modern cosmology

ingredient 1: general relativity

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outline

cosmology: topics and aims

physical cosmology

cosmology aims to describe the dynamics of the universe and the formation of structures such as galaxies and clusters, as well as their properties, using physical models

- physical cosmology has 3 main building blocks
  1. general relativity: dynamics of the universe
  2. fluid mechanics: formation of structures by self gravity
  3. statistics: description of structures
- cosmology is an observational science, and uses a number of techniques: galaxy surveys, lensing surveys, primary and secondary CMB anisotropies, supernovae

cosmology and philosophy

physical cosmology

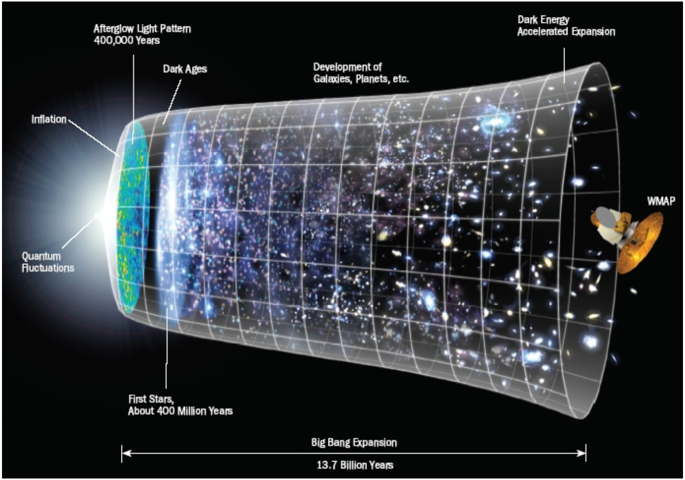
is cosmology a branch of science?

- repeatability of the experiments partly possible
- fundamental assumptions can **never** be tested
- observations replace experiments, change of setup not possible
- observation of random processes, questions of ergodicity
- fundamental statistical limitations (cosmic variance, finite observable volume)

aims of this course

1. understand the main ingredients of modern cosmology
2. understand the types of observations, and how cosmological models are tested
3. introduce the standard model of cosmology ΛCDM
4. understand the basic parameter set, and how the values are derived
5. understand the need of certain properties of the standard model
6. get an idea of future developments and experiments

history of the universe



expansion history of the universe

typical numbers...

- distant galaxies seem to fly away from the observer
- recession velocity is proportional to distance $\vec{\upsilon} = H\vec{r}$, constant of proportionality: Hubble constant $H = 100h$ km/s/Mpc, with $h = 0.72$
- get typical scales for length, time and density with the natural constants c and G

$$t_H = \frac{1}{H_0}, \quad x_H = \frac{c}{H_0}, \quad \rho_{crit} = \frac{3H_0^2}{8\pi G} \tag{1}$$

question

compute the numerical values!

question

is general relativity needed for the dynamics of the universe? compare Schwarzschild radius $r_s = 2GM/c^2$ and x_H!

history of cosmology

- W. Herschel: star counts, rough idea of the shape of the milky way
- E. Hubble: resolves stars in spiral nebulae: galaxies on their own
- E. Hubble: recession velocity of galaxies: dynamic cosmology
- A. Einstein: general relativity
- G. Lemaître: cosmological models based on general relativity
- A. Friedmann: expanding universes
- J. Peebles: structure formation by gravitational amplification
- A. Guth: inflation, initial fluctuations in the density field
- M. Rees, S. White: dark matter, ΛCDM paradigm

(physical) cosmology is a very young discipline!

Newtonian cosmology

- use Newtonian gravity for describing dynamics of the universe
- basic assumptions
  1. Euclidean (flat) space
  2. homogeneous distribution of matter
  3. isotropic expansion
- consider a test particle on the surface of a sphere

$$\ddot{r} = -\frac{GM}{r^2} \quad \text{with} \quad M = \frac{4\pi}{3}\rho r^3 \tag{2}$$

- results from Newton's law

$$\ddot{r} = -\frac{\partial}{\partial 4}\Phi \quad \text{with} \quad \Delta\Phi = 4\pi G\rho \tag{3}$$

- comoving coordinate: $r = ax$, a: scale-factor

$$\ddot{a} = -\frac{4\pi G}{3}\rho a \quad \rightarrow \quad \frac{\dot{a}^2}{2} = \frac{GM}{a} + E \tag{4}$$

critical density

- E: integration constant, 3 possible types of solutions
  1. $E > 0$: elliptic
  2. $E < 0$: hyperbolic
  3. $E = 0$: parabolic
- evolution of scale-factor a:

$$E = \frac{\dot{a}^2}{2} - \frac{GM}{a} \quad \rightarrow \quad \frac{E}{a^2} = \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 - \frac{GM}{a^3} = \frac{H^2}{2} - \frac{4\pi G}{3} \tag{5}$$

- conditions for parabolic solution:

$$E = 0 \leftrightarrow \rho_{crit} = \frac{3H_0^2}{8\pi G} \tag{6}$$

- observationally: $\rho_{obs} = \Omega_m \rho_{crit}$ with $\Omega_m = 0.25$

question

what is the numerical value of ρ_{crit}?

concepts of general relativity

- **metric**: distance between two points (example: light travel time)
  - symmetry $d(x, y) = d(y, x)$
  - positive definiteness $d(x, y) > 0$, $d(x, x) = 0$
  - triangle inequality $d(x, y) + d(y, z) \geq d(x, z)$
- **line element**: distances are defined $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
  - position dependent metric tensor $g_{\mu\nu}$
  - recover Minkowski-metric $\eta_{\mu\nu}$ for empty space
- **Einstein field equation**: energy momentum tensor $T_{\mu\nu}$ sources $g_{\mu\nu}$
  - fancy Poisson equation of the type $\Delta\Phi \propto \rho$
  - nonlinear field equation
  - cosmological constant
- **geodesics**: trajectories are influenced by the metric
  - photons follow $ds^2 = 0$
  - massive particles: geodesic equation

Friedmann-Lemaître-Robertson-Walker universes

- relativistic world model: metric is a solution to the field equation
    - homogeneous distribution of matter (Copernican principle)
    - work with a maximally symmetric metric
    - solve for a size-time relation $a(t)$
- solutions due to Friedmann and Lemaitre
- Einstein field equation

$$G_{\mu\nu} = R_{\mu\nu} + \frac{R}{2} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu}$$

- source: energy momentum tensor $T_{\mu\nu}$, with 4-velocity of the fluid u_μ

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu}$$

- Λ: introduced for constructing static solutions, but follows naturally from a variational principle (Einstein-Hilbert Lagrangian)

symmetry of the metric

- Robertson-Walker metric: $g_{\mu\nu}$ of an isotropic matter distribution

$$ds^2 = c^2dt^2 + g_{ij}dx^i dx^j = c^2dt^2 - a^2(t)d\vec{r}^2$$

with scale-factor $a(t)$

- use spherical coordinates χ, θ, φ

$$ds^2 = c^2dt^2 - a^2\left[d\chi^2 + f_K^2(\chi)(d\theta^2 + \sin^2\theta d\varphi^2)\right]$$

- global curvature of the metric K:
  - spherical $(K > 0)$: $f_K(\chi) = \frac{1}{\sqrt{K}}\sin\left(\frac{\chi}{\sqrt{K}}\right)$
  - euclidean/flat $(K = 0)$: $f_K(\chi) = \chi$
  - hyperbolical $(K < 0)$: $f_K(\chi) = \frac{1}{\sqrt{K}}\sinh\left(\frac{\chi}{\sqrt{K}}\right)$

cosmological redshift

- 2 equivalent interpretations
  - light waves are stretched as they propagate through a non-static metric, stretching $\propto a$
  - distance objects move away with the Hubble flow and the light emitted is Doppler-redshifted
- wave length λ, redshift z and scale factor $a = \lambda_e/\lambda_o$

$$z = \frac{\lambda_o}{\lambda_e} - 1 \rightarrow a = \frac{1}{1+z}$$

- two effects: each photon looses energy and the photon number flux is decreased
- big bang has infinite redshift (in principle unobservable)
- CMB: photons are generated at 3000K at $z = 10^3 \rightarrow$ CMB temperature of 3K

Friedmann equations

- solve Einstein-equation with a homogeneous fluid and the RW-line element ($ds \to g_{\mu\nu} \to R_{\mu\nu\rho\sigma} \to R_{\mu\nu} \to R$)
- keep cosmological constant Λ
- 2 Friedmann equations (temporal and spatial)
  - define Hubble function $H = \dot{a}/a = d(\ln a)/dt$

$$\frac{\dot{a}}{a} = \frac{8\pi G}{3}\rho - K\frac{c^2}{a^2} + \frac{\Lambda}{3}$$

  - acceleration parameter $q = \ddot{a}a/\dot{a}^2$

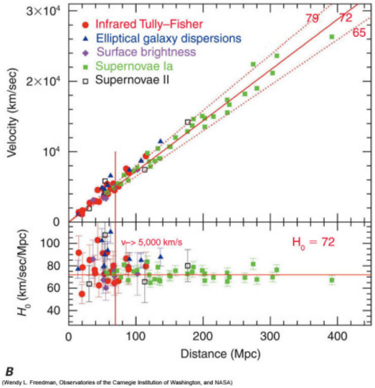
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right) + \frac{\Lambda}{3}$$

- critical density $\rho_{crit} = \frac{3H_0^2}{8\pi G} \simeq 10^{-29} g/cm^3 \simeq 3 \times 10^{11} M_\odot/Mpc^3$
- flatness: total density adds up to critical density
- define density parameters $\Omega = \rho/\rho_{crit}$

misconceptions about relativity and cosmology

- where does the universe expand into?
  → only the metric changes
- where did the big bang happen?
  → the RW-metric is homogeneous - so it happened at every point! more exactly - every observer would have experienced the big bang at the same time along his world line
- can we observe the big bang?
  → no! it is infinitely redshifted
- are recession velocities close to c unphysical?
  → no! there is no Lorentz-boost that transforms from our inertial frame to that of a receeding galaxy
- is $c/H_0 = 3$ Gpc the size of the universe?
  → no! it is a size scale, and the light horizon. the universe is infinite, but we only observe a spherical region of the size c/H_0

Hubble expansion



Hubble diagramme

- recession velocity of distant objects: need for dynamical cosmology
- redshift was originally interpreted as a galaxy evolution parameter!

Friedmann eqns: evolution of a in FLWR-universes

- substitution of RW-line element into field equation yields:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \qquad (7)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad (8)$$

for a homogeneous ideal fluid with density ρ and pressure p

- Hubble function $H(a)$ and deceleration parameter

$$H(a) = \frac{\dot{a}}{a} = \frac{d}{dt}\ln a \quad \text{and} \quad q(a) = -\frac{\ddot{a}a}{\dot{a}^2} \qquad (9)$$

question

the two Friedmann equations are equivalent, but why does curvature appear in the $\dot{a}$-equation, but not in the expression for $\ddot{a}$?

cosmological fluids and equation of state

- adiabatic equation: combine the two Friedmann equations

$$\frac{d}{da}\left(a^3\rho(a)\right)-p\frac{d}{da}\left(a^3\right)=0 \quad \text{or, equivalently} \quad 3H(a)\left(p+\rho\right)+\dot{\rho}=0. \tag{10}$$

- introduce equation of state parameter w

$$p = w\rho. \tag{11}$$

- adiabatic equation describes the change of energy density in Hubble expansion

question

show that for a universe with no curvature the relation between deceleration and eos parameter is given by: $q = \frac{3(1+w)}{2} - 1$.

equation of state: overview

fluid	$\rho(a)$	$H(a)$	w	q
radiation	$\propto a^{-4}$	$\propto a^{-2}$	$+1/3$	1
matter	$\propto a^{-3}$	$\propto a^{-3/2}$	0	$1/2$
curvature	$\propto a^{-2}$	$\propto a^{-1}$	$-1/3$	0
dark energy	$\propto a^{-2\ldots0}$	$\propto a^{-1\ldots0}$	$-1/3\ldots-1$	$0\ldots-1$
Λ	$=\mathrm{const}$	$=\mathrm{const}$	-1	-1

question

fluids with $w < -1$ are called phantom dark energy. what is so weird about them?

negative equation of state

- fluids with negative eos w are very important (negative pressure)
  - the cosmological constant Λ has $w = -1$
  - dark energy is constructed to have **time-varying** $w = -1/3 \ldots -1$
- Hubble function for a multi-component universe with dark energy and matter, but critical density:

$$\frac{H^2(a)}{H_0^2} = \frac{\Omega_m}{a^3} + \Omega_\varphi \exp\left(3\int_a^1 \mathrm{d}\ln a\, \left[1 + w(a)\right]\right) \tag{12}$$

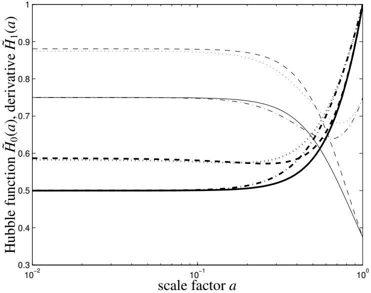
question

show that $w < -1/3$ implies accelerated expansion and that $w = -1$ implies a constant Hubble function

question

simplify eqn. (12) to a constant dark energy eos w, and to

Hubble function H(a): expansion velocity



scaled Hubble function $a^{3/2}H(a)/H_0$ **and derivative** $a^{5/2}dH(a)/da/H_0$

- Hubble function is monotonically decreasing and infinite at $a = 0$
- representation: $a^{3/2}H(a)/H_0$, because $H(a) \propto a^{-3/2}$ in $\Omega_m = 1$

question

if dark energy dominates: at what redshift is $q = 0$ or $\Omega_m = \Omega_\Lambda$?

curvature

- curvature is a nonlinearity in the field equation
- formally $w = -1/3$, although curvature is not a physical substance!
- solutions (fully curved, empty universe, $\Omega_k = 1$) imply:
  - deceleration vanishes, $q = 0$
  - Hubble expansion is constant, $\dot{a} = \mathrm{const}$ (but not $H(a)$!)
- **distinguish carefully** between geometry and dynamics
  - an matter-underdense universe is hyperbolic and expands forever
  - a matter-overdense universe is spherical and recollapses
  - multicomponent fluids are more complicated! construction of critical universes is possible, with accelerating dynamics (ΛCDM)
- curvature is special: it is the only energy density, which can be negative $\Omega_k < 0$, in which case the curvature is hyperbolic

dark energy

- matter and radiation are physical fluids with $w = 0$ and $w = +1/3$
- curvature and cosmological constant are GR phenomena with $w = -1/3$ and $w = -1$
- is it possible to construct a fluid with **varying negative eos**?
- consider a scalar field φ with self-interaction $V(\varphi)$
  - total energy $\rho = \dot{\varphi}^2 + V(\varphi)$
  - pressure $p = \dot{\varphi}^2 - V(\varphi)$

$$w = \frac{p}{\rho} = \frac{\dot{\varphi}^2 - V(\varphi)}{\dot{\varphi}^2 + V(\varphi)} \tag{13}$$

- **slow roll**: consider the limit $\dot{\varphi}^2 \ll V(\varphi)$

$$w \rightarrow -1 + \varepsilon \tag{14}$$

- fluids with low kinetic and high potential energy have negative w

why dark energy and Λ are two different things

- Λ is part of the gravitational theory
- slow-roll ($w = -1$) is perfectly fulfilled and holds always. dark energy is driven by $V(\varphi)$ and naturally builds up $\dot{\varphi}$, so that w moves away from -1
- part of the vacuum equations, no external (scalar) field needed
- naturally appears when deriving the field equation from the Einstein-Hilbert action in a variational approach (see lecture of M. Bartelmann, Lovelock-theorem for constructing S_{grav})
- any dark energy theory still would need to explain why Λ is zero

never think Λ is just dark energy with $w = -1$!

- dark energy is necessarily dynamic and changes its eos w with time
- CPL-parameterisation:

observations in FLRW-cosmologies

- 2 things are in principle observable in (homogeneous) cosmology
  - Hubble function $H(a)$, with **geometrical** probes
  - formation of structure $D_+(a)$, with **structure formation** probes
- geometrical probes measure cosmological distances, while taking care of the evolving metric
- distance measures are **not unique**, 4 different sensible definitions
- assumptions:
  - Copernican principle (isotropic and homogeneous metric)
  - general relativity is the gravitational theory
  - homogeneous, ideal fluids
- observations have **degeneracies** between the parameters, especially in multi-component fluids

distance measures: proper distance

- proper distance p is the light travel time of a photon $dp = -cdt$ emitted at a_e and absorbed at a_a
- $dp = -cda/(aH)$ with $da/dt = aH(a) \leftrightarrow dt = da/(aH)$, and therefore

$$p = c\int_{a_e}^{a_a} \frac{da}{aH(a)} \qquad (16)$$

- unit of p given in Hubble distance $d_H = c/H_0 \simeq 3$ Gpc/h

question

proper distance is related to lookback time. how much time has passed since the light of a quasar at redshift $z = 5$ was emitted?

distance measures: comoving distance

- comoving distance χ is the distance on a spatial hypersurface between the world lines of a source and the observer moving with the Hubble flow
- photon geodesics are defined by $ds = 0$ (Fermat's principle)
- therefore $cdt = -ad\chi$ (from metric), $d\chi = -cda/(a^2H)$

$$\chi = c\int_{a_e}^{a_a} \frac{da}{a^2H(a)} \tag{17}$$

- complete analogy to conformal time $d\eta = da/(a^2H)$, such that $\chi = c\eta$

question

compute the comoving distance in ΛCDM to a high redshift quasar ($z = 5$), and to the CMB ($z = 1098$). compare to SCDM

distance measures: angular diameter distance

- angular diameter distance d is the distance infered from the angle under which a physical object appears
- physical cross section ΔA, solid angle $\Delta\Omega$:

$$\frac{\Delta A}{4\pi a_e^2 \chi} = \frac{\Delta\Omega}{4\pi} \quad (18)$$

- define d:

$$d \equiv \sqrt{\frac{\Delta A}{\Delta\Omega}} = a_e \chi \quad (19)$$

distance measures: luminosity distance

- luminosity distance l is measured from the lumunisity of an object and the flux received by the observer
- definition

$$l = \left(\frac{a_a}{a_e}\right)^2 d = \frac{a_a^2}{a_e}\chi \qquad (20)$$

- two redshifts decrease the **energy flux**
  - each photon is redshifted individually by the Hubble flow
  - the arrival time between two subsequent photons is stretched

question

what would the luminosity distance be if a detector just counts photons and would not measure energy fluxes?

distance measures: peculiarities

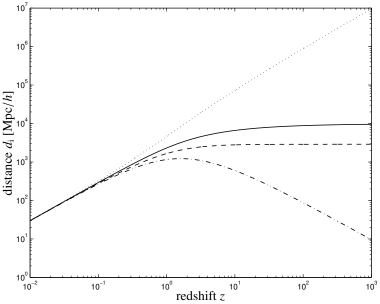
- evolving metric → 4 sensible distance definitions
- distances carry same information, with known cosmology they can be transformed into each other
- distance measures are useful cosmological probes
  - luminosity of distant objects
  - angular size of distant objects
- all definitions agree at small redshifts, but diverge at $z \simeq 1$:

$$\text{distance} \simeq \frac{cz}{H_0} + \mathcal{O}(z^2) \qquad (21)$$

question

which distance measures are additive? monotonic in z?

relation between distance and redshift

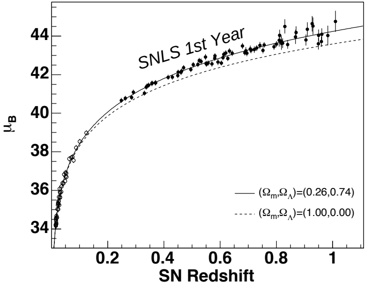


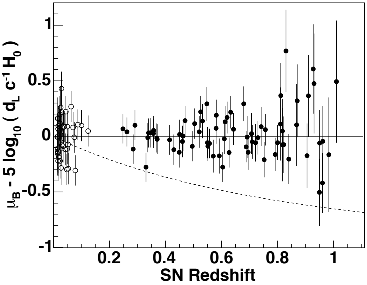
cosmological distances vs. redshift z

question

angular diameter distance decreases at $z > 1$ - does that mean that an object starts to appear larger with increasing distance?

supernovae: standard candles



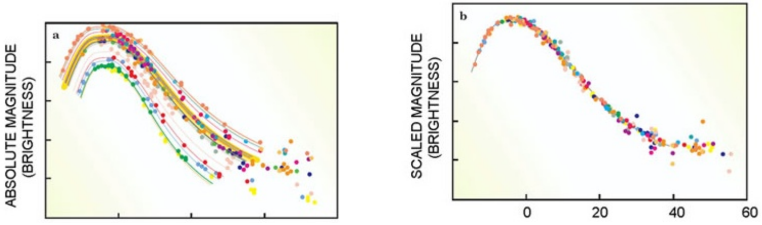


cosmological distances vs. redshift z **cosmological distances vs. redshift z**

- supernovae of the type Ia have very similar intrinsic absolute luminosities, corresponding to their released energy of 10^{44} Joule
- idea: measure apparent magnitude and redshift z of the host galaxy

→ luminosity distance

relation between distance and redshift

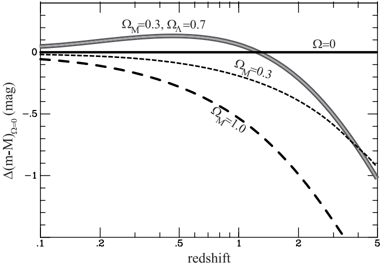


uncorrected lightcurves

calibrated lightcurves

- correlation between peak brightness and width of the light curve
- theoretically understood (amount of Nickel production), but **empirically** corrected
- assumption: high-redshift supernovae follow the same physics (metallicity?), dust extinction can be controlled

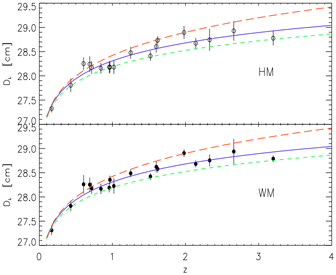
bounds on cosmology



fit of cosmological models to supernova data

- degeneracy: difficult to distinguish between curvature and Λ

γ-ray bursts: standard candles



fit of cosmological models to supernova data

- a number of empirical (badly understood) calibrations needed, relation not as tight as supernovae
- reaches out to considerable redshift, but low statistics

Tully-Fisher and Faber-Jackson distances

- if the luminosity of a galaxy can be inferred, and its redshift measured, it can be used as a cosmological probe
  - **Tully-Fisher relation**: in spiral galaxies, the luminosity L depends on circular velocity v

$$L \propto v^{3\ldots4.2} \tag{22}$$

  - **Faber-Jackson relation**: in elliptical galaxies, the luminosity L depends on velocity dispersion σ

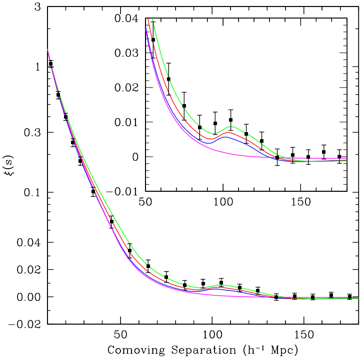
$$L \propto \sigma^4 \tag{23}$$

- assumption: parameters measured from a local galaxy sample, and luminosity depends positively on mass

question

derive the FJ-relation: virial theorem requires $\sigma^2 \propto M/R$, assume $M \propto L$ and a constant surface brightness

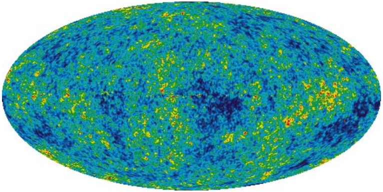
baryon acoustic oscillations: standard ruler



pair density $\xi(r)$ of galaxies as a function of separation r

- baryon acoustic oscillations: the (pair) density of galaxies is enhanced at a separation of about 100Mpc/h comoving
- idea: angle under which this scale is viewed depends on redshift

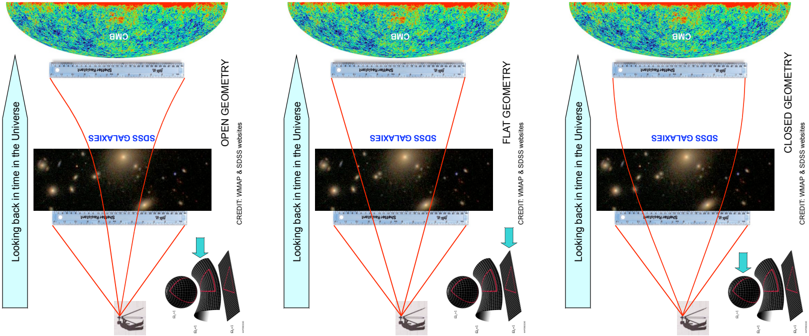
cosmic microwave background: standard ruler



all-sky map of the cosmic microwave background, WMAP

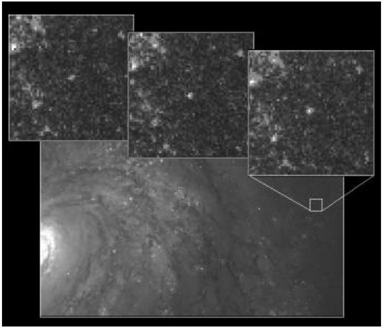
- hot and cold patches of the CMB have a typical physical size, related to the horizon size at the time of formation of hydrogen atoms
- idea: physical size and apparent angle are related, redshift of decoupling known

standard ruler: measurement principle (Eisenstein)



- curvature can be well constrained
- assumption: galaxy bias understood, nonlinear structure formation not too important

Hubble keystone project: determination of h



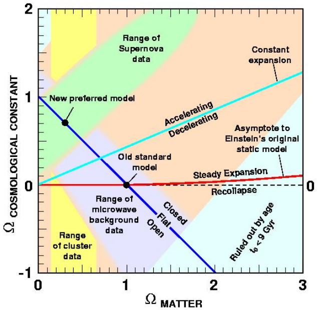
cepheid star in the galaxy M100

- original motivation for HST: determination of h
- idea: observe Cepheid stars in distant galaxies ($\simeq$ 20Mpc/h)
- Cepheid stars are variable and have a tight relation between variability and total luminosity

cosmological standard model

- FLRW-models are based on
  - general relativity
  - with time-homogeneous isotropic metric (RW-line element)
  - sourced by ideal (inviscid), homogeneous fluids
- time-evolution of the metric is described by the two Friedmann equations
- relevant parameters are:
  - density of fluids
  - curvature (density smaller or larger than critical density?)
  - equation of state of all fluids (fluids with negative eos?)
  - value of the Hubble-constant (today's expansion velocity)

ΛCDM concordance model and parameter choices



constraints on Ω_m and Ω_Λ

- each measurement has different degeneracies
- combination yields a flat universe, with nonzero Λ