QCD and Monte Carlo simulation II

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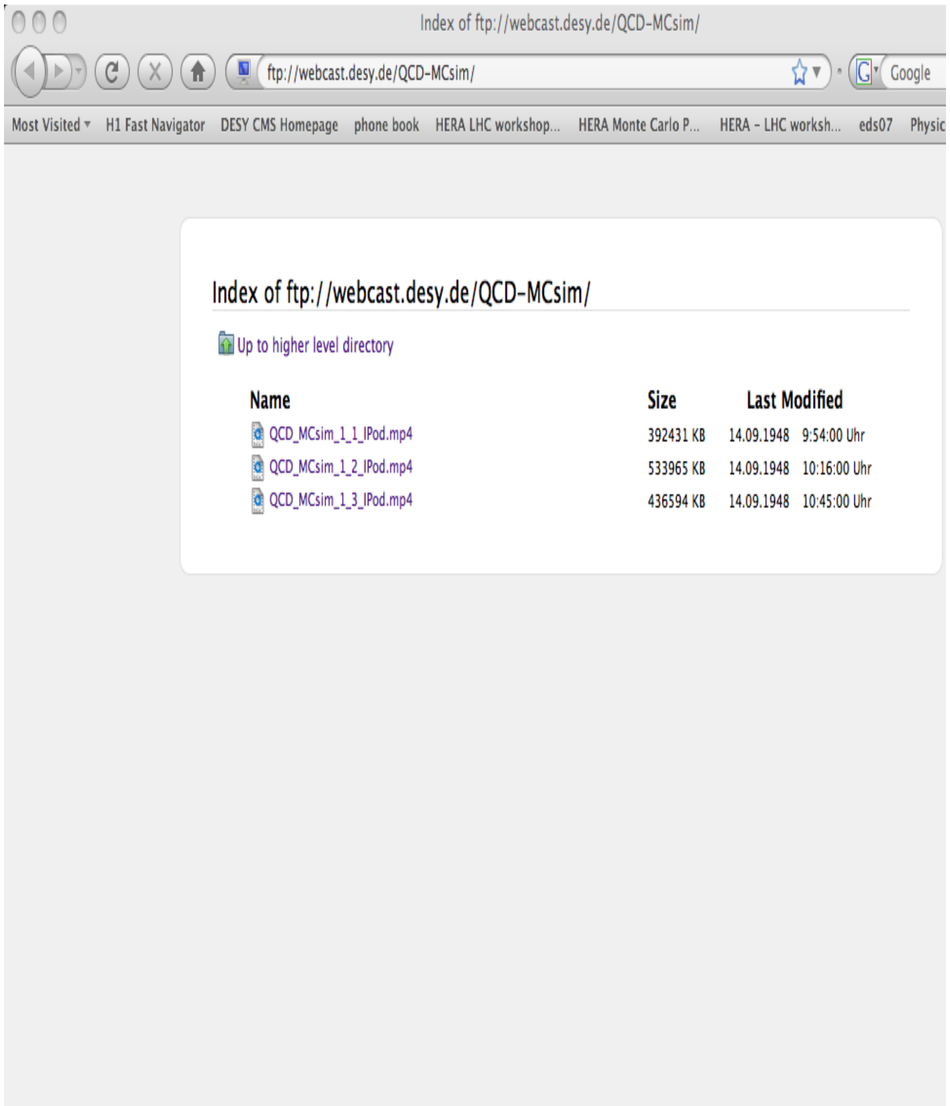
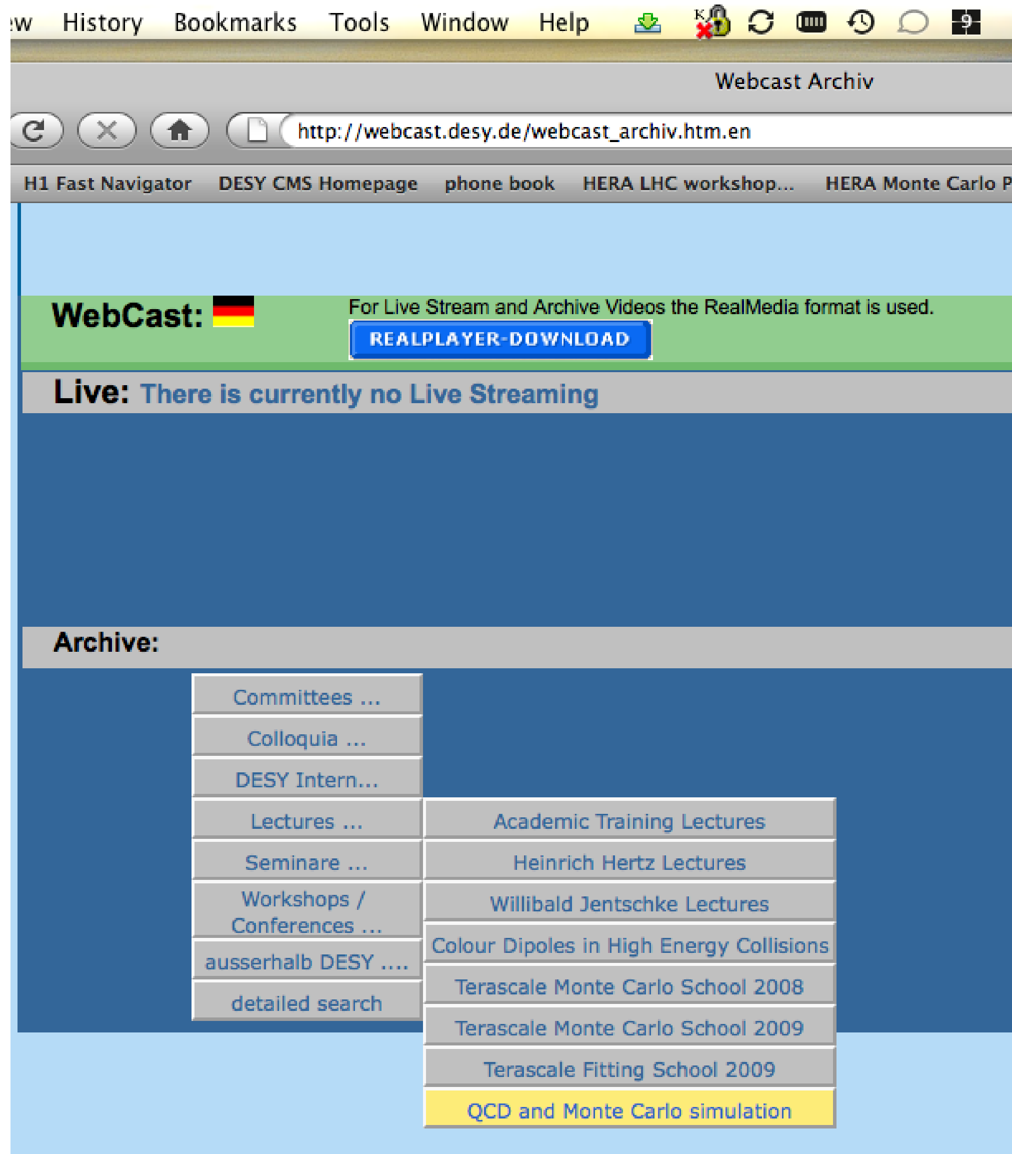
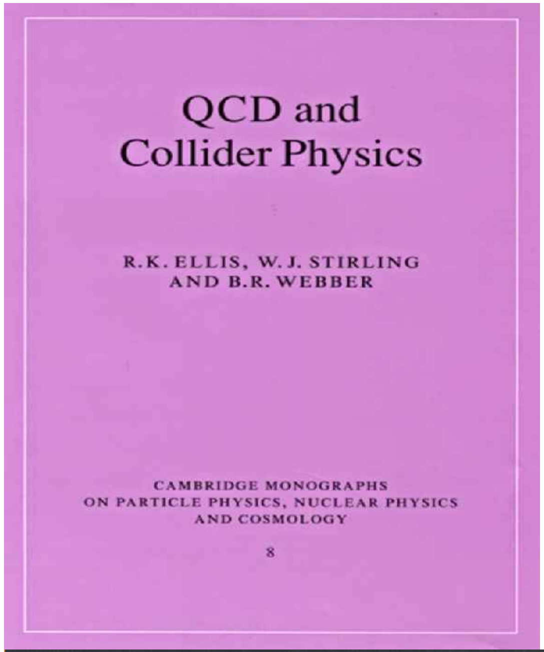
http://www.desy.de/~jung/qcd_and_mc_2009

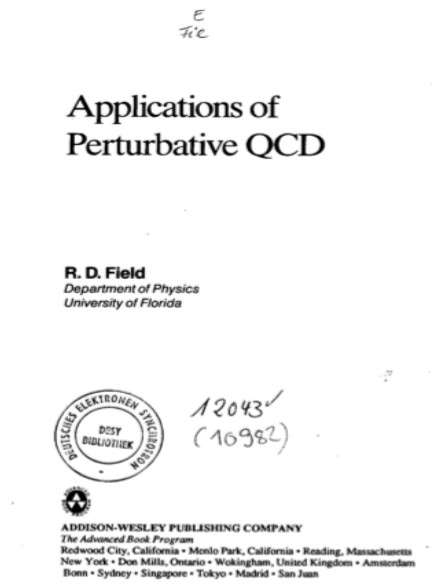
Video recordings

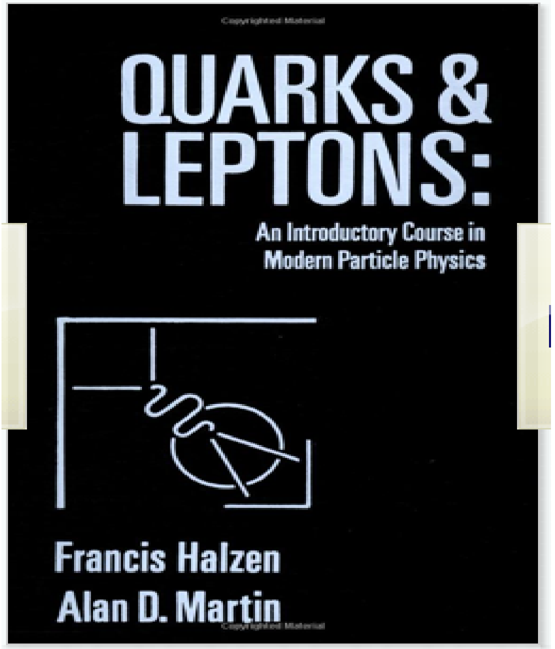


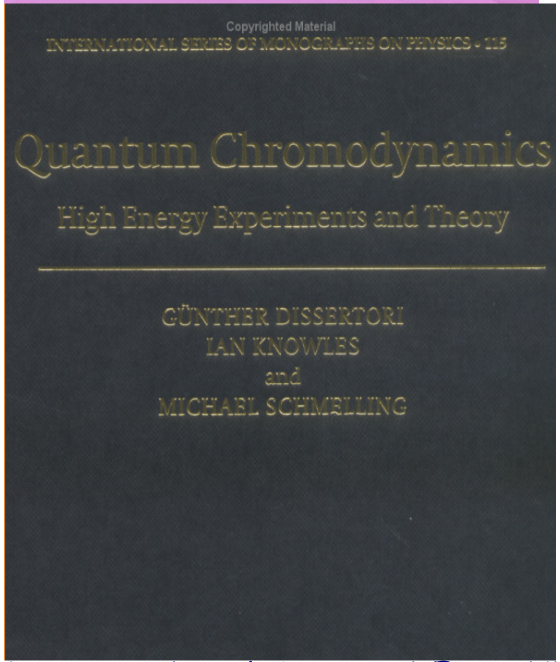


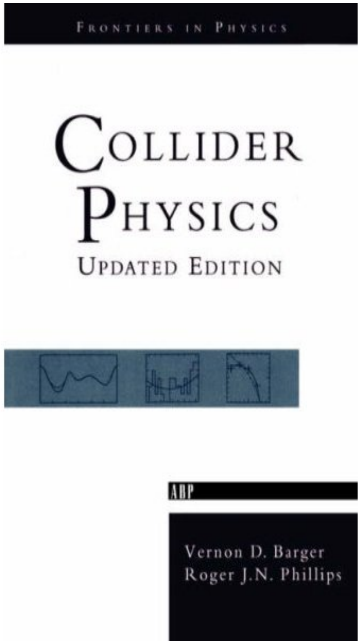
Literature

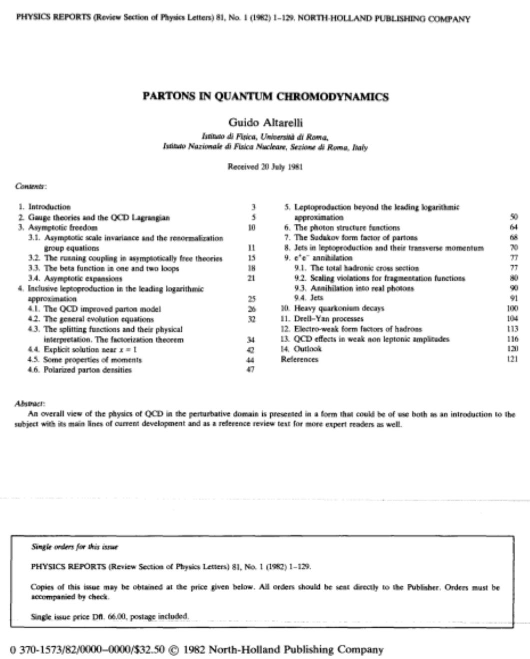












PHYSICS REPORTS (Review Section of Physics Letters) 81, No. 1 (1982) 1–129. NORTH-HOLLAND PUBLISHING COMPANY

PARTONS IN QUANTUM CHROMODYNAMICS

Guido Altarelli
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Received 20 July 1981

Contents:

Abstract:
An overall view of the physics of QCD in the perturbative domain is presented in a form that could be of use both as an introduction to the subject with its main lines of current development and as a reference review text for more expert readers as well.

Single orders for this issue

PHYSICS REPORTS (Review Section of Physics Letters) 81, No. 1 (1982) 1–129.

Copies of this issue may be obtained at the price given below. All orders should be sent directly to the Publisher. Orders must be accompanied by check.

Single issue price Dfl. 66.00, postage included.

Outline of the lectures

- 10 July Intro to Monte Carlo techniques and structure of matter
- 17 July DGLAP and small x: solution with MCs
- 23 July Small x, CCFM and BFKL
- 24 July W/Z production in pp and soft gluon resummation

- Lectures will be recorded and made available immediately .....

- Exercises in the afternoons: 14:00 – 16:30 in sem 1
  Assistant: A. Grebenyuk

- Discussion forum online:
  see link from web page or

http://www.terascale.de/research_topics/rt1_physics_analysis/monte_carlo_generators/discussion_forum/discussion_forum_lecture_2_monte_carlos

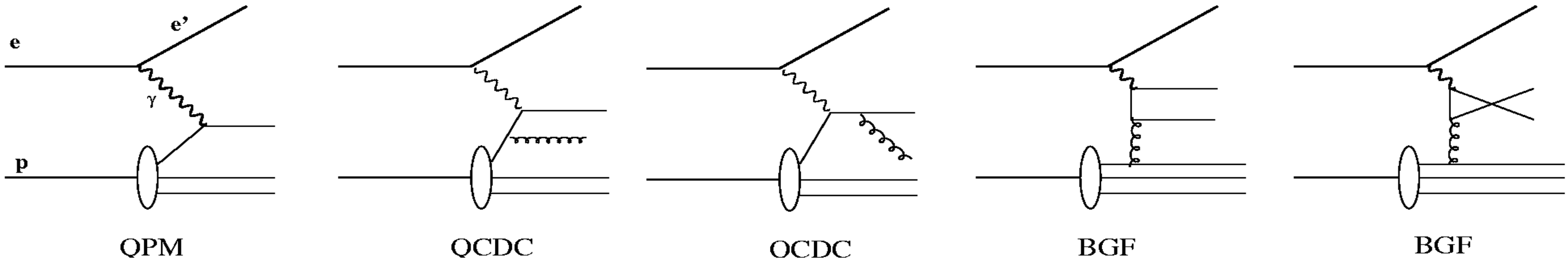
Requests to you ...

- If things go wrong .. lecture is too easy... too trivial ... too complicated, too chaotic or too boring ...
- **PLEASE complain immediately !**
- PLEASE ask questions any time !

Questions from last lecture and last exercise ?

Recap from last lecture …

recap: Higher order corrections to DIS



- lowest order: $e + q \to e' + q' \quad \mathcal{O}(\alpha_s^0)$
- higher order: $e + q \to e' + q' + g, \quad e + g \to e' + q + \bar{q} \quad \mathcal{O}(\alpha_s^1)$
  - What is the dominant part of the x-section ?
    - Investigate full x-section of QCDC and BGF
    - dominant part comes from small transverse momenta ...
    - rewrite x-section in terms of $k_\perp$
    - use small t limit:

$$\frac{d\sigma}{dk_\perp} = \frac{d\sigma}{dt}\frac{1}{(1-z)} = \frac{1}{(1-z)}\frac{1}{F}dLips|ME|^2$$

$$= \frac{1}{(1-z)}\frac{1}{16\pi}\frac{1}{\hat{s}+Q^2}\frac{1}{\hat{s}}|ME|^2$$

recap: Inelastic Scattering: x-section,phase space, ME

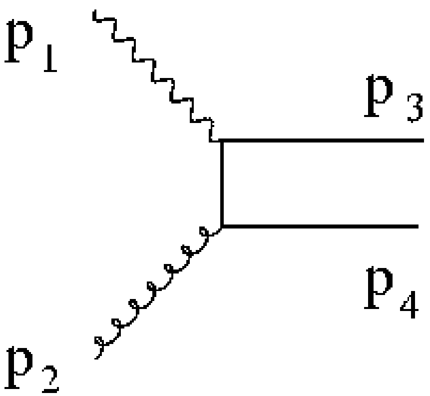
- Cross section definition: $$d\sigma = \frac{1}{F} d\text{Lips} \, |M|^2$$
- with initial flux $$F = 4\sqrt{(p_1 p_2)^2 - m_1^2 m_2^2}$$
- and Lorentz invariant phase space

$$\text{dLips} = (2\pi)^4 \delta^4(-p_1 - p_2 + \sum_i p_i) \sum_i \frac{d^4 p_i}{(2\pi)^3} \delta(p_i^2 - m_i^2)$$

$$\text{dLips} = (2\pi)^4 \delta^4(-p_1 - p_2 + \sum_i p_i) \sum_i \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

$$\text{dLips} = (2\pi)^4 \delta^4(-p_1 - p_2 + \sum_i p_i) \sum_i \frac{1}{(2\pi)^3} \frac{dp_i^+}{p_i^+} d^2 p_{t\,i}$$

recap: partonic cross sections



$$\hat{s} = (p_1 + p_2)^2 = Q^2 \frac{1-z}{z}$$

$$\hat{t} = k^2 = (p_1 - p_3)^2$$

$$\hat{u} = (p_2 - p_3)^2$$

- Flux for virtual photons:

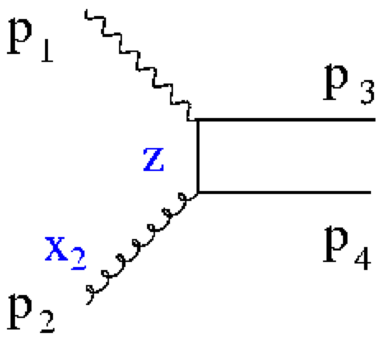
$$F = 4\sqrt{(p_1.p_2)^2 + m_1^2 m_2^2} = 2(\hat{s} + Q^2)$$

- x-section with virtual photons:

$$\frac{d\sigma}{dt} = \frac{1}{16\pi}\frac{1}{\hat{s}^2}|ME|^2 \rightarrow \frac{1}{16\pi}\frac{1}{\hat{s}+Q^2}\frac{1}{\hat{s}}|ME|^2$$

real photons

recap: kinematics



$$\hat{s} = (p_1 + p_2)^2 = Q^2 \frac{1-z}{z}$$

$$\hat{t} = k^2 = (p_1 - p_3)^2$$

$$\hat{u} = (p_2 - p_3)^2$$

Define:

$$z = \frac{Q^2}{2p_1p_2}$$

$$x_{bj} = zx_2$$

- Using s+t+u=-Q^2 gives:

$$k_\perp^2 = \frac{\hat{t}\hat{u}\hat{s}}{(\hat{s}+Q^2)^2}$$

- and for $\hat{t} \ll \hat{s}$

$$k_\perp^2 = \frac{-\hat{t}\hat{s}}{\hat{s}+Q^2} = -t\,(1-z)$$

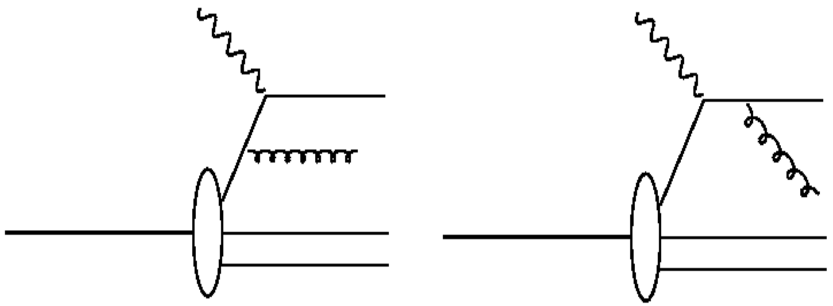
recap: QCDC - contribution

Blackboard

$$|M|^2 = 32\pi^2 \left(e_q^2 \alpha \alpha_s\right) \frac{4}{3} \left[\frac{-\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} + \frac{2\hat{u}Q^2}{\hat{s}\hat{t}}\right]$$

$$= 32\pi^2 \left(e_q^2 \alpha \alpha_s\right) \frac{4}{3} \frac{-1}{t} \left[\frac{Q^2(1+z^2)}{z(1-z)} + \cdots\right]$$

- integrate over *kt* generates *log*, BUT what is the lower limit



$$\frac{d\sigma}{dk_\perp^2} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi} \frac{1}{k_\perp^2} \left[P_{qq}(z) + \cdots\right]$$

$$P_{qq}(z) = \frac{4}{3} \frac{1+z^2}{1-z} \qquad \sigma_0 = \frac{4\pi^2\alpha}{\hat{s}}$$

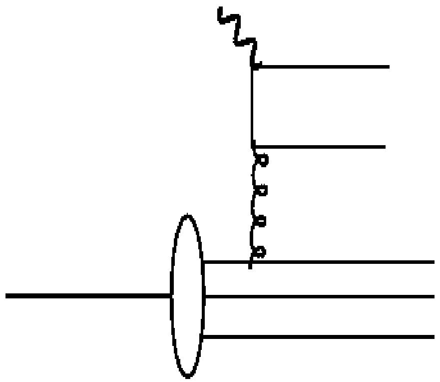
$$\sigma^{QCDC} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi} \left[P_{qq}(z) \log\left(\frac{Q^2(1-z)}{\chi^2 z}\right) + \cdots\right]$$

recap: boson gluon fusion

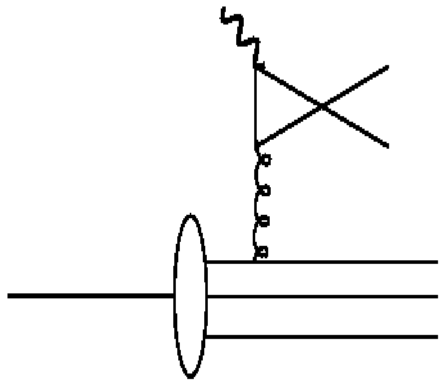
$$|M|^2 = 32\pi^2 \left(e_q^2 \alpha \alpha_s\right) \frac{1}{2}\left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - \frac{2\hat{s}Q^2}{\hat{t}\hat{u}}\right]$$

Blackboard

$$= 32\pi^2 \left(e_q^2 \alpha \alpha_s\right) \frac{-1}{t}\frac{Q^2}{z}\frac{1}{2}\left(z^2 + (1-z)^2 + \cdots\right)$$



BGF



BGF

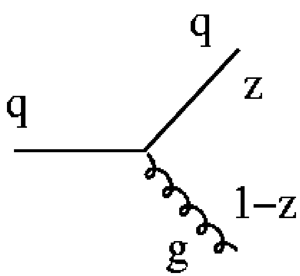
$$\frac{d\sigma}{dk_\perp^2} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi}\frac{1}{k_\perp^2}\left[P_{qg}(z) + \cdots\right]$$

$$P_{qg}(z) = \frac{1}{2}\left(z^2 + (1-z)^2\right)$$

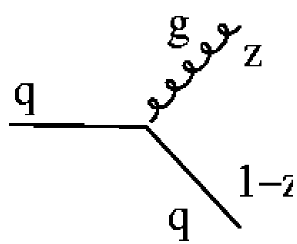
- integrate over kt generates log, BUT what is the lower limit

$$\sigma^{BGF} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi}\left[P_{qg}(z)\log\left(\frac{Q^2(1-z)}{\chi^2 z}\right) + \cdots\right]$$

Splitting functions in lowest order

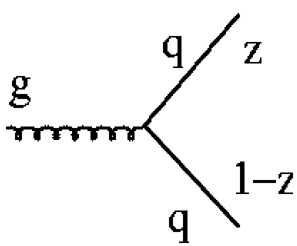


$$P_{qq} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

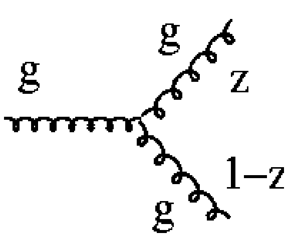


$$P_{gq} = \frac{4}{3}\left(\frac{1+(1-z)^2}{z}\right)$$

similarity to EPA...



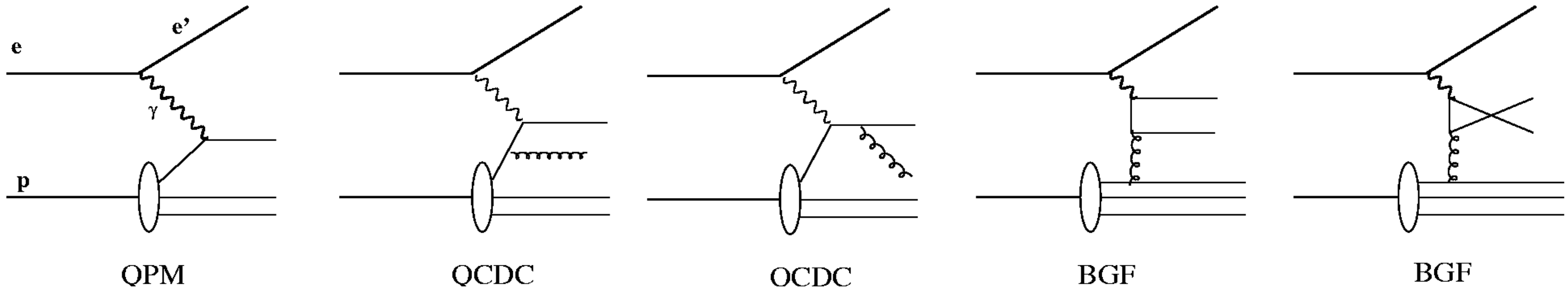
$$P_{qg} = \frac{1}{2}\left(z^2 + (1-z)^2\right)$$



$$P_{gg} = 6\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right)$$

recap: O(α_s) contribution to F_2

$$\frac{F_2}{x} =$$



- divergency for $k_\perp \to 0$ or $\chi \to 0$

$$\frac{F_2}{x} = \sum e_q^2 \int \frac{dx_2}{x_2} q_i(x_2) \delta\left(1 - \frac{x}{x_2}\right)$$

$$+$$

$$q_i(x_2) \left[\frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{x_2}\right) \left[\log\left(\frac{Q^2}{\chi^2}\right) + \log\left(\frac{1-z}{z}\right) + \ldots\right] + C_q(z, ..)\right]$$

$$+$$

$$g(x_2) \left[\frac{\alpha_s}{2\pi} P_{qg}\left(\frac{x}{x_2}\right) \left[\log\left(\frac{Q^2}{\chi^2} + \log\left(\frac{1-z}{z}\right) + \ldots\right] + C_g(z, \ldots)\right)\right]$$

Collinear factorization: DGLAP

- introduce new scale $\mu^2 \gg \chi^2$ and include soft, non-perturbative physics into renormalized parton density:

$$q_i(x,\mu^2) = q_i^0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i^0(\xi)P_{qq}\left(\frac{x}{\xi}\right) + g^0(\xi)P_{qg}\left(\frac{x}{\xi}\right)\right]\log\left(\frac{\mu^2}{\chi^2}\right)$$

- Dokshitzer Gribov Lipatov Altarelli Parisi equation:

  V.V. Gribov and L.N. Lipatov Sov. J. Nucl. Phys. 438 and 675 (1972) 15, L.N. Lipatov Sov. J. Nucl. Phys 94 (1975) 20, G. Altarelli and G. Parisi Nucl.Phys.B 298 (1977) 126, Y.L. Dokshitser Sov. Phys. JETP 641 (1977) 46

$$\frac{dq_i(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i(\xi,\mu^2)P_{qq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{qg}\left(\frac{x}{\xi}\right)\right]$$

- BUT there are also gluons....

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{gq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{gg}\left(\frac{x}{\xi}\right)\right]$$

- DGLAP is the analogue to the beta function for running of the coupling

Collinear factorization ....

... generalisation of
parton model result
to QCD !!!

Collinear factorization

see handbook of pQCD, chapter IV, B

$$F_2^{(Vh)}(x,Q^2) = \sum_{i=f,\bar{f},G} \int_0^1 d\xi C_2^{(Vi)} \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \frac{\mu_f^2}{\mu^2}, \alpha_s(\mu^2) \right) \otimes f_{i/h}(\xi, \mu_f^2, \mu^2)$$

- Factorization Theorem in DIS (Collins, Soper, Sterman, (1989) in Pert. QCD, ed. A.H. Mueller, Wold Scientific, Singapore, p1.)
  - hard-scattering function $C_2^{(Vi)}$ is infrared finite and calculable in pQCD, depending only on vector boson *V*, parton *i*, and renormalization and factorization scales. It is independent of the identity of hadron *h*.
  - pdf $f_{i/h}(\xi, \mu_f^2, \mu^2)$ contains all the infrared sensitivity of cross section, and is specific to hadron h, and depends on factorization scale.
- Generalization: applies to any DIS cross section defined by a sum over hadronic final states .... but be careful what it really means....
- explicit factorization theorems exist for:
  - inclusive and diffractive DIS (... see above....)
  - Drell Yan (in hadron hadron collisions)
  - single particle inclusive cross sections (fragmentation functions)

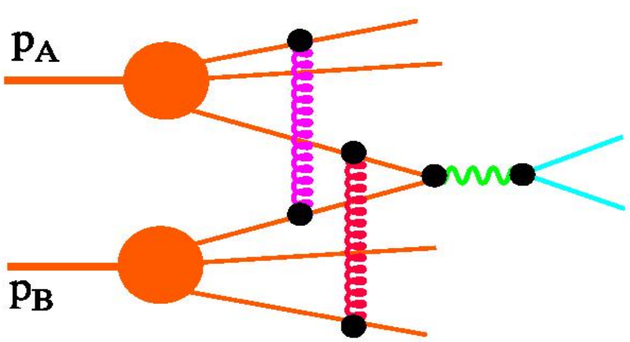
Factorization proofs and all that ...

- About factorization proofs (Wu-Ki Tung, pQCD and the parton structure of the nucleon, 2001, In *Shifman, M. (ed.): At the frontier of particle physics, vol. 2* 887-971)

tions $F_a^\lambda(x, \frac{Q}{m}, \alpha_s(\mu))$ (a = all parton flavors). Although the underlying physical ideas are relatively simple, as emphasized in the last two sections, the mathematical proofs are technically very demanding.[7,15,19] For this reason, actual proofs of factorization only exist for a few hard processes; and certain proofs (e.g. that for the Drell-Yan process) stayed controversial for some time before a consensus were reached.[15] Because of the general character of the physical ideas and the mathematical methods involved, however, it is generally *assumed* that the attractive *quark-parton model does apply to all high energy interactions* with at least one large energy scale.

$$\frac{d\sigma}{dy} = \sum_{a,b} \int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B f_A^a(\xi_A, \mu) f_B^b(\xi_B, \mu) \frac{d\hat{\sigma}_{ab}(\mu)}{dy} + \mathcal{O}\left(\left(\frac{m}{P}\right)^p\right)$$

- The problem with Drell-Yan: initial state interactions...
- factorization here does not hold graph-by-graph but only for all ....



Collinear factorization ....

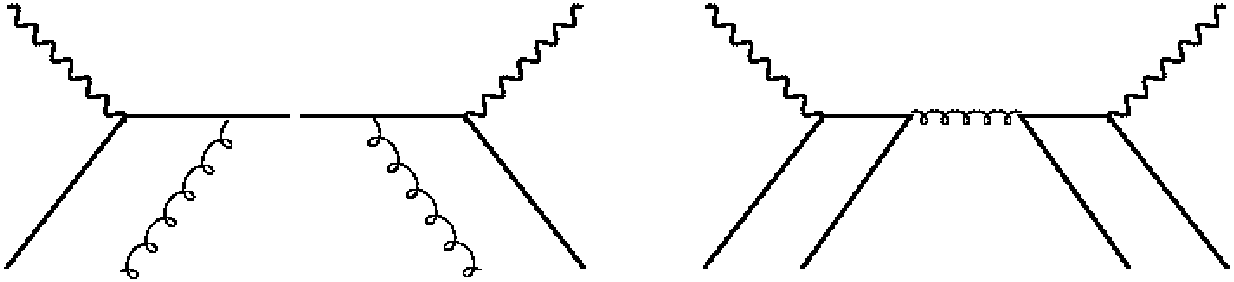
- So far considered only *"leading twist"*

Ellis, Webber, Stirling, 123
Roberts 108

twist = dimension (spin) of operators in Operator Product Expansion (OPE)

$$F_2(x,Q^2) = \sum_i C_{2\,i} \otimes f_i + \text{non-leading power of } Q$$

- Factorization theorem (Collins hep-ph/9709499):
- in general:



$$F_2(x,Q^2) = \sum_n \frac{B_n(x,Q^2)}{Q^{2n}}$$

n>0 higher twists
non-leading powers ...

- NOT covered by factorization theorem.... but contributions can be large ?!?

Warning on Factorization:

- The limits are factorization (i.e., the universality) of h h $\rightarrow$ h + X is not yet fully explored!
- You must surely sum over (i.e., not ask questions about) the soft stuff (as we do with jets)
- Some limits are becoming "clear" in h h $\rightarrow$ h h (b-to-b) + X
  See, e.g., J. Collins, hep-ph/0708.4410
- The INTRO discussion in
  G. Sterman, hep-ph/0807.5118
- The application of SCET (Soft Collinear Effective Theory)
  C. W. Bauer, et al., hep-ph/0808.2191
- See also, M. Seymour, et al., hep-ph/0808.1269

But even this is not the full story...

J. Collins, J.W. Qiu hep-ph 0705.2141

- factorization breaking in $pp \to j_1 j_2 X$

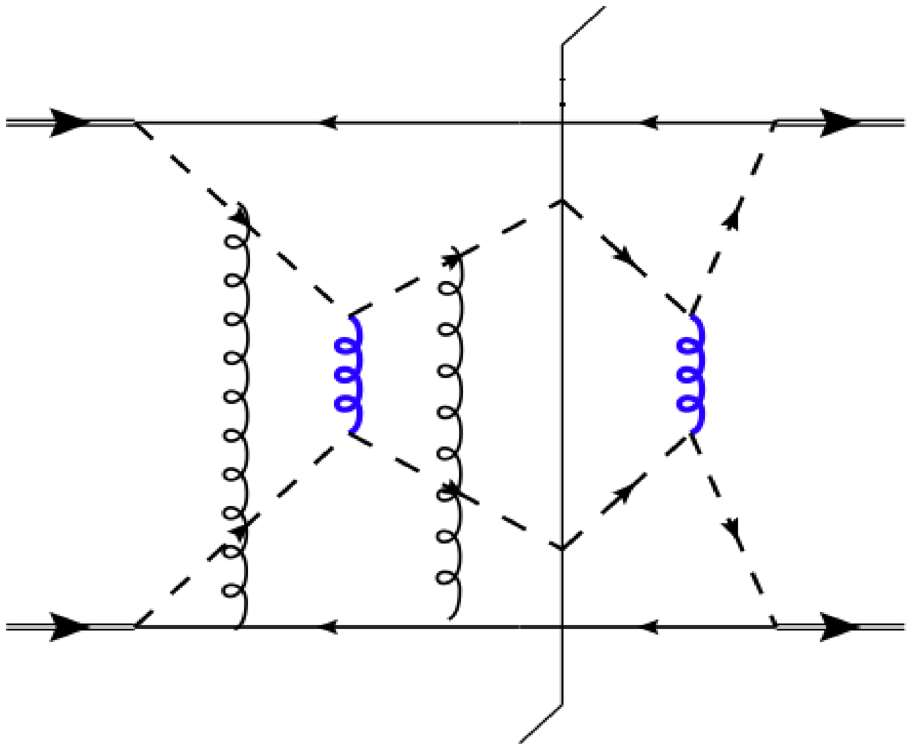


FIG. 8 (color online). The exchange of two extra gluons, as in this graph, will tend to give nonfactorization in unpolarized cross sections.

Collinear factorization schemes

- DIS scheme: absorbing all finite contributions C_q into quark densities, with no finite $\mathcal{O}(\alpha_s)$ corrections:

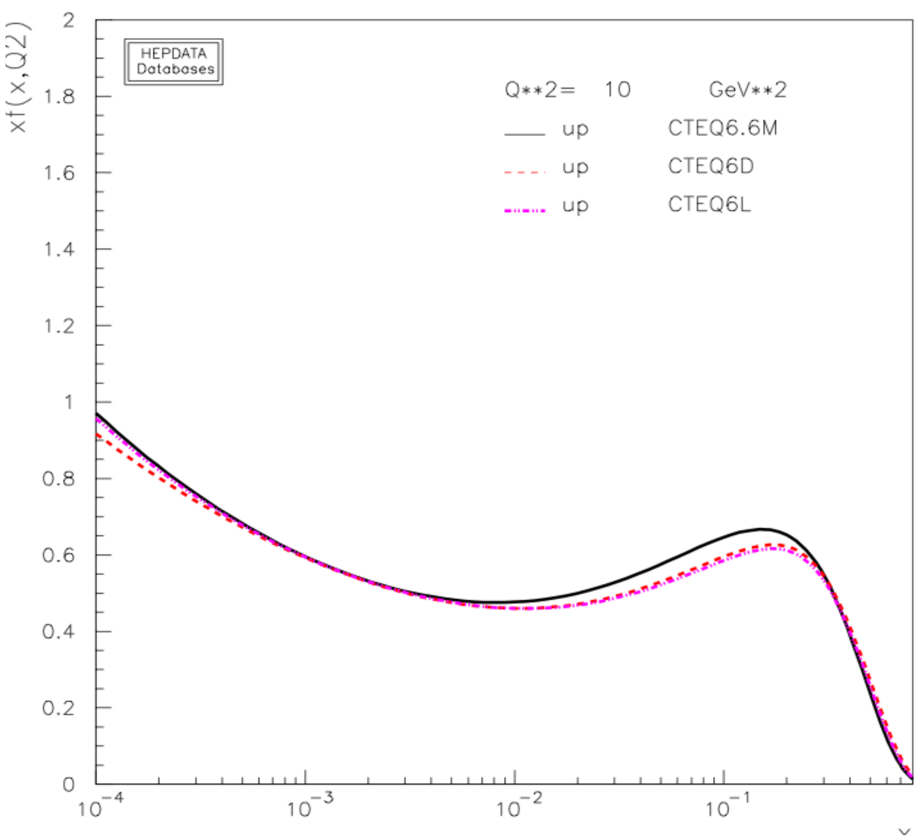
$$F_2^{DIS}(x,Q^2) = x\sum e_q^2 q(x,Q^2)$$

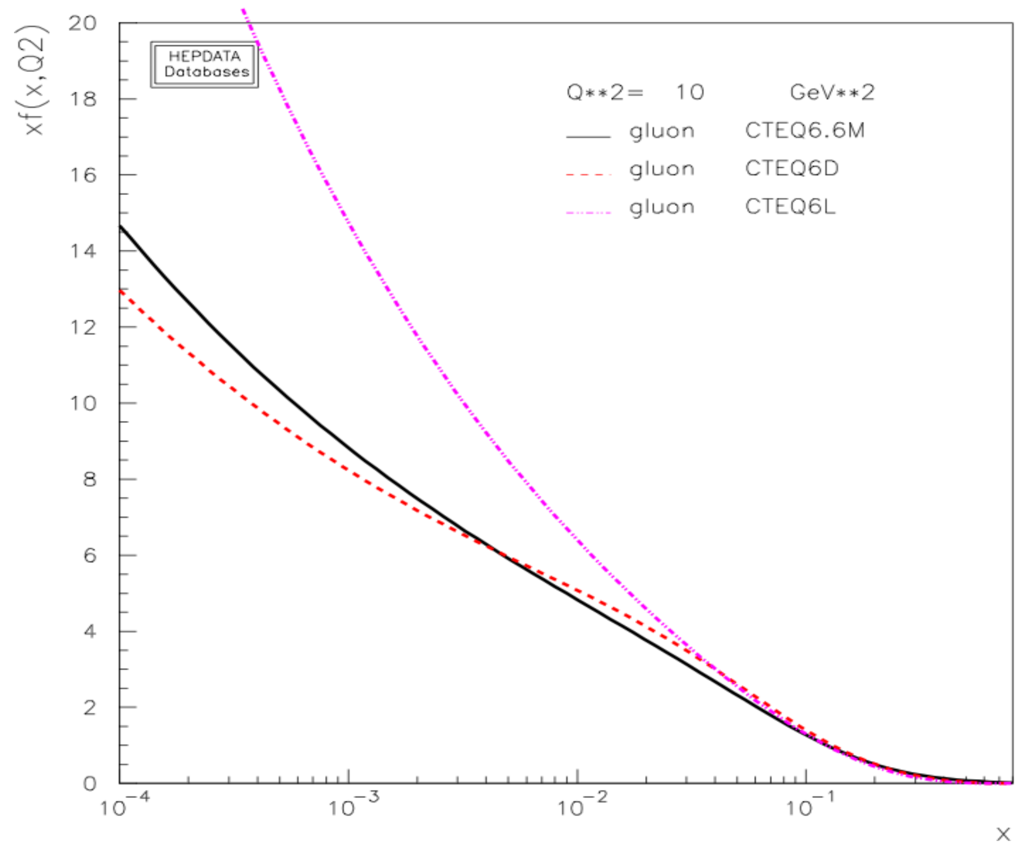
- $\overline{\mathrm{MS}}$ scheme, where only minimal contributions from the finite parts are absorbed in the quark distributions:

$$F_2^{\overline{MS}}(x,Q^2) = x\sum e_q^2 \int \frac{dx_2}{x_2} q^{\overline{MS}}(x,Q^2)\left[\delta\left(1-\frac{x}{x_2}\right)+\frac{\alpha_s}{2\pi}C^{\overline{MS}}\left(\frac{x}{x_2}\right)+\ldots\right]$$

- once the scheme is chosen, it **MUST** be used in all other cross section calculations
- higher order corrections will of course depend on the chosen scheme...
- **BUT**.... there are still other contributions to be included... gluon induced processes

PDFs in different fact. schemes





- differences between LO and NLO DIS, $\overline{MS}$ scheme in quark and gluon densities
- can make significant effects for x-sections

But back to the evolution equation

Evolution kernels - splitting fcts

- Splitting functions have perturbative expansion in the running coupling:

$$P(z, \alpha_s) = P^{(0)}(z) + \frac{\alpha_s}{2\pi} P^{(1)}(z) + \ldots$$

- including more and more loops ....

Splitting functions at higher orders

S. Moch, HERA-LHC workshop, June 2004

$$P(z,\alpha_s) = P^{(0)}(z) + \frac{\alpha_s}{2\pi} P^{(1)}(z) + \ldots$$

The calculation (in a nut shell)

– Calculate anomalous dimensions (Mellin moments of splitting functions)
$\longrightarrow$ divergence of Feynman diagrams in dimensional regularization $D = 4 - 2\varepsilon$

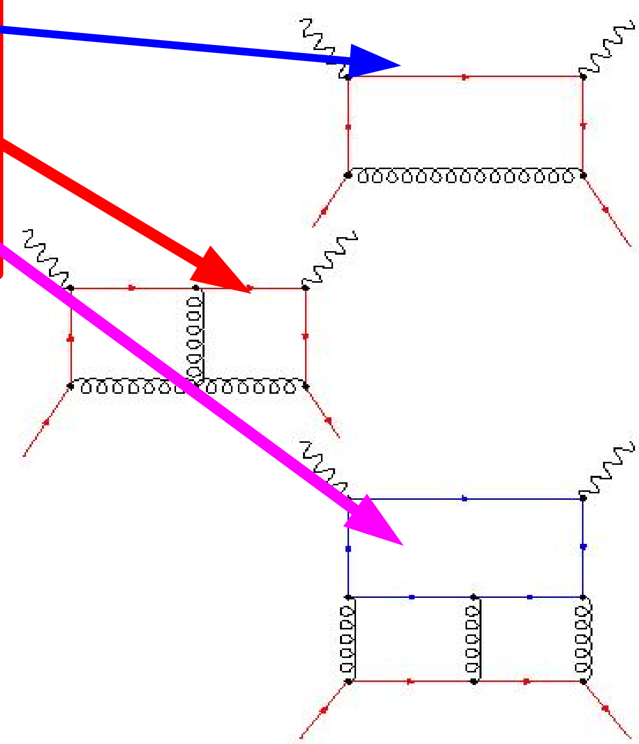
$$\gamma_{ij}^{(n)}(N) = - \int^{1} dx\, x^{N-1} P_{ij}^{(n)}(x)$$

loops again:
1-loop
2-loops
3-loops

– **One-loop** Feynman diagrams
$\longrightarrow$ in total 18 for $\gamma_{ij}^{(0)}$ / $P_{ij}^{(0)}$
(pencil + paper)

– **Two-loop** Feynman diagrams
$\longrightarrow$ in total 350 for $\gamma_{ij}^{(1)}$ / $P_{ij}^{(1)}$
(simple computer algebra)

– **Three-loop** Feynman diagrams
$\longrightarrow$ in total 9607 for $\gamma_{ij}^{(2)}$ / $P_{ij}^{(2)}$
(cutting edge technology $\longrightarrow$ computer algebra system FORM Vermaseren '89-'04)



Splitting functions (cont'd)

S. Moch, HERA-LHC workshop, June 2004

LO and NLO singlet splitting functions

$$P(z,\alpha_s) = P^{(0)}(z) + \frac{\alpha_s}{2\pi} P^{(1)}(z) + \ldots$$

$$\begin{aligned}
P^{(0)}_{\rm ps}(x) &= 0 \\
P^{(0)}_{\rm qg}(x) &= 2 n_f\, p_{\rm qg}(x) \\
P^{(0)}_{\rm gq}(x) &= 2 C_F\, p_{\rm gq}(x) \\
P^{(0)}_{\rm gg}(x) &= C_A \Big(4 p_{\rm gg}(x) + \frac{11}{3}\delta(1-x) \Big) - \frac{2}{3} n_f\, \delta(1-x)
\end{aligned}$$

$$P^{(1)}_{\rm ps}(x) = 4 C_F n_f \Big(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4{\rm H}_0 + x^2 \Big[\frac{8}{3}{\rm H}_0 - \frac{56}{9} \Big] + (1+x)\Big[5{\rm H}_0 - 2{\rm H}_{0,0} \Big] \Big)$$

$$\begin{aligned}
P^{(1)}_{\rm qg}(x) &= 4 C_A n_f \Big(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2 p_{\rm qg}(-x){\rm H}_{-1,0} - 2 p_{\rm qg}(x){\rm H}_{1,1} + x^2 \Big[\frac{44}{3}{\rm H}_0 - \frac{218}{9} \Big] \\
&+4(1-x)\Big[{\rm H}_{0,0} - 2{\rm H}_0 + x{\rm H}_1 \Big] - 4\zeta_2 x - 6{\rm H}_{0,0} + 9{\rm H}_0 \Big) + 4 C_F n_f \Big(2 p_{\rm qg}(x) \Big[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 \\
&-\zeta_2 \Big] + 4x^2 \Big[{\rm H}_0 + {\rm H}_{0,0} + \frac{5}{2} \Big] + 2(1-x)\Big[{\rm H}_0 + {\rm H}_{0,0} - 2x{\rm H}_1 + \frac{29}{4} \Big] - \frac{15}{2} - {\rm H}_{0,0} - \frac{1}{2}{\rm H}_0 \Big)
\end{aligned}$$

$$\begin{aligned}
P^{(1)}_{\rm gq}(x) &= 4 C_A C_F \Big(\frac{1}{x} + 2 p_{\rm gq}(x) \Big[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 - \frac{11}{6}{\rm H}_1 \Big] - x^2 \Big[\frac{8}{3}{\rm H}_0 - \frac{44}{9} \Big] + 4\zeta_2 - 2 \\
&-7{\rm H}_0 + 2{\rm H}_{0,0} - 2{\rm H}_1 x + (1+x)\Big[2{\rm H}_{0,0} - 5{\rm H}_0 + \frac{37}{9} \Big] - 2 p_{\rm gq}(-x){\rm H}_{-1,0} \Big) - 4 C_F n_f \Big(\frac{2}{3}x \\
&- p_{\rm gq}(x) \Big[\frac{2}{3}{\rm H}_1 - \frac{10}{9} \Big] \Big) + 4 C_F^2 \Big(p_{\rm gq}(x) \Big[3{\rm H}_1 - 2{\rm H}_{1,1} \Big] + (1+x)\Big[{\rm H}_{0,0} - \frac{7}{2} + \frac{7}{2}{\rm H}_0 \Big] - 3{\rm H}_{0,0} \\
&+1 - \frac{3}{2}{\rm H}_0 + 2{\rm H}_1 x \Big)
\end{aligned}$$

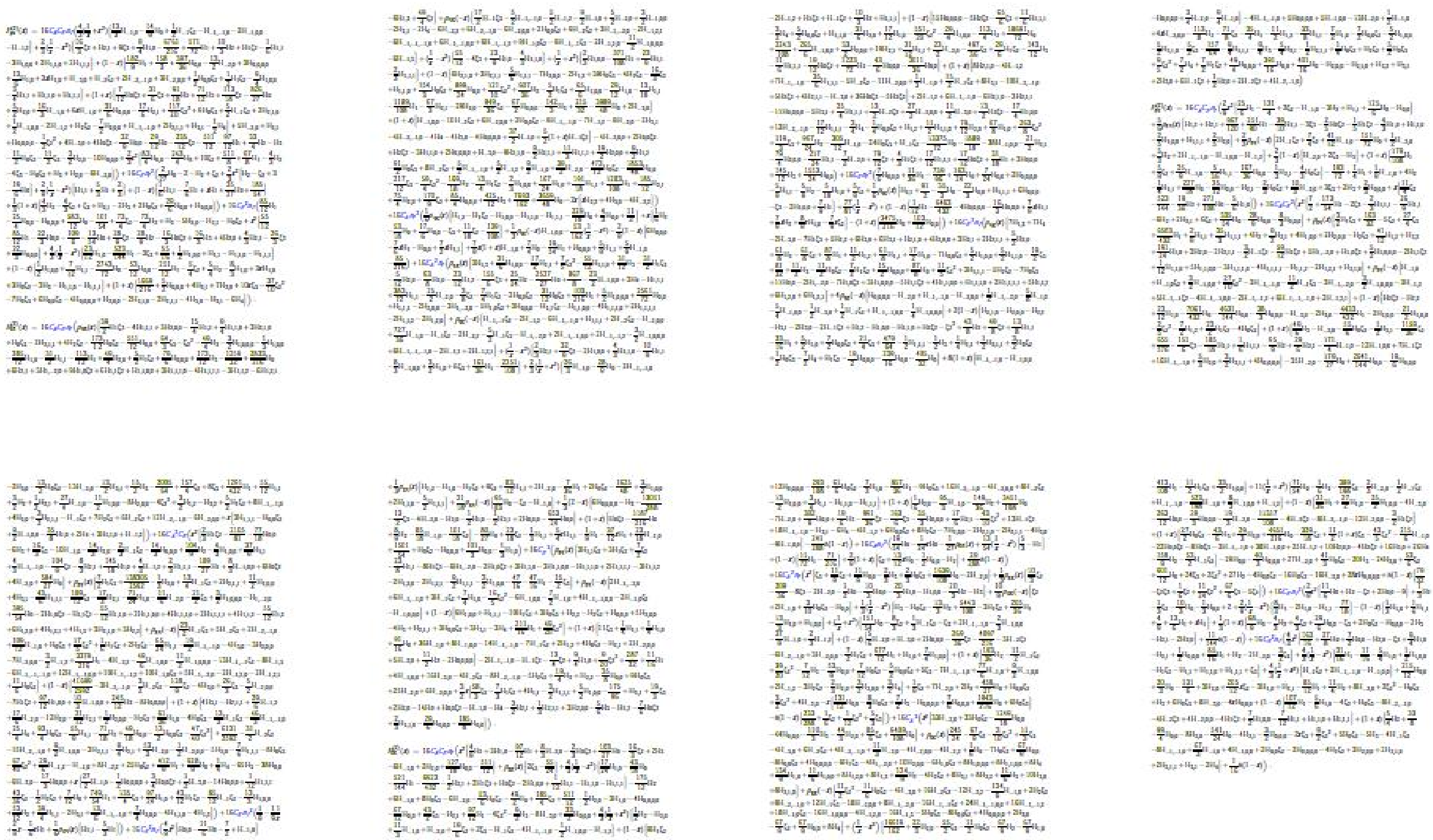
$$\begin{aligned}
P^{(1)}_{\rm gg}(x) &= 4 C_A n_f \Big(1 - x - \frac{10}{9} p_{\rm gg}(x) - \frac{13}{9}\Big(\frac{1}{x} - x^2 \Big) - \frac{2}{3}(1+x){\rm H}_0 - \frac{2}{3}\delta(1-x) \Big) + 4 C_A^2 \Big(27 \\
&+(1+x)\Big[\frac{11}{3}{\rm H}_0 + 8{\rm H}_{0,0} - \frac{27}{2} \Big] + 2 p_{\rm gg}(-x) \Big[{\rm H}_{0,0} - 2{\rm H}_{-1,0} - \zeta_2 \Big] - \frac{67}{9}\Big(\frac{1}{x} - x^2 \Big) - 12{\rm H}_0 \\
&- \frac{44}{3}x^2{\rm H}_0 + 2 p_{\rm gg}(x) \Big[\frac{67}{18} - \zeta_2 + {\rm H}_{0,0} + 2{\rm H}_{1,0} + 2{\rm H}_2 \Big] + \delta(1-x)\Big[\frac{8}{3} + 3\zeta_3 \Big] \Big) + 4 C_F n_f \Big(2{\rm H}_0 \\
&+ \frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\Big[4 - 5{\rm H}_0 - 2{\rm H}_{0,0} \Big] - \frac{1}{2}\delta(1-x) \Big) .
\end{aligned}$$

Splitting functions (cont'd)

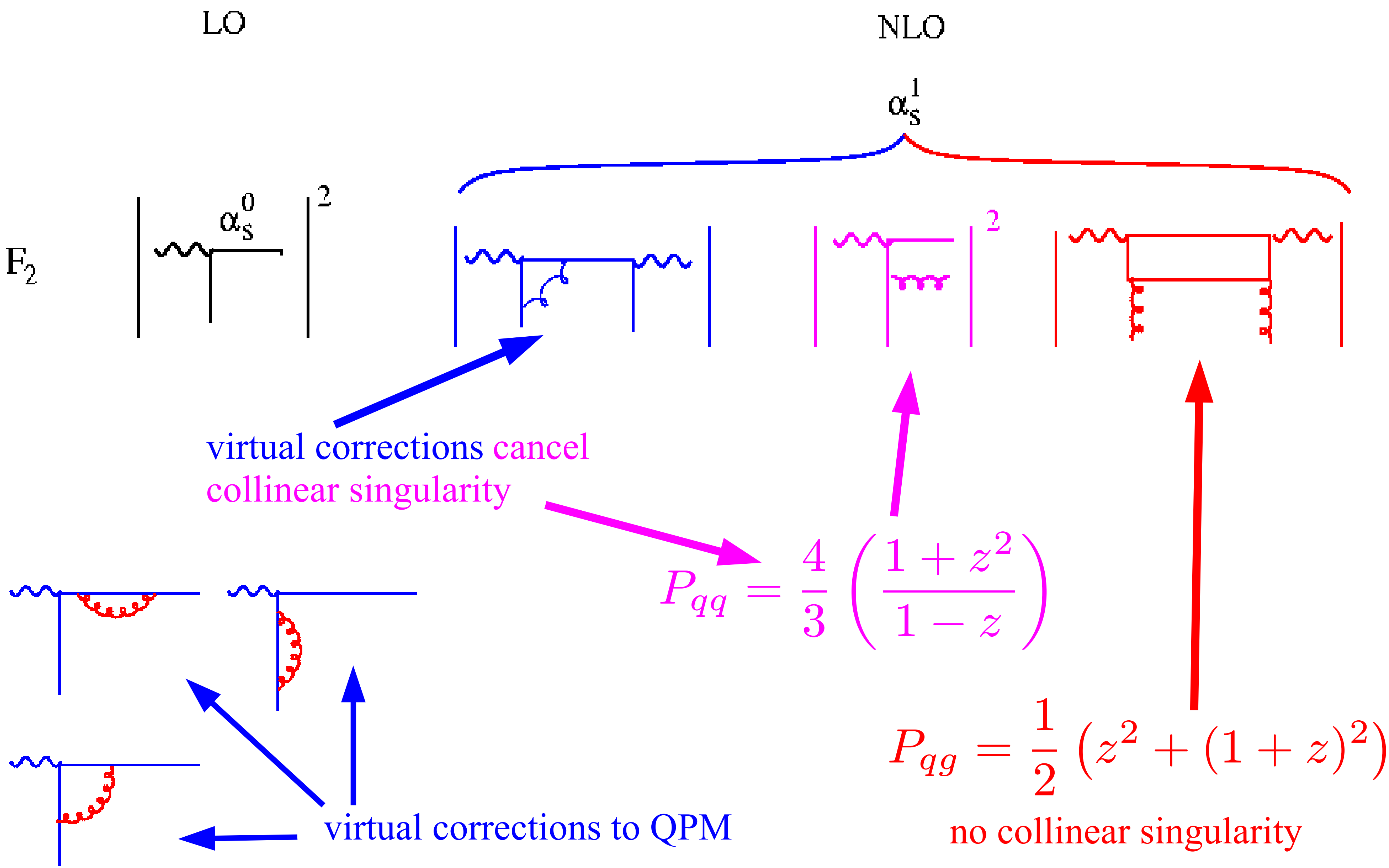
$$P(z,\alpha_s) = P^{(0)}(z) + \frac{\alpha_s}{2\pi}P^{(1)}(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 P^{(2)}(z) + \ldots$$

S. Moch, HERA-LHC workshop, June 2004

NNLO singlet splitting functions



NLO contributions to $F_2(x,Q^2)$



Evolution kernels – splitting fcts

- some of the splitting functions are also divergent... $\frac{1}{1-z}$
- use *plus-distribution* to avoid dangerous region:

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{1-z}$$

- divergence cancelled by virtual corrections ...
- use splitting functions with plus-distribution

Blackboard

virtual contributions, again ...

- we should have: $P(z) \to P(z) + K\delta(1-z)$

- conservation of quark (baryon) number:

$$\mathcal{P}_{qq}(z,Q^2) = \delta(1-z) + \frac{\alpha_s}{2\pi}\hat{P}^{+}_{qq}\log\frac{Q^2}{\mu^2} + \ldots$$

$$\int_0^1 \mathcal{P}_{qq}(z,Q^2)dz = 1$$

$$\int_0^1 \frac{dz}{z}P_{qq}(z) = 0$$

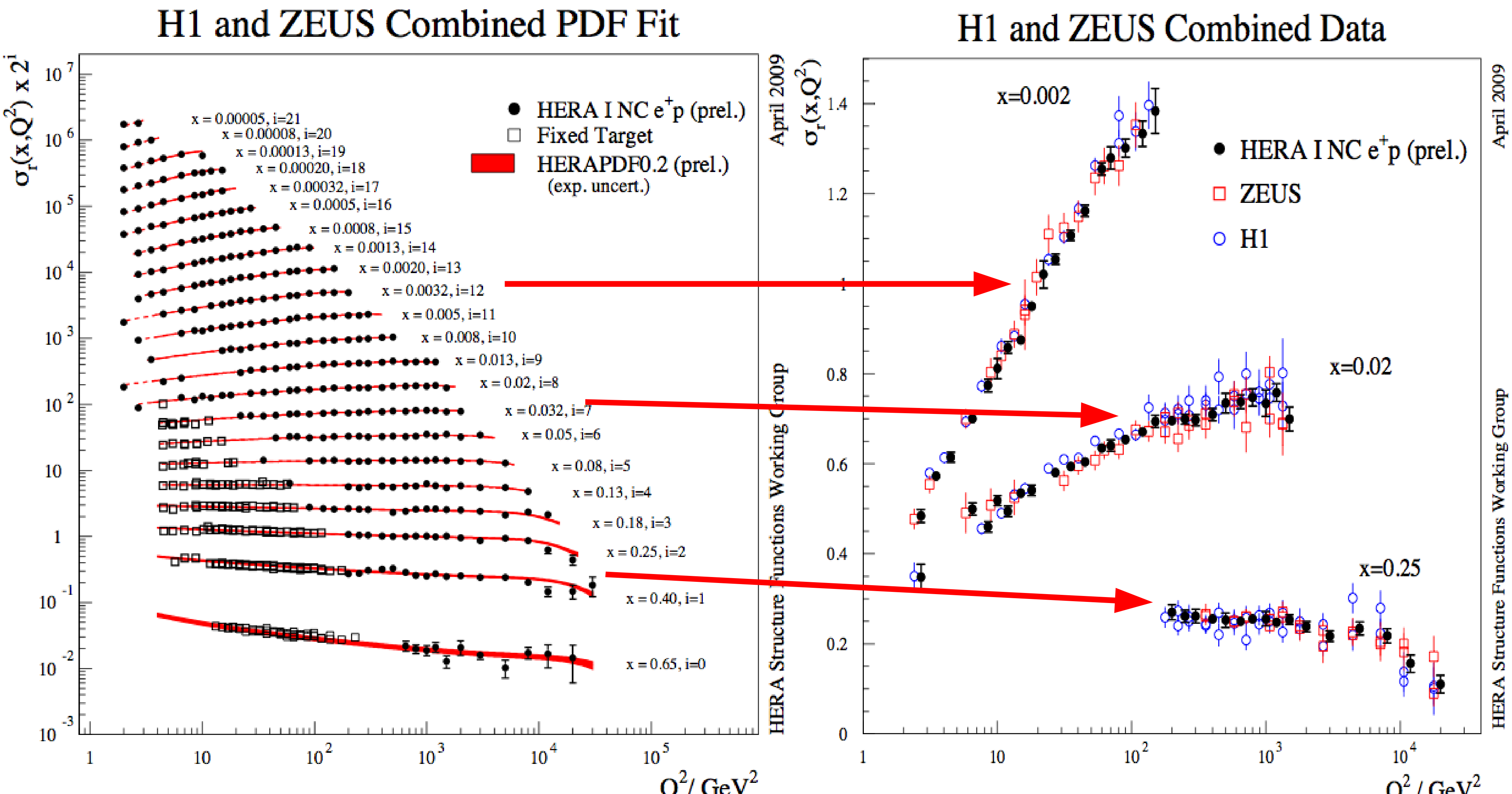
Blackboard

- changed splitting function:

$$P_{qq} = \frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)$$

How to apply these results

Applying DGLAP to DIS data ...

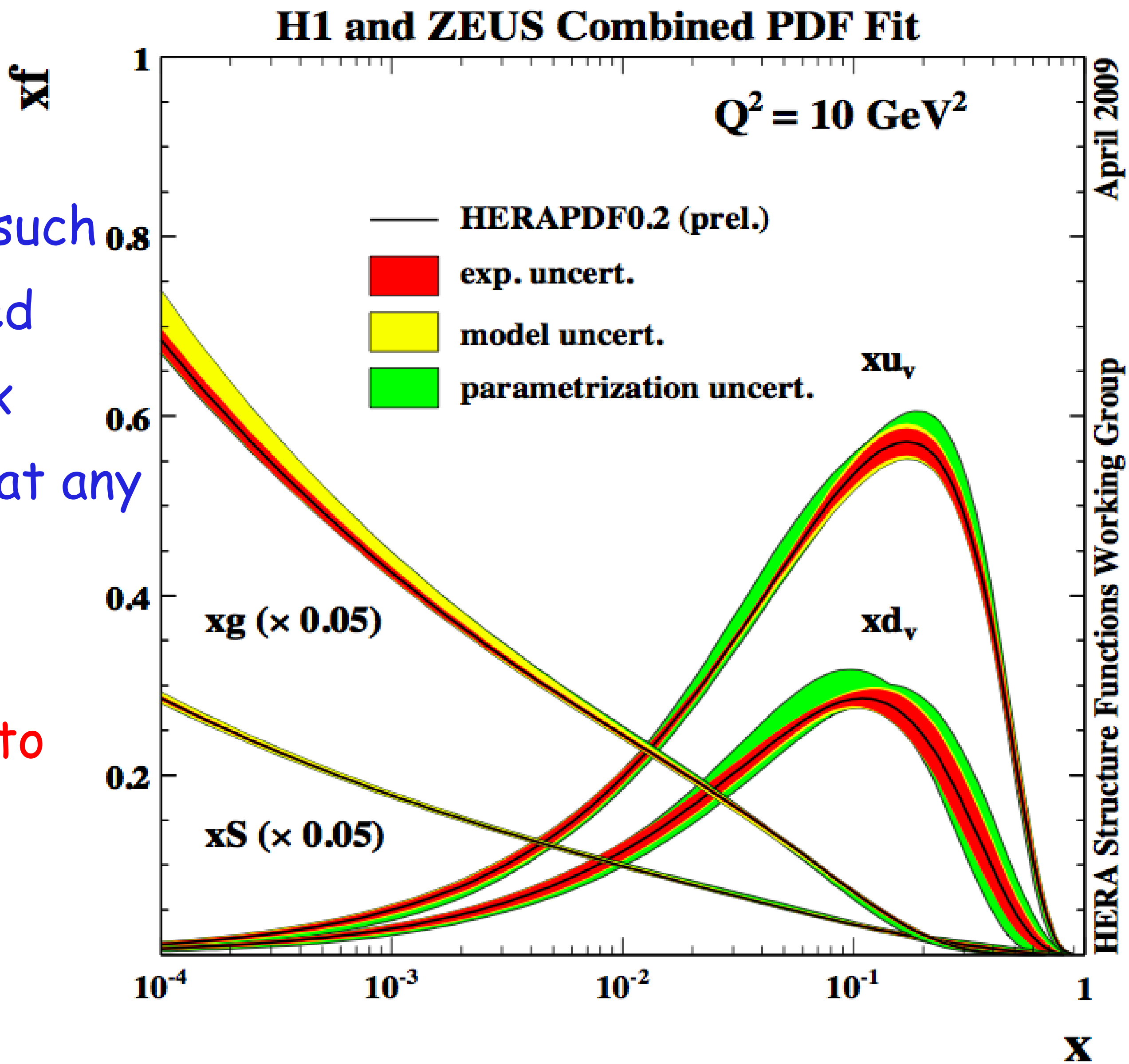


- Theory describes measurement over huge range in x and Q^2
- Success of theory (DGLAP)

Extraction of PDFs from DGLAP fits

- Solve DGLAP equations
- adjust input parameters (starting distributions) such that F2 is best described
- extract PDFs as fct of x
- then DGLAP gives PDFs at any Q^2

- Sum rules are essential to constrain starting distributions



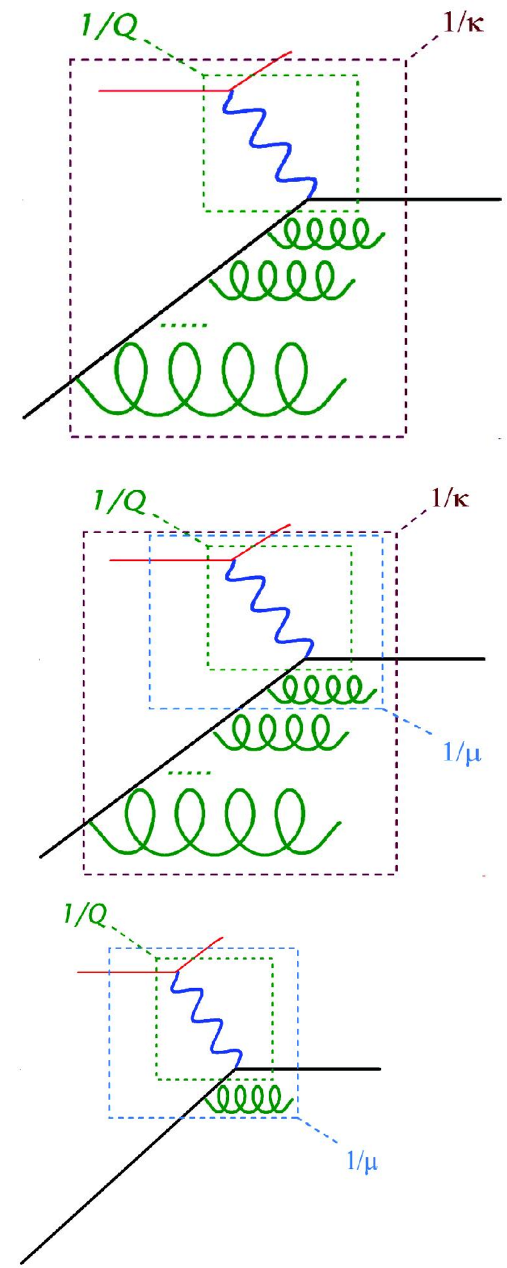
Meaning of evolution equations

- order emissions by "size"
- limits on the included emissions are provided by the scales (Q^2) for the short distance and χ for the long distance

- next separate contributions above and below factorization scale μ

- factor scales χ to μ into renormalized parton distributions. All above μ can be calculated perturbatively

From S. Ellis, Lecture 2, 2003



Solving DGLAP equations ...

- Different methods to solve integro-differential equations
  - brute-force (BF) method (M. Miyama, S. Kumano CPC 94 (1996) 185)

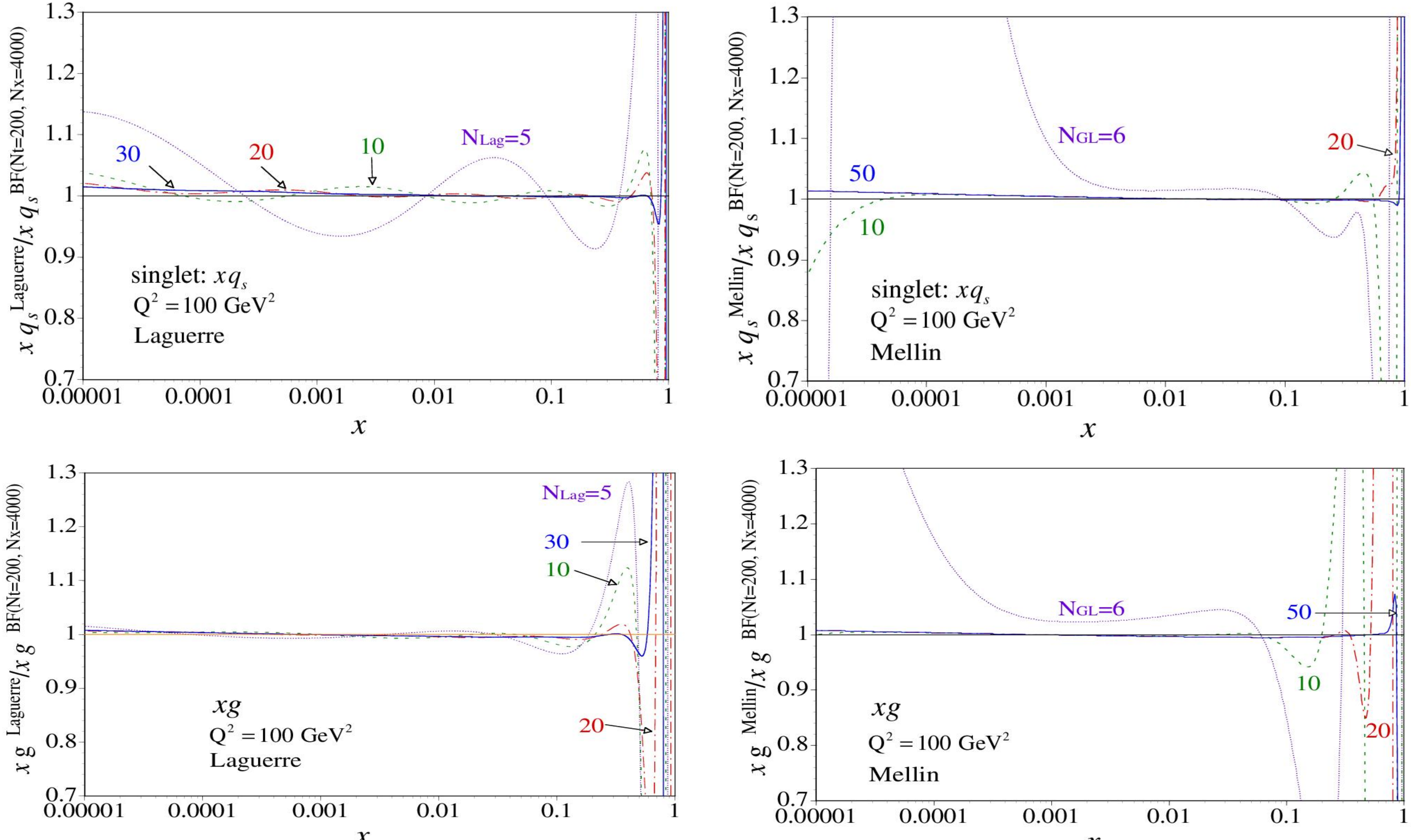
$$\frac{df(x)}{dx} = \frac{f(x)_{m+1} - f(x)_m}{\Delta x_m} \qquad \int f(x)dx = \sum f(x)_m \Delta x_m$$

  - Laguerre method (S. Kumano J.T. Londergan CPC 69 (1992) 373, and L. Schoeffel Nucl.Instrum.Meth.A423:439-445,1999)
  - Mellin transforms (M. Glueck, E. Reya, A. Vogt Z. Phys. C48 (1990) 471)
  - QCDNUM: calculation in a grid in x,Q2 space (M. Botje Eur.Phys.J. C14 (2000) 285-297)
  - CTEQ evolution program in x,Q2 space: http://www.phys.psu.edu/~cteq/
  - QCDFIT program in x,Q2 space (C. Pascaud, F. Zomer, LAL preprint LAL/94-02, H1-09/94-404,H1-09/94-376)
  - MC method using Markov chains (S. Jadach, M. Skrzypek hep-ph/0504205)
  - Monte Carlo method from iterative procedure
- brute-force method and MC method are best suited for detailed studies of branching processes !!!

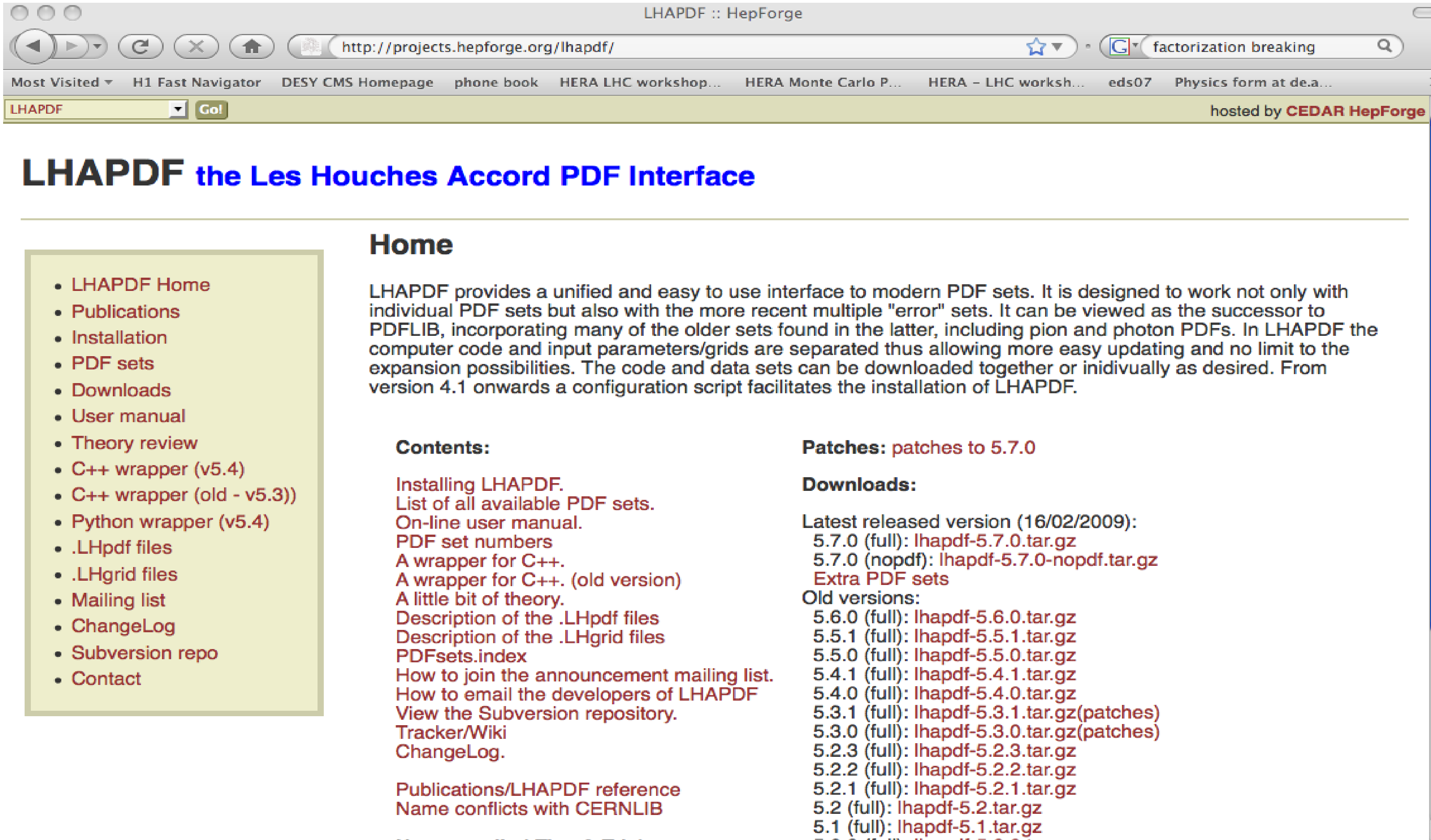
Comparison of different methods

- Compare Laguerre and Mellin method with *brute-force (BF)* method

S. Kumano, T-H Nagai hep-ph/0405160



Evolution code in LHAPDF


LHAPDF :: HepForge

http://projects.hepforge.org/lhapdf/

factorization breaking

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Home

LHAPDF provides a unified and easy to use interface to modern PDF sets. It is designed to work not only with individual PDF sets but also with the more recent multiple "error" sets. It can be viewed as the successor to PDFLIB, incorporating many of the older sets found in the latter, including pion and photon PDFs. In LHAPDF the computer code and input parameters/grids are separated thus allowing more easy updating and no limit to the expansion possibilities. The code and data sets can be downloaded together or inidivually as desired. From version 4.1 onwards a configuration script facilitates the installation of LHAPDF.

Contents:

Installing LHAPDF.
List of all available PDF sets.
On-line user manual.
PDF set numbers
A wrapper for C++.
A wrapper for C++. (old version)
A little bit of theory.
Description of the .LHpdf files
Description of the .LHgrid files
PDFsets.index
How to join the announcement mailing list.
How to email the developers of LHAPDF
View the Subversion repository.
Tracker/Wiki
ChangeLog.

Publications/LHAPDF reference
Name conflicts with CERNLIB

Patches: patches to 5.7.0

Downloads:

Latest released version (16/02/2009):
5.7.0 (full): lhapdf-5.7.0.tar.gz
5.7.0 (nopdf): lhapdf-5.7.0-nopdf.tar.gz
Extra PDF sets
Old versions:
5.6.0 (full): lhapdf-5.6.0.tar.gz
5.5.1 (full): lhapdf-5.5.1.tar.gz
5.5.0 (full): lhapdf-5.5.0.tar.gz
5.4.1 (full): lhapdf-5.4.1.tar.gz
5.4.0 (full): lhapdf-5.4.0.tar.gz
5.3.1 (full): lhapdf-5.3.1.tar.gz(patches)
5.3.0 (full): lhapdf-5.3.0.tar.gz(patches)
5.2.3 (full): lhapdf-5.2.3.tar.gz
5.2.2 (full): lhapdf-5.2.2.tar.gz
5.2.1 (full): lhapdf-5.2.1.tar.gz
5.2 (full): lhapdf-5.2.tar.gz
5.1 (full): lhapdf-5.1.tar.gz

Can

- CTEQ, QCDNUM, and other evolution packages...

From evolution equation to parton branching ...

How ?

Divergencies again...

- collinear divergencies factored into renormalized parton distributions $z \to 1$
- what about soft divergencies ?

treated with "plus" prescription

$$\frac{1}{1-z} \to \frac{1}{1-z}_+$$

with $\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{(1-z)}$

- soft divergency treated with Sudakov form factor:

$$\Delta(t) = \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \int^{z_{max}} dz \frac{\alpha_s}{2\pi} \tilde{P}(z)\right]$$

Blackboard

DGLAP evolution again....

- differential form: $$t\frac{\partial}{\partial t} f(x,t) = \int \frac{dz}{z}\frac{\alpha_s}{2\pi} P_+(z)\, f\left(\frac{x}{z},t\right)$$

- differential form using f/Δ_s with

$$\Delta_s(t) = \exp\left(- \int_x^{z_{max}} dz \int_{t_0}^{t} \frac{\alpha_s}{2\pi}\frac{dt'}{t'}\tilde{P}(z)\right)$$

$$t\frac{\partial}{\partial t}\frac{f(x,t)}{\Delta_s(t)} = \int \frac{dz}{z}\frac{\alpha_s}{2\pi}\frac{\tilde{P}(z)}{\Delta_s(t)} f\left(\frac{x}{z},t\right)$$

- integral form

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z}\int \frac{dt'}{t'}\cdot\frac{\Delta_s(t)}{\Delta_s(t')}\tilde{P}(z) f\left(\frac{x}{z},t'\right)$$

no – branching probability from t_0 to t

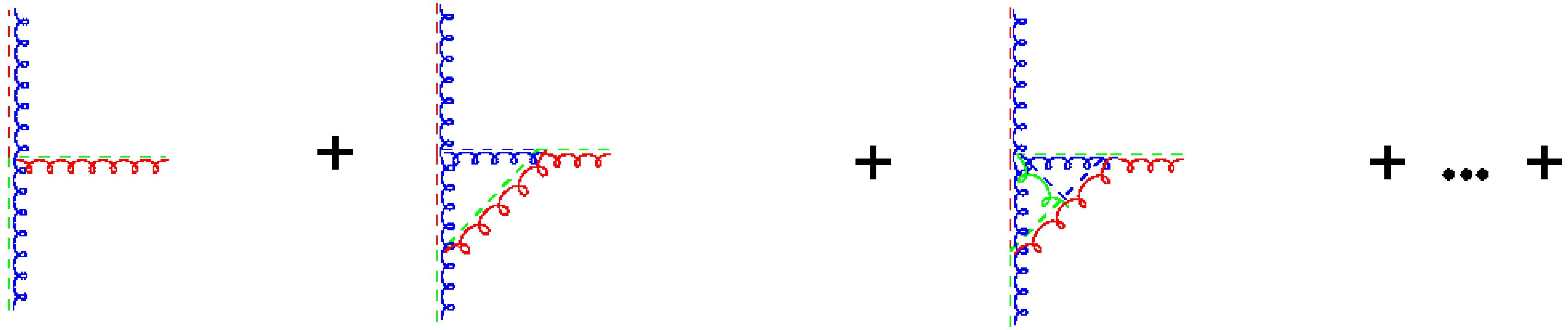
Sudakov form factor: all loop resum...

$g \to gg$ Splitting Fct $\tilde{P}(z) = \frac{\bar{\alpha}_s}{1-z} + \frac{\bar{\alpha}_s}{z} + \ldots$

- Sudakov form factor .... all loop resummation

$$\Delta_S = \exp\left(- \int dz \int \frac{dq'}{q'} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)$$

$$\Delta_S = 1 + \left(- \int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^1 + \frac{1}{2!} \left(- \int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^2 \ldots$$



$$\tilde{P}(z) \left[1 - \int \int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) + \frac{1}{2!} \left(- \int \int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^2 - \ldots - \right]$$

Sudakov form factor

- what is the limit on z- integration ?
  - resolvable branching ?
  - $$z < 1 - \frac{Q_0^2}{Q_b^2}$$ with Q_0 a soft cutoff

Blackboard

- probability of no -radiation between Q_a and Q_b
  - $$\Pi(Q_a^2, Q_b^2) = \frac{\Delta(Q_a^2)}{\Delta(Q_b^2)} = \exp\left[- \int_{Q_b^2}^{Q_a^2} \frac{dq^2}{q^2} \int_0^{z_{cut}} dz \frac{\alpha_s}{2\pi} P(z) \right]$$

Sudakov form factors

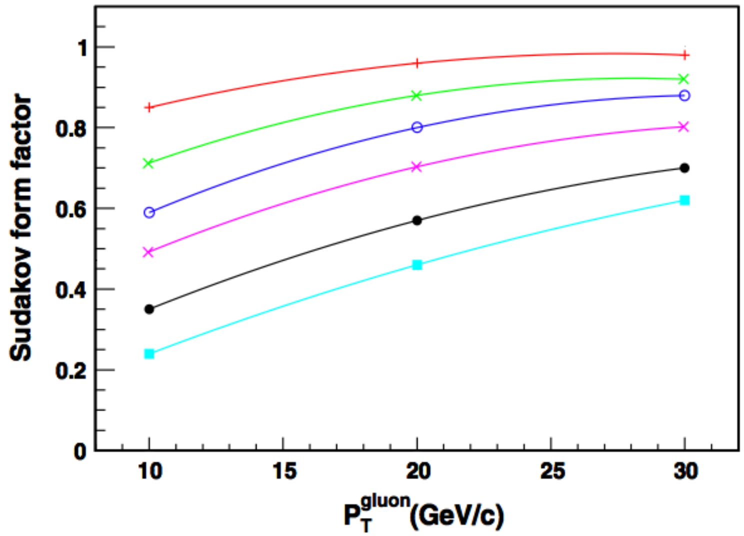


Figure 21. The Sudakov form factors for initial-state gluons at a hard scale of 100 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton x values of 0.3, 0.1, 0.03, 0.01, 0.001 and 0.0001.

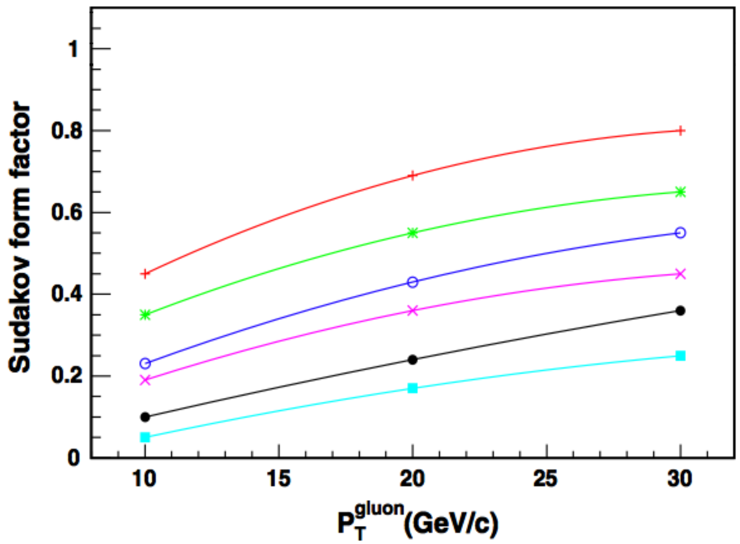


Figure 22. The Sudakov form factors for initial-state gluons at a hard scale of 500 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton x values of 0.3, 0.1, 0.03, 0.01, 0.001 and 0.0001.

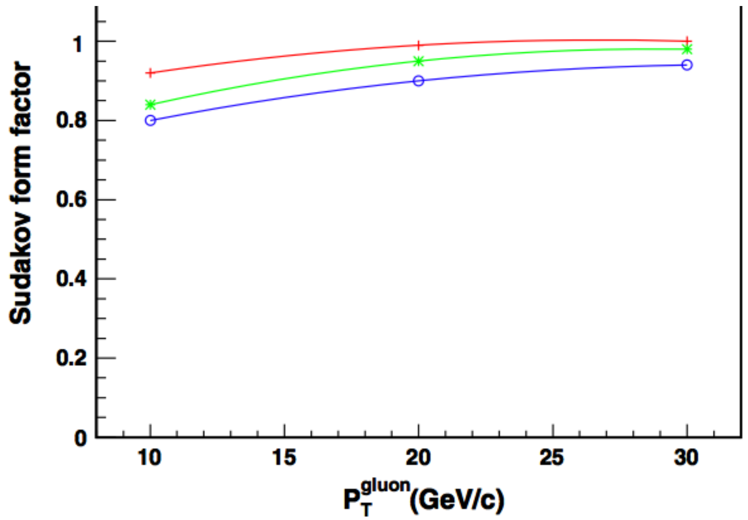


Figure 23. The Sudakov form factors for initial-state quarks at a hard scale of 100 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton x values of 0.3, 0.1 and 0.03.

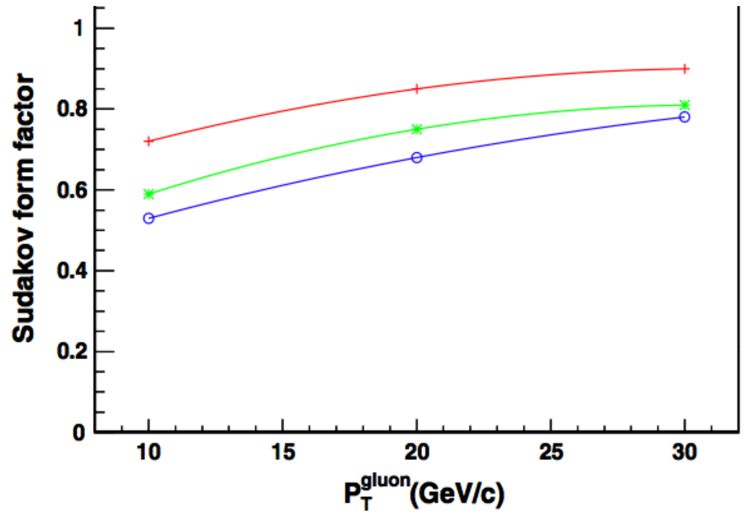


Figure 24. The Sudakov form factors for initial-state quarks at a hard scale of 500 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton x values of 0.3, 0.1 and 0.03.

Solving integral equations

- Integral equation of Fredholm type: $\phi(x) = f(x) + \lambda \int_a^b K(x,y)\phi(y)dy$
- solve it by iteration (Neumann series):

Blackboard

$$
\begin{aligned}
\phi_0(x) &= f(x) \\
\phi_1(x) &= f(x) + \lambda \int_a^b K(x,y) f(y) dy \\
\phi_2(x) &= f(x) + \lambda \int_a^b K(x,y_1) f(y_1) dy_1 + \lambda^2 \int_a^b \int_a^b K(x,y_1) K(y_1,y_2) f(y_2) dy_2 dy_1 \\
\phi_n(x) &= \sum_{i=0}^{n} \lambda^i u_i(x) \\
u_0(x) &= f(x) \\
u_1(x) &= \int_a^b K(x,y) f(y) dy \\
u_n(x) &= \int_a^b \int_a^b K(x,y_1) K(y_1,y_2) \cdots K(y_{n-1},y_n) f(y_n) dy_2 \cdots dy_n
\end{aligned}
$$

with the solution: $\phi(x) = \lim_{n\to\infty} q_n(x) = \lim_{n\to\infty} \sum_{i=0}^{n} \lambda^i u_i(x)$

DGLAP re-sums leading logs...

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

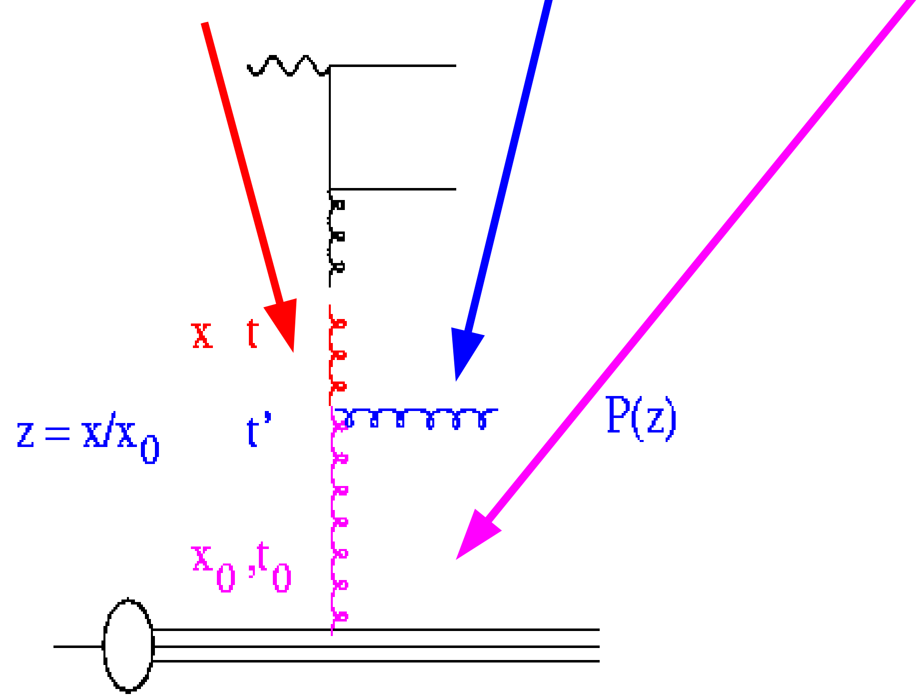
from t' to t w/o branching

branching at t'

from t_0 to t' w/o branching

$$f_0(x,t) = f(x,t_0)\Delta(t)$$

$$f_1(x,t) = f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0)\Delta(t')$$



DGLAP re-sums leading logs...

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

from t' to t w/o branching

branching at t'

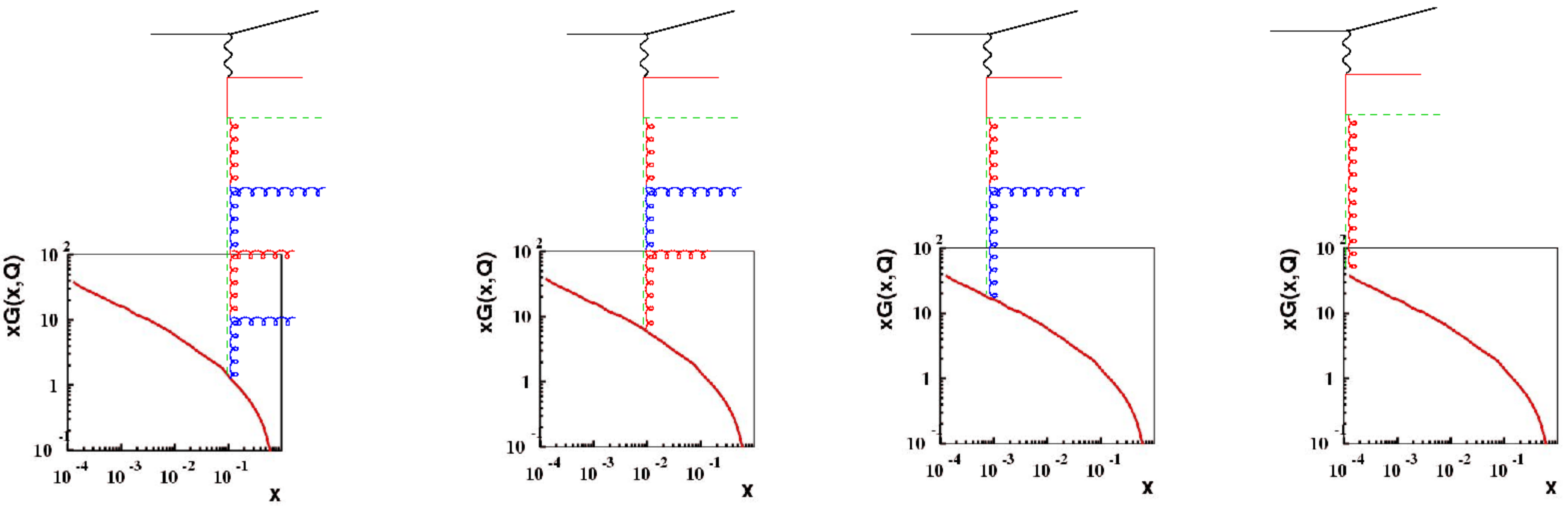
from t_0 to t' w/o branching

$$\begin{aligned}
f_0(x,t) &= f(x,t_0)\Delta(t) \\
f_1(x,t) &= f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z,t_0)\Delta(t') \\
&= f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A \otimes \Delta(t) f(x/z,t_0) \\
f_2(x,t) &= f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A \otimes \Delta(t) f(x/z,t_0) + \\
&\quad \frac{1}{2}\log^2\frac{t}{t_0} A \otimes A \otimes \Delta(t) f(x/z,t_0) \\
f(x,t) &= \lim_{n\to\infty} f_n(x,t) = \lim_{n\to\infty} \sum_n \frac{1}{n!} \log^n\left(\frac{t}{t_0}\right) A^n \otimes \Delta(t) f(x/z,t_0)
\end{aligned}$$

DGLAP re-sums $\log t$ **to all orders !!!!!!!!!!!!!!!!**

DGLAP evolution equation... again...

- for fixed x and Q^2 chains with different branchings contribute
- iterative procedure, spacelike parton showering



$$f(x,t) = f_0(x,t_0)\Delta_s(t) + \sum_{k=1}^{\infty} f_k(x_k,t_k)$$

What is happening at small x ?

- For $x \to 0$ only gluon splitting function matters:

$$P_{gg} = 6\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right) = 6\left(\frac{1}{z} - 2 + z(1-z) + \frac{1}{1-z}\right)$$

$$P_{gg} \sim 6\frac{1}{z} \text{ for } z \to 0$$

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} g(\xi,\mu^2) P_{gg}\left(\frac{x}{\xi}\right)$$

Blackboard

- evolution equation is then:

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} g(\xi,\mu^2) P_{gg}\left(\frac{x}{\xi}\right)$$

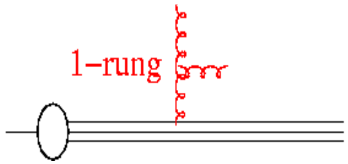
$$xg(x,t) = \frac{3\alpha_s}{\pi}\int_{t_0}^{t} d\log t' \int_x^1 \frac{d\xi}{\xi}\xi g(\xi,t') \quad \text{with} \quad t = \mu^2$$

Estimates at small x: DLL

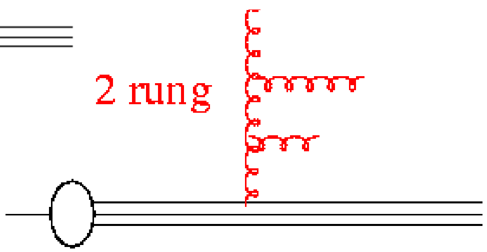
$$xg(x,t) = \frac{3\alpha_s}{\pi}\int_{t_0}^{t} d\log t' \int_x^1 \frac{d\xi}{\xi}\xi g(\xi,t') \quad \text{with} \quad t=\mu^2$$

- use constant starting distribution at small t: $xg_0(x) = C$

$$xg_1(x,t) = \frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}C$$

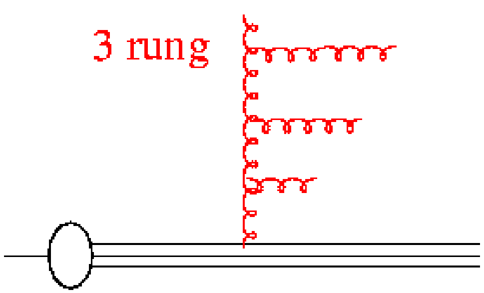


$$xg_2(x,t) = \left(\frac{3\alpha_s}{\pi}\frac{1}{2}\log\frac{t}{t_0}\frac{1}{2}\log\frac{1}{x}\right)^2 C$$



$$\vdots$$

$$xg_n(x,t) = \frac{1}{n!}\frac{1}{n!}\left(\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}\right)^n C$$



$$xg(x,t) = \sum_n \frac{1}{n!}\frac{1}{n!}\left(\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}\right)^n C$$

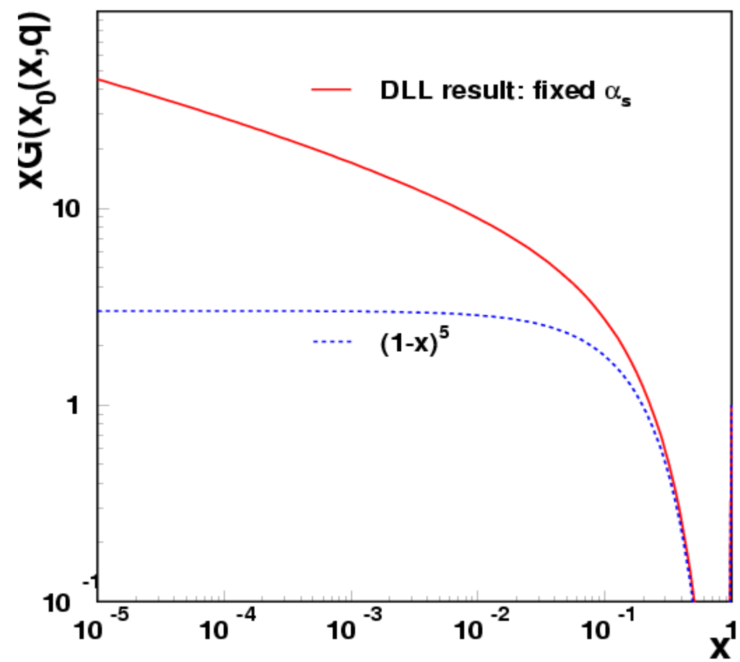
$$xg(x,t) \sim C\exp\left(2\sqrt{\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}}\right)$$

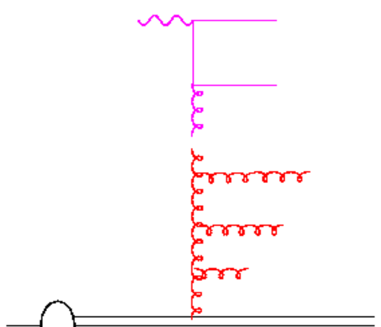
double leading log approximation (DLL)

Results from DLL approximation

- DLL arise from taking small x limit of splitting fct:
  - $\log 1/x$ from small *x* limit of splitting fct
  - $\log t/t_0$ from *t* integration
  - strong ordering in *x* from small *x* limit
  - strong ordering in *t* from small *t* limit of ME...
- DLL gives rapid increase of gluon density from a flat starting distribution
- gluons are coupled to F_2... strong rise of F_2 at small x:

$$xg(x,t) \sim C \exp\left(2\sqrt{\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}}\right)$$



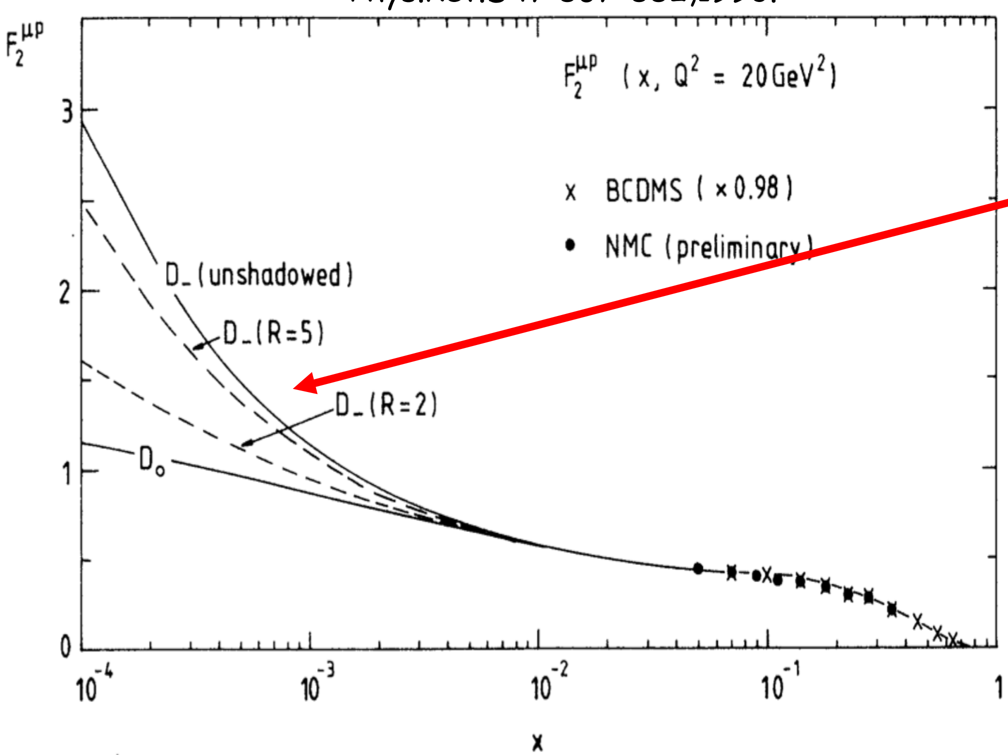


- consequences:
- rise continues forever ???
- what happens when too high gluon density ?
-

Remember the pre-HERA times

- Just before HERA started in 1992, new PDF fits (NLO DGLAP) were released, using all existing high precision data

Martin, Stirling, Roberts Apr 1992.
Phys.Rev.D47:867-882,1993.



- 1st HERA data 1992

H1 Nucl. Phys. B407 (1993) 515