ISAPP 2019 Lecture

→ Preliminaries: Ex? Cosmo? TH? Astro?

→ please ask!

Goal of lecture

- repeat/recall/introduce SM
- explain why we are not happy with it
- Higgs implications
- [simple additions to the SM]

I The Standard Model

I 1) Basics

QFT in $d=4=3+1$ dimensions

fermions:	ψ	$[\psi]=3/2$	energy dimension in natural units
vectors:	A_μ	$[A_\mu]=1$	
scalars:	ϕ	$[\phi]=1$	

described by equations of motion, obtained from Lagrange density ("Lagrangian")

$$\boxed{\mathcal{L}=\mathcal{L}(\psi,\partial_\mu\psi)}$$

Action: $S=\int d^4x\,\mathcal{L} \Rightarrow [\mathcal{L}]=4$ ($\leftrightarrow$ renormalizability)

principle of least action $\delta S=0$

$$\Rightarrow \boxed{\partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}=\frac{\partial\mathcal{L}}{\partial\psi}}$$ Euler-Lagrange

Examples

∘) $\mathcal{L} = \bar\psi (i\not\partial - m)\psi$ (*)

$\equiv \bar\psi (\not p - m)\psi$

$\xrightarrow{EL} (\not p - m)\psi = 0$ Dirac

∘) $\mathcal{L} = \frac{1}{2}\left[(\partial_\mu\phi)(\partial^\mu\phi) - m^2\phi^2\right]$

$\xrightarrow{EL} (\partial_\mu\partial^\mu + m^2)\phi = 0$ Klein-Gordon

∘) $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j_\mu A^\mu$ ($F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$)

$\xrightarrow{EL} \partial_\mu F^{\mu\nu} = j^\nu$ Maxwell

∘) $\mathcal{L} = -\frac{1}{2}G_{\mu\nu}G^{\mu\nu *} + m^2 B_\mu B^{*\mu}$

($G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$)

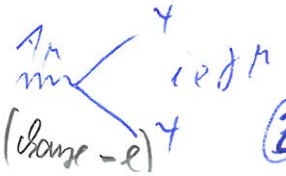
$\xrightarrow{EL} \partial_\mu G^{\mu\nu} + m^2 B^\nu = 0$ Proca

Remarks: ∘) $\mathcal{L} = f(\psi, \partial_\mu\psi)$, no higher derivatives (energy unbounded from below, solutions not determined by initial field values, ...)

∘) $\mathcal{L} = T - V$

∘) $\mathcal{L}$ not unique

∘) Feynman rules: $\mathcal{L} = \bar\psi(\not p + eA_\mu)\psi$



(charge $-e$)

I 2) Gauge Invariance

a) Global

$(*)$ is invariant under $\psi \to \psi' = e^{i\alpha}\psi$
$\approx (1+i\alpha)\psi$
$\Rightarrow \delta\psi = \psi' - \psi = i\alpha\psi$

"gauge trafo"

$\underline{\alpha = \text{const}}$

$$\delta\mathcal{L} \overset{!}{=} 0 \Rightarrow 0 = \frac{\partial\mathcal{L}}{\partial\psi}\delta\psi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\delta(\partial_\mu\psi)$$

$$= \frac{\partial\mathcal{L}}{\partial\psi}(i\alpha\psi) + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)} i\alpha(\partial_\mu\psi)$$

$$= i\alpha\underbrace{\left(\frac{\partial\mathcal{L}}{\partial\psi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\right)}_{=0}\psi + i\alpha\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi\right)$$

$$\Rightarrow \partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\psi\right] = 0 \Rightarrow \boxed{\partial_\mu\bar{\psi}\gamma^\mu\psi = 0}$$

Conserved Current

Noether theorem $\leftrightarrow$ global symmetry $\underline{U(1)}$

◦) can use $e^{i\alpha\hat{Q}}$; $\hat{Q}\psi = q\psi$ charge operator

◦) infinitesimal rotation is enough

b) local $U(1)$

now $\alpha = \alpha(x) \Rightarrow \psi \to \psi' = e^{i\alpha(x)}\psi$

$\mathcal{L}$ is not invariant due to $\partial_\mu \psi' = (\partial_\mu \psi) e^{i\alpha} + i\psi(\partial_\mu \alpha) e^{i\alpha}$

$\Rightarrow$ introduce „covariant derivative" $\partial_\mu \to D_\mu$

$$\bar{\psi}(i\not{\partial} - m)\psi \to \bar{\psi}(i\not{D} - m)\psi$$

effect: $\mathcal{L}' = \underbrace{\bar{\psi} e^{-i\alpha}}_{\bar{\psi}'}(i\not{D}' - m)\underbrace{e^{i\alpha}\psi}_{\psi'}$

is invariant if $D'_\mu \psi' = e^{i\alpha} D_\mu \psi$

achieved for $D_\mu = \partial_\mu - ieA_\mu$ with $A'_\mu = A_\mu + \frac{1}{e}(\partial_\mu \alpha)$

proof: $D'_\mu \psi' = (\partial_\mu - ieA_\mu - i(\partial_\mu \alpha)) e^{i\alpha}\psi$

$= \underline{i(\partial_\mu \alpha) e^{i\alpha}\psi} + e^{i\alpha}\partial_\mu \psi - ieA_\mu e^{i\alpha}\psi \underline{- i(\partial_\mu \alpha) e^{i\alpha}\psi}$

$= e^{i\alpha}(\partial_\mu - ieA_\mu)\psi = e^{i\alpha} D_\mu \psi$

∘) new $\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi$

$$= \bar{\psi}(i\not{\partial} - m)\psi + e \underbrace{\bar{\psi}\gamma_\mu\psi}_{j^\mu} A^\mu$$!coupling to photon!

∘) $\exists$ another gauge invariant term:

$$F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu = \ldots = F_{\mu\nu}$$

$\Rightarrow F_{\mu\nu}F^{\mu\nu}$ is allowed term of dim-4

($\leftrightarrow$ gauge, Lorentz, dim-4)

can also write: $F_{\mu\nu} = \frac{i}{e}[D_\mu, D_\nu]$

∘) $m_\gamma^2 A_\mu A^\mu$ term <u>not</u> gauge invariant

$\Rightarrow m_\gamma = 0$ and stable

$\Rightarrow$ total $\mathcal{L}$

$$\boxed{\mathcal{L}_{tot} = \overline{\Psi}(\not{p} - m)\Psi + e\overline{\Psi}\gamma_\mu A^\mu \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}}$$

free   interaction   kinetic term of gauge field

„QED", tested to $10^{-(10+x)}$ precision

$\Rightarrow$ gauge principle works!

gives $+\frac{1}{2}\sum_{i=1}^{3} \dot{A}_i^2$

this was Abelian gauge symmetry $e^{i\alpha}e^{i\beta} = e^{i\beta}e^{i\alpha}$

generalize to Non-Abelian G:

I 3) Non-Abelian Gauge Theories

We need $SU(N)$, group of unitary $N \times N$ matrices with $\det = +1$ (Lie group like $O(N)$, rotations in N-dim real or complex space)

$$\boxed{U = e^{i \vec{\alpha} \vec{\tau}} \simeq 1 + i \vec{\alpha} \vec{\tau} = 1 + i \alpha^a \tau^a}$$

$(N^2 - 1)$ τ^a traceless and hermitean generators

α^a infinitesimal real parameters

$$\boxed{[\tau^a, \tau^b] = i f^{abc} \tau^c}$$

antisymmetric structure constants f_{abc}

Lie algebra $su(N)$ of Lie group $SU(N)$
vector space spanned by generators, easier to work with

-) $U(1)$ has generator 1
-) $SU(2)$ has 3 Pauli matrices $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

  $[\sigma_i/2, \sigma_j/2] = i \varepsilon_{ijk} \frac{\sigma_k}{2}$

-) $SU(3)$ has 8 Gell-Mann matrices

•) adjoint representation (N^2-1) dimensional

$$(\tau^a)_{jk} = -i f_{ajk}$$

e.g. $\tau^1_{ad} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$, $\tau^2_{ad} = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}$, $\tau^3_{ad} = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

actually: gauge field transforms in adjoint of gauge group, not in adjoint rep. of group of gauge trafos...

•) N - dimensional rep. is called „fundamental"

•) 3- dim rep. of $SU(2)$

$\tau_1^{(3)} = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, $\tau_2^{(3)} = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

$\tau_3^{(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ with $[\tau_i^{(3)}, \tau_j^{(3)}] = i\varepsilon_{ijk}\, \tau_k^{(3)}$

apply gauge invariance to Dirac equation:

$\Psi \to \mathcal{U}\Psi \Rightarrow \Psi$ must be (at least) an N-component object!

e.g. $L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ or $Q = \begin{pmatrix} u \\ d \end{pmatrix}_L$ for $SU(2)_L$

$q = \begin{pmatrix} q_r \\ q_g \\ q_b \end{pmatrix}$ for $SU(3)_c$

trafo: $\boxed{\Psi_i \to \Psi_i' = \mathcal{U}_{ij}\Psi_j}$ with $\mathcal{U}_{ij} = (1 + i\alpha^a \tau^a)_{ij}$

gives $\mathcal{L}' = \bar{\Psi}\,\mathcal{U}^+ (\not{D}' - m)\,\mathcal{U}\Psi$

is invariant when $\mathcal{U}^+ D_\mu' \mathcal{U} = D_\mu \Rightarrow D_\mu' \mathcal{U}\Psi = \mathcal{U} D_\mu \Psi$

or $D_\mu' = \mathcal{U} D_\mu \mathcal{U}^+$

is achieved for $D_\mu = \partial_\mu - ig A_\mu^a \tau^a = \partial_\mu - ig \vec{A}_\mu \vec{\tau}$

$\equiv \partial_\mu - ig \tilde{A}_\mu$

with $\tilde{A}_\mu \to \tilde{A}_\mu' = \frac{-i}{g}(\partial_\mu \mathcal{U})\,\mathcal{U}^{-1} + \mathcal{U}\tilde{A}_\mu \mathcal{U}^{-1}$

cf. with $A_\mu \xrightarrow{\mathrm{QED}} A_\mu + \frac{1}{e}(\partial_\mu \alpha)$

proof: $D'_\mu \psi' = \left[\partial_\mu - ig\left(\frac{-i}{g}(\partial_\mu U)U^{-1} + U\tilde{A}_\mu U^{-1}\right)\right] U\psi$

$$= \underline{(\partial_\mu U)\psi} + U(\partial_\mu \psi) \underline{-(\partial_\mu U)\psi} - ig\, U\tilde{A}_\mu \psi$$

$$= U(\partial_\mu - ig\tilde{A}_\mu)\psi = U D_\mu \psi \qquad ☺$$

Note: there are N^2-1 gauge fields, one for each generator

infinitesimal trafo: $A_\mu^a \tau^a \to (1 + i\Theta^b\tau^b) A_\mu^a \tau^a (1 - i\Theta^b \tau^b)$

$$- \frac{i}{g}\left[\partial_\mu (1 + i\Theta^a\tau^a)\right](1 - i\Theta^b\tau^b)$$

$$= A_\mu^a \tau^a - i\Theta^b A_\mu^a [\tau^a, \tau^b] + \frac{1}{g}(\partial_\mu \Theta^a)\tau^a$$

$$= A_\mu^a \tau^a + f_{abc}\,\Theta^b A_\mu^c \tau^a + \frac{1}{g}(\partial_\mu \Theta^a)\tau^a$$

$$A \to U A U^{-1}$$

•) kinetic term: $F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu] = F^a_{\mu\nu}\, \tau^a$

$$= \dots = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu)\, \tau^a$$

gauge invariant? $[D'_\mu, D'_\nu] = [\mathcal{U} D_\mu \mathcal{U}^+, \mathcal{U} D'_\nu \mathcal{U}^+]$

$$= \mathcal{U}[D_\mu, D_\nu]\mathcal{U}^+$$

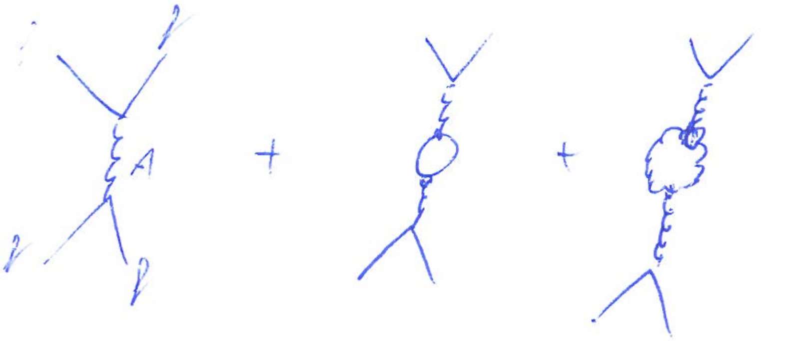
$\Rightarrow \mathrm{Tr}\{F_{\mu\nu} F^{\mu\nu}\}$ is invariant

<u>Note</u>: •) $m_W^2 A_\mu A^\mu$ still forbidden

•) $F_{\mu\nu}^2 \sim (\partial A + g A^2)^2 \sim g A^3 + g^2 A^4$

self-interactions

measurable effect of self-interacting gauge bosons



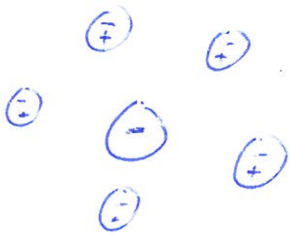
$\alpha = \alpha(Q^2)$

e.g. QED: $\alpha(Q^2) = \dfrac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} N_f \log Q^2/\mu^2}$ $(*)$

$\alpha \nearrow$ for $Q^2 \nearrow$

„Landau pole"

$\alpha(0) = \frac{1}{137} \longleftrightarrow \alpha(M_Z^2) = \frac{1}{128}$



„screening"

$Q^2 \nearrow$ see more charge

$\Rightarrow \alpha \nearrow$

useful: <u>β-function</u> $\quad \dfrac{d\alpha}{d\log Q^2} = \beta_\alpha = \dfrac{\alpha^2}{3\pi}$

reference measurement at μ^2 $\longleftrightarrow$ initial condition of differential equation

$\alpha(\log Q^2) = \dfrac{-1}{c + \frac{1}{3\pi}\log Q^2} \Rightarrow \alpha(\log \mu^2) = \dfrac{-1}{c + (\log \mu^2)/3\pi}$; insert in $\alpha(\log Q^2)$ to get $(*)$

for <u>positive</u> β_α: $\alpha \nearrow$ for $Q^2 \nearrow$

SU(N) result: $\beta(\alpha_s) = -\left(\frac{11}{3}N - \frac{2}{3}N_f\right)\frac{\alpha_s^2}{4\pi}$

with $N=3$: $\alpha_s \downarrow$ for $Q^2 \nearrow$ as long as $N_f \leq 16$

SM: $N_f = 6$

„asymptotic freedom"

also: $\alpha_s \nearrow$ for $Q^2 \searrow$ „confinement"

$\alpha_s(M_Z^2) = 0.12 \leftrightarrow \alpha_s(2m_p) = 0.3$

$\alpha_s(\Lambda_{QCD}) = \infty \Rightarrow \Lambda_{QCD} \simeq 100$ MeV

I 4) SM - Content

$\boxed{SU(2)_L \times U(1)_Y}$ $\Rightarrow 2^2 - 1 + 1 = 4$ gauge bosons

$SU(2)_L$ → weak IA
$U(1)_Y$ → hypercharge

not enough to choose symmetries! need particles (= reps)

$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \sim (\underset{\text{doublet of } SU(2)_L}{2_L}, \underset{\text{hypercharge } Y_L}{Y_L})$

$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (2_L, Y_Q)$

$u_R \sim (1_L, Y_u)$, $d_R \sim (1_L, Y_d)$, $e_R \sim (1_L, Y_e)$

chiral theory! LH transform differently than RH (→ QED is vector theory)

gauge trafos
$$\boxed{\begin{aligned} \exp\left\{\tfrac{i}{2} g' \hat{Y} \beta(x)\right\} &\equiv U_Y \\ \exp\left\{\tfrac{i}{2} g\, \sigma_i\, \alpha_i(x)\right\} &\equiv U_L \end{aligned}}$$
; $\hat{Y} u_R = Y_u u_R$, $\hat{Y} L = Y_L L$

hypercharge operator

note: $\frac{1}{2}$ in U_Y is convention!

need also: $\hat{I}_3$ with $\hat{I}_3 u_L = \frac{1}{2} u_L$, $\hat{I}_3 e_R = 0$
$\hat{I}_3 d_L = -\frac{1}{2} d_L$

$\hat{I}_3 \leftrightarrow \frac{\sigma_3}{2}$

„isospin" operator

now construct Lagrangians

$$\left.\begin{aligned}
\bar{L}\,\not{D}\,L &= \bar{L}\underbrace{\left(\partial_\mu + ig\frac{\sigma_i}{2}W_\mu^i + \frac{i}{2}g'\hat{Y}B_\mu\right)}\gamma^\mu L \\
\bar{Q}\,\not{D}\,Q &= \bar{Q}\qquad\qquad\qquad \ldots \qquad\qquad\qquad\quad Q \\
\bar{u}_R\,\not{D}\,u_R &= \bar{u}_R\left(\partial_\mu + \frac{i}{2}g'\hat{Y}B_\mu\right)\gamma^\mu u_R
\end{aligned}\right\}(**)$$

$$\vdots$$

mass terms? $\bar{\psi} m \psi = \overline{\psi_L + \psi_R}\; m(\psi_L + \psi_R)$

$= \bar{\psi}_L m \psi_R + h.c. \Leftrightarrow$ both helicities needed!

$\bar{L}\, m\, e_R$ and $\bar{e}_L\, m\, e_R$ are not gauge invariant

$\Rightarrow$ no masses possible!

unless Higgs mechanism is introduced

I 5) Spontaneous Symmetry Breaking

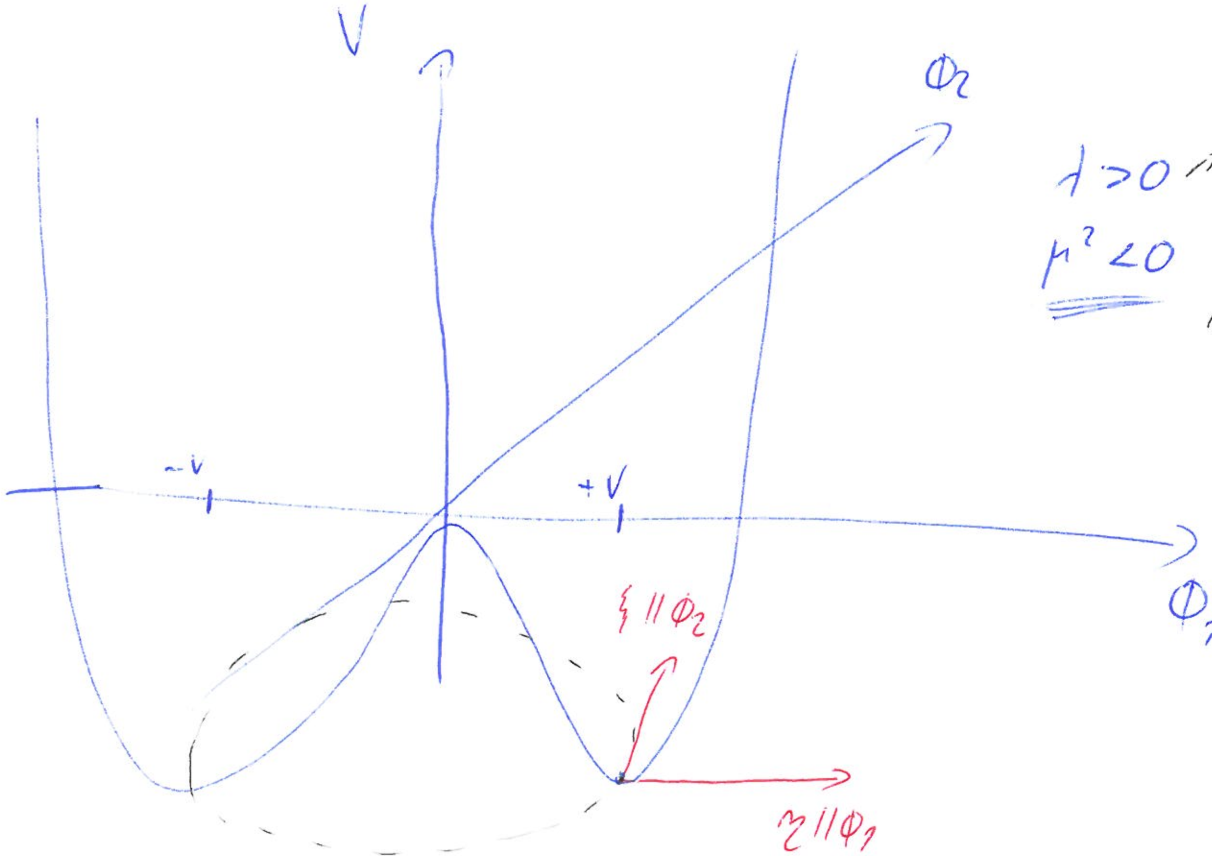
SSB: $\mathcal{L}$ has symmetry which ground state does not have

a) Global $U(1)$

$$(*) \quad \mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi)^* - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 = T - V$$

$$\phi = \sqrt{\tfrac{1}{2}}(\phi_1 + i\phi_2)$$

Symmetry: $\phi \to \phi' = e^{i\alpha}\phi \qquad \alpha = \text{const}$



$\lambda > 0$ → bounded from below...

$\mu^2 < 0$ not a mass term

minimum at $v^2 = \phi_1^2 + \phi_2^2 = -\mu^2/\lambda$

we can only perform perturbation theory around minimum

w.l.o.g. ~~$\Phi = v$~~, $\Phi = \sqrt{\tfrac{1}{2}}\left(v + \eta(x) + i\xi(x)\right)$

$\langle \Phi \rangle = v$ "vacuum expectation value"

insert back in $(*)$: $\mathcal{L} = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 + \mu^2 \eta^2$ + const, cubic, quadratic

$\Rightarrow$ $\boxed{\text{correct mass term for } \eta \text{, massless } \xi}$

"flat direction"

<u>Goldstone-boson</u>

> If there is a global symmetry generated by N generators, and the vacuum is invariant under $M < N$ generators, then there are $N - M$ massless Goldstone Bosons (**GB**)

"Goldstone theorem"

<u>Example:</u> $O(3)$ scalar theory $\Phi = (\varphi_1, \varphi_2, \varphi_3)^T$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi_i)(\partial^\mu \varphi_i) - \mu^2 \varphi_i \varphi_i - \lambda(\varphi_i \varphi_i)^2 \xrightarrow{\min} \varphi_i^2 = v^2 = -\mu^2/\lambda$$

choose $\langle \Phi \rangle = (0, 0, v)^T \Rightarrow O(2)$ invariance

$\left.\begin{array}{l} O(3): \text{ 3 generators} \\ O(2): \text{ 1 generator} \end{array}\right\} \Rightarrow 3 - 1 = 2$ massless GB

~~Note that~~ If in addition to SSB a small explicit breaking

$$(*) \to (*) - \tfrac{1}{2}\varepsilon\,\phi_1^2 = \mathcal{L}_0 + \delta\mathcal{L}$$

One finds: $m_\eta^2 = 2(-\mu^2+\varepsilon)$

$m_\xi^2 = \varepsilon$ Pseudo-Goldstone boson

Current would no longer be conserved: $\partial_\mu J^\mu \propto \delta\mathcal{L}$

Example: QCD: $\mathcal{L} = \bar\psi_L\, i\not{D}\,\psi_L + \bar\psi_R\, i\not{D}\,\psi_R$

with $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$ massless quarks

invariant under global $SU(2)_L \times SU(2)_R$ or $SU(2)_V \times SU(2)_A$

$SU(2)_V \to \exp\{-i\,\frac{\vec\tau}{2}\vec\Theta_V\} \Rightarrow j_\mu = \bar\psi\gamma_\mu\frac{\tau^i}{2}\psi$ conserved (CVC)

$SU(2)_A \to \exp\{-i\,\frac{\vec\tau}{2}\vec\Theta_A\gamma_5\} \Rightarrow \bar\psi\gamma_\mu\gamma_5\frac{\tau^i}{2}\psi$ conserved (PCAC)

However $m\bar\psi\psi$ breaks $SU(2)_A$ explicitely
$\Rightarrow$ partially conserved axial current

$$m_u\bar u u + m_d\bar d d = \underbrace{\tfrac{1}{2}(m_u+m_d)\bar\psi\psi}_{\substack{\text{breaks } SU(2)_A \\ \text{conserves } SU(2)_V}} + \underbrace{\tfrac{1}{2}(m_u-m_d)\bar\psi\tau_3\psi}_{\substack{\text{breaks } SU(2)_V \\ \leftrightarrow \text{ isospin;} \\ \text{small effect}}}$$

$\partial_\mu j_V^\mu \propto (m_u - m_d)$

$\partial_\mu j_A^\mu \propto (m_u + m_d)$

$\boxed{\text{Pseudo-Goldstones are PIONS}}$

b) Local $U(1)$

(*) invariant under $\Phi \to e^{i\alpha(x)}\Phi \Rightarrow \partial_\mu \to \partial_\mu - ieA_\mu \equiv D_\mu$

trafo for A: $A_\mu \to A_\mu + \frac{1}{e}(\partial_\mu \alpha)$

insert $\Phi = \sqrt{\tfrac{1}{2}}(v + \eta + i\xi)$ in $\mathcal{L}$

$$\mathcal{L}' = \underbrace{\tfrac{1}{2}(\partial_\mu \xi)^2}_{GB} + \tfrac{1}{2}(\partial_\mu \eta)^2 + \underbrace{\mu^2\eta^2}_{\text{mass of } \eta} + \underbrace{\tfrac{1}{2}e^2v^2 A_\mu A^\mu}_{\substack{m_A = ev \\ \text{photon mass!}}} - \underbrace{ev A_\mu (\partial^\mu \xi)}_{\text{WTF?}} + \dots$$

also terms $A_\mu \xi(\partial_\mu\eta)\xi$ a $A_\mu(\partial^\mu\xi)v$

attention: before SSB: $\Phi, A_\mu^{m=0}$: $2+2 = 4$ d.o.f

after SSB: $\eta, \xi, A_\mu^{m\neq 0}$: $1+1+3 = 5$ d.o.f.

$\Rightarrow$ need to use gauge freedom..

note that: ·) $\Phi \simeq \sqrt{\tfrac{1}{2}}(v+\eta)e^{i\xi/v}$

·) ~~~ terms can be written as

$$\tfrac{1}{2}e^2v^2\left(A_\mu - \tfrac{1}{ev}\partial_\mu \xi\right)^2$$

$\Rightarrow$ make gauge choice: $\alpha = -\xi/v$

$$\boxed{\begin{aligned} &\Phi \to \Phi' = e^{-i\xi/v}\Phi = \sqrt{\tfrac{1}{2}}(v+\eta) \\ &A_\mu \to A'_\mu = A_\mu - \tfrac{1}{ev}\partial_\mu \xi \end{aligned}}$$

unitary gauge ($w \to \infty$) instead of adding $\mathcal{L} = \frac{1}{\xi}(\partial_\mu A^\mu - wM_A\xi)^2$

insert in $\mathcal{L} \Rightarrow \mathcal{L} = \frac{1}{2}(\partial_\mu \eta)^2 + \frac{1}{2} e^2 v^2 A'_\mu A'^\mu + \mu^2 \eta^2$

$[D'_\mu \phi'][D'^\mu \phi']^*$

$= \frac{1}{2}[\partial_\mu \eta - ie A'_\mu (v+\eta)][\partial^\mu \eta + ie A'^\mu (v+\eta)]$

$= \frac{1}{2}(\partial_\mu \eta)^2 + e^2 A'^2_\mu (v+\eta)^2$

$$+ \frac{1}{2} e^2 A'_\mu A'^\mu (\eta^2 + 2v\eta) - \lambda v \eta^3 - \frac{\lambda}{4} \eta^4$$

$$- \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu}$$

Great! $\rightarrow$ ξ disappeared in A_μ

$\rightarrow$ A'_μ massive

$\rightarrow$ massive η „Higgs boson"

$\rightarrow$ A'_μ interacts with Higgs

$\rightarrow$ Higgs interacts with itself

c) local SU(2) ~~SSB~~

$\Phi \sim (2_L, Y_\Phi)$

$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$

$$\Phi = \sqrt{\tfrac{1}{2}} \begin{pmatrix} \Phi_1 + i\Phi_2 \\ \Phi_3 + i\Phi_4 \end{pmatrix}$$

minimum at $\Phi^\dagger \Phi = -\mu^2/2\lambda$

w.l.o.g. $\Phi_3^2 = -\mu^2/\lambda = v^2$

since $\exp\{i \alpha_a \sigma_a\} \begin{pmatrix} 0 \\ b \end{pmatrix} \simeq \begin{pmatrix} \alpha_2 + i\alpha_1 \\ b - i\alpha_3 \end{pmatrix}$, I can write:

$$\Phi = \sqrt{\tfrac{1}{2}} \exp\left\{ i \vec{\sigma} \frac{\vec{\Theta}}{2} \right\} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$\Rightarrow$ the 3 would-be GB $\Theta_{1,2,3}$ can be gauged away and disappear in the simultaneously transformed gauge bosons $W_\mu^{1,2,3}$

then mass terms come from $(D_\mu \Phi)^\dagger (D^\mu \Phi)$

with $D_\mu = \partial_\mu + i g \frac{\sigma^a}{2} W_\mu^a$

$$\Rightarrow \left| i g \frac{\vec{\sigma}}{2} \vec{W}_\mu \sqrt{\tfrac{1}{2}} \binom{0}{v} \right|^2 = \frac{g^2 v^2}{8} \left[(W_\mu^1)^2 + (W_\mu^2)^2 + (W_\mu^3)^2 \right]$$

~~A)~~ I 7) The SM, finally

consider charge-conserving currents a la $(**)$ (page 14)

$$\bar{Q} \not{D} Q \quad c\Big(\overset{I_3 u_L}{\tfrac{1}{2}} \bar{u}_L \gamma_\mu u_L W^{\mu,3} \overset{I_3 d_L}{-\tfrac{1}{2}} \bar{d}_L \gamma_\mu d_L W^{\mu,3} \Big) g$$

$$+ \frac{Y_Q}{2} (\bar{u}_L \gamma_\mu u_L + \bar{d}_L \gamma_\mu d_L) B^\mu g'$$

$$\bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R = \overset{I_3}{0} \cdot g \bar{u}_R \gamma_\mu u_R W^{\mu,3} + \frac{Y_u}{2} \bar{u}_R \gamma_\mu u_R B^\mu g'$$

$$+ 0 \cdot g \bar{d}_R \gamma_\mu d_R W^{\mu,3} + \frac{Y_d}{2} \bar{d}_R \gamma_\mu d_R B^\mu g'$$

compare with QED: $(\bar{u}_L \gamma_\mu u_L + \bar{u}_R \gamma_\mu u_R) A^\mu e \cdot q_u$

$\Rightarrow$ with $\hat{Q} u = q_u u$: $\boxed{\hat{I}_3 + \frac{\hat{Y}}{2} = \hat{Q}}$ $\Rightarrow$ vacuum should be invariant under $\hat{Q}$, then photon is massless

and $g W_\mu^3 + g' B_\mu = e A_\mu$

$$\Rightarrow \hat{Q}\begin{pmatrix}0\\v\end{pmatrix} \overset{!}{=} 0 \Rightarrow -\frac{1}{2}+\frac{Y_\Phi}{2}=0 \Rightarrow Y_\Phi = 1$$

$$e^{i\alpha\hat{Q}}\begin{pmatrix}0\\v\end{pmatrix}=\begin{pmatrix}0\\v\end{pmatrix}$$

note: $\sigma_i\begin{pmatrix}0\\v\end{pmatrix} \neq 0$ and $\hat{Y}\begin{pmatrix}0\\v\end{pmatrix}\neq 0 \Rightarrow$ both broken

masses of gauge bosons:

$$(D_\mu \Phi)(D^\mu\Phi)^\dagger \subset \left|\left(g\frac{\vec{\sigma}}{2}\vec{W}_\mu + \frac{g'}{2}B_\mu\right)\begin{pmatrix}0\\v\end{pmatrix}\right|^2$$

$$= \dots = \frac{1}{8}v^2g^2\left[(W_\mu^1)^2+(W_\mu^2)^2\right] \cancel{+\frac{v^2}{8}}$$

$$+\frac{v^2}{8}(W_\mu^3, B_\mu)\begin{pmatrix}g^2 & -gg'\\ -gg' & g'^2\end{pmatrix}\begin{pmatrix}W^{\mu 3}\\ B^\mu\end{pmatrix}$$

$$\equiv 1 + 2$$

$$1 = \left(\frac{1}{2}vg\right)^2 W_\mu^+ W^{\mu -} \quad \text{with } W_\mu^\mp = \sqrt{\tfrac{1}{2}}(W_\mu^1 \pm iW_\mu^2)$$

because $\bar{Q}\,\not{D}\,Q \subset \frac{1}{2}g\Big(\bar{u}_L\underbrace{(W_\mu^1 - iW_\mu^2)}_{\sqrt{2}W_\mu^+}d_L + \bar{d}_L\underbrace{(W_\mu^1+iW_\mu^2)}_{\sqrt{2}W_\mu^-}u_L$

$$m_W^2 = \left(\frac{1}{2}vg\right)^2$$

$$② = \frac{v^2}{8}(W_\mu^3, B_\mu)\, R R^T \underbrace{\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}}\, R R^T \begin{pmatrix} W^{\mu 3} \\ B^\mu \end{pmatrix}$$

with $R = \begin{pmatrix} c_w & s_w \\ -s_w & c_w \end{pmatrix}$ $\quad c_w = \cos\theta_w$, $s_w = \sin\theta_w$

Weinberg angle

$$\tan\theta_w = g'/g$$

$$\left.\begin{array}{l} Tr(\ ) = g^2 + g'^2 = \lambda_1 + \lambda_2 \\ Det(\ ) = 0 \qquad = \lambda_1 \lambda_2 \end{array}\right\} \Rightarrow 1 \text{ massless state} \Rightarrow \text{photon}$$

$$\boxed{\begin{aligned} A_\mu &= \cos\theta_W\, B_\mu + \sin\theta_W\, W_\mu^3 \\ Z_\mu &= c_w\, W_\mu^3 - s_w\, B_\mu \end{aligned}}$$

with $M_Z^2 = \frac{v^2}{4}(g^2 + g'^2)$

$$\Rightarrow \frac{M_W^2}{M_Z^2} = \cos^2\theta_W$$

$$\theta_w = 29°$$

$$s_w^2 \simeq 0.23$$

isospin of gauge bosons

need Casimir operator (commutes with all generators, $\exists N-1$ for $SU(N)$

in $SU(2)$: $\frac{1}{4}(\sigma_1^2+\sigma_2^2+\sigma_3^2) \equiv C$

for $L = \begin{pmatrix}\nu\\ e\end{pmatrix}_L$: $CL = \frac{1}{2}(\frac{1}{2}+1)L \Rightarrow$ isospin $1/2$

$\frac{\sigma_3}{2} L = \frac{1}{2}\begin{pmatrix}\nu\\ -e\end{pmatrix}_L \Rightarrow I_3(\nu_L) = \frac{1}{2}$

$I_3(e_L) = -1/2$

adjoint rep has $I_W = 1$: $\sum_i \tau_{ad}^{(i)} = 1(1+1)$

eigenvectors of τ_{ad}^3 are : $x_1 = \begin{pmatrix}1\\ i\\ 0\end{pmatrix}$, $x_2 = \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}$, $x_3 = \begin{pmatrix}1\\ -i\\ 0\end{pmatrix}$

$\tau_{ad}^3 x_1 = x_1 \qquad \tau_{ad}^3 x_2 = 0 \qquad \tau_{ad}^3 x_3 = -x_3$

$\Rightarrow I_3 = 1, 0, -1$ triplet

with $W_\mu^\pm = \sqrt{\frac{1}{2}}(W_\mu^1 \mp W_\mu^2)$, or $\begin{pmatrix}W_1\\ W_2\\ W_3\end{pmatrix} = \frac{W^+}{\sqrt{2}}\begin{pmatrix}1\\ i\\ 0\end{pmatrix} + \frac{W^-}{\sqrt{2}}\begin{pmatrix}1\\ -i\\ 0\end{pmatrix} + W^3\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}$

it follows that W^+, W^-, W^3 have $I_3 = +1, -1, 0$

I8) Fermion Masses and Couplings

a) Couplings to gauge bosons

write $\bar{L} \not{D} L$, $\bar{Q} \not{D} Q$, $\bar{u}_R \not{D} u_R$ etc.
in terms of $W_\mu^\pm$, Z_μ, A_μ

e.g. $Z_\mu \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f \frac{e}{2 c_w s_w}$

	u	d	ν	e	
$2v_f$	$1 - \frac{8}{3} s_w^2$	$-1 + \frac{4}{3} s_w^2$	1	$-1 + 4 s_w^2$	$2 \cdot I_3 - 4 s_w^2 Q$
$2a_f$	1	-1	1	-1	

for $W^\pm$ couplings:

$$\mathcal{L} = \frac{g}{2\sqrt{2}} \left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu + \bar{\nu}_e \gamma_\mu (1-\gamma_5) e \right] W^{\mu,+}$$

$$+ m_w^2 W_\mu^+ W^{\mu,-}$$

@ low energy: $\partial_\mu W^{\mu,+} = 0 \Rightarrow \frac{\partial \mathcal{L}}{\partial W_\mu^+} = 0$

$$\Rightarrow W_\mu^- = \frac{g}{2\sqrt{2} m_w^2} \left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu + \bar{\nu}_e \gamma_\mu (1-\gamma_5) e \right]$$

insert back in $\mathcal{L}$

$$\Rightarrow \mathcal{L}_{eff} = \frac{g^2}{8 m_w^2} \left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu \right] \left[\bar{e} \gamma^\mu (1-\gamma_5) \nu_e \right]$$

$\Rightarrow \dfrac{g^2}{8 m_W^2} = \dfrac{G_F}{\sqrt{2}}$ Fermi-theory

"effective theory", W heavy and integrated out

vs.

b) Fermion Masses and Couplings to the Higgs

$\phi \sim (2_L, 1)$ $\quad Q = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (2_L, 1/3)$

$u_R \sim (1_L, 4/3)$; $d_R \sim (1_L, -2/3)$

$\Rightarrow \bar{Q}\, \phi\, d_R$ is invariant $\left(\begin{matrix} Q \to U \psi \\ \phi \to U \phi \end{matrix} \right.$, hypercharges add to zero $\Big)$

$\Rightarrow \mathcal{L} = -g_d\, \bar{Q}\, \phi\, d_R$ with Yukawa coupling g_d

$$\mathcal{L}_{SSB} = -g_d\, \bar{Q} \sqrt{\tfrac{1}{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} d_R = -\underbrace{\frac{g_d v}{\sqrt{2}}}_{\text{mass of down}} \bar{d}_L d_R - \underbrace{\sqrt{\tfrac{1}{2}}\, g_d}_{\text{coupling to Higgs}} h\, \bar{d}_L d_R$$

$g_d = m_d / v$

$\propto$ mass

what about up-quarks? answer: $\bar{Q}\,\tilde{\Phi}\,u_R$ with $\tilde{\Phi} = i\sigma_2\phi^*$

hypercharges add up to zero, what about $SU(2)$?

$$\bar{Q}\,\tilde{\Phi} \to \bar{Q}\,U^\dagger i\sigma_2 U^* \phi^*$$

with $U \simeq 1 + i\frac{\sigma_i}{2}\alpha_i$, use that $(-i\sigma_i)(i\sigma_2)(-i\sigma_i^*) = i\sigma_2 \;\forall i$

$\Rightarrow$ term is invariant and can be used for up-quark mass!

Note: all masses in SM $\propto v$:

- $m_f = g_f\, v/\sqrt{2}$
- $m_W = \frac{1}{2} v g$, $\quad m_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$
- $m_h = \sqrt{2\lambda v^2}$

I 9) The SM summarized

•) only 1 energy scale : μ^2 or v

•) 3 Generations of Fermions

$L_1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$; $L_2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$; $L_3 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$ $\qquad e_R, \mu_R, \tau_R$

$Q_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L$; $Q_2 = \begin{pmatrix} c \\ s \end{pmatrix}_L$; $L_3 = \begin{pmatrix} t \\ b \end{pmatrix}_L$ $\qquad u_R, d_R, c_R, s_R, b_R, t_R$

!) no $\nu_R \sim (1,0)$ ⇒ no neutrino masses...

!) Yukawa terms, e.g. $\overline{Q}_1 t_R$, can be off-diagonal ; diagonalize ⇒ CKM-matrix with 3+1 parameters

•) 3 gauge couplings

•) 2 parameters in Higgs potential

⇒

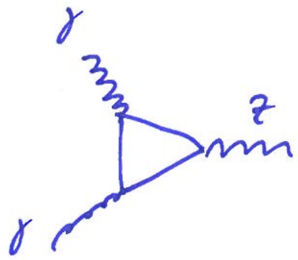
Quarks	10
Leptons	3 (→ 12 for m_ν)
Higgs	2
Gauge	3
(strong CP)	1

⇒ 19 free parameters

$\Rightarrow$ Questions:

- Why so many free parameters?
- Why 3 generations?
- DM, DE, Y_B, m_ν, ... ?
- B, L accidentaly conserved (B−L ...)

Anomalies:



needs to vanish (among other diagrams)

axial part of Z ($\leftrightarrow a_f$)

contribution gives
$$\sum Q_f^2 a_f = \overset{u_L}{\tfrac{1}{2}\left(\tfrac{2}{3}\right)^2 \cdot 3} + \overset{d_L}{\left(-\tfrac{1}{2}\right)\left(-\tfrac{1}{3}\right)^2 \cdot 3}$$
$$+ \overset{\nu_L}{0} + \overset{e_L}{(-1)^2\left(-\tfrac{1}{2}\right)}$$
$$= \tfrac{2}{3} - \tfrac{1}{6} + 0 - \tfrac{1}{2} = 0$$

needs quarks + leptons!

$\Rightarrow$ if 4th generation, need both types

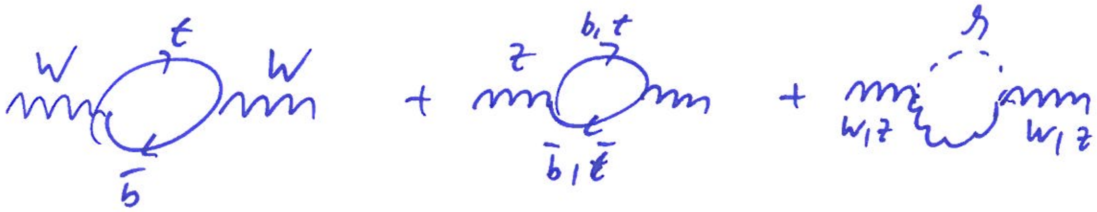
$\Rightarrow$ if new BSM $U(1)'$: important test!

II Higgs and its Implications

$m_h = 125$ GeV, behaves as expected, $(\lambda = 0.13)$

II 1) Custodial symmetry

$$\rho = \frac{m_W^2}{m_Z^2 c_W^2} = 1$$, radiative corrections small. Why?



$$\Delta \rho \propto \Big[m_t^2 + m_b^2 - 2\,\frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \log m_t^2/m_b^2$$

$$- \frac{11}{9} m_Z^2 s_W^2 \log m_h^2/m_Z^2 \Big]$$

$\Delta\rho = 0$ for $m_t = m_b$ and $g' = 0$

related to <u>custodial symmetry</u>

$\leftrightarrow$ Higgs potential V has global $O(4)$ symmetry

$$\sqrt{2}\,\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \quad \alpha \quad \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix} : \quad \Phi^\dagger \Phi \propto \varphi^T \varphi$$

define $\Phi = (\tilde{\phi}, \phi)$ with $\tilde{\phi} = i\sigma_2 \phi^*$
(2x2 matrix)

$$\langle \Phi \rangle = \sqrt{\tfrac{1}{2}} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \quad ; \quad \mathrm{Tr}\{\Phi^\dagger \Phi\} \propto \phi^\dagger \phi$$

V has $SU(2)_L \times SU(2)_R$ symmetry $\quad \Phi \to U_L \Phi U_R^\dagger$

vacuum is invariant for $U_L = U_R^\dagger$ „diagonal subgroup"

$SU(2)_L$ is global SM symmetry

~~custodial~~

<u>Yukawas:</u> $\quad L = \begin{pmatrix} t \\ b \end{pmatrix}_L \quad R = \begin{pmatrix} t \\ b \end{pmatrix}_R$

$$\Rightarrow \mathcal{L} = g_t \bar{L} \tilde{\phi} t_R + g_b \bar{L} \phi b_R$$
$$= \bar{L} \Phi \begin{pmatrix} g_t & 0 \\ 0 & g_b \end{pmatrix} R$$
$$= \underbrace{\frac{g_t + g_b}{2} \bar{L} \Phi R}_{\text{invariant}} + \underbrace{\frac{g_t - g_b}{2} \bar{L} \Phi \sigma_3 R}_{\text{not invariant under } SU(2)_R}$$

<u>Hypercharge:</u> $\quad \mathrm{Tr}\{(D_\mu \Phi)^\dagger (D^\mu \Phi)\} \propto (D_\mu \phi^\dagger)(D^\mu \phi)$

with $D_\mu \Phi = \partial_\mu \Phi + ig \vec{W}_\mu \vec{\sigma} \Phi + ig' B_\mu \Phi \sigma_3$

$\tilde{W} \to U_L \tilde{W} U_L^\dagger$

$Y(\phi) = 1$
$Y(\tilde{\phi}) = -1$

$\mathcal{L}$ invariant only if $g' = 0$, breaks $SU(2)_R$

$\Rightarrow$ new physics should have custodial symmetry ...

e.g. $\mathcal{L} = \frac{1}{\Lambda^2}\left(\phi^+ D_\mu \phi\right)^2 = \frac{1}{\Lambda^2} \operatorname{Tr}\{\sigma_3 \Phi^+ D_\mu \Phi\}$,

Higgs-triplet, ...

In general: $$\rho = \sum \frac{v_i^2\left[T_i(T_i+1) - Y_i^2/4\right]}{\sum_i 1/2\, Y_i^2 v_i^2}$$

SM: $T_i = \frac{1}{2}$ $Y_i = 1 \Rightarrow \rho = \frac{3/4 - 1/4}{1/2} = 1$

e.g. triplet: $T = 1 \Rightarrow Y = 2\sqrt{2/3}$ to have $\rho = 1$

$\Rightarrow Q = T_3 - \frac{Y}{2} = 0$ not possible...

would work for $T = 3$, $Y = 4$

$Q = T_3 - 2 = 0$ possible

Higgs septuplet with 11 Higgses...

(→) Oblique Parameters

1 loop corrections to vector boson self-energies (assumed to be dominant NP effect... ↔ no effect on light fermions)

$$i\left(\Pi_{VV'}(q^2)g^{\mu\nu} - \Delta_{VV'}q^\mu q^\nu\right]$$

$$\downarrow$$

$$q^\mu j_\mu \propto \bar{f} q f \propto \bar{f} m_f f \to 0$$

for light fermions

$$m_V^2 \to m_V^2 + \Pi_{VV}(q^2 = m_V^2)$$

only six functions matter: $\Pi'_{\gamma\gamma}$, $\Pi'_{Z\gamma}$, Π_{ZZ}, Π'_{ZZ}, Π_{WW}, Π'_{WW} (momentum derivative)

$\Pi = \Pi(0) + q^2\Pi'(0) + \ldots$

(QED Ward Identities: $\Pi_{\gamma\gamma} = \Pi_{Z\gamma} = 0$) 3 absorbed in other observables α, G_F, M_Z

$$S = \cancel{\frac{4 s_W^2 c_W^2}{\alpha}\left(-\Pi'_{\gamma\gamma} + \Pi'_{ZZ} - \Pi'_{\gamma Z}\frac{c_W^2 - s_W^2}{c_W s_W}\right)} \; \cancel{\frac{4 s^2 c^2}{\alpha}\Pi_{ZZ}}$$

$$T = \frac{1}{\alpha}\left(\Pi_{WW} - \Pi_{ZZ}\right)$$

$U = \ldots$ typically tiny (↔ dim 8)

S: shift of Z mass

$T = \alpha\Delta\rho$

Example $\Delta S = \frac{N_c}{6\pi}\left(1 - 2Y \log m_t^2/m_b^2\right)$

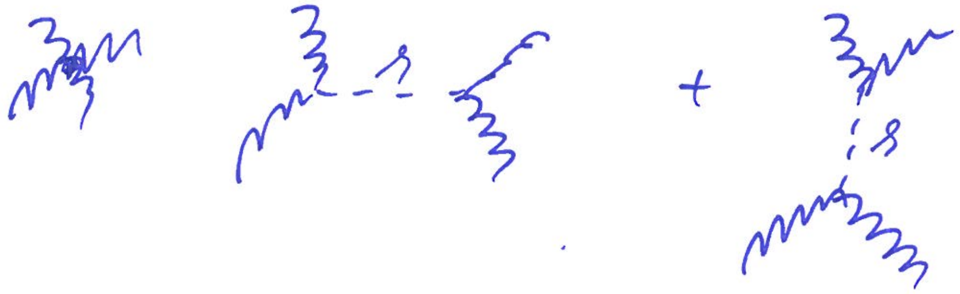
~~⇒ 4th generation~~

$$\Delta T \propto \left(m_t^2 + m_b^2 - 2\frac{m_t^2 m_b^2}{m_t^2 - m_b^2}\log\frac{m_t^2}{m_b^2}\right)$$

⇒ 4th generation with $m_{t'} = m_{b'}$ would hide in T, but would appear in S...

(⇒ new physics difficult to hide..)

II 2) Equivalence Theorem

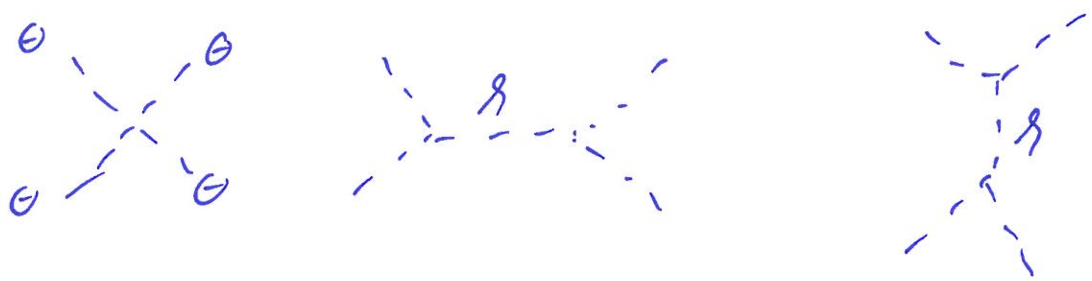


$W^+W^- \to W^+W^-$

$$\varepsilon_\mu^{\lambda=0} = \frac{1}{m_W}\left(|\vec{p}|, 0, 0, E\right) \quad ; \quad \varepsilon_\mu^{\lambda=\pm 1} = \sqrt{\tfrac{1}{2}}\left(0, \mp 1, -i, 0\right)$$

dominated @ high energies by $\lambda = 0$ mode

$\Rightarrow$ Goldstone bosons !



$\Rightarrow$ write $V = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$ with $\Phi = \sqrt{\tfrac{1}{2}} \begin{pmatrix} -\theta_2 - i\theta_1 \\ v + h + i\theta_3 \end{pmatrix}$

$$= \frac{m_h^2}{2v^2}\, \theta_+ \theta_- \,\theta_+ \theta_- + \frac{m_h^2}{v}\, h\, \theta_+ \theta_-$$

$\left[\theta_\pm = \sqrt{\tfrac{1}{2}}\,(\theta_1 \pm i\theta_2)\right]$

$\Rightarrow i\mathcal{M}(W^+W^- \to W^+W^-) = \ldots$

$$\sigma_{tot} = \frac{1}{s}\, \mathrm{Im}\{\mathcal{M}(\theta = 0)\}$$ (optical theorem)

$$\mathcal{M} = c \cdot \sum (2l+1)\, P_l(\cos\theta)\, a_l$$ (partial wave decomposition)

Legendre

partial waves

$$\Rightarrow \mathrm{Re}\{a_0\} < 1/2$$

in our case: $a_0 = \dfrac{m_h^2}{8\pi v^2} \Rightarrow m_h^2 < 4\pi v^2 = (870\ \mathrm{GeV})^2$

„unitarity bound"

something should regularize $W_L W_L \to W_L W_L$
$\Leftrightarrow$ LHC

II 3) RG-Analysis

+ gauge

$\lambda = \lambda(Q^2) = f(\lambda, g, g', m_t/v) = \dots$ lengthy

a) large λ

$$\frac{d\lambda}{d\log Q^2} = \frac{3}{4\pi^2}\lambda^2$$

$\lambda \nearrow$ for $Q^2 \nearrow$

Landau pole, only avoidable for $\lambda = 0 \dots$: triviality bound

$\lambda(\text{low energy}) \overset{!}{<} \lambda_{max} \Rightarrow m_h < m_h^{max}$

b) small λ

$$\frac{d\lambda}{d\log Q^2} = -\frac{3}{16\pi^2} y_t^4 \quad : \lambda \downarrow \text{ for } Q^2 \nearrow$$

shouldn't be negative...

"vacuum stability"

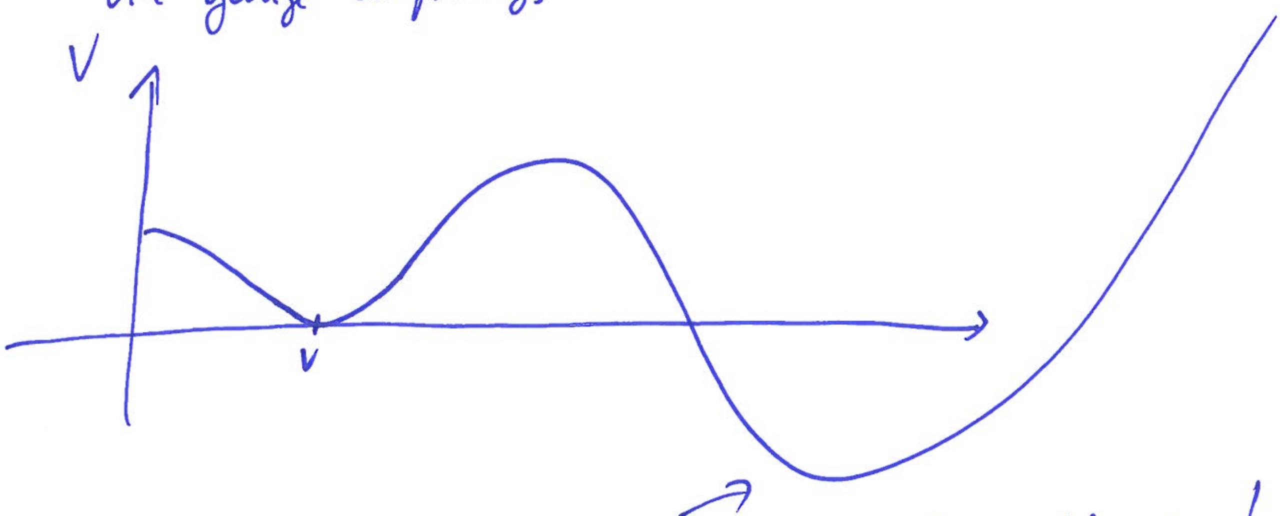
$\lambda(\text{low energy}) > \lambda_{min} \Rightarrow m_h > m_h^{min}$

further corrections, e.g. $V_{eff} \propto y_t^4 \, \Phi^4 \ln g^2\Phi^2/\mu^2$

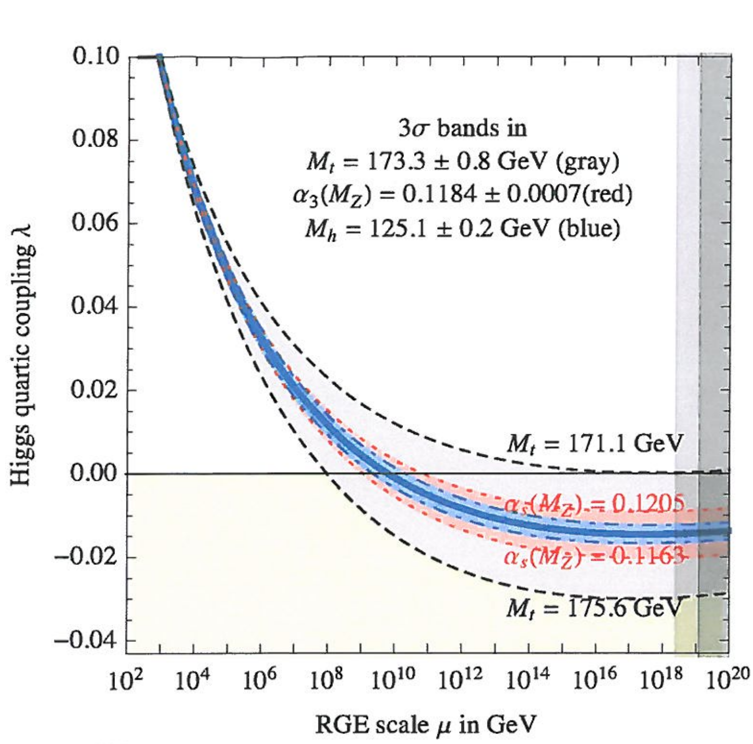
from summing terms  ...

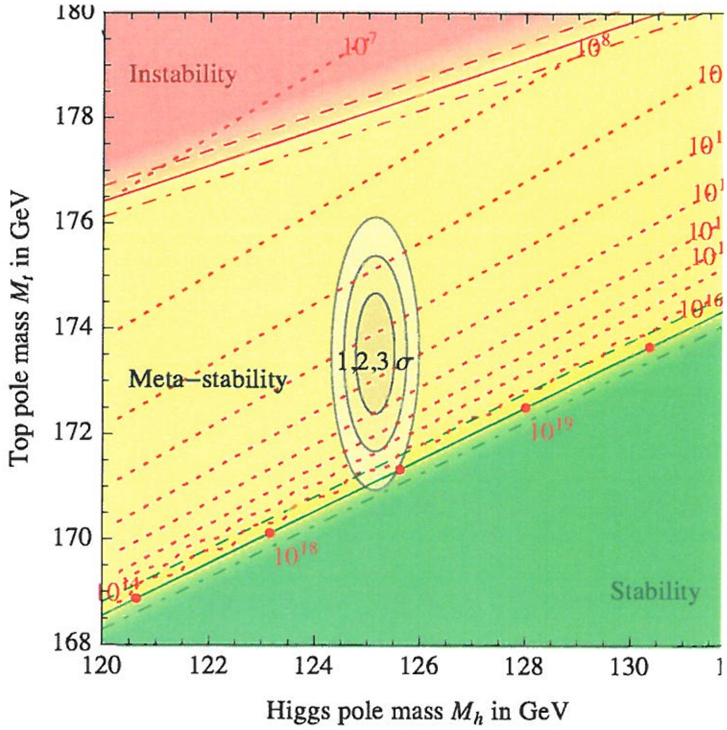
[note: if no μ^2-term: "conformal symmetry"
minimum generated by V_{eff} ! "dimensional transmutation"
Coleman-Weinberg mechanism, doesn't work in SM-only, but in extensions]

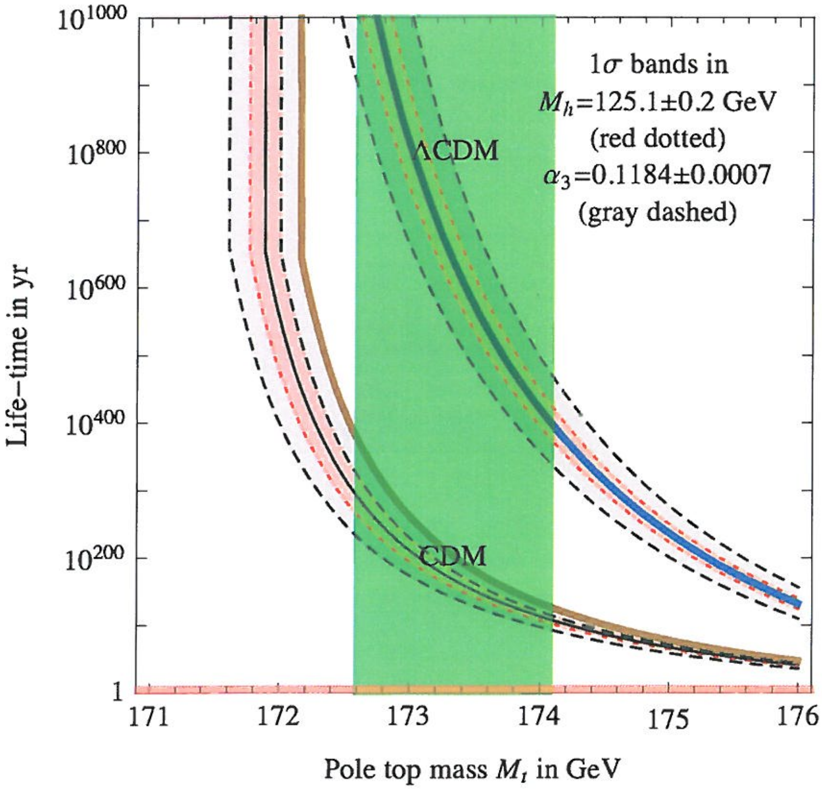
at large Φ-values V becomes positive again ($y_t \downarrow$)
via gauge couplings

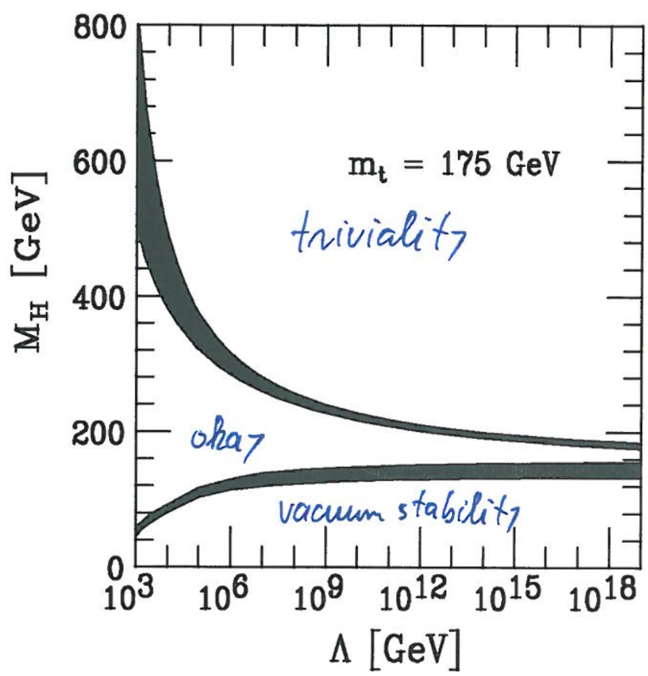


true minimum deeper than v!









See: xxx.lanl.gov/abs/1307.3536

$\Leftrightarrow$ „vacuum decay" •) tunneling
•) bubble formation

seems we are safe...

new physics potentially helping or dangerous...
(e.g. m_D of type I seesaw $\bar{L} m_D N_R$)

II B4) Hierarchy Problem

embedding of SM in extended theory

$$m_{fermion} = m_0 (1 + c \log \Lambda/\mu) \quad \text{mild...}$$

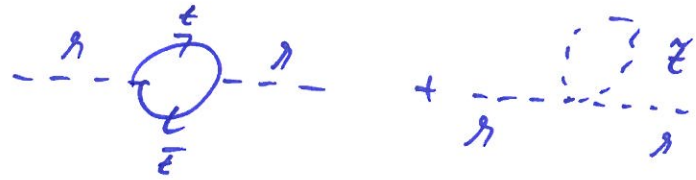
$$m_h^2 = (m_h^0)^2 + \frac{3}{8\pi^2 v^2} (4 m_t^2 - 2 m_W^2 - 4 m_Z^2 - m_h^2) \Lambda^2$$

quadratic!

want $\Delta m_h \lesssim m_h^0$, but for $\Lambda = M_{Pl} = \sqrt{\frac{1}{G_N}}$

$$m_h^2 = (m_h^0)^2 + 10^{37}\ \text{GeV}^2$$

$\Rightarrow 10^{-33}$ tuning... ☹

e.g. SUSY: 

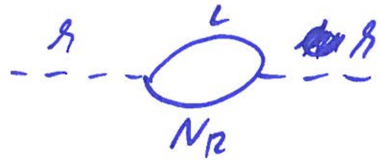
"stop"

$$\Delta m_h^2 \propto (-\lambda_t^2 + \lambda_S)\,\Lambda^2 = 0 \quad \text{for exact SUSY...}$$

if broken: $\Delta m_h^2 \simeq \frac{1}{16\pi^2}\, m_{SUSY}^2 \log \Lambda/m_{SUSY}$

(must be zero for $m_{SUSY} = 0$, should be $\Lambda^2 \to \log \Lambda/m_{SUSY}$)

$\Rightarrow$ expect new particles @ TeV for $\Delta m_h^2 \lesssim 10^2$ GeV

<u>Example:</u> 

type I seesaw

$$\delta m_h^2 = c\, y^2 M_N^2 = c\,\frac{m_D^2 M_N^2}{v^2} = c\,\frac{m_\nu M_N^3}{v^2}$$