Neutrino physics – theory

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Plan of the lectures

- Weyl, Dirac and Majorana fermions
- Neutrino masses in simplest extensions of the Standard Model. The seesaw mechanism(s).
- Neutrino oscillations in vacuum
  - Same E or same p ?
  - QM uncertainties and coherence issues
  - Wave packet approach to neutrino oscillations
  - Lorentz invariance of oscillation probabilities
  - 2f and 3f neutrino mixing schemes and oscillations
  - Implications of CP, T and CPT
- Coherent elastic neutrino nucleus scattering (CEvNS)

Weyl, Dirac and Majorana neutrino femions

Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$$

The chiral (Weyl) representation of the Dirac γ-matrices:

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix},$$

LH and RH chirality projector operators:

$$P_L = \frac{\mathbb{1} - \gamma_5}{2}, \qquad P_R = \frac{\mathbb{1} + \gamma_5}{2}.$$

They have the following properties:

$$P_L^2 = P_L\,, \quad P_R^2 = P_R\,, \quad P_L P_R = P_R P_L = 0\,, \quad P_L + P_R = \mathbb{1}$$

LH and RH spinor fields: $\Psi_{R,L} = \frac{\mathbb{1} \pm \gamma_5}{2}\Psi\,, \qquad \Psi = \Psi_L + \Psi_R\,.$

Why LH and RH chirality? For relativistic particles chirality almost coincides with helicity (projection of the spin of the particle on its momentum).

$$P_{\pm} = \frac{1}{2}\left(1 \pm \frac{\boldsymbol{\sigma}\mathbf{p}}{|\mathbf{p}|}\right).$$

At $E \gg m$ positive-energy solutions satisfy

$$\Psi_R \simeq \Psi_+\,, \qquad\qquad \Psi_L \simeq \Psi_-\,.$$

N.B.: Helicity of a free particle is conserved; chirality is not (unless $m = 0$).

Particle - antiparticle conjugation operation $\hat{C}$:

$$\hat{C}: \qquad \psi \to \psi^c = \mathcal{C}\bar{\psi}^T$$

where $\bar{\psi} \equiv \psi^\dagger \gamma^0$ and $\mathcal{C}$ satisfies

$$\mathcal{C}^{-1}\gamma_\mu \mathcal{C} = -\gamma_\mu^T\,, \qquad \mathcal{C}^\dagger = \mathcal{C}^{-1} = -\mathcal{C}^* \quad (\Rightarrow\ \mathcal{C}^T = -\mathcal{C})\,.$$

In the Weyl representation: $\mathcal{C} \; = \; i\gamma^2\gamma^0$.

Some useful relations:

◊ $(\psi^c)^c = \psi\,, \quad \overline{\psi^c} = -\psi^T \mathcal{C}^{-1}\,, \quad \overline{\psi_1}\psi_2^c = \overline{\psi_2}\psi_1^c\,, \quad \overline{\psi_1} A \psi_2 = \overline{\psi_2^c}(\mathcal{C} A^T \mathcal{C}^{-1})\psi_1^c\,.$

(A – an arbitrary 4×4 matrix).

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$$\diamondsuit \qquad (\psi_L)^c = (\psi^c)_R\,, \qquad (\psi_R)^c = (\psi^c)_L\,,$$

i.e. the antiparticle of a left-handed fermion is right-handed.

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From the expression for γ_5:

$$\psi_L = \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \qquad \psi_R = \begin{pmatrix} 0 \\ \xi \end{pmatrix},$$

$\Rightarrow$ Chiral fields are 2-component rather than 4-component objects.

Dirac *vs.* Majorana neutrino masses

Dirac equation in terms of 2-spinors ϕ and ξ:

$$(i\partial_0 - i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})\phi - m\xi = 0\,,$$

$$(i\partial_0 + i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})\xi - m\phi = 0\,.$$

Fermion mass couples LH and RH components of ψ. For $m = 0$ eqs. for ϕ and ξ decouple (Weyl equations; Weyl fermions).

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Dirac Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi\,.$$

The fermion mass Lagrangian:

$$-\mathcal{L}_m \;=\; m\,\bar{\psi}\psi \;=\; m\,(\bar{\psi}_L + \bar{\psi}_R)(\psi_L + \psi_R) \;=\; m\,(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)\,,$$

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LH and RH fields are necessary to make up a fermion mass.

Dirac fermions: ψ_L and ψ_R are completely independent fields

For Majorana fermions: $\psi_R \;=\; (\psi_L)^c$, where $(\psi)^c \;\equiv\; \mathcal{C}\,\bar{\psi}^T$.

Dirac *vs.* Majorana neutrino masses

Acting on a chiral field, particle-antiparticle conjugation flips its chirality:

$$(\psi_L)^c = (\psi^c)_R\,, \qquad (\psi_R)^c = (\psi^c)_L$$

(the antiparticle of a left handed fermion is right handed) $\Rightarrow$
one can construct a massive fermion field out of ψ_L and $(\psi_L)^c$:

$$\chi \;=\; \psi_L \;+\; (\psi_L)^c$$

$\Rightarrow$ Majorana field:

$$\chi^c \;=\; \chi$$

Majorana mass term:

$$-\mathcal{L}_m^{Maj} \;=\; \frac{m}{2}\,\overline{(\psi_L)^c}\,\psi_L \;+\; h.c. \;=\; -\,\frac{m}{2}\,\psi_L^T \mathcal{C}^{-1}\,\psi_L \;+\; h.c. \;=\; \frac{m}{2}\,\bar{\chi}\chi\,.$$

Breaks all charges (electric, lepton, baryon) – can only be written for entirely neutral fermions $\Rightarrow$ Neutrinos are the only known candidates!

D. and M. fields: plane wave decomposition

Plane-wave decomposition of a Dirac field:

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_s \left[b_s(\vec{p})u_s(\vec{p})e^{-ipx} + d_s^\dagger(\vec{p})v_s(\vec{p})e^{ipx}\right]$$

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For Majorana fields:

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The spinors $u_s(\vec{p})$ and $v_s(\vec{p})$ satisfy

$$\mathcal{C}\,\overline{u}^T = v\,, \qquad \mathcal{C}\,\overline{v}^T = u \qquad \Rightarrow$$

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$$\boxed{\chi^c \equiv \mathcal{C}\bar{\chi}^T = \chi}$$

♢ Majorana particles are genuinely neutral (coincide with their antiparticles).

Fermion masses in the Standard Model

Come from Yukawa interactions of fermions with the Higgs field:

$$-\mathcal{L}_Y = h^u_{ij}\overline{Q}_{Li}u_{Rj}\tilde{H} + h^d_{ij}\overline{Q}_{Li}d_{Rj}H + f^e_{ij}\bar{l}_{Li}e_{Rj}H + h.c.$$

$$Q_{Li} = \binom{u_{Li}}{d_{Li}}, \qquad l_{Li} = \binom{\nu_{Li}}{e_{Li}}, \qquad H = \binom{H^+}{H^0}, \qquad \tilde{H} = i\tau_2 H^*$$

$u_{Ri},\ d_{Ri},\ e_{Ri}$ – $SU(2)_L$ - singlets.

EWSB: $\langle H^0\rangle = v \simeq 174$ GeV $\Rightarrow$ fermion mass matrices are generated:

$\diamondsuit$ $\quad (m_u)_{ij} = h^u_{ij}v\,, \qquad (m_d)_{ij} = h^d_{ij}v\,, \qquad (m_e)_{ij} = f^e_{ij}v\,.$

No RH neutrinos were introduced in the SM!

Why is $m_\nu = 0$ in the Standard Model ?

- No RH neutrinos N_{Ri} – Dirac mass terms cannot be introduced
- Operators of the kind $l\,l H H$, which could could produce Majorana neutrino mass after $H \to \langle H \rangle$, are dimension 5 and so cannot be present at the Lagrangian level in a renormalizable theory
- These operators cannot be induced in higher orders either (even nonperturbatively) because they would break not only lepton number L but also $B-L$, which is exactly conserved in the SM

<u>In the Standard Model:</u>

B and L are accidental symmetries at the Lagrangian level. Get broken at 1-loop level due the axial (triangle) anomaly. <u>But:</u> their difference $B-L$ is still conserved and is an exact symmetry of the model

Diagonalization of fermion mass matrices

I. Dirac fermions (e.g. charged leptons):

$$-\mathcal{L}_m \;=\; \sum_{a,b=1}^{N_f} m'_{ab}\,\bar{\Psi}'_{aL}\Psi'_{bR} + h.c. \;=\; \bar{\Psi}'_L m' \Psi'_R + \bar{\Psi}'_R m'^{\dagger} \Psi'_L$$

Rotate Ψ'_L and Ψ'_R by unitary transformations:

$$\Psi'_L = V_L \Psi_L\,, \quad \Psi'_R = V_R \Psi_R\,; \qquad\qquad m = V_L^{\dagger} m' V_R \;=\; diag.$$

Diagonalized mass term:

$$-\mathcal{L}_m \;=\; \bar{\Psi}_L (V_L^{\dagger} m' V_R) \Psi_R + h.c. \;=\; \sum_{i=1}^{N_f} m_i \bar{\Psi}_{iL} \Psi_{Ri} + h.c.$$

Mass eigenstate fields:

$$\Psi_i \;=\; \Psi_{iL} + \Psi_{iR}; \qquad\qquad -\mathcal{L}_m = \sum_{i=1}^{N_f} m_i\,\bar{\Psi}_i \Psi_i$$

Invariant w.r.t. $U(1)$ transfs. $\Psi_i \to e^{i\alpha_i}\Psi_i$ – conservs individual ferm. numbers

Diagonalization of fermion mass matrices

II. Majorana fermions:

$$\mathcal{L}_m \;=\; -\frac{1}{2}\sum_{a,b=1}^{N_f} m'_{ab}\,\overline{(\Psi'_{aL})^c}\,\Psi'_{bL} + h.c. \;=\; \frac{1}{2}\Psi'^T_L C^{-1}\,m'\Psi'_L + h.c.$$

Matrix m' is symmetric: $m'^T = m'$. $\diamond$ Problem: prove this.

Unitary transformation of Ψ'_L:

$$\Psi'_L = U_L \Psi_L\,, \qquad\qquad m = U_L^T m'\,U_L \;=\; diag.$$

Diagonalized mass term:

$$\mathcal{L}_m \;=\; \frac{1}{2}[\Psi_L^T C^{-1}(U_L^T m'\,U_L)\Psi_L + h.c. \;=\; \frac{1}{2}\sum_{i=1}^{N_f} m_i \Psi_{Li}^T\,C^{-1}\,\Psi_{Li} + h.c.$$

Mass eigenstate fields:

$$\chi_i \;=\; \Psi_{iL} + (\Psi_{iL})^c; \qquad\qquad \mathcal{L}_m = -\frac{1}{2}\sum_{i=1}^{N_f} m_i\,\bar{\chi}_i\chi_i$$

Not invariant w.r.t. $U(1)$ transfs. $\Psi_{Li} \to e^{i\alpha_i}\Psi_{Li}$

Neutrino masses and lepton flavour violation

For Dirac neutrinos the relevant terms in the Lagrangian are

$$-\mathcal{L}_{w+m} \;=\; \frac{g}{\sqrt{2}}(\bar{e}'_{La}\gamma^{\mu}\,\nu'_{La})\,W^{-}_{\mu} + (m'_l)_{ab}\,\bar{e}'_{Ra}e'_{Lb} + (m'_{\nu})_{ab}\,\bar{\nu}'_{Ra}\nu'_{Lb} \;+\; h.c.$$

Diagonalization of mass matrices:

$$e'_L \;=\; V_L\,e_L\,, \quad e'_R \;=\; V_R\,e_R\,, \quad \nu'_L \;=\; U_L\,\nu_L\,, \quad \nu'_R \;=\; U_R\,\nu_R$$

$$V_L^{\dagger}m'_lV_R \;=\; m_l\,, \qquad U_L^{\dagger}m'_{\nu}U_R \;=\; m_{\nu} \qquad (m_{l,\nu} \;-\; \text{diagonal mass matrices})$$

$$-\mathcal{L}_{w+m} \;=\; \frac{g}{\sqrt{2}}(\bar{e}_L\gamma^{\mu}\,V_L^{\dagger}U_L\,\nu_L)\,W^{-}_{\mu} \;+\; \text{diag. mass terms} + h.c.$$

For $m'_{\nu}=0$: without loss of generality one can consider both CC term and m_l term diagonal $\Rightarrow$ the Lagrangian is invariant w.r.t. three separate $U(1)$ transformations:

$$\diamondsuit \quad e_{La,Ra} \to e^{i\phi_a}e_{La,Ra}\,, \quad \nu_{La,Ra} \to e^{i\phi_a}\nu_{La,Ra} \qquad (a=e,\mu,\tau)$$

Neutrino masses and lepton flavour violation

⇒ For massles neutrinos three individual lepton numbers (lepton flavours) $L_e,\ L_\mu,\ L_\tau$ conserved.

For massive Dirac neutrinos $L_e,\ L_\mu,\ L_\tau$ are violated ⇒ ν oscillations and $\mu \to e\gamma$, $\mu \to 3e$, etc. allowed.

But: the total lepton number $L = L_e + L_\mu + L_\tau$ is conserved.

For massive Majorana neutrinos: individual lepton flavours $L_e,\ L_\mu,\ L_\tau$ and the total lepton number L are violated.

In addition to neutrino oscillations and LFV decays $2\beta0\nu$ decay ($\Delta L = 2$ process) is allowed.

Why are neutrinos so light ?

In the minimal SM: $m_\nu = 0$. Add 3 RH ν's N_{Ri}:

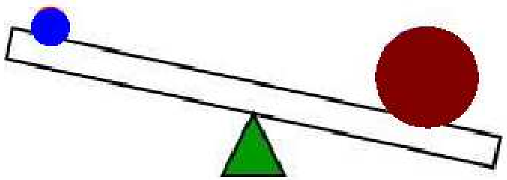
$$-\mathcal{L}_Y \supset Y_\nu \bar{l}_L N_R H + h.c., \qquad l_{Li} = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}$$

$\langle H^0 \rangle = v = 174\ \mathrm{GeV} \quad \Rightarrow \quad m_\nu = m_D = Y_\nu v$
$m_\nu < 1\ \mathrm{eV} \quad \Rightarrow \quad Y_\nu < 10^{-11}$ – Not natural !

Is it a problem? $Y_e \simeq 3 \times 10^{-6}$. But: with $m_\nu \neq 0$, huge disparity between the masses within each fermion generation !

A simple and elegant mechanism – seesaw
(Minkowski, 1977; Gell-Mann, Ramond & Slansky, 1979; Yanagida, 1979; Glashow, 1979; Mohapatra & Senjanović, 1980)

Heavy N_{Ri}'s make ν_{Li}'s light :



$$-\mathcal{L}_{Y+m} = Y_\nu \bar{l}_L N_R \tilde{H} + \frac{1}{2} M_R N_R N_R + h.c.,$$

In the $n_L = (\nu_L,\ (N_R)^c)^T$ basis: $-\mathcal{L}_m = \frac{1}{2} n_L^T C \mathcal{M}_\nu n_L + h.c.,$

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix}$$

N_{Ri} are EW singlets $\Rightarrow$ M_R can be $\sim M_{\rm GUT}(M_I) \gg m_D \sim v$.

Block diagonalization: $M_N \simeq M_R$,

♢ $\boxed{m_{\nu_L} \simeq -m_D^T\, M_R^{-1}\, m_D}$ $\Rightarrow$ $m_\nu \sim \frac{(174\ \mathrm{GeV})^2}{M_R}$

For $m_\nu \lesssim 0.05\ \mathrm{eV}$ $\Rightarrow$ $M_R \gtrsim 10^{15}$ GeV$\sim M_{GUT} \sim 10^{16}$ GeV !

The (type I) seesaw mechanism

Consider the case of n LH and k RH neutrino fields:

$$\mathcal{L}_m = \frac{1}{2}\nu_L'^T\, \mathcal{C}^{-1}\, m_L\, \nu_L' - \overline{N_R'}\, m_D\, \nu_L' + \frac{1}{2}N_R'^T\, \mathcal{C}^{-1}\, M_R^*\, N_R' + h.c.$$

m_L and M_R – $n \times n$ and $k \times k$ symmetric matrices, m_D – an $k \times n$ matrix.

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m_L and M_R – $n\times n$ and $k\times k$ symmetric matrices, m_D – an $k\times n$ matrix.
Introduce an $n+k$ - component LH field

$$n_L = \begin{pmatrix} \nu_L' \\ (N_R')^c \end{pmatrix} = \begin{pmatrix} \nu_L' \\ N_L'^c \end{pmatrix} \qquad \Rightarrow$$

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$$\mathcal{L}_m = \frac{1}{2}\,n_L^T\,\mathcal{C}^{-1}\mathcal{M}\,n_L + h.c.\,,$$

where

$$\mathcal{M} = \begin{pmatrix} m_L & m_D^T \\ m_D & M_R \end{pmatrix} \qquad (\mathcal{M}:\ \text{matrix}\ (n+k)\times(n+k))$$

Problem: prove these formulas.

Block-diagonalization of $\mathcal{M}$

$$n_L = V \chi'_L \,, \qquad V^T \mathcal{M} V = V^T \begin{pmatrix} m_L & m_D^T \\ m_D & M_R \end{pmatrix} V = \begin{pmatrix} \tilde{m}_L & 0 \\ 0 & \tilde{M}_R \end{pmatrix}$$

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Look for the unitary matrix V in the form

$$V = \begin{pmatrix} \sqrt{1-\rho\rho^\dagger} & \rho \\ -\rho^\dagger & \sqrt{1-\rho^\dagger\rho} \end{pmatrix}$$

(ρ: matrix $n \times k$)

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Assume that characteristic scales of neutrino masses satisfy

$$m_L, m_D \ll M_R \qquad \Rightarrow \quad \rho \ll 1.$$

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$$m_L, m_D \ll M_R \qquad \Rightarrow \quad \rho \ll 1.$$

Treat ρ as perturbation $\Rightarrow$

$$\rho^* \simeq m_D^T M_R^{-1}\,, \qquad \tilde{M}_R \simeq M_R\,,$$

$$\boxed{\tilde{m}_L \simeq m_L - m_D^T M_R^{-1} m_D}$$

Type I seesaw mechanism – 1-gener. case

A simple 1-flavour case ($n = k = 1$). Notation change: $M_R \to m_R$, $N_R \to \nu_R$.

$$\mathcal{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \qquad (m_L,\, m_D,\, m_R \ - \ \text{real positive numbers})$$

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Can be diagonalized as $O^T \mathcal{M} O = \mathcal{M}_d$ where O is real orthogonal 2×2 matrix and $\mathcal{M}_d = diag(m_1, m_2)$. Introduce the fields χ_L through $n_L = O\chi_L$:

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$$n_L = \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix} \qquad (\chi_{1L},\, \chi_{2L} - \text{LH comp. of } \chi_{1,2})$$

Type I seesaw mechanism – 1-gener. case

A simple 1-flavour case ($n = k = 1$). Notation change: $M_R \to m_R$, $N_R \to \nu_R$.

$$\mathcal{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \qquad (m_L,\, m_D,\, m_R \; - \; \text{real positive numbers})$$

Can be diagonalized as $O^T \mathcal{M} O = \mathcal{M}_d$ where O is real orthogonal 2×2 matrix and $\mathcal{M}_d = diag(m_1,\, m_2)$. Introduce the fields χ_L through $n_L = O\chi_L$:

$$n_L = \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix} \qquad (\chi_{1L},\, \chi_{2L} \; - \; \text{LH comp. of } \chi_{1,2})$$

Rotation angle and mass eigenvalues:

$$\tan 2\theta = \frac{2 m_D}{m_R - m_L}\,,$$

$$m_{1,2} = \frac{m_R + m_L}{2} \mp \sqrt{\left(\frac{m_R - m_L}{2}\right)^2 + m_D^2}\,.$$

$m_1,\, m_2$ real but can be of either sign

1-generation case – contd.

$$\begin{aligned}\mathcal{L}_m &= \frac{1}{2}\, n_L^T\, \mathcal{C}^{-1}\mathcal{M}\, n_L + h.c. = \frac{1}{2}\, \chi_L^T\, \mathcal{C}^{-1}\mathcal{M}_d\, \chi_L + h.c.\\ &= \frac{1}{2}(m_1\, \chi_{1L}^T\, \mathcal{C}^{-1}\chi_{1L} + m_2\, \chi_{2L}^T\, \mathcal{C}^{-1}\chi_{2L}) + h.c. = \frac{1}{2}(\,|m_1|\, \overline{\chi}_1\chi_1 + |m_2|\, \overline{\chi}_2\chi_2\,)\end{aligned}$$

1-generation case – contd.

$$\mathcal{L}_m = \frac{1}{2}\, n_L^T\, \mathcal{C}^{-1} \mathcal{M}\, n_L + h.c. = \frac{1}{2}\, \chi_L^T\, \mathcal{C}^{-1} \mathcal{M}_d\, \chi_L + h.c.$$

$$= \frac{1}{2}(m_1\, \chi_{1L}^T\, \mathcal{C}^{-1} \chi_{1L} + m_2\, \chi_{2L}^T\, \mathcal{C}^{-1} \chi_{2L}) + h.c. = \frac{1}{2}(\,|m_1|\,\overline{\chi}_1 \chi_1 + |m_2|\,\overline{\chi}_2 \chi_2\,)$$

Here

$$\chi_1 = \chi_{1L} + \eta_1 (\chi_{1L})^c\,, \qquad \chi_2 = \chi_{2L} + \eta_2 (\chi_{2L})^c\,.$$

with $\eta_i = 1$ or -1 for $m_i > 0$ or < 0 respectively.

1-generation case – contd.

$$\mathcal{L}_m = \frac{1}{2}\, n_L^T\, \mathcal{C}^{-1} \mathcal{M}\, n_L + h.c. = \frac{1}{2}\, \chi_L^T\, \mathcal{C}^{-1} \mathcal{M}_d\, \chi_L + h.c.$$

$$= \frac{1}{2}(m_1\, \chi_{1L}^T\, \mathcal{C}^{-1} \chi_{1L} + m_2\, \chi_{2L}^T\, \mathcal{C}^{-1} \chi_{2L}) + h.c. = \frac{1}{2}(\,|m_1|\,\overline{\chi}_1 \chi_1 + |m_2|\,\overline{\chi}_2 \chi_2\,)$$

Here

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♢ Mass eigenstates χ_1, χ_2 are Majorana states!

1-generation case – contd.

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$$= \frac{1}{2}(m_1\, \chi_{1L}^T\, \mathcal{C}^{-1} \chi_{1L} + m_2\, \chi_{2L}^T\, \mathcal{C}^{-1} \chi_{2L}) + h.c. = \frac{1}{2}(\,|m_1|\, \overline{\chi}_1 \chi_1 + |m_2|\, \overline{\chi}_2 \chi_2\,)$$

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Interesting limiting cases:

(a) $m_R \gg m_L,\, m_D$ (seesaw limit)

$$m_1 \approx m_L - \frac{m_D^2}{m_R} \to -\frac{m_D^2}{m_R} \quad \text{for} \quad m_L = 0$$
$$m_2 \approx m_R$$

1-generation case – contd.

(b) $m_L = m_R = 0$ (Dirac case)

$$\mathcal{M} = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix} \quad \rightarrow \quad \mathcal{M}_d = \begin{pmatrix} -m & 0 \\ 0 & m \end{pmatrix} .$$

1-generation case – contd.

(b) $m_L = m_R = 0$ (Dirac case)

$$\mathcal{M} = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix} \quad \rightarrow \quad \mathcal{M}_d = \begin{pmatrix} -m & 0 \\ 0 & m \end{pmatrix}.$$

Diagonalized by rotation with angle $\theta = 45°$. We have $\eta_2 = -\eta_1 = 1$;

$$\chi_1 + \chi_2 = \sqrt{2}(\nu_L + \nu_R)\,, \quad \chi_1 - \chi_2 = -\sqrt{2}(\nu_L^c + \nu_R^c) = -(\chi_1 + \chi_2)^c.$$

$$\Downarrow$$

1-generation case – contd.

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$$\Downarrow$$

$$\frac{1}{2}\,m\,(\overline{\chi}_1\chi_1 + \overline{\chi}_2\chi_2) = \frac{1}{4}\,m\,[\overline{(\chi_1+\chi_2)}(\chi_1+\chi_2) + [\overline{(\chi_1-\chi_2)}(\chi_1-\chi_2)] = m\,\bar{\nu}_D\nu_D\,,$$

where

$$\nu_D \equiv \nu_L + \nu_R\,.$$

1-generation case – contd.

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$$\Downarrow$$

$$\frac{1}{2}\,m\,(\overline{\chi}_1\chi_1 + \overline{\chi}_2\chi_2) = \frac{1}{4}\,m\,[\overline{(\chi_1 + \chi_2)}(\chi_1 + \chi_2) + [\overline{(\chi_1 - \chi_2)}(\chi_1 - \chi_2)] = m\,\bar{\nu}_D\nu_D\,,$$

where

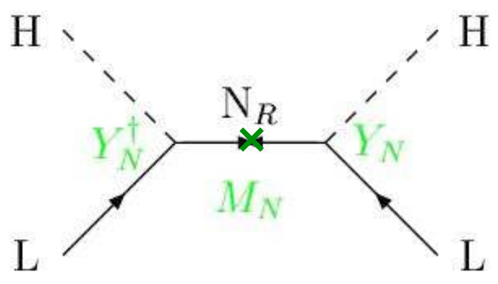
$$\nu_D \equiv \nu_L + \nu_R\,.$$

(c) $m_L, m_R \ll m_D$ (pseudo-Dirac neutrino): $|m_{1,2}| \approx m_D \pm \frac{m_L + m_R}{2}$.

The 3 basic seesaw models

i.e. tree level ways to generate the dim 5 $\frac{\lambda}{M} LLHH$ operator

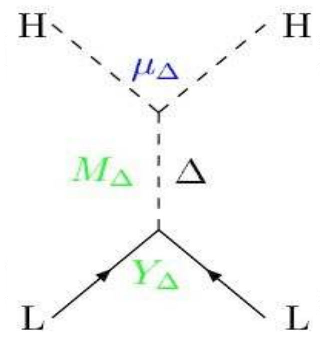
Right-handed singlet: (type-I seesaw)



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

m_ν small if M_N large (or if Y_ν small)

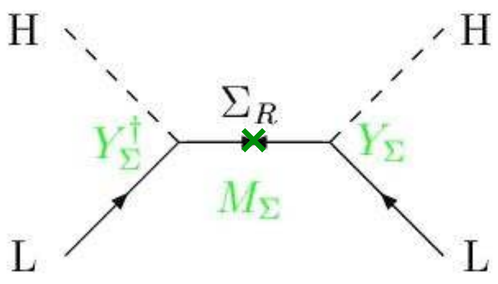
Scalar triplet: (type-II seesaw)



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

m_ν small if M_Δ large (or if Y_Δ, μ small)

Fermion triplet: (type-III seesaw)

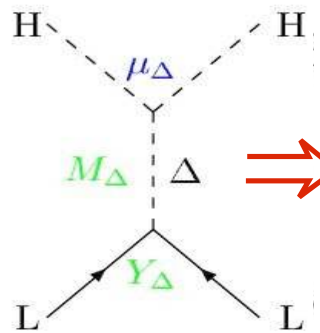


$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

m_ν small if M_Σ large (or if Y_Σ small)

Access to the seesaw parameters from ν mass matrix data

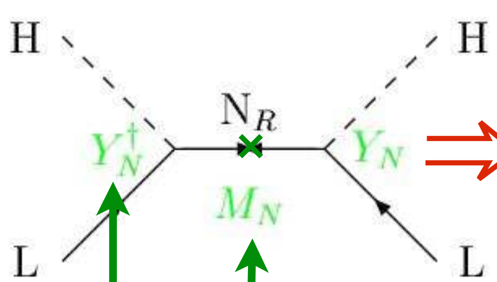
- Type II seesaw:



$$\Longrightarrow \quad m_{\nu ij} = Y_{\Delta ij} \frac{\mu_\Delta}{M_\Delta^2} v^2 \quad \Longrightarrow$$

ν mass matrix data gives full access to type II flavour structure

- Type I or III seesaw model:



$$\Longrightarrow \quad m_{\nu ij} = Y_{Nik}^T \frac{1}{M_{Nk}} Y_{Nkj} v^2 \quad \Longrightarrow$$

ν mass matrix data: gives access to 9 parameter combinations of Y_N and M_N

3 masses of the N

15 parameters in Yukawa matrix → 9 real parameters, 6 phases

} 18 parameters

Neutrino oscillations

Neutrinos can oscillate !

A periodic change of neutrino flavour (identity):

$$\nu_e \to \nu_\mu \to \nu_e \to \nu_\mu \to \nu_e \ \ \ldots$$

Happens without any external influence!

Dr. Jekyll / Mr. Hyde kind of story

Neutrinos have two-sided (or even 3-sided) personality !

$$P(\nu_e \to \nu_\mu; L) = \sin^2 2\theta \cdot \sin^2\left(\frac{\Delta m^2}{4p} L\right)$$

Hints of oscillations of solar neutrinos seen since the 1960s

First unambiguous evidence – oscillations of atmospheric neutrinos (The Super-Kamiokande Collaboration, 1998)

A bit of history...

Idea of neutrino oscillations: First put forward by Pontecorvo in 1957. Suggested possibility of $\nu \leftrightarrow \bar{\nu}$ oscillations by analogy with $K^0 \bar{K}^0$ oscillations.

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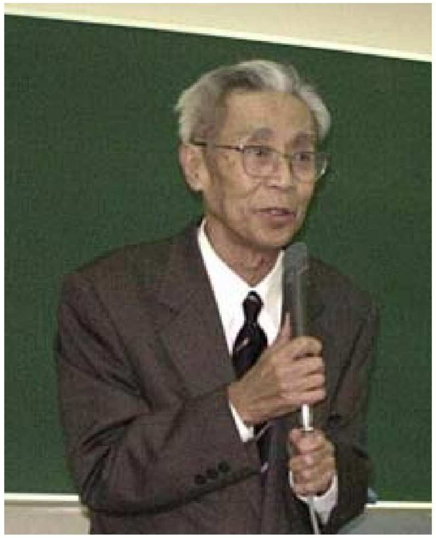
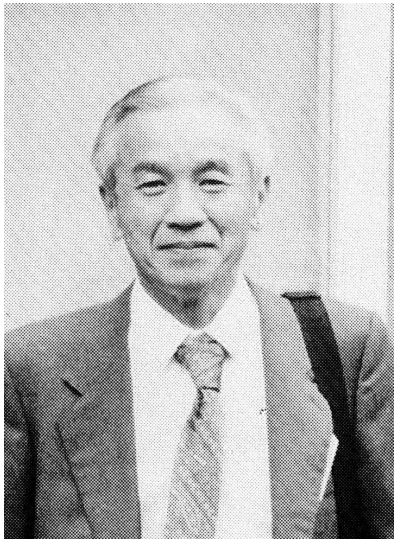
B. Pontecorvo
1913 - 1993

S. Sakata
1911 – 1970



Z. Maki
1929 – 2005



M. Nakagawa
1932 – 2001

Neutrino revolution

Neutrino mass had been unsuccessfully looked for for almost 40 years (several wrong discovery claims)

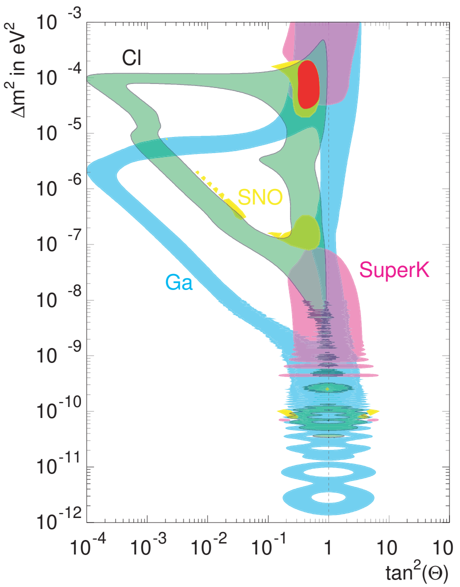
Since 1998 – an avalanche of discoveries :

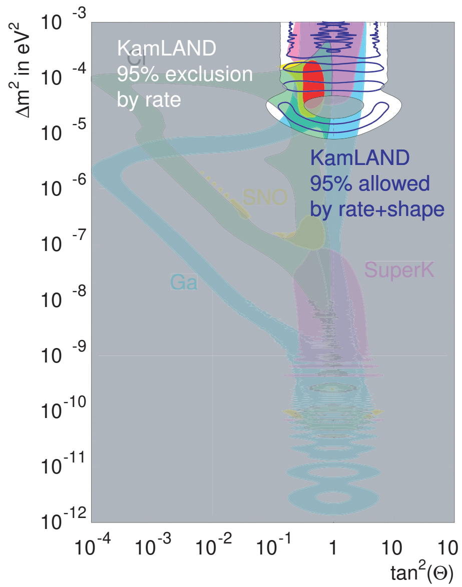
Oscillations of atmospheric, solar, reactor and accelerator neutrinos

Neutrino oscillations imply that neutrinos are massive

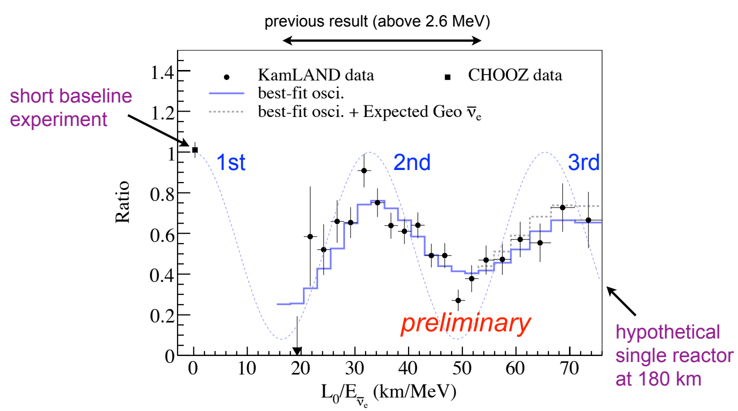
In the standard model neutrinos are massless $\Rightarrow$ we have now the first compelling evidence of physics beyond the standard model !

Oscillations discovered experimentally !





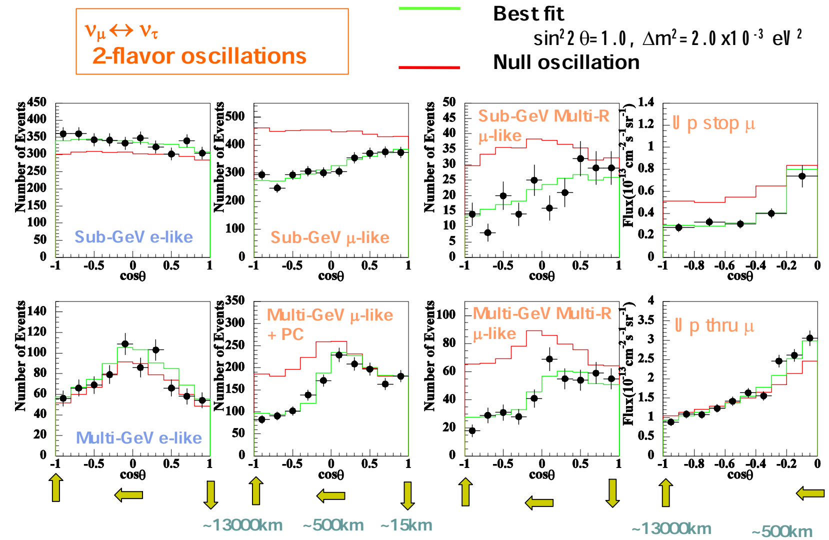
Neutrino Oscillation



KamLAND covers the 2nd and 3rd maximum

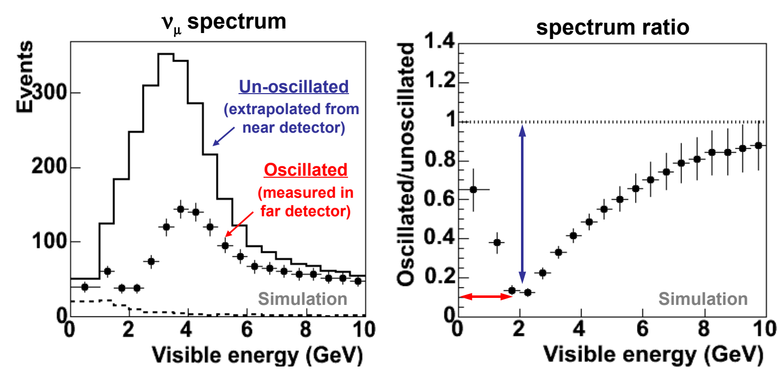
→ **characteristic of neutrino oscillation**

Zenith angle distributions



ν_μ Disappearance Measurement

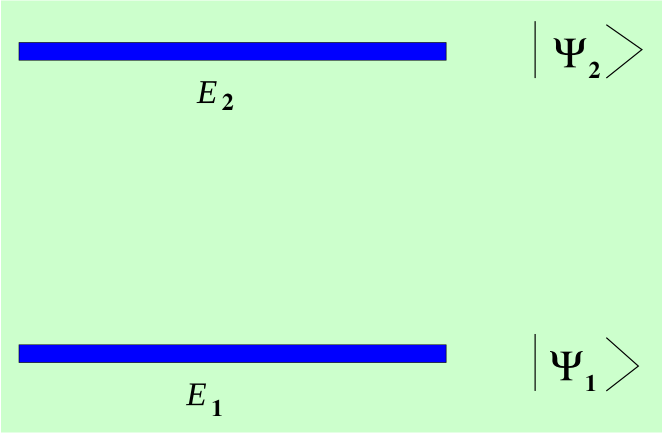
Look for ν_μ deficit : $P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{E}\right)$



Andy Blake, Cambridge University

The MINOS Experiment, slide 7

Oscillations: a well known QM phenomenon



$$\Psi_1(t) = e^{-i\,E_1\,t}\,\Psi_1(0)$$

$$\Psi_2(t) = e^{-i\,E_2\,t}\,\Psi_2(0)$$

$$\Psi(0) = a\,\Psi_1(0) + b\,\Psi_2(0) \quad (|a|^2 + |b|^2 = 1)\,; \qquad \Rightarrow$$

$$\Psi(t) = a\,e^{-i\,E_1\,t}\,\Psi_1(0) + b\,e^{-i\,E_2\,t}\,\Psi_2(0)$$

Probability to remain in the same state $|\Psi(0)\rangle$ after time t:

♢ $$P_{\rm surv} = |\langle\Psi(0)|\Psi(t)\rangle|^2 = \Big||a|^2\,e^{-i\,E_1\,t} + |b|^2\,e^{-i\,E_2\,t}\Big|^2$$

$$= 1 - 4|a|^2|b|^2\sin^2[(E_2 - E_1)\,t/2]$$

Neutrino oscillations: theory

Leptonic mixing

For $m_\nu \neq 0$ weak eigenstate neutrinos ν_e, ν_μ, ν_τ do not coincide with mass eigenstate neutrinos ν_1, ν_2, ν_3

Diagonalization of leptonic mass matrices:

$$e'_L \rightarrow V_L\, e_L\,, \qquad \nu'_L \rightarrow U_L\, \nu_L\, \ldots \qquad \Rightarrow$$

$$-\mathcal{L}_{w+m} = \frac{g}{\sqrt{2}}(\bar{e}_L \gamma^\mu\, V_L^\dagger U_L\, \nu_L)\, W_\mu^- \;+\; \text{diag. mass terms} + h.c.$$

Leptonic mixing matrix: $U = V_L^\dagger U_L$

♢ $$\nu_{\alpha L} = \sum_i U_{\alpha i}\, \nu_{iL} \qquad \Rightarrow \qquad |\nu_{\alpha L}\rangle = \sum_i U^*_{\alpha i}\, |\nu_{iL}\rangle$$

$$(\alpha = e\,,\, \mu\,,\, \tau, \qquad i = 1\,,\, 2\,,\, 3)$$

Master formula for ν oscillations

The standard formula for the oscillation probability of relativistic or quasi-degenerate in mass neutrinos in vacuum:

$$\diamondsuit \qquad \boxed{P(\nu_\alpha \to \nu_\beta; L) = \left| \sum_i U_{\beta i}\, e^{-i\frac{\Delta m^2_{ij}}{2p}L}\, U^*_{\alpha i} \right|^2}$$

$$(\hbar = c = 1)$$

Problem: prove that the RHS does not depend on the index j.

Oscillation disappear when either

- $U = \mathbb{1}$, i.e. $U_{\alpha i} = \delta_{\alpha i}$ (no mixing) or
- $\Delta m^2_{ij} = 0$ (massless or mass-degenerate neutrinos).

How is it usually derived?

Assume at time $t = 0$ and coordinate $x = 0$ a flavour eigenstate $|\nu_\alpha\rangle$ is produced:

$$|\nu(0,0)\rangle \;=\; |\nu_\alpha^{\rm fl}\rangle \;=\; \sum_i U_{\alpha i}^*\,|\nu_i^{\rm mass}\rangle$$

After time t at the position x, for plane-wave particles:

$$|\nu(t,\vec{x})\rangle \;=\; \sum_i U_{\alpha i}^*\, e^{-ip_i x}|\nu_i^{\rm mass}\rangle$$

Mass eigenstates pick up the phase factors $e^{-i\phi_i}$ with

$$\phi_i \;\equiv\; p_i\,x \;=\; Et \;-\; \vec{p}\vec{x}$$

$$P(\nu_\alpha \to \nu_\beta) \;=\; \left|\langle\nu_\beta^{\rm fl}|\nu(t,x)\rangle\right|^2$$

How is it usually derived?

Consider $\vec{x}\,||\,\vec{p} \quad \Rightarrow \quad \vec{p}\vec{x} = \mathrm{px} \quad (\mathrm{p} = |\vec{p}|, \quad \mathrm{x} = |\vec{x}|)$

Phase differences between different mass eigenstates:

$$\Delta\phi = \Delta E \cdot t - \Delta \mathrm{p} \cdot \mathrm{x}$$

<u>Shortcuts to the standard formula</u>

1. Assume the emitted neutrino state has a well defined momentum (same momentum prescription) $\quad \Rightarrow \quad \Delta\mathrm{p} = 0.$

   For ultra-relativistic neutrinos $E_i = \sqrt{\mathrm{p}^2 + m_i^2} \simeq p + \frac{m_i^2}{2\mathrm{p}} \quad \Rightarrow$

$$\Delta E \simeq \frac{m_2^2 - m_1^2}{2E} \equiv \frac{\Delta m^2}{2E}\,; \qquad t \approx x \qquad (\hbar = c = 1)$$

$\Rightarrow$ The standard formula is obtained

How is it usually derived?

2. Assume the emitted neutrino state has a well defined energy (same energy prescription) $\Rightarrow$ $\Delta E = 0$.

$$\Delta\phi = \Delta E \cdot t - \Delta \mathrm{p} \cdot \mathrm{x} \quad \Rightarrow \quad - \Delta \mathrm{p} \cdot \mathrm{x}$$

For ultra-relativistic neutrinos $\mathrm{p}_i = \sqrt{E^2 - m_i^2} \simeq E - \frac{m_i^2}{2\mathrm{p}}$ $\Rightarrow$

$$-\Delta \mathrm{p} \equiv \mathrm{p}_1 - \mathrm{p}_2 \approx \frac{\Delta m^2}{2E}\,;$$

$\Rightarrow$ The standard formula is obtained

Stand. phase $\Rightarrow$ $$(l_{\rm osc})_{ik} = \frac{4\pi E}{\Delta m_{ik}^2} \simeq 2.5\ m\ \frac{E\,(\mathrm{MeV})}{\Delta m_{ik}^2\ \mathrm{eV}^2}$$

Same E and same p approaches

Same E and same p approaches

Very simple and transparent

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Allow one to quickly arrive at the desired result

Same E and same p approaches

Very simple and transparent

Allow one to quickly arrive at the desired result

Trouble: they are both wrong

Kinematic constraints

Same momentum and same energy assumptions: contradict kinematics!

Pion decay at rest $(\pi^+ \to \mu^+ + \nu_\mu, \quad \pi^- \to \mu^- + \bar{\nu}_\mu)$:
For decay with emission of a massive neutrino of mass m_i:

$$E_i^2 = \frac{m_\pi^2}{4}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_i^2}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

$$p_i^2 = \frac{m_\pi^2}{4}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_i^2}{2}\left(1 + \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_i^4}{4m_\pi^2}$$

For massless neutrinos: $E_i = p_i = E \equiv \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 30\ \text{MeV}$

To first order in m_i^2:

$$E_i \simeq E + \xi \frac{m_i^2}{2E}, \qquad p_i \simeq E - (1-\xi)\frac{m_i^2}{2E}, \qquad \xi = \frac{1}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \approx 0.2$$

Kinematic constraints

Same momentum or same energy would require
$\xi = 1$ or $\xi = 0$ – not the case!

Also: would violate Lorentz invariance of the oscillation probability

How can wrong assumptions lead to the correct oscillation formula ?

Problems with the plane-wave approach

- Same momentum $\Rightarrow$ oscillation probabilities depend only on time. Leads to a paradoxical result – no need for a far detector ! "Time-to-space conversion" (??) – assumes neutrinos to be point-like particles (notion opposite to plane waves).
- Same energy – oscillation probabilities depend only on coordinate. Does not explain how neutrinos are produced and detected at certain times. Correspponds to a stationary situation.

Plane wave approach $\Leftrightarrow$ exact energy-momentum conservation. Neutrino energy and momentum are fully determined by those of external particles $\Rightarrow$ only one mass eigenstate can be emitted!

- Consistent approaches:

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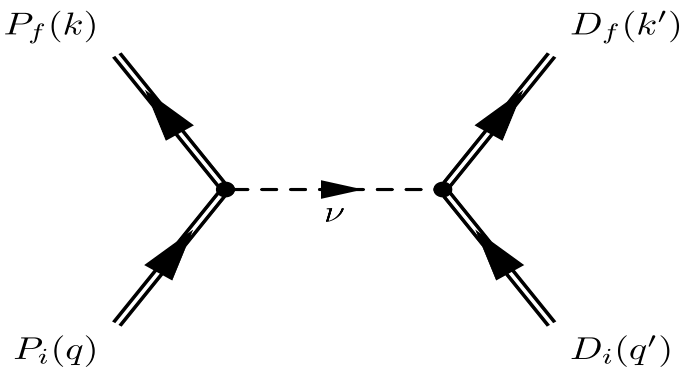
  - QM wave packet approach – neutrinos described by wave packets rather than by plane waves

♢ Consistent approaches:

- QM wave packet approach – neutrinos described by wave packets rather than by plane waves
- QFT approach: neutrino production and detection explicitly taken into account. Neutrinos are intermediate particles described by propagators

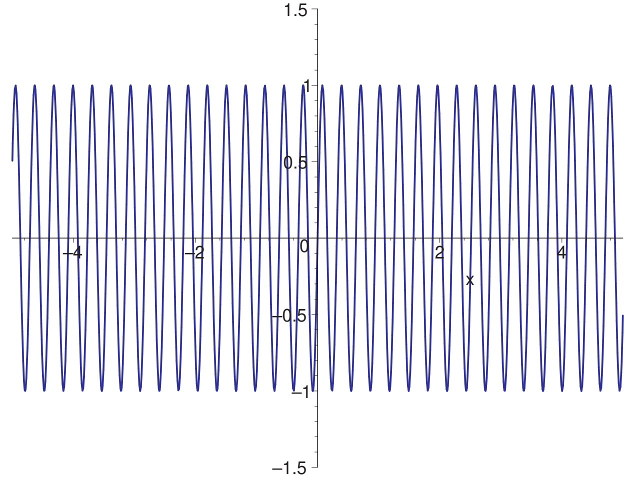


QM wave packet approach

In QM propagating particles are described by wave packets!

– Finite extensions in space and time.

Plane waves: the wave function at time $t = 0$ $\quad \Psi_{\vec{p}_0}(\vec{x}) \;=\; e^{i\vec{p}_0\vec{x}}$



Wave packets: superpositions of plane waves with momenta in an interval of width σ_p around mom. p_0 $\Rightarrow$ constructive interference in a spatial interval of width σ_x around some point x_0 and destructive interference outside it.

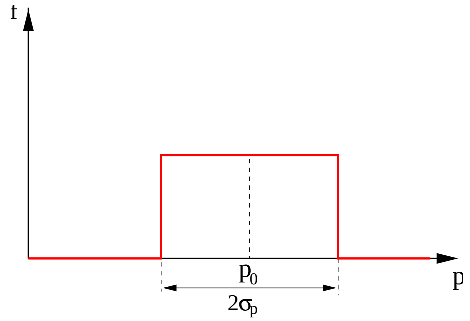
$$\sigma_x\,\sigma_p \geq 1/2 \quad \text{– QM uncertainty relation}$$

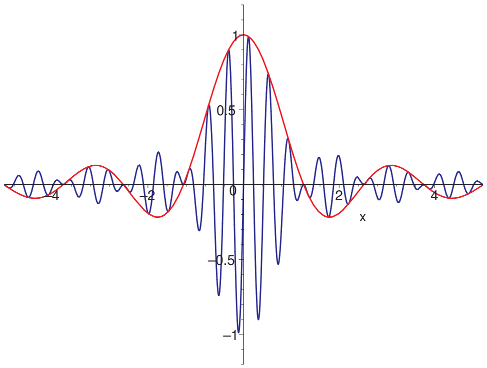
Wave packets

W. packet centered at $\vec{x}_0 = 0$ at time $t = 0$:

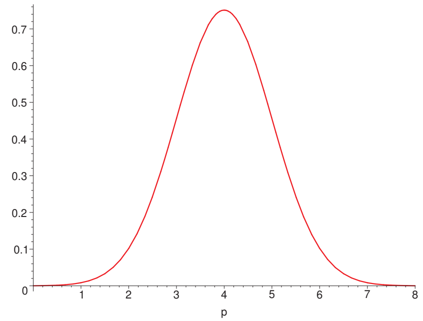
$$\Psi(\vec{x};\, \vec{p}_0, \sigma_{\vec{p}}) \;=\int \frac{d^3p}{(2\pi)^3}\, f(\vec{p}-\vec{p}_0)\, e^{i\vec{p}\,\vec{x}}$$

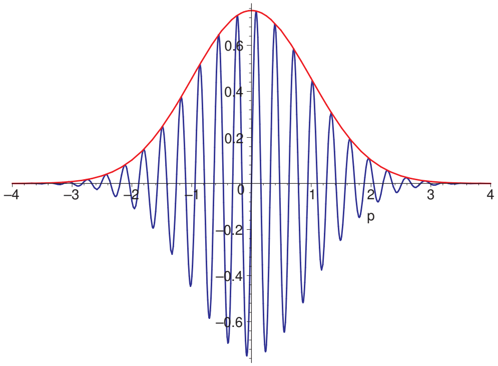
Rectangular mom. space w. packet:





Gaussian mom. space w. packet:





$\sigma_x \sigma_p \;=\; 1/2$ – minimum uncertainty packet

Propagating wave packets

Include time dependence:

$$\Psi(\vec{x},\,t) \;=\; \int \frac{d^3p}{(2\pi)^3}\, f(\vec{p}-\vec{p}_0)\, e^{i\vec{p}\vec{x}-iE(p)t}$$

Example: Gaussian wave packets

Momentum-space distribution:

$$f(\vec{p}-\vec{p}_0) \;=\; \frac{1}{(2\pi\sigma_p^2)^{3/4}} \exp\left\{-\frac{(\vec{p}-\vec{p}_0)^2}{4\sigma_p^2}\right\}$$

Momentum dispersion: $\langle \vec{p}^{\,2}\rangle - \langle \vec{p}\,\rangle^2 = \sigma_p^2$.

Coordinate-space wave packet (neglecting spreading):

$$\Psi(\vec{x},\,t) \;=\; e^{i\vec{p}_0\vec{x}-iE(p_0)t}\; \frac{1}{(2\pi\sigma_x^2)^{3/4}} \exp\left\{-\frac{(\vec{x}-\vec{v}_g t)^2}{4\sigma_x^2}\right\}, \qquad \sigma_x^2 = 1/(4\sigma_p^2)$$

$$\langle \vec{x}\,\rangle = \vec{v}_g t\,; \qquad \langle \vec{x}^{\,2}\rangle - \langle \vec{x}\,\rangle^2 = \sigma_x^2\,.$$

QM wave packet approach

The evolved produced state:

$$|\nu_\alpha^{\rm fl}(\vec{x},t)\rangle \;=\; \sum_i U_{\alpha i}^*\,|\nu_i^{\rm mass}(\vec{x},t)\rangle \;=\; \sum_i U_{\alpha i}^*\,\Psi_i^S(\vec{x},t)|\nu_i^{\rm mass}\rangle$$

The coordinate-space wave function of the ith mass eigenstate (w. packet):

$$\boxed{\Psi_i^S(\vec{x},t) \;=\; \int \frac{d^3p}{(2\pi)^3}\, f_i^S(\vec{p})\, e^{i\vec{p}\vec{x}-iE_i(p)t}}$$

Momentum distribution function $f_i^S(\vec{p})$: sharp maximum at $\vec{p}=\vec{P}$ (width of the peak $\sigma_{pP} \ll P$).

$$E_i(p) \;=\; E_i(P) \,+\, \frac{\partial E_i(p)}{\partial \vec{p}}\bigg|_{\vec{P}} (\vec{p}-\vec{P}) \,+\, \frac{1}{2}\frac{\partial^2 E_i(p)}{\partial \vec{p}^2}\bigg|_{\vec{p}_0} (\vec{p}-\vec{P})^2 \,+\, \ldots$$

$$\vec{v}_i \;=\; \frac{\partial E_i(p)}{\partial \vec{p}} \;=\; \frac{\vec{p}}{E_i}\,, \qquad\qquad \alpha \;\equiv\; \frac{\partial^2 E_i(p)}{\partial \vec{p}^2} \;=\; \frac{m_i^2}{E_i^2}$$

Evolved neutrino state

$$\Psi_i^S(\vec{x},\,t)\;\simeq\;e^{-iE_i(P)t+i\vec{P}\vec{x}}\,g_i^S(\vec{x}-\vec{v}_i t)\qquad(\alpha\;\to\;0)$$

$$g_i^S(\vec{x}-\vec{v}_i t)\;\equiv\;\int\frac{d^3q}{(2\pi)^3}\,f_i^S(\vec{q}+\vec{P})\,e^{i\vec{q}(\vec{x}-\vec{v}_g t)}$$

Problem: derive this result

Center of the wave packet: $\vec{x}-\vec{v}_i t=0$. Spatial length: $\sigma_{xP}\;\sim\;1/\sigma_{pP}$
(g_i^S decreases quickly for $|\vec{x}-\vec{v}_i t|\gtrsim\sigma_{xP}$).

Detected state (centered at $\vec{x}=\vec{L}$):

$$|\nu_\beta^{\rm fl}(\vec{x})\rangle\;=\;\sum_k U_{\beta k}^*\,\Psi_k^D(\vec{x})|\nu_i^{\rm mass}\rangle$$

The coordinate-space wave function of the ith mass eigenstate (w. packet):

$$\Psi_i^D(\vec{x})\;=\;\int\frac{d^3p}{(2\pi)^3}\,f_i^D(\vec{p})\,e^{i\vec{p}(\vec{x}-\vec{L})}$$

Oscillation probability

Transition amplitude:

$$\mathcal{A}_{\alpha\beta}(T,\vec{L}) \;=\; \langle \nu_\beta^{\rm fl}|\nu_\alpha^{\rm fl}(T,\vec{L})\rangle \;=\; \sum_i U_{\alpha i}^* U_{\beta i}\, \mathcal{A}_i(T,\vec{L})$$

$$\mathcal{A}_i(T,\vec{L}) \;=\; \int \frac{d^3p}{(2\pi)^3}\, f_i^S(\vec{p})\, f_i^{D*}(\vec{p})\, e^{-iE_i(p)T+i\vec{p}\vec{L}}$$

Strongly suppressed unless $|\vec{L}-\vec{v}_i T| \lesssim \sigma_x$. E.g., for Gaussian wave packets:

$$\mathcal{A}_i(T,\vec{L}) \;\propto\; \exp\left[-\frac{(\vec{L}-\vec{v}_i T)^2}{4\sigma_x^2}\right], \qquad \sigma_x^2 \;\equiv\; \sigma_{xP}^2+\sigma_{xD}^2$$

Oscillation probability:

◇ $$P(\nu_\alpha\to\nu_\beta;T,\vec{L}) \;=\; |\mathcal{A}_{\alpha\beta}|^2 = \sum_{i,k} U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^*\, \mathcal{A}_i(T,\vec{L})\mathcal{A}_k^*(T,\vec{L})$$

Phase difference

Oscillations are due to phase differences of different mass eigenstates:

$$\Delta\phi = \Delta E \cdot T - \Delta p \cdot L \qquad (E_i = \sqrt{p_i^2 + m_i^2})$$

Consider the case $\Delta E \ll E$ (relativistic or quasi-degenerate neutrinos) $\Rightarrow$

$$\Delta E = \frac{\partial E}{\partial p}\Delta p + \frac{\partial E}{\partial m^2}\Delta m^2 = v_g\,\Delta p + \frac{1}{2E}\,\Delta m^2$$

$$\Delta\phi = (v_g\,\Delta p + \frac{1}{2E}\Delta m^2)\,T - \Delta p \cdot L$$

$$= -\,(L - v_g\,T)\Delta p + \frac{\Delta m^2}{2E}\,T$$

In the center of wave packet $(L - v_g\,T) = 0\,!$ In general, $|L - v_g\,T| \lesssim \sigma_x$;
if $\sigma_x \ll l_{\rm osc}$, $|L - v_g\,T|\Delta p \ll 1 \quad \Rightarrow$

$$\Delta\phi = \frac{\Delta m^2}{2E}\,T\,, \qquad\qquad L \simeq v_g T \simeq T$$

– the result of the “same momentum” approach recovered!

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$$\diamondsuit \qquad \Delta\phi = -\frac{1}{v_g}(L - v_g\,T)\Delta E + \frac{\Delta m^2}{2p}\,L \qquad \Rightarrow \qquad \frac{\Delta m^2}{2p}\,L$$

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- Neutrinos are relativistic or quasi-degenerate with $\Delta E \ll E$

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– the result of the "same energy" approach recovered!

The reasons why wrong assumptions give the correct result:

- Neutrinos are relativistic or quasi-degenerate with $\Delta E \ll E$
- The size of the neutrino wave packet is small compared to the oscillation length: $\sigma_x \ll l_{\rm osc}$ (more precisely: energy uncertainty $\sigma_E \gg \Delta E$)

Oscillation probability in WP approach

$$P(\nu_\alpha \to \nu_\beta; T, \vec{L}) \;=\; \left|\mathcal{A}_{\alpha\beta}\right|^2 = \sum_{i,k} U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^* \, \mathcal{A}_i(T, \vec{L}) \mathcal{A}_k^*(T, \vec{L})$$

$$\mathcal{A}_i(T, \vec{L}) \;=\; \int \frac{d^3p}{(2\pi)^3} \, f_i^S(\vec{p}) \, f_i^{D*}(\vec{p}) \, e^{-iE_i(p)T + i\vec{p}\vec{L}}$$

Oscillation probability in WP approach

Neutrino emission and detection times are not measured (or not accurately measured) in most experiments $\Rightarrow$ integration over T:

$$P(\nu_\alpha \to \nu_\beta; L) = \int dT\, P(\nu_\alpha \to \nu_\beta; T, L) = \sum_{i,k} U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^*\, e^{-i\frac{\Delta m_{ik}^2}{2P}L}\, \tilde{I}_{ik}$$

$$\tilde{I}_{ik} = N \int \frac{dq}{2\pi} f_i^S(r_k q - \Delta E_{ik}/2v + P_i) f_i^{D*}(r_k q - \Delta E_{ik}/2v + P_i)$$

$$\times f_k^{S*}(r_i q + \Delta E_{ik}/2v + P_k) f_k^D(r_i q + \Delta E_{ik}/2v + P_k)\, e^{i\frac{\Delta v}{v}qL}$$

Here: $v \equiv \frac{v_i + v_k}{2}$, $\Delta v \equiv v_k - v_i$, $r_{i,k} \equiv \frac{v_{i,k}}{v}$, $N \equiv 1/[2E_i(P)2E_k(P)v]$,

Problem: derive this result. Hint: use $\Delta E_{ik} \simeq v\Delta p_{ik} + \Delta m_{ik}^2/2E$ and go to the shifted integration variable $q \equiv p - P$ where $P \equiv (P_i + P_k)/2$.

When are neutrino oscillations observable?

Keyword: *Coherence*

Neutrino flavour eigenstates ν_e, ν_μ and ν_τ are coherent superpositions of mass eigenstates ν_1, ν_2 and ν_3 $\Rightarrow$ oscillations are only observable if

- neutrino production and detection are coherent
- coherence is not (irreversibly) lost during neutrino propagation.

Possible decoherence at production (detection): If by accurate E and p measurements one can tell (through $E = \sqrt{p^2 + m^2}$) which mass eigenstate is emitted, the coherence is lost and oscillations disappear!

Full analogy with electron interference in double slit experiments: if one can establish which slit the detected electron has passed through, the interference fringes are washed out.

When are neutrino oscillations observable?

Another source of decoherence: wave packet separation due to the difference of group velocities Δv of different mass eigenstates.

If coherence is lost: Flavour transition can still occur, but in a non-oscillatory way. E.g. for $\pi \to \mu\nu_i$ decay with a subsequent detection of ν_i with the emission of e:

$$P \;\propto\; \sum_i P_{\rm prod}(\mu\,\nu_i) P_{\rm det}(e\,\nu_i) \;\propto\; \sum_i |U_{\mu i}|^2 |U_{ei}|^2$$

– the same result as for averaged oscillations.

How are the oscillations destroyed? Suppose by measuring momenta and energies of particles at neutrino production (or detection) we can determine its energy E and momentum p with uncertainties σ_E and σ_p. From $E_i = \sqrt{p_i^2 + m_i^2}$:

$$\sigma_{m^2} \;=\; \left[(2E\sigma_E)^2 \;+\; (2p\sigma_p)^2\right]^{1/2}$$

When are neutrino oscillations observable?

If $\sigma_{m^2} < \Delta m^2 = |m_i^2 - m_k^2|$ – one can tell which mass eigenstate is emitted.

$\sigma_{m^2} < \Delta m^2$ implies $2p\sigma_p < \Delta m^2$, or $\sigma_p < \Delta m^2/2p \simeq l_{\rm osc}^{-1}$.

But: To measure p with the accuracy σ_p one needs to measure the momenta of particles at production with (at least) the same accuracy $\Rightarrow$ uncertainty of their coordinates (and the coordinate of ν production point) will be

$$\sigma_{\rm x,\,prod} \gtrsim \sigma_p^{-1} > l_{\rm osc}$$

$\Rightarrow$ Oscillations washed out. Similarly for neutrino detection.

Natural necessary condition for coherence (observability of oscillations):

$$L_{\rm source} \ll l_{\rm osc}\,, \qquad L_{\rm det} \ll l_{\rm osc}$$

No averaging of oscillations in the source and detector

Satisfied with very large margins in most cases of practical interest

Wave packet separation

Wave packets representing different mass eigenstate components have different group velocities v_{gi} $\Rightarrow$ after time $t_{\rm coh}$ (coherence time) they separate $\Rightarrow$ Neutrinos stop oscillating! (Only averaged effect observable).

Coherence time and length:

$$\Delta v \cdot t_{\rm coh} \simeq \sigma_x\,; \qquad l_{\rm coh} \simeq v t_{\rm coh}$$

$$\Delta v = \frac{p_i}{E_i} - \frac{p_k}{E_k} \simeq \frac{\Delta m^2}{2E^2}$$

$$\boxed{l_{\rm coh} \simeq \frac{v}{\Delta v}\sigma_x = \frac{2E^2}{\Delta m^2}\, v\sigma_x}$$

The standard formula for $P_{\rm osc}$ is obtained when the decoherence effects are negligible.

A manifestation of neutrino coherence

Even non-observation of neutrino oscillations at distances $L \ll l_{\rm osc}$ is a consequence of and an evidence for coherence of neutrino emission and detection! Two-flavour example (e.g. for ν_e emission and detection):

$$A_{\rm prod/det}(\nu_1) \sim \cos\theta\,, \qquad A_{\rm prod/det}(\nu_2) \sim \sin\theta \qquad \Rightarrow$$

$$A(\nu_e \to \nu_e) = \sum_{i=1,2} A_{\rm prod}(\nu_i) A_{\rm det}(\nu_i) \sim \cos^2\theta + e^{-i\Delta\phi}\sin^2\theta$$

Phase difference $\Delta\phi$ vanishes at short L $\Rightarrow$

$$P(\nu_e \to \nu_e) = (\cos^2\theta + \sin^2\theta)^2 = 1$$

If ν_1 and ν_2 were emitted and absorbed incoherently) $\Rightarrow$ one would have to sum probabilities rather than amplitudes:

$$P(\nu_e \to \nu_e) \sim \sum_{i=1,2} |A_{\rm prod}(\nu_i) A_{\rm det}(\nu_i)|^2 \sim \cos^4\theta + \sin^4\theta \; < \; 1$$

Are coherence constraints compatible?

Observability conditions for ν oscillations:

- Coherence of ν production and detection
- Coherence of ν propagation

Both conditions put upper limits on neutrino mass squared differences Δm^2 :

$$(1) \quad \Delta E_{jk} \sim \frac{\Delta m^2_{jk}}{2E} \ll \sigma_E; \qquad\qquad (2) \quad \frac{\Delta m^2_{jk}}{2E^2} L \ll \sigma_x \simeq v_g/\sigma_E$$

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But: The constraints on σ_E work in opposite directions:

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Are they compatible? – Yes, if LHS $\ll$ RHS $\Rightarrow$

$$\boxed{2\pi \frac{L}{l_{\rm osc}} \ll \frac{v_g}{\Delta v_g} \; (\gg 1)}$$

– fulfilled in all cases of practical interest

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Sterile neutrinos: hints from SBL accelerator experiments (LSND, MiniBooNE), reactor neutrino anomaly, keV sterile neutrinos, pulsar kicks, leptogenesis via ν oscillations, SN r-process nucleosynthesis, unconventional contributions to $2\beta0\nu$ decay ...

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Production/detection coherence has to be re-checked – important implications for some neutrino experiments!

Neutrino oscillations: *Coherence at macroscopic distances –*
$L > 10{,}000$ km in atmospheric neutrino experiments !

Oscillation probability in WP approach

Neutrino emission and detection times are not measured (or not accurately measured) in most experiments $\Rightarrow$ integration over T:

$$P(\nu_\alpha \to \nu_\beta; L) = \int dT \, P(\nu_\alpha \to \nu_\beta; T, L) = \sum_{i,k} U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^* \, e^{-i \frac{\Delta m_{ik}^2}{2\bar{P}} L} \, \tilde{I}_{ik}$$

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- For $(\Delta v/v)\sigma_p L \ll 1$ (i.e. $L \ll l_{\rm coh} = (v/\Delta v)\sigma_x$) $\tilde{I}_{ik}$ is approximately independent of L; in the opposite case $\tilde{I}_{ik}$ is strongly suppressed

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- $\tilde{I}_{ik}$ is also strongly suppressed unless $\Delta E_{ik}/v \ll \sigma_p$, i.e. $\Delta E_{ik} \ll \sigma_E$ – coherent production/detection condition

The standard osc. probability?

The standard formula for the oscillation probability corresponds to $\tilde{I}_{ik} = 1$.

If the two above conditions are satisfied, $\tilde{I}_{ik}$ is not suppressed and is L-, E- and i, k-independent (i.e. a constant).

The standard probability is obtained when this constant is 1 (normalization necessary!)

Normaliz. condition:

$$\int \frac{d^3p}{(2\pi)^3} |f_i^S(\vec{p})|^2 |f_i^D(\vec{p})|^2 = 1$$

The normalization prescription

Oscillation probability calculated in QM w. packet approach is not automatically normalized ! Can be normalized “by hand” by imposing the unitarity condition:

$$\sum_{\beta} P_{\alpha\beta}(L) \;=\; 1\,.$$

This gives

$$\int dT|\mathcal{A}_i(L,T)|^2 = 1 \quad \Rightarrow \quad \tilde{I}_{ii} = N_1 \int \frac{dp}{2\pi v}|f_i^S(p)|^2\,|f_i^D(p)|^2 \;=\; 1$$

– important for proving Lorentz invariance of the oscillation probability.

Depends on the overlap of $f_i^S(p)$ and $f_i^S(p)$ $\Rightarrow$ no independent normalization of the produced and detected neutrino wave function would do!

In QFT approach the correctly normalized $P_{\alpha\beta}(L)$ is automatically obtained and the meaning of the normalization procedure adopted in the w. packet approach clarified

Oscillations and QM uncertainty relations

Neutrino oscillations – a QM interference phenomenon, owe their existence to QM uncertainty relations

Neutrino energy and momentum are characterized by uncertainties σ_E and σ_p related to the spatial localization and time scale of the production and detection processes. These uncertainties

- allow the emitted/absorbed neutrino state to be a coherent superposition of different mass eigenstates
- determine the size of the neutrino wave packets $\Rightarrow$ govern decoherence due to wave packet separation

σ_E – the effective energy uncertainty, dominated by the smaller one between the energy uncertainties at production and detection. Similarly for σ_p.

Universal oscillation formula?

The complete process: production – propagation – detection: factorization

$$\Gamma_{ab}(L,E) \;=\; j_a(E)\,P_{ab}^{\rm prop}(L,E)\,\sigma_b(E)$$

with a universal $P_{ab}^{\rm prop}(L,E)$ is only possible when all 3 processes are independent

In general not true, and production – propagation – detection should be considered as a single inseparable process!

To get the standard formula one assumes for the emitted and absorbed states

$$|\nu_a^{\rm fl}\rangle \;=\; \sum_i U_{ai}^*\,|\nu_i^{\rm mass}\rangle$$

The weights of the mass eigenstaes are just U_{ai}^* – do not depend on the masses of ν_i $\Rightarrow$ only true when the phase space volumes at production and detection do not depend on the mass of ν_i.

Universal oscillation formula?

This is only true if the charact. energy E at production (and detection) is large compared to all m_i (relativistic neutrinos), or compared to all $|m_i - m_k|$ (quasi-degenerate neutrinos).

$\Rightarrow$ Neutrino oscillations can be described by a universal probability only when neutrinos are relativistic or quasi-degenerate

Also: loss of coherence of propagating neutrino state depends on the coherence of the production and detection processes

$\Rightarrow$ The standard formula for the oscillation probability is only valid when all decoherence effects are negligible !

Lorentz invariance of oscillation probability

1. "Paradox" of neutrino w. packet length

For neutrino production in decays of unstable particles at rest (e.g. $\pi \to \mu\nu_\mu$):

$$\sigma_E \simeq \tau^{-1} = \Gamma_\pi \,, \qquad \sigma_x \simeq \frac{v_g}{\sigma_E} \simeq \frac{v_g}{\Gamma_\pi} \;\; (= v_g \tau)$$

Lorentz invariance of oscillation probability

1. "Paradox" of neutrino w. packet length

For neutrino production in decays of unstable particles at rest (e.g. $\pi \to \mu\nu_\mu$):

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<u>The solution:</u> pion decay takes finite time. During the decay time the pion moves over distance $l = u\tau'$ ("chases" the neutrino if $u > 0$).

$$\sigma'_x \simeq v'_g/\Gamma' - l = v'_g\tau' - u\tau' = (v'_g - u)\gamma_u\tau = \frac{v_g\tau}{\gamma_u(1+v_g u)} \,,$$

[the relativ. law of addition of velocities: $v'_g = (v_g + u)/(1 + v_g u)$].

Lorentz invariance issues – contd.

That is

$$\sigma'_x = \frac{\sigma_x}{\gamma_u(1+v_g u)}$$

For relativistic neutrinos $v_g \approx v'_g \approx 1 \quad \Rightarrow$

$$\sigma'_x = \sigma_x \sqrt{\frac{1-u}{1+u}}$$

$\Rightarrow$ when the pion is boosted in the direction of neutrino emission $(u > 0)$ the neutrino wave packet gets contracted; when it is boosted in the opposite direction $(u < 0)$ – the wave packet gets dilated.

Lorentz invariance issues – contd.

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The stand. osc. formula results when (i) production and detection and (ii) propagation are coherent; for neutrinos from conventional sources (i) implies $\sigma_x \ll l_{\rm osc}$ $\Rightarrow$ one can consider neutrinos pointlike and set $L = v_g t$.

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$\Rightarrow$ $$\boxed{L'/p' = L/p}$$

Lorentz invariance issues – contd.

A more general argument (applies also to Mössbauer neutrinos which are not pointlike): Consider the phase difference

$$\diamondsuit \qquad \Delta\phi = -\frac{1}{v_g}(L - v_g\, t)\Delta E + \frac{\Delta m^2}{2p}\, L$$

– a Lorentz invariant quantity, though the two terms are in not in general separately Lorentz invariant.

But: If the 1st term is negligible in all Lorentz frames, the second term is Lorentz invariant by itself $\Rightarrow$ L/p is Lorentz invariant.

The 1st term can be neglected when the production/detection coherence conditions are satisfied. In particular, it vanishes in the limit of pointlike neutrinos $L = v_g t$. N.B.:

$$L' - v_g' t' = \gamma_u \left[(L + ut) - \frac{v_g + u}{1 + v_g u}(t + uL)\right] = \frac{L - v_g t}{\gamma_u (1 + v_g u)}\,,$$

i.e. the condition $L = v_g t$ is Lorentz invariant. MB neutrinos: $\Delta E \simeq 0$.

Lorentz invariance issues – contd.

The oscillation probability must be Lorentz invariant even when the coherence conditions are not satisfied !
Lorentz invariance is enforced by the normalization condition.

$$P_{ab}(L) \;=\; \sum_{i,k} U_{ai} U_{bi}^* U_{ak}^* U_{bk}\, I_{ik}(L)\,, \qquad \text{where}$$

$$I_{ik}(L) \;\equiv\; \int dT\, \mathcal{A}_i(L,T) \mathcal{A}_k^*(L,T) e^{-i\Delta\phi_{ik}}$$

From the norm. cond. $\int dT\, |\mathcal{A}_i(L,T)|^2 \;=\; 1 \qquad \Rightarrow$

$$|\mathcal{A}_i|^2 dT \;=\; inv. \quad \Rightarrow \quad |\mathcal{A}_i||\mathcal{A}_k| dT \;=\; inv. \quad \Rightarrow \quad \mathcal{A}_i \mathcal{A}_k^* dT \;=\; inv.$$

The phase difference $\Delta\phi_{ik} = \Delta E_{ik} T - \Delta p_{ik} L$ is also Lorentz invariant $\quad\Rightarrow$
so is $I_{ik}(L)$, and consequently $P_{ab}(L)$.

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<u>But:</u> Conditions for partial decoherence are difficult to realize

They may still be realized if relatively heavy sterile neutrinos exist

Phenomenology of neutrino oscillations

Neutrino mixing schemes

I. Dirac case

$$-\mathcal{L}_{w+m} = \frac{g}{\sqrt{2}}(\bar{e}_L \gamma^\mu V_L^\dagger U_L \nu_L) W_\mu^- + \sum_{\alpha=1}^{n} m_{l\alpha} \bar{e}_\alpha e_\alpha + \sum_{i=1}^{n} m_i \bar{\nu}_i \nu_i + h.c.$$

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Osc. probability: the same expression

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III. Dirac + Majorana mass term (n LH and k RH neutrinos)

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Index a can take $n+k$ values; denote collectively the first n of them with α and the last k with σ $\Rightarrow$

D + M mass term – contd.

Active and sterile LH neutrino fields in terms of LH components of mass eigenstates:

$$\nu_{\alpha L} = \sum_{i=1}^{n+k} \mathcal{U}_{\alpha i} \chi_{iL} \,, \qquad (\nu_{\sigma R})^c = \sum_{i=1}^{n+k} \mathcal{U}_{\sigma i} \chi_{iL} \,.$$

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The usual oscillations described by the standard f-la with $U \to \mathcal{U}$ and summation over i up to $n+k$. In addition: new types of oscillations possible.

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Sterile - sterile neutrino oscillations:

$$P(\nu^c{}_{\sigma L} \to \nu^c{}_{\rho L}; L) = \left| \sum_{i=1}^{n+k} \mathcal{U}_{\rho i}\, e^{-i\frac{\Delta m_{ij}^2}{2p}L}\, \mathcal{U}^*_{\sigma i} \right|^2 .$$

An important example: 2-flavour case

$$
\begin{aligned}
|\nu_e\rangle &= \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle \\
|\nu_\mu\rangle &= -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle
\end{aligned}
$$

$\Rightarrow$

$$
U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \equiv \begin{pmatrix} c & s \\ -s & c \end{pmatrix}
$$

***

$\diamondsuit$ $$P_{\rm tr} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4p}L\right)$$

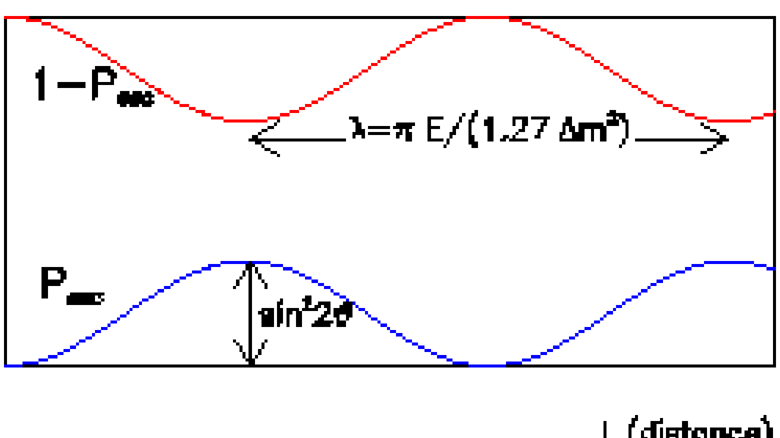
- Problem: Derive this formula from the general expression for $P_{\alpha\beta}$.
- Problem: Write this formula in the usual units, reinstating all factors of $\hbar$ and c. Find its classical and non-relativistic limits.

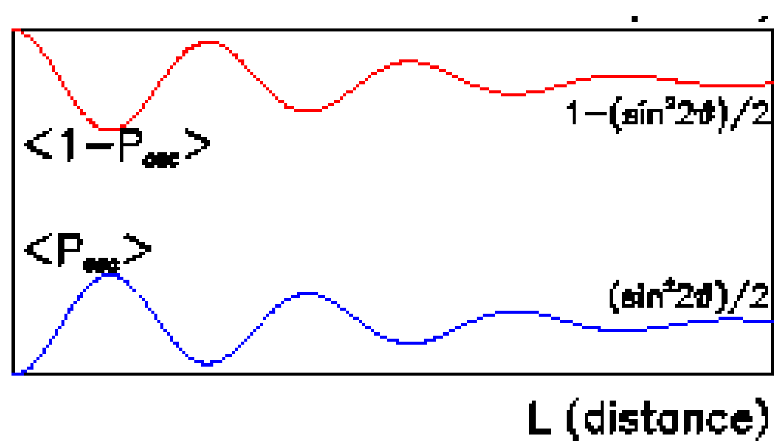
Oscillation amplitude: $\sin^2 2\theta$. Oscillation phase:

$$\frac{\Delta m^2}{4p} L = \pi \, \frac{L}{l_{\rm osc}} \,, \qquad\qquad l_{\rm osc} \equiv \frac{4\pi p}{\Delta m^2} \;\simeq\; 2.48\,{\rm m}\,\frac{p\,({\rm MeV})}{\Delta m^2\,({\rm eV}^2)}\,.$$

For large oscillation phase $\Rightarrow$ averaging regime (due to finite E-resolution of detectors and/or finite size of ν source/detector):

$$P_{\rm tr} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4p} L\right) \quad \rightarrow \quad \frac{1}{2}\sin^2 2\theta$$





3f neutrino mixing and oscillations

General case of n flavours – parameter counting

$(n \times n)$ unitary mixing matrix $\tilde{U}$ $\Rightarrow$ n^2 real parameters:

$$\begin{pmatrix} n \\ 2 \end{pmatrix} = \frac{n(n-1)}{2} \text{ mixing angles}, \qquad \frac{n(n+1)}{2} \text{ phases}$$

For leptonic mixing matrix n phases can be absorbed into re-defenition of the phases of LH charged fields: $e_{\alpha L} \to e^{i\phi_\alpha} e_{\alpha L}$ (e.g., 1st line of $\tilde{U}$ can be made real). This can be compensated in the mass term of charged leptons by rephasing $e_{\alpha R} \to e^{i\phi_\alpha} e_{\alpha R}$, so that $\bar{e}_{\alpha L} e_{\alpha R} = inv$.

Similarly, for Dirac neutrinos phases of one column can be fixed by absorbing $n-1$ phases into a redefinition of ν_{iL} (RH neutrino fields can be rephased analogously, so that $\bar{\nu}_{iL}\nu_{iR} = inv.$) $\Rightarrow$ In Dirac ν case $n + (n-1) = 2n-1$ phases are unphysical – can be rotated away by redefining charged lepton and neutrino fields.

N.B.: Kinetic terms of e_L, e_R and ν_L, ν_R are also invariant w.r.t. rephasing.!

Physical phases

Number of physical phases:

$$\frac{n(n+1)}{2} - (2n-1) \;=\; \frac{(n-1)(n-2)}{2}.$$

Phys. phases responsible for CP violation! $\Rightarrow$ No Dirac-type CPV for $n<3$.

In Majorana case:

$$\mathcal{L}_m \propto \nu_L^T C \nu_L \;+\; h.c.$$

Rephasing of ν_L is not possible (cannot be compensated in $\mathcal{L}_m$)

Only n phases can be removed from $\tilde{U}$ (by redefinition of $e_{\alpha L}$ fields) $\Rightarrow$ In addition to Dirac-type phases there are $(n-1)$ physical Majorana-type CP-violating phases.

Majorana phases do not affect oscillations

Majorana-type phases can be factored out in the mixing matrix:

$$\tilde{U} = UK$$

U contains Dirac-type phases, K – Majorana-type phases σ_i:

$$K = \mathrm{diag}(1\,, e^{i\sigma_1}\,, ...\,, e^{i\sigma_{n-1}})$$

Neutrino evolution equation: $i\frac{d}{dt}\,\nu = H_{\rm eff}\,\nu$

$$H_{\rm eff} = UK \begin{pmatrix} E_1 & & & \\ & E_2 & & \\ & & . & \\ & & & . \end{pmatrix} K^\dagger U^\dagger = U \begin{pmatrix} E_1 & & & \\ & E_2 & & \\ & & . & \\ & & & . \end{pmatrix} U^\dagger$$

Does not depend on the matrix of Majorana $\not{C}\not{P}$ phases K $\Rightarrow$
ν oscillations are insensitive to Majorana phases. Also true for osc. in matter.

3f oscillation parameters

Three neutrino species $(\nu_e, \nu_\mu, \nu_\tau)$ – linear superpositions of three mass eigenstates (ν_1, ν_2, ν_3). Mixing matrix U – 3×3 unitary matrix. Depends on 3 mixing angles and one Dirac-type $\not{C}\not{P}$ phase $\delta_{\rm CP}$.

Experiment: 2 mixing angles large (in the standard parameterization – θ_{12} and θ_{23}), one (θ_{13}) is relatively small.

Three neutrinos species – 2 independent mass squared differences, e.g. Δm^2_{21} and Δm^2_{31}.

$$\Delta m^2_{21} \ll \Delta m^2_{31}$$

What do we know about neutrino parameters

From atmsopheric and LBL accelerator neutrino experiments:

- ♢ $\Delta m^2_{31} \simeq 2.5 \times 10^{-3}\,\mathrm{eV}^2\,, \qquad \theta_{23} \sim 45^\circ$

From solar neutrino experiments and KamLAND:

- ♢ $\Delta m^2_{21} \simeq 7.5 \times 10^{-5}\,\mathrm{eV}^2\,, \qquad \theta_{12} \simeq 33^\circ$

From T2K + Double Chooz, Daya Bay and Reno reactor neutrino experiments:

- ♢ $\theta_{13} \simeq 9^\circ \quad$ (previosly from Chooz $\lesssim 12^\circ$)

CP-violating phase δ_{CP} practically unconstrained at the moment.

Leptonic mixing and 3f osc. in vacuum

Relation between flavour and mass eigenstates:

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i}\, \nu_i$$

ν_α – fields of flavour eigenstates, ν_i – of mass eigenstates.

3f mixing matrix:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Leptonic mixing and 3f osc. in vacuum

Relation btween flavour and mass eigenstates:

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i}^{*}\,|\nu_i\rangle$$

Oscillation probability in vacuum:

$$P(\nu_\alpha \to \nu_\beta; L) \;=\; \left|\sum_{i=1}^{3} U_{\beta i}\, e^{-i\frac{\Delta m_{i1}^2}{2p}L}\, U_{\alpha i}^{*}\right|^2 \;=\; \left|\left[U\, e^{-i\frac{\Delta m^2}{2p}L}\, U^\dagger\right]_{\beta\alpha}\right|^2$$

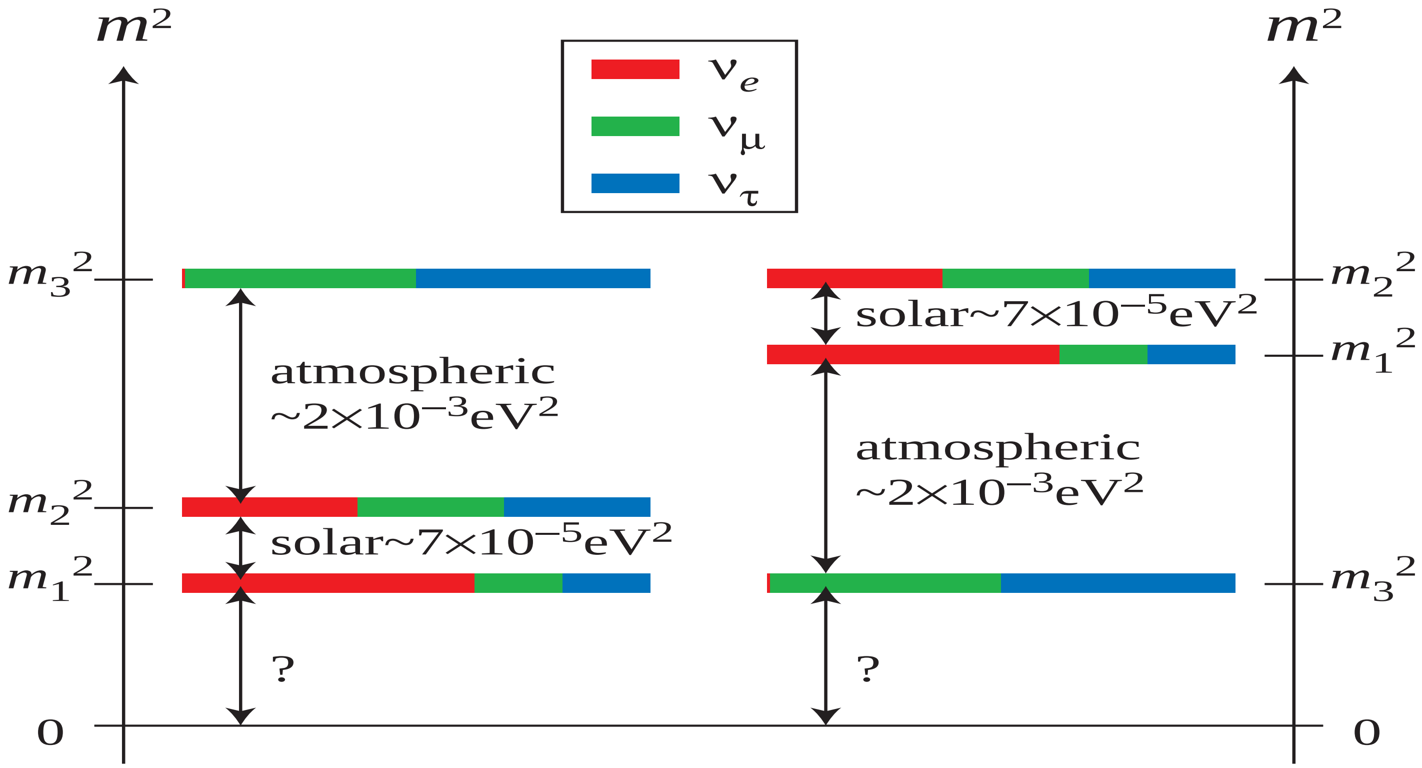
3f mixing matrix in the standard parameterization ($c_{ij} = \cos\theta_{ij},\ s_{ij} = \sin\theta_{ij}$):

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\rm CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= O_{23}\,(\Gamma_\delta\, O_{13}\, \Gamma_\delta^\dagger)\, O_{12}\,, \qquad \Gamma_\delta \equiv {\rm diag}(1\,,\ 1\,,\ e^{i\delta_{\rm CP}})$$

3f neutrino mixing

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix}$$



2f oscillations: physical ranges of parameters

$$|\nu_e\rangle \;=\; \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

$$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

In general, $\theta \in [0, 2\pi]$. But: there are transformations that leave ν mixing formulas unchanged:

- $\theta \to \theta + \pi, \quad |\nu_1\rangle \to -|\nu_1\rangle, \quad |\nu_2\rangle \to -|\nu_2\rangle \quad \Rightarrow \quad \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
- $\theta \to -\theta, \quad |\nu_2\rangle \to -|\nu_2\rangle, \quad |\nu_\mu\rangle \to -|\nu_\mu\rangle \quad \Rightarrow \quad \theta \in [0, \frac{\pi}{2}]$
- $\theta \to \frac{\pi}{2} - \theta, \quad |\nu_1\rangle \leftrightarrow |\nu_2\rangle, \quad |\nu_\mu\rangle \to -|\nu_\mu\rangle \quad \Rightarrow \quad \Delta m^2 \to -\Delta m^2$

One can always choose $\Delta m^2 > 0$ by choosing appropriately θ within $[0, \frac{\pi}{2}]$.

For vacuum oscillations: $P_{\rm tr}$, $P_{\rm surv}$ depend only on $\sin^2 2\theta \;\Rightarrow$ one can choose θ to be in $[0, \frac{\pi}{4}]$. Not true for oscillations in matter!

Similar considerations in the 3f case: all $\theta_{ij} \in [0, \frac{\pi}{2}]$; $\delta_{\rm CP} \in [0, 2\pi]$.

$\not{CP}$ and $\not{T}$ in ν osc. in vacuum

$\nu_a \to \nu_b$ oscillation probability:

$$\diamondsuit \quad P(\nu_\alpha, t_0 \to \nu_\beta; t) = \left| \sum_i U_{\beta i}\, e^{-i\frac{\Delta m_{i1}^2}{2E}(t-t_0)}\, U_{\alpha i}^* \right|^2$$

- CP: $\nu_{\alpha,\beta} \leftrightarrow \bar{\nu}_{\alpha,\beta} \quad \Rightarrow \quad U_{\alpha i} \to U_{\alpha i}^* \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$
- T: $t \rightleftarrows t_0 \quad \Leftrightarrow \quad \nu_\alpha \leftrightarrow \nu_\beta$

$$\Rightarrow \quad U_{\alpha i} \to U_{\alpha i}^* \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$$

T-reversed oscillations (“backwards in time”) $\Leftrightarrow$ oscillations between interchanged initial and final flavours

$\diamond$ *$\not{CP}$ and $\not{T}$ – absent in 2f case, pure $N \geq 3$f effects!*

$\diamond$ No $\not{CP}$ and $\not{T}$ for survival probabilities ($\beta = \alpha$).

CP and T violation in vacuum – contd.

- CPT: $\nu_{\alpha,\beta} \leftrightarrow \bar{\nu}_{\alpha,\beta}$ & $t \rightleftarrows t_0$ $(\nu_\alpha \leftrightarrow \nu_\beta)$

  $\diamond$ $P(\nu_\alpha \to \nu_\beta) \to P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$

The standard formula for $P_{\alpha\beta}$ in vacuum is CPT invariant!

$\not{CP} \Leftrightarrow \not{T}$ – *consequence of CPT*

Measures of $\not{CP}$ and $\not{T}$ – probability differences:

$$\Delta P_{\alpha\beta}^{\rm CP} \equiv P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

$$\Delta P_{\alpha\beta}^{\rm T} \equiv P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$$

From CPT:

$\diamond$ $\Delta P_{\alpha\beta}^{\rm CP} = \Delta P_{\alpha\beta}^{\rm T}\,;$ $\Delta P_{\alpha\alpha}^{\rm CP} = 0$

3f case

One $\not{CP}$ Dirac-type phase $\delta_{\rm CP}$ *(Majorana phases do not affect ν oscillations!)* $\Rightarrow$ one $\not{CP}$ and $\not{T}$ observable:

$$\diamond \quad \Delta P^{\rm CP}_{e\mu} = \Delta P^{\rm CP}_{\mu\tau} = \Delta P^{\rm CP}_{\tau e} \equiv \Delta P$$

$$\Delta P = -4 s_{12}\, c_{12}\, s_{13}\, c_{13}^2\, s_{23}\, c_{23}\, \sin\delta_{\rm CP}$$
$$\times \left[\sin\left(\frac{\Delta m^2_{12}}{2E}\,L\right) + \sin\left(\frac{\Delta m^2_{23}}{2E}\,L\right) + \sin\left(\frac{\Delta m^2_{31}}{2E}\,L\right)\right]$$

Vanishes when

- At least one $\Delta m^2_{ij} = 0$
- At least one $\theta_{ij} = 0$ or $90°$
- $\delta_{\rm CP} = 0$ or $180°$
- In the averaging regime
- In the limit $L \to 0$ (as L^3)

Very difficult to observe!

Small parameters

Approximate formulas for probabilities can be obtained using expansions in small parameters:

$$(1) \quad \frac{\Delta m_\odot^2}{\Delta m_{\rm atm}^2} = \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \sim 1/30$$

$$(2) \quad |U_{e3}| = |\sin\theta_{13}| \sim 0.16$$

In the limits $\Delta m_{21}^2 = 0$ or $U_{e3} = 0$ – probabilities take an effective 2f form.

(N.B.: $P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$)

Coherent elastic neutrino-nucleus scattering

Coherent elastic neutrino-nucleus scattering

NC – mediated neutrino-nucleus scattering:

$$\nu + A \to \nu + A$$

Incoherent scattering – Probabilities of scattering on individual nucleons add:

$\diamondsuit$ $\sigma \propto (\#$ of scatterers$)$

Coherent scattering on nucleus as a whole – Amplitudes of scattering on individual nucleons add

$\diamondsuit$ $\sigma \propto (\#$ of scatterers$)^2$

Significant increase of the cross sections (but requires small momentum transfer, $q \lesssim R^{-1}$)

(D.Z. Freedman, 1974)

Coherent neutrino nucleus scattering: Predictions & Implications

Coherent effects of a weak neutral current

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National Accelerator Laboratory, Batavia, Illinois 60510
and Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11790
(Received 15 October 1973; revised manuscript received 19 November 1973)

If there is a weak neutral current, then the elastic scattering process $\nu + A \to \nu + A$ should have a sharp coherent forward peak just as $e + A \to e + A$ does. Experiments to observe this peak can give important information on the isospin structure of the neutral current. The experiments are very difficult, although the estimated cross sections (about 10^{-38} cm^2 on carbon) are favorable. The coherent cross sections (in contrast to incoherent) are almost energy-independent. Therefore, energies as low as 100 MeV may be suitable. Quasi-coherent nuclear excitation processes $\nu + A \to \nu + A^*$ provide possible tests of the conservation of the weak neutral current. Because of strong coherent effects at very low energies, the nuclear elastic scattering process may be important in inhibiting cooling by neutrino emission in stellar collapse and neutron stars.

- Implications for neutrino transport in supernovae

- Large cross section important for understanding how neutrinos emerge from supernovae

THE WEAK NEUTRAL CURRENT AND ITS EFFECTS IN STELLAR COLLAPSE

Daniel Z. Freedman
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790

David N. Schramm[1] *and David L. Tubbs*[2]
Enrico Fermi Institute (LASR), University of Chicago, Chicago, Illinois 60637

NC-induced neutrino-nucleus scattering: flavour blind.

◊ $$\left[\frac{d\sigma_{\nu A}}{d\Omega}\right]_{\rm coh} \simeq \frac{G_F^2}{16\pi^2} E_\nu^2 [Z(4\sin^2\theta_W - 1) + N]^2 \,(1+\cos\theta)|F(\vec{q}^{\,2})|^2$$

$F(\vec{q}^{\,2})$ is nuclear formfactor:

$$F_{N(Z)}(\vec{q}^{\,2}) = \frac{1}{N(Z)} \int d^3x \rho_{N(Z)}(\vec{x}) e^{i\vec{q}\vec{x}}, \qquad \vec{q} = \vec{k} - \vec{k}'.$$

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For $q \ll R^{-1} \quad \Rightarrow \quad F(\vec{q}^{\,2}) = 1, \quad \left[d\sigma_{\nu A}/d\Omega\right]_{\rm coh} \propto N^2$.

For $q \gg R^{-1}$: $\quad F(\vec{q}^{\,2}) \ll 1$.

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By Heisenberg uncertainty relation: for $q \lesssim R^{-1}$ the uncertainty of the coordinate of the sctatterer $\delta x \gtrsim R \quad \Rightarrow$ it is in principle impossible to find out on which nucleon the neutrino has scattered. Also: neutrino waves scattered off different nucleons of the nucleus are in phase with each other.

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The necessary conditions for coherent scattering!

$$R \simeq 1.2 \text{ fm} \, A^{1/3}; \quad A \sim 130 \quad \Rightarrow \quad R^{-1} \sim 30 \text{ MeV}.$$

Recoil energy of the nucleus:

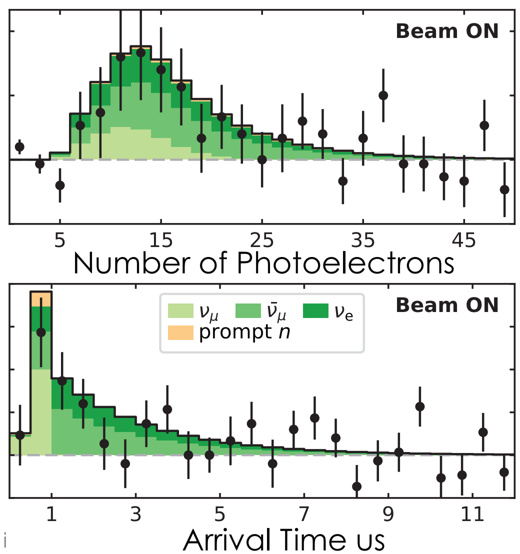
$$E_{rec} \simeq \frac{\vec{q}^{\,2}}{2M_A}, \qquad E_{rec}^{max} = \frac{2E_\nu^2}{M_A + 2E_\nu} \simeq \frac{2E_\nu^2}{M_A}.$$

For $q \sim 30$ MeV: $E_{rec} \sim 5$ keV.

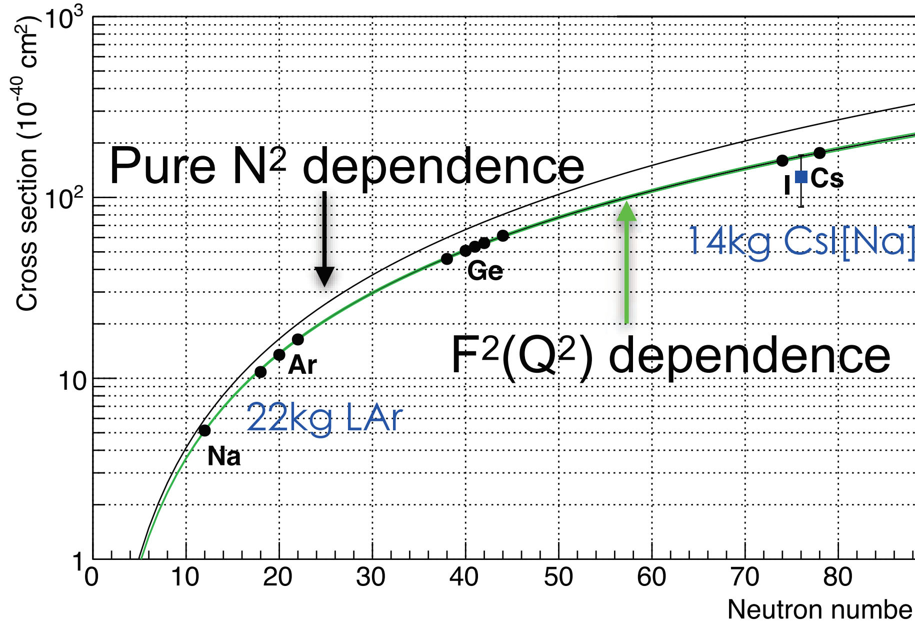
Need to detect very low recoil energies $\Rightarrow$ requires

- Very low detection thresholds
- Low backgrounds
- Intense neutrino fluxes

First Observation of CEvNS



Akimov et al. *Science*
Vol 357, Issue 6356
15 September 2017



First light detectors deployed to measure neutron-squared dependence. (Na, Ge in 2019)

High precision measurements enable the full potential of CEvNS scientific impact.

Oak Ridge National Laboratory Jason Newby, Magnificent CEvNS Workshop 2018

COHERENT experiment

Neutrino energies: $E_\nu \sim 16 - 53\ \mathrm{MeV}$. Nuclear recoil energy: keV - scale.

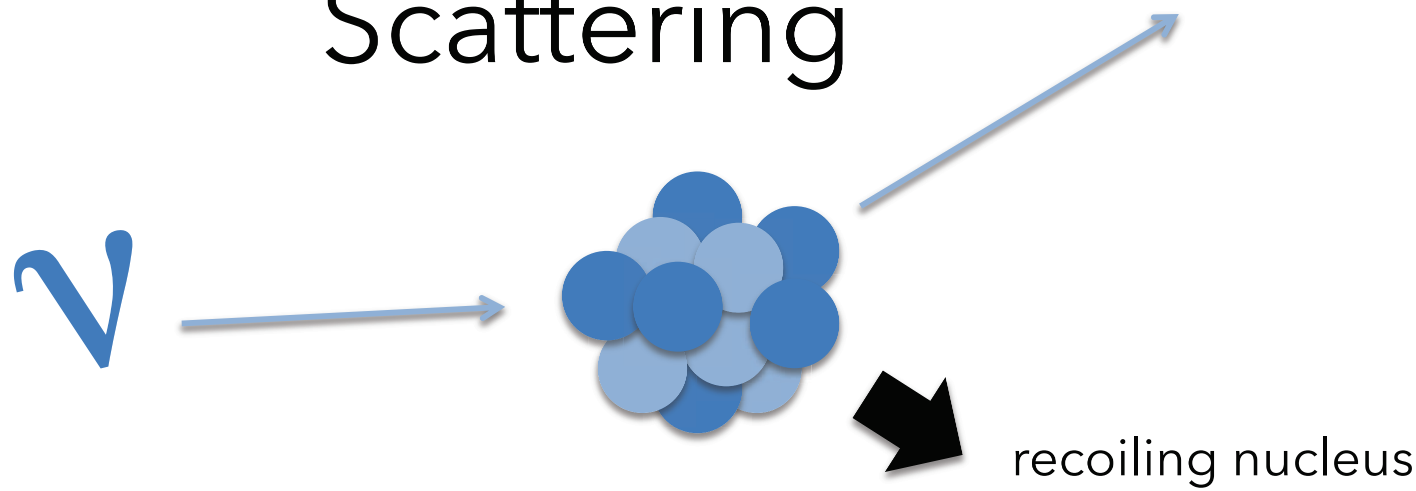
# of events expected (SM): 173 ± 48

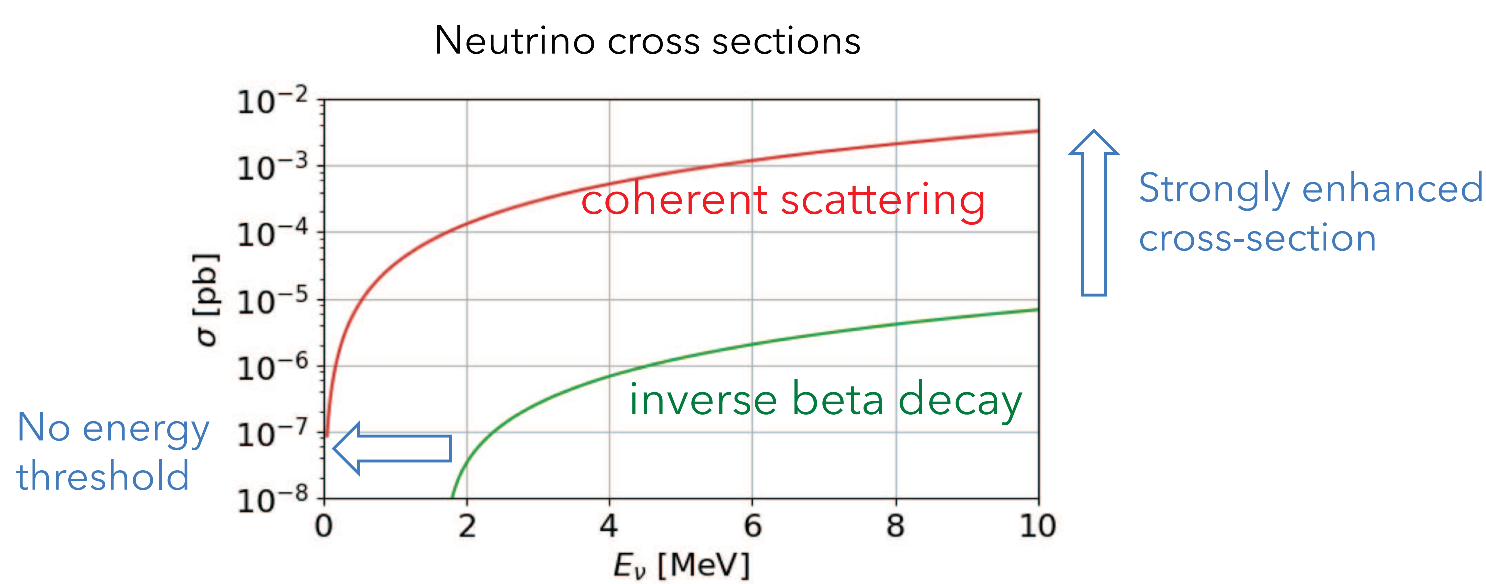
# of events detected: 134 ± 22

"We report a 6.7 sigma significance for an excess of events, that agrees with the standard model prediction to within 1 sigma"
$\sim 2 \times 10^{23}$ POT; $\sigma \sim 10^{-38}$ cm^2.

D. Akimov et al., Science 10.1126/science.aao0990 (2017).

Coherent Neutrino-Nucleus Scattering



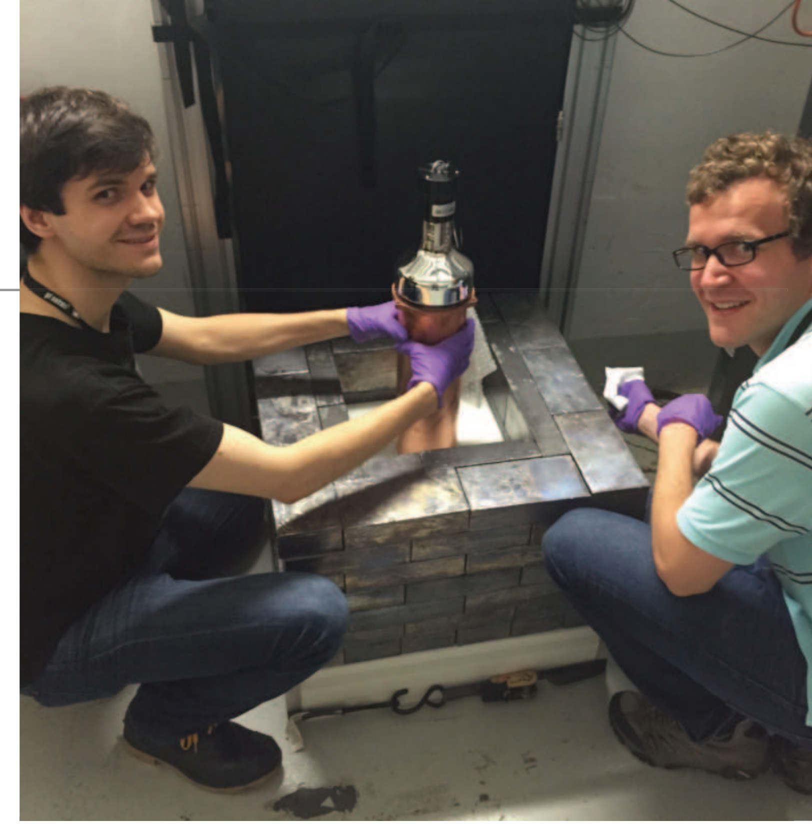


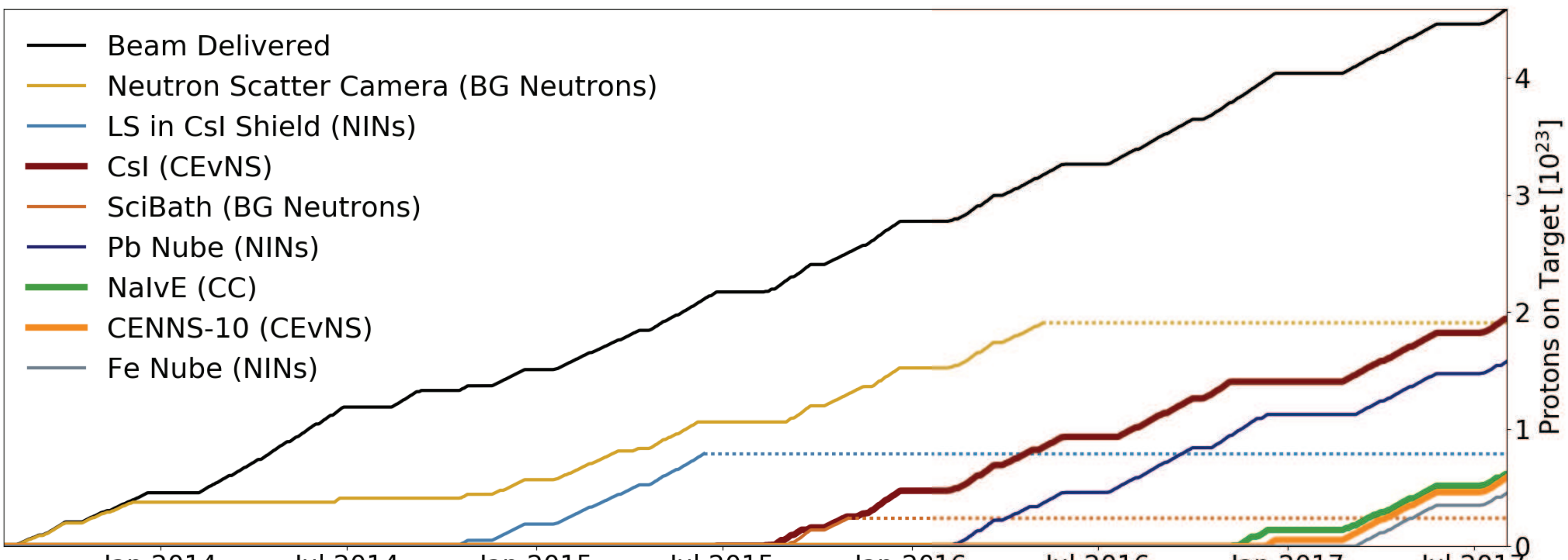
Magnificent CEvNS, Raimund Strauss

A hand-held neutrino detector

- 14.6 kg low-background CsI[Na] detector deployed to a basement location of the SNS in the summer of 2015

- ~ $2\text{x}10^{23}$ POT delivered and recorded since CsI began taking data





6

Why is CEvNS interesting?

- Large cross sections – small detectors
- Very clean SM predictions for cross sections – sensitivity to NSI
- Sensitivity to μ_ν and $\langle r_\nu^2 \rangle$
- Possibility to measure $\sin^2\theta_W$ at low energies
- Masurements of neutron formfactors (nuclear structure)
- Nuclear reactor monitoring (non-proliferation)
- Precision flavor-independent neutrino flux measurements for oscillation experiments
- Sterile neutrino searches
- Energy transport in SNe
- SN neutrino detection
- Input for DM direct detection (neutrino floor)
- Detection of solar neutrinos

Why is CEvNS interesting?

Many experiments planned or under way – CONUS, TEXONO, Ricochet, Connie, ν-cleus, RED100, MINER, νGEN, ...

Many theoretical studies

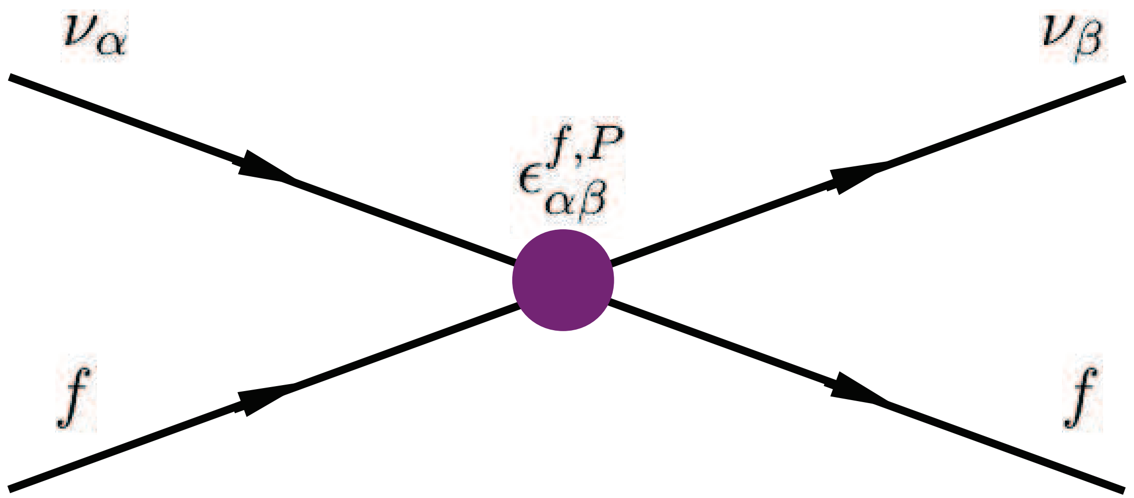
A very active field!

Backup slides

NSI parameterization

P. Coloma. P.B. Denton, M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz,
”Curtailing the Dark Side in Non-Standard Neutrino Interactions”, arXiv:1701.04828

$$\mathcal{L}_{\rm NSI} = -2\sqrt{2}G_F \sum_{f,P,\alpha,\beta} \epsilon_{\alpha\beta}^{f,P} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta)(\bar{f}\gamma_\mu P f)$$



Assuming heavy NSI mediators

CEvNS cross section and NSI

J. Barranco, O.G. Miranda, T.I. Rashba,
"Probing new physics with coherent neutrino scattering off nuclei", arXiv:hep-ph/0508299

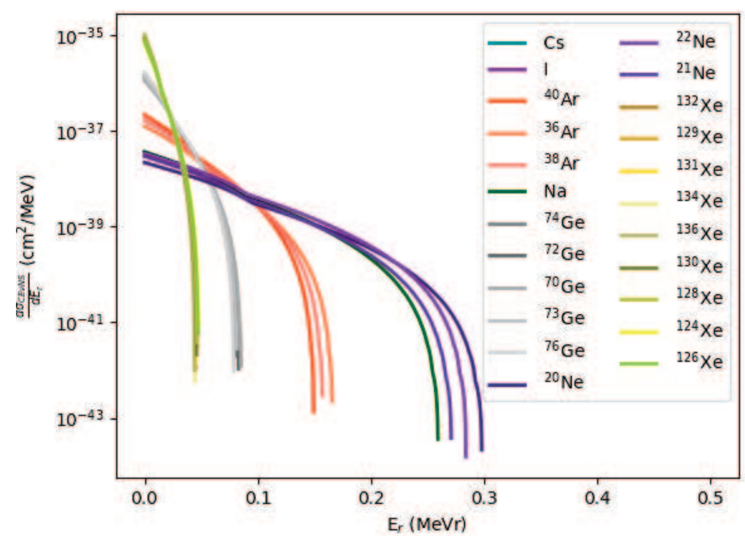
$$\frac{d\sigma}{dT} = \frac{G_F^2 M}{2\pi} F^2(Q) \left[(G_V + G_A)^2 + (G_V - G_A)^2 \left(1 - \frac{T}{E_\nu}\right)^2 - (G_V^2 - G_A^2) \frac{MT}{E_\nu^2} \right]$$

$$G_V = (g_V^p + \boxed{2\varepsilon_{ee}^{uV} + \varepsilon_{ee}^{dV}})Z + (g_V^n + \boxed{\varepsilon_{ee}^{uV} + 2\varepsilon_{ee}^{dV}})N$$

NSI terms

$$G_A = (g_A^p + 2\varepsilon_{ee}^{uA} + \varepsilon_{ee}^{dA})(Z_+ - Z_-) + (g_A^n + \varepsilon_{ee}^{uA} + 2\varepsilon_{ee}^{dA})(N_+ - N_-) \approx 0$$

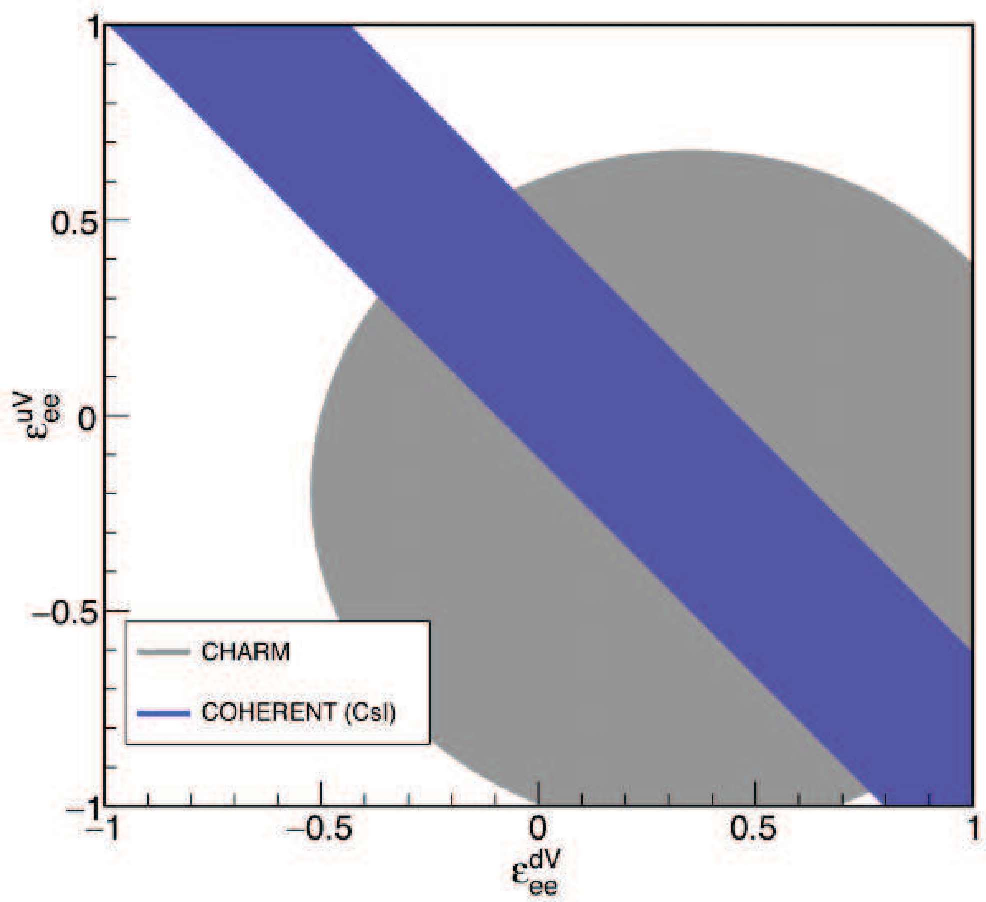
- Modification = $\dfrac{\sigma(\varepsilon)}{\sigma^{SM}}$



SM diff σ weighted by piDAR spectra

COHERENT NSI constraint

- August 2017 result
- 14.6 kg CsI[Na]
- ~2 years running
  - 308.1 live-days
- Events
  - 134 ± 22 observed
  - 173 ± 48 predicted



Why straight lines for SM rate?

J. Barranco, O.G. Miranda, T.I. Rashba,
"Probing new physics with coherent neutrino scattering off nuclei", arXiv:hep-ph/0508299

$$\frac{d\sigma}{dT} = \frac{G_F^2 M}{2\pi} F^2(Q) \left[(G_V + G_A)^2 + (G_V - G_A)^2 \left(1 - \frac{T}{E_\nu}\right)^2 - (G_V^2 - G_A^2) \frac{MT}{E_\nu^2} \right]$$

$$G_V = (g_V^p + 2\varepsilon_{ee}^{uV} + \varepsilon_{ee}^{dV})Z + (g_V^n + \varepsilon_{ee}^{uV} + 2\varepsilon_{ee}^{dV})N \qquad G_A \approx 0$$

SM rate: $G_V^{SM} = g_V^p Z + g_V^n N$

$$\frac{d\sigma^{SM}}{dT} = \frac{d\sigma}{dT}(G_V^{SM}) \quad \rightarrow \quad {G_V^{SM}}^2 = G_V^2$$

$$(g_V^p + 2\varepsilon_{ee}^{uV} + \varepsilon_{ee}^{dV})Z + (g_V^n + \varepsilon_{ee}^{uV} + 2\varepsilon_{ee}^{dV})N = \pm (g_V^p Z + g_V^n N)$$

Generating two straight lines in NSI-coupling space with SM rate

Magnificent CEvNS 2018/11/02 Gleb Sinev, Duke Constraining NSI with Multiple Targets 13

Including magnetic moment scattering

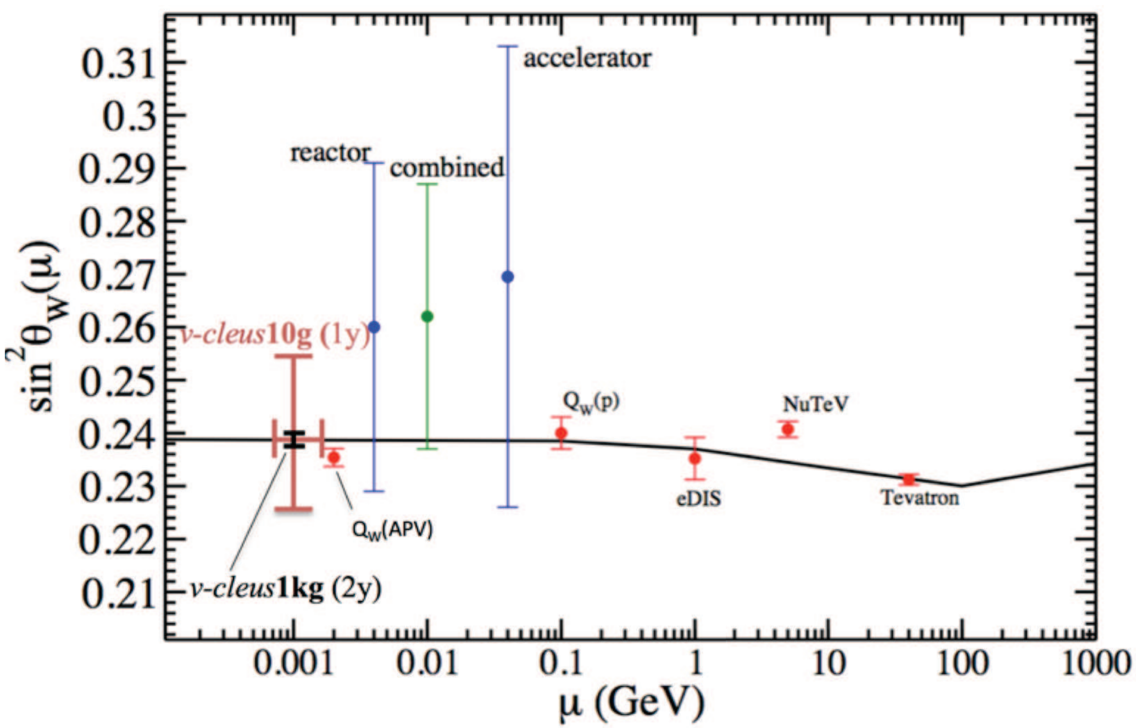
$$\frac{d\sigma}{dT} = \frac{G_F^2}{8\pi} M \left[2 - \frac{2T}{T_{max}} + \left(\frac{T}{E}\right)^2\right] Q_W^2 [F_Z(Q^2)]^2 + \frac{\pi\alpha^2\mu_{\mathrm{eff}}^2 Z^2}{m_e^2}\left[\frac{1}{T} - \frac{1}{E}\right][F_\gamma(Q^2)]^2$$

$$\mu_{\mathrm{eff}}^2 = \sum_i \left|\sum_j U_{(e\ \mathrm{or}\ \mu)j}\ e^{-iE_jL}\mu_{ji}\right|^2$$

Note that this is a different combination at CEνNS than what is measured at reactors or solar neutrino experiments!

Weinberg Angle

"Running" of Weinberg Angle



$$\left(\frac{d\sigma}{dE}\right)_{\nu_\alpha A} = \frac{G_F^2 M}{\pi} F^2(2ME)\left[1-\frac{ME}{2k^2}\right] \times$$
$$\{[Z(g_V^p + 2\varepsilon_{\alpha\alpha}^{uV} + \varepsilon_{\alpha\alpha}^{dV}) + N(g_V^n + \varepsilon_{\alpha\alpha}^{uV} + 2\varepsilon_{\alpha\alpha}^{dV})]^2$$

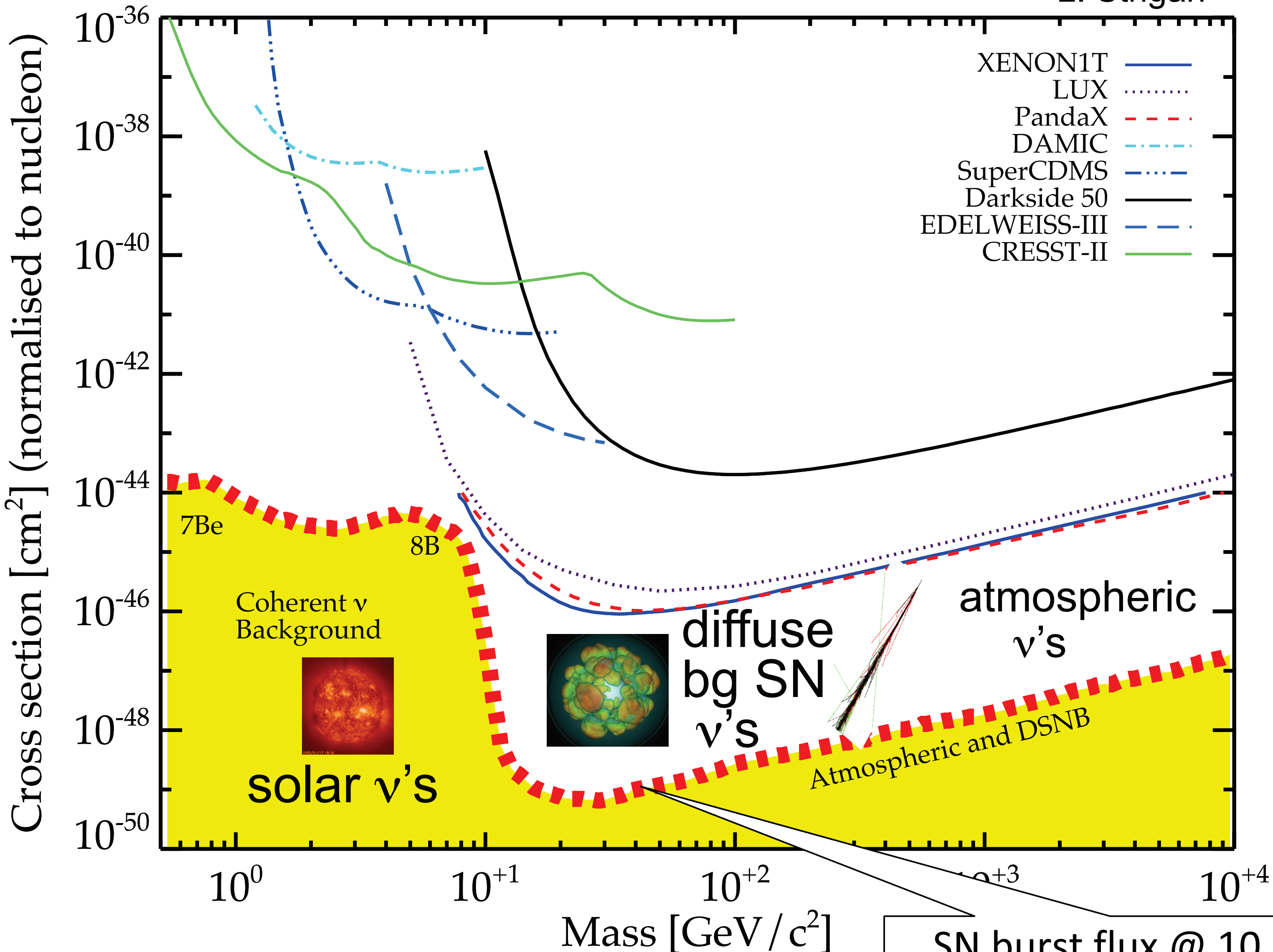
With $g_V^p = (\frac{1}{2} - 2\sin^2\theta_W)$ and $g_V^n = -\frac{1}{2}$

First determination of the Weinberg angle at q = 1MeV/c after 2-3 weeks of measurement with 10g!

The so-called “**neutrino floor**” for DM experiments

J. Billard, E. Figueroa-Feliciano, and L. Strigari, arXiv:1307.5458v2 (2013).

L. Strigari

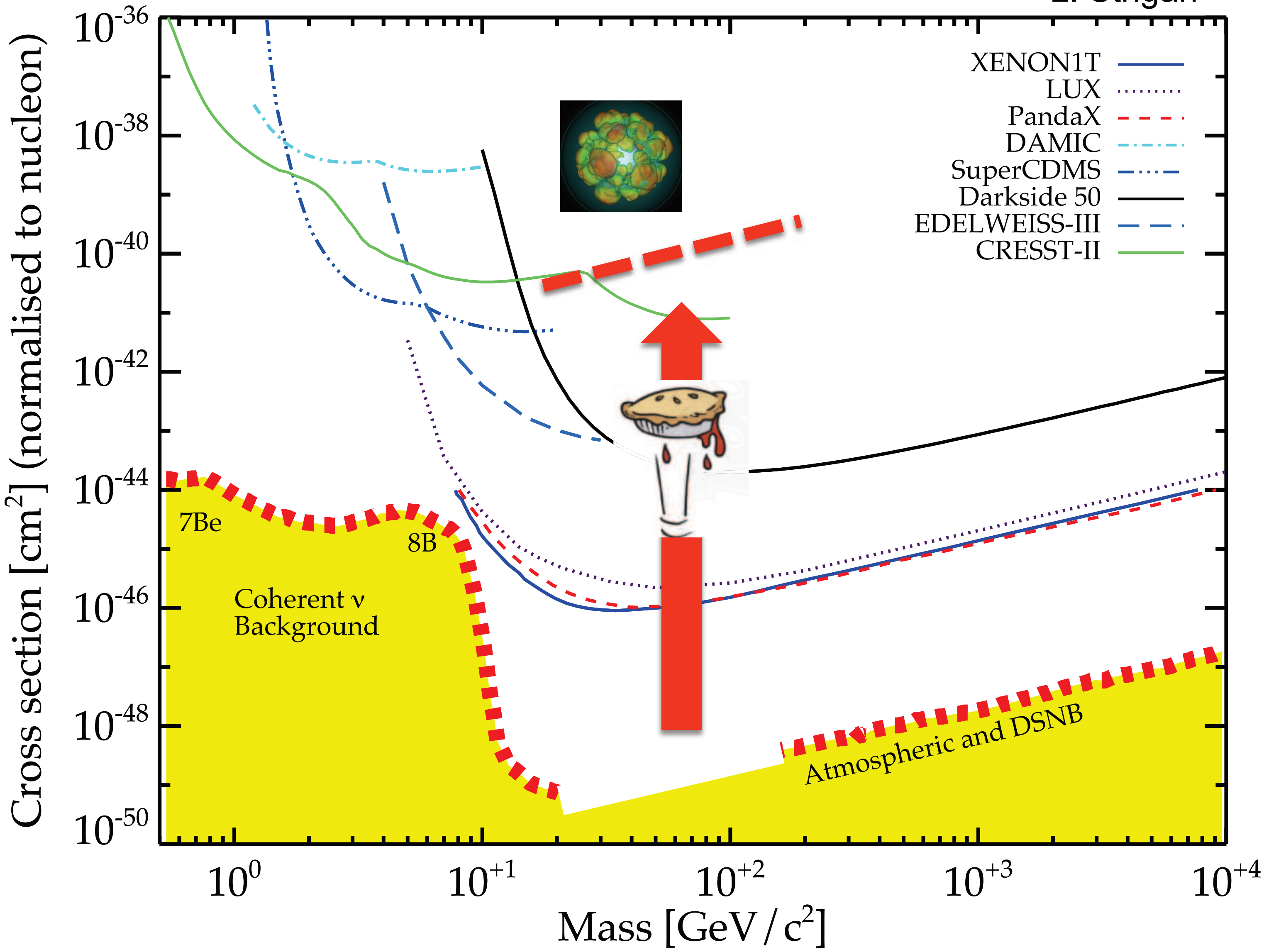


SN burst flux @ 10 kpc is 9-10 orders of magnitude greater than DSNB flux

Think of a SN burst as "**the ν floor coming up to meet you**"

J. Billard, E. Figueroa-Feliciano, and L. Strigari, arXiv:1307.5458v2 (2013).

L. Strigari



Backup slides

A brief *Curriculum Vitae* of neutrino

- ♢ Suggested by W. Pauli in 1930 to explain the continuous electron spectra in β-decay and nuclear spin/statistics
- ♢ Discovered by F. Reines and C. Cowan in 1956 in experiments with reactor $\bar{\nu}_e$ (Nobel prize to F. Reines in 1995)
- ♢ 1957 – the idea of neutrino oscillations put forward by B. Pontecorvo ($\nu \leftrightarrow \bar{\nu}$)
- ♢ 1957 – Chiral nature of ν_e established by Goldhaber, Grodzins & Sunyar
- ♢ 1962 – Discovery of the second neutrino type – ν_μ (Nobel prize to Lederman, Schwartz & Steinberger in 1988)
- ♢ 1962 – the idea of neutrino flavour oscillations put forward by Maki, Nakagawa & Sakata

- ♢ 1968 – First observation of solar neutrinos by R. Davis and collaborators
- ♢ 1975 – Discovery of the third lepton flavour – τ lepton
  (Nobel prize to M. Perl in 1995)
- ♢ 1985 – Theoretical discovery of resonant ν oscillations in matter by Mikheyev and Smirnov based on an earlier work of Wolfenstein
  (the MSW effect)
- ♢ 1987 – First observation of neutrinos from supernova explosion (SN 1987A)
- ♢ 1998 – "Evidence for oscillations of atmospheric neutrinos" by the Super-Kamiokande Collaboration
- ♢ 2000 – Discovery of the third neutrino species – ν_τ by the DONUT Collaboration (Fermilab)

- ♢ 2002 – "Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory" – flavor transformations of solar neutrinos confirmed

- ♢ 2002 – Discovery of oscillations of reactor neutrinos by KamLAND Collaboration; identification of the solution of the solar neutrino problem

- ♢ 2002 – Confirmation of oscillations of atmospheric neutrinos by K2K accelerator neutrino experiment

- ♢ 2002 – Nobel prize to R. Davis and M. Koshiba for "detection of cosmic neutrinos"

(2002 – "Annus Mirabilis" of neutrino physics)

- ♢ 2004 – Evidence for oscillatory nature of ν disappearance by Super-Kamiokande (atmospheric ν's) and KamLAND.

- 2006 – Independent confirmation of oscillations of atmospheric neutrinos by MINOS accelerator neutrino experiment
- 2007 – First real-time detection of solar $^7\mathrm{Be}$ neutrinos by Borexino
- 2011/12 – Measurement of the last leptonic mixing angle θ_{13} by T2K, Double Chooz, Daya Bay and Reno
- 2012/14 – Detection of solar pep and pp neutrinos by Borexino
- 2015 – Nobel prize to Takaaki Kajita and Arthur McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have mass"
- 2017 – First observation of coherent neutrino scattering on nuclei by the COHERENT Collaboration

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More to come !