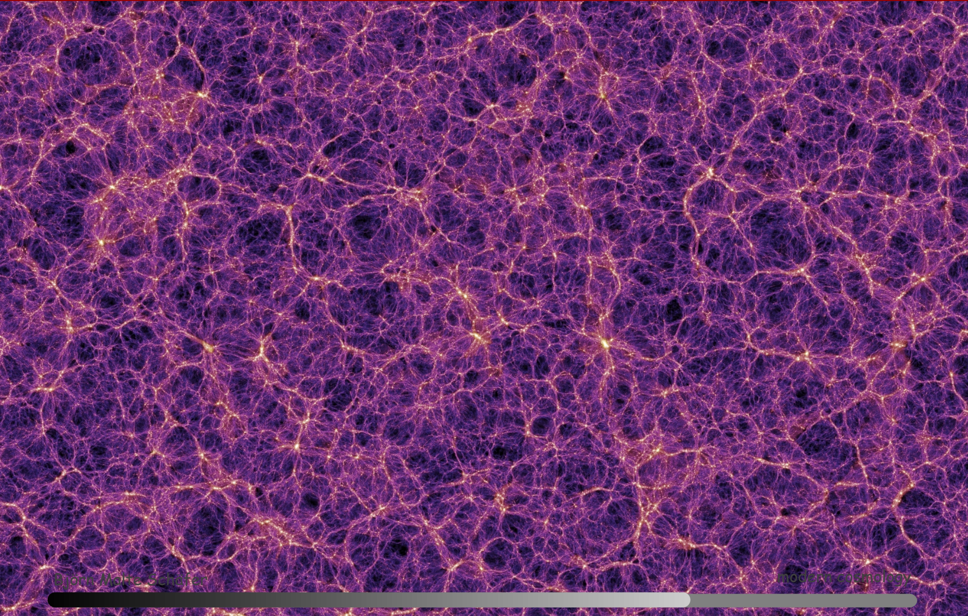
CMB axis of evil: multipole alignment

- CMB-sky: weird (unprobable) alignment between low multipoles

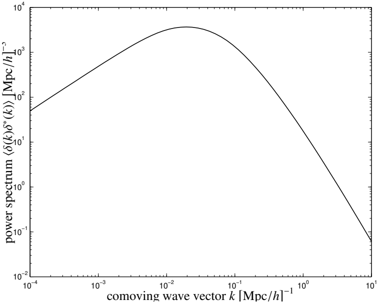
weak and strong Gaussianity

- differentiate weak and strong Gaussianity
- strong Gaussianity: Gaussian distributed amplitudes of Fourier modes
  - implies Gaussian amplitude distribution in real space
  - argumentation: via cumulants
- weak Gaussianity: central limit theorem
  - assume independent Fourier modes, but arbitrary amplitude distribution in Fourier space
  - Fourier transform: many elementary waves contribute to amplitude at a given point
  - central limit theorem: sum over a large number of independent random numbers is Gaussian distributed
  - field in real space is approximately Gaussian, even though the Fourier modes can be arbitrarily distributed

the cosmic web (Millenium simulation)

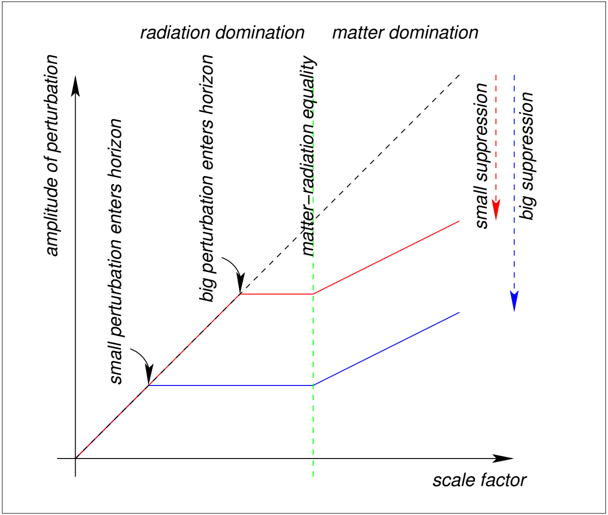


CDM spectrum P(k) and the transfer function T(k)



- ansatz for the CDM power spectrum: $P(k) = k^{n_s} T(k)^2$
- small scales suppressed by radiation driven expansion $\rightarrow$ **Meszaros-effect**
- asymptotics: $P(k) \propto k$ on large scales, and $\propto k^{-3}$ on small scales

Meszaros effect 1



Meszaros effect 2

- perturbation grows $\propto a^2$ outside horizon in the radiation-dominated era (really difficult to understand, need covariant perturbation theory)
- when entering the horizon, fast radiation driven expansion keeps perturbation from growing, dynamical time-scale $t_{dyn} \gg t_{Hubble} = 1/H(a)$
- all perturbations start growing at the time of matter-radiation equality ($z \simeq 7000$, $\Omega_M(z) = \Omega_R(z)$), growth $\propto a$
- size of the perturbation corresponds to scale factor of the universe at horizon entry
- total suppression is $\propto k^{-2}$, power spectrum $\propto k^{-4}$
- exact solution of the problem: numerical solution for transfer function $T(k)$, with shape parameter Γ, which reflects the matter density

CDM shape parameter Γ

- exact shape of $T(k)$ follows from Boltzmann codes
- express wave-vector k in units of the shape parameter:

$$q \equiv \frac{k/\mathrm{Mpc}^{-1}h}{\Gamma} \qquad (32)$$

- Bardeen-fitting formula for low-Ω_m cosmologies

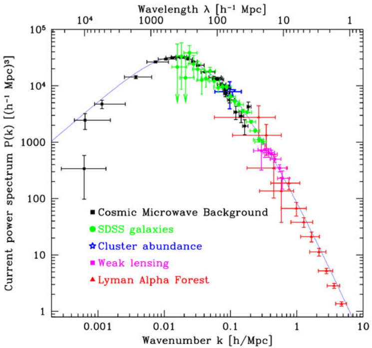
$$T(q) = \frac{\ln(1+eq)}{eq} \times \left[1 + aq + (bq)^2 + (cq)^3 + (dq)^4\right]^{-\frac{1}{4}},$$

- to good approximation $\Gamma = \Omega_m h$
- small $\Gamma \rightarrow$ skewed to left, big $\Gamma \rightarrow$ skewed to right

question

verify the asymptotic behaviour of $T(q)$ for $q \ll 1$ and $q \gg 1$

observational constraints on $P(k)$



data for $P(k)$ from observational probes

- many observational channels are sensitive to $P(k)$
- amazing agreement for the shape

normalisation of the spectrum: σ_8

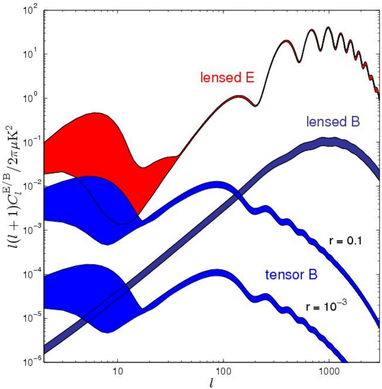
- CDM power spectrum $P(k)$ needs to be normalised
- observations: fluctuations in the galaxy counts on 8 Mpc/h-scales are approximately constant and $\simeq 1$ (Peebles)
- introduced filter function $W(\vec{x})$
- convolve density field $\delta(\vec{x})$ with filter function $W(\vec{x})$ in real space $\rightarrow$ multiply density field $\delta(\vec{k})$ with filter function $W(\vec{k})$ in Fourier space
- convention: σ_8, $R = 8$ Mpc/h

$$\sigma_8^2 = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}k\, k^2 P(k) W^2(kR) \tag{33}$$

with a spherical top-hat filter $W(kR)$

- least accurate cosmological parameter, discrepancy between WMAP, lensing and clusters

lensing and CMB constraints on σ_8



constraints on Ω_m and σ_8

- some tension between best-fit values
- possibly related to measurement of galaxy shapes in lensing

cosmological standard model

cosmology + structure formation are described by:

- dark energy density Ω_φ
- cold dark matter density Ω_m
- baryon density Ω_b
- dark energy density equation of state parameter w
- Hubble parameter h
- primordial slope of the CDM spectrum n_s, from inflation
- normalisation of the CDM spectrum σ_8

cosmological standard model: 7 parameters

known to few percent accuracy, amazing predictive power

properties of dark matter

current paradigm:

structures from by gravitational instability from inflationary fluctuations in the cold dark matter (CDM) distribution

- collisionless → very small interaction cross-section
- cold → negligible thermal motion at decoupling, no cut-off in the spectrum $P(k)$ on a scale corresponding to the diffusion scale
- dark → no interaction with photons, possible weak interaction
- matter → gravitationally interacting

main conceptual difficulties

- collisionlessness → hydrodynamics, no pressure or viscosity
- non-saturating interaction (gravity) → extensivity of binding energy

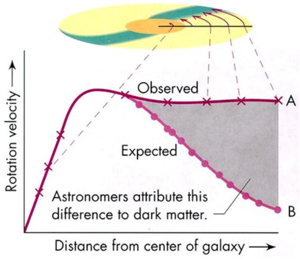
dark matter and the microwave background

- fluctuations in the density field at the time of decoupling are $\simeq 10^{-5}$
- long-wavelength fluctuations grow proportionally to a
- if the CMB was generated at $a = 10^{-3}$, the fluctuations can only be 10^{-2} today
- large, supercluster-scale objects have $\delta \simeq 1$

cold dark matter

need for a **nonbaryonic** matter component, which is not interacting with photons

galaxy rotation curves



- balance centrifugal and gravitational force
- difficulty: measured in low-surface brightness galaxies
- galactic disk is embedded into a larger halo composed of CDM

question

show that the density profile of a galaxy needs to be $\rho \propto 1/r^2$

structure formation equations

cosmic structure formation

cosmic structures are generated from tiny inflationary seed fluctuations, as a fluid mechanical, self-gravitating phenomenon (with Newtonian gravity), on an expanding background

- continuity equation: no matter ist lost or generated

$$\frac{\partial}{\partial t}\rho + \mathrm{div}(\rho\vec{\upsilon}) = 0 \tag{34}$$

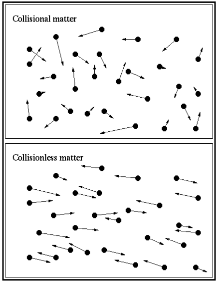
- Euler-equation: evolution of velocity field due to gravitational forces

$$\frac{\partial}{\partial t}\vec{\upsilon} + \vec{\upsilon}\nabla\vec{\upsilon} = -\nabla\Phi \tag{35}$$

- Poisson-equation: potential is sourced by the density field

$$\Delta\Phi = 4\pi G\rho \tag{36}$$

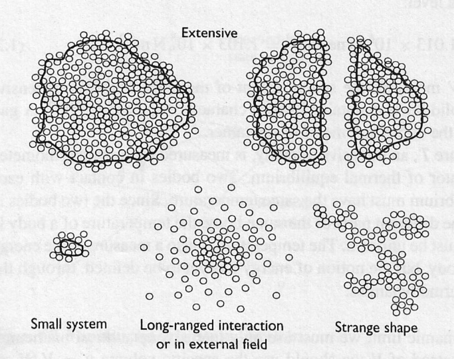
collisionlessness of dark matter



source: P.M. Ricker

- CDM is collisionless (elastic collision cross section ≪ neutrinos)
  - why can galaxies rotate and how is vorticity generated?
  - why do galaxies form from their initial conditions without viscosity?
  - how can one stabilise galaxies against gravity without pressure?
  - is it possible to define a temperature of a dark matter system?

non-extensivity of gravity



source: Kerson Huang, statistical physics

- gravitational interaction is long-reached
- gravitational binding energy per particle is not constant for $n \to \infty$