MBL, Topology, and DMRG-X

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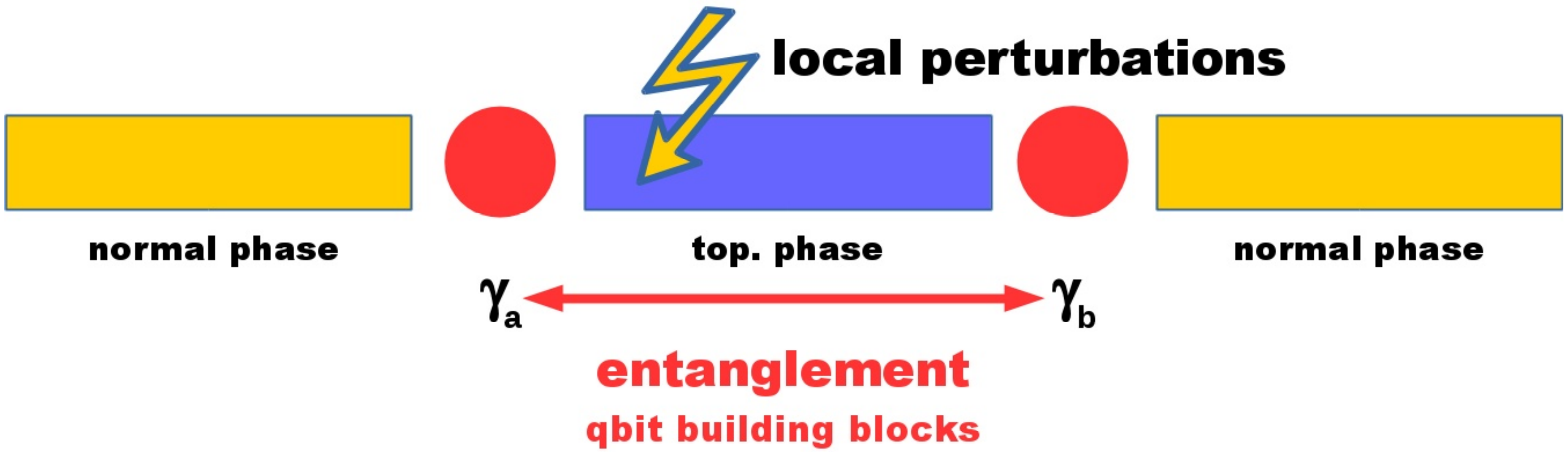
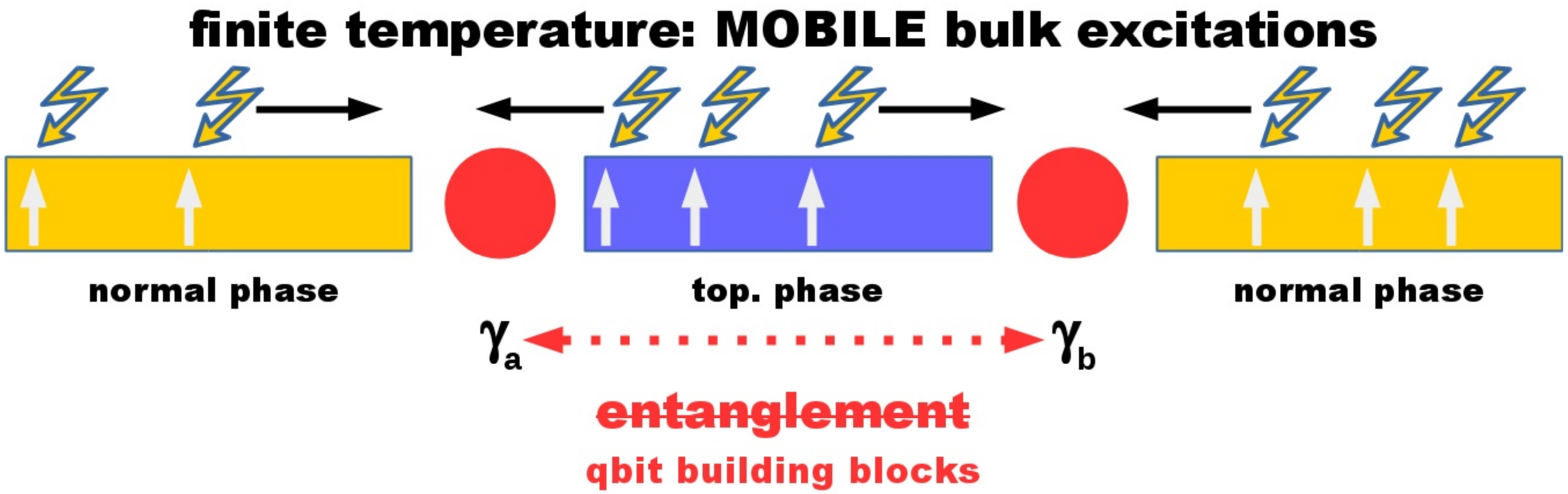
CRC183
ENTANGLED STATES OF MATTER

Topology & Disorder



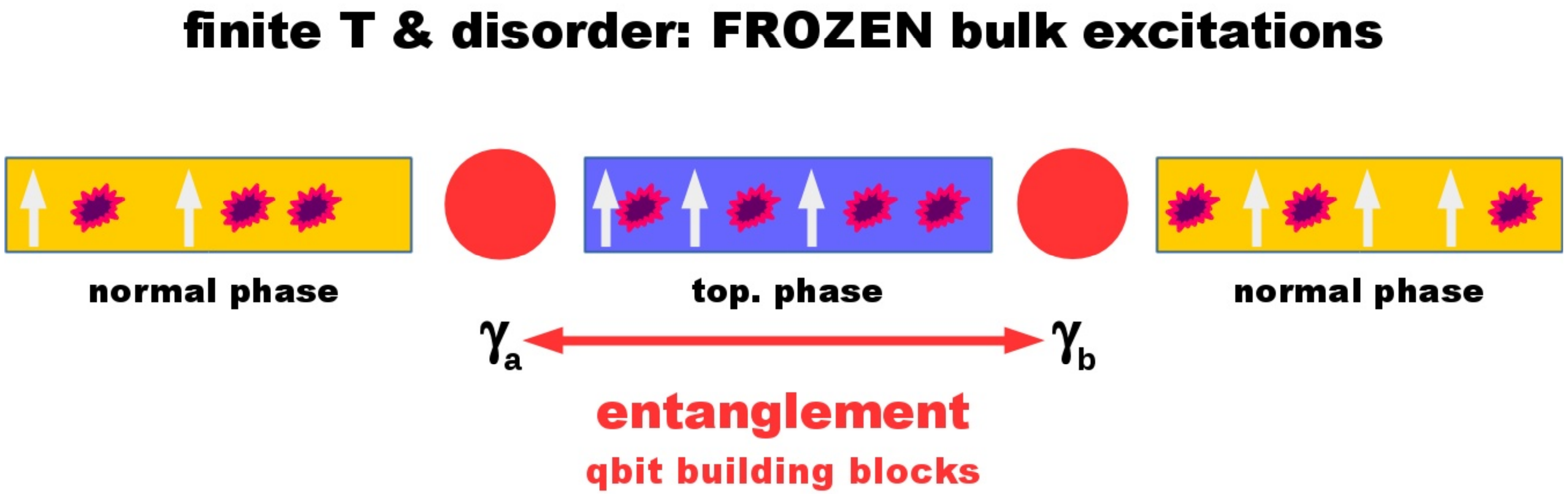
protection by topology!

Topology & Disorder



loss of coherence!

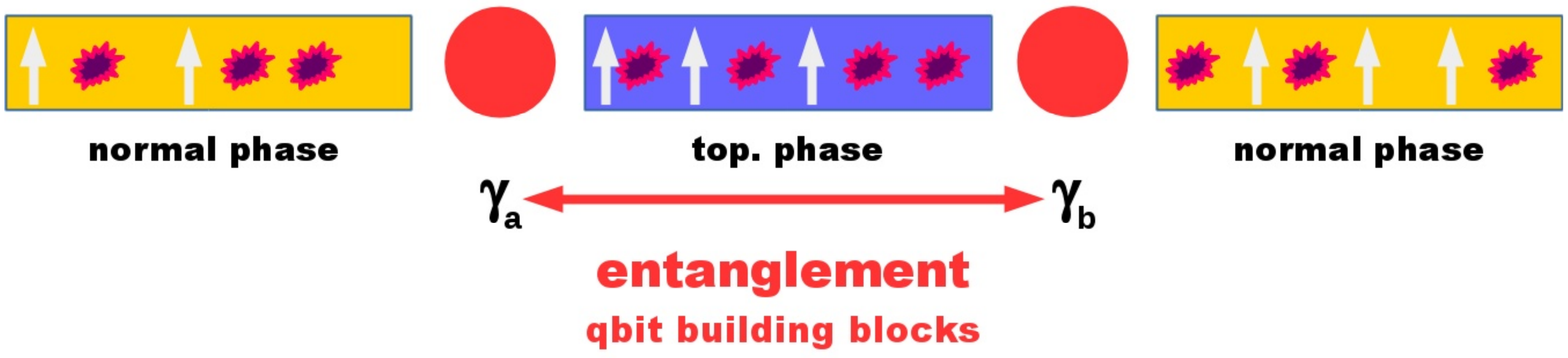
Topology & Disorder



protection by MBL!

Topology & Disorder

finite T & disorder: FROZEN bulk excitations



Many-body localization in a disordered quantum Ising chain

Jonas A. Kjäll,[1] Jens H. Bardarson,[1] and Frank Pollmann[1]

Localization protected quantum order

David A. Huse,[1,2] Rahul Nandkishore,[1] Vadim Oganesyan,[3,4] Arijeet Pal,[5] and S. L. Sondhi[2]

Localization and topology protected quantum coherence at the edge of hot matter

Yasaman Bahri[1], Ronen Vosk[2], Ehud Altman[1,2] & Ashvin Vishwanath[1]

starting point: 1d system that has a ground state topological phase

goal: compute phase diagram for

- finite energy
- disorder
- interactions

using the 'gold standard' (DMRG)

Kitaev chain $\Rightarrow$ failure $\Rightarrow$ toy model used in prior papers

The Toy Model

- $H = \sum_i \left(\lambda_i \sigma_{i-1}^z \sigma_i^x \sigma_{i+1}^z + h_i \sigma_i^x + V_i \sigma_i^x \sigma_{i+1}^x\right)$
- random couplings drawn from normal distribution $\sigma_\lambda = 1$, σ_h, σ_V

start with simple limit:

- $h_i = V_i = 0 \Rightarrow H = \sum_i \lambda_i \underbrace{\sigma_{i-1}^z \sigma_i^x \sigma_{i+1}^z}_{O_i}$ with $[O_i, O_j] = 0$
- all eigenstates are MPS with bond dimension 2 $\Rightarrow$ localized
- OBC: edge spins $\Sigma_L^x = \sigma_1^x \sigma_2^z$, $\Sigma_L^y = \sigma_1^y \sigma_2^z$, $\Sigma_L^z = \sigma_1^z$ $\Rightarrow$ topological

use DMRG-X to determine phase diagram at $h_i, V_i > 0$!?

The Method: DMRG-X

- MBL: excited states have low entanglement
- How to find their MPS representation?

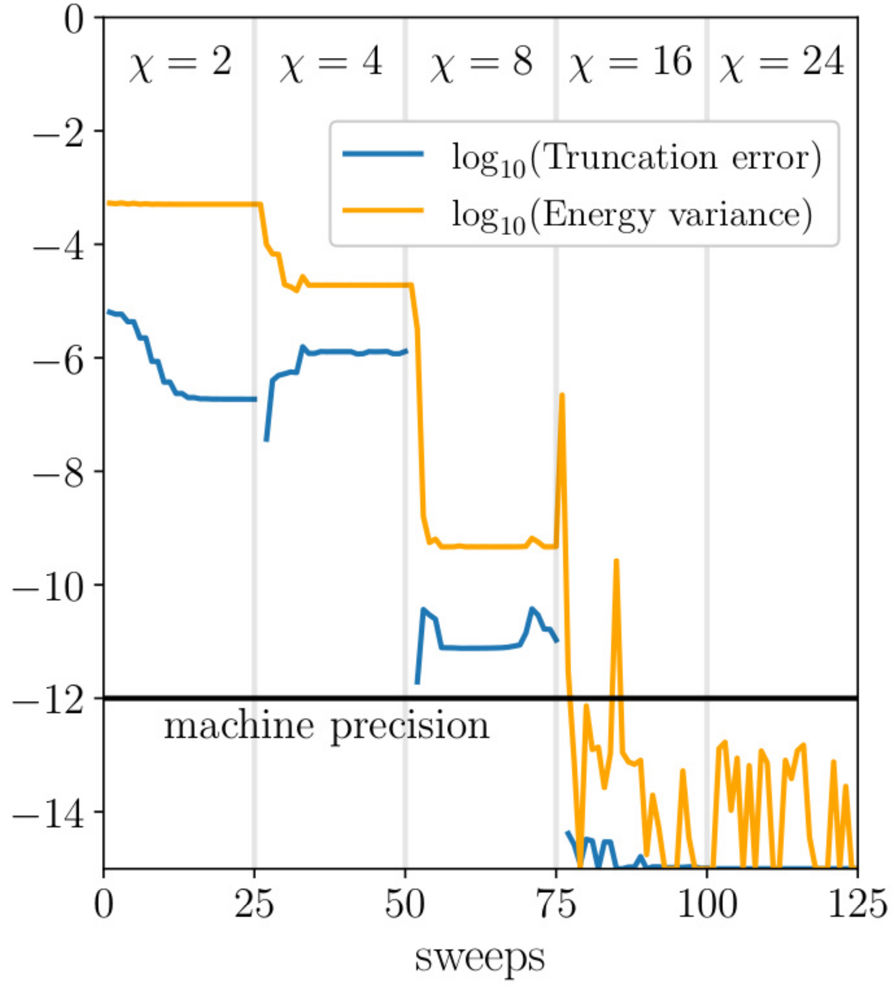
  (Khemani et al.'16, Lim/Sheng'16, Kennes&CK'16, Yu et al.'17)
- GS DMRG: take MPS, sweep, update matrices to minimize energy

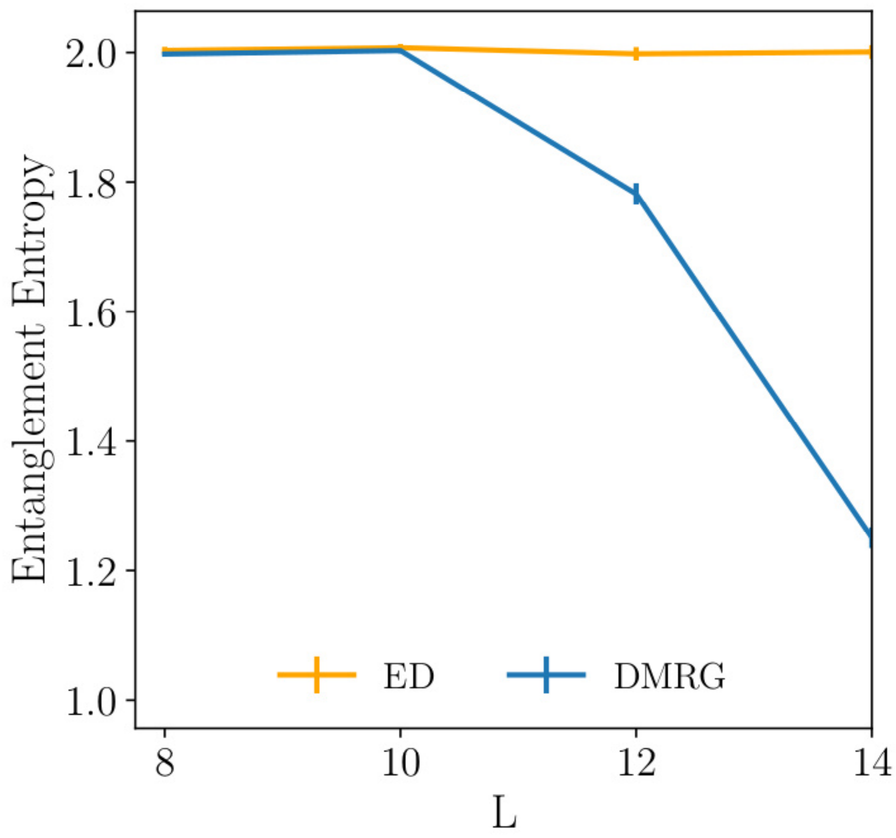
DMRG-X approach:

- XXZ chain $H = \sum_i h_i \sigma_i^z + \text{pert.} = H_0 + \text{pert.}$
- start from random eigenstate of H_0: $|\uparrow\downarrow\downarrow\downarrow\uparrow\cdots\rangle$
- states close in energy differ vastly in their spatial structure!
- sweep, update MPS, pick state with max overlap to previous state
- here: start from eigenstate of $H_0 = \sum_i \lambda_i \sigma_{i-1}^z \sigma_i^x \sigma_{i+1}^z$ (bond dim 2)

The Method: DMRG-X

- can converge into excited eigenstate for large $L = 50$!
- compute physical quantities: behavior unexpected
- comparison with ED for small L: something is wrong

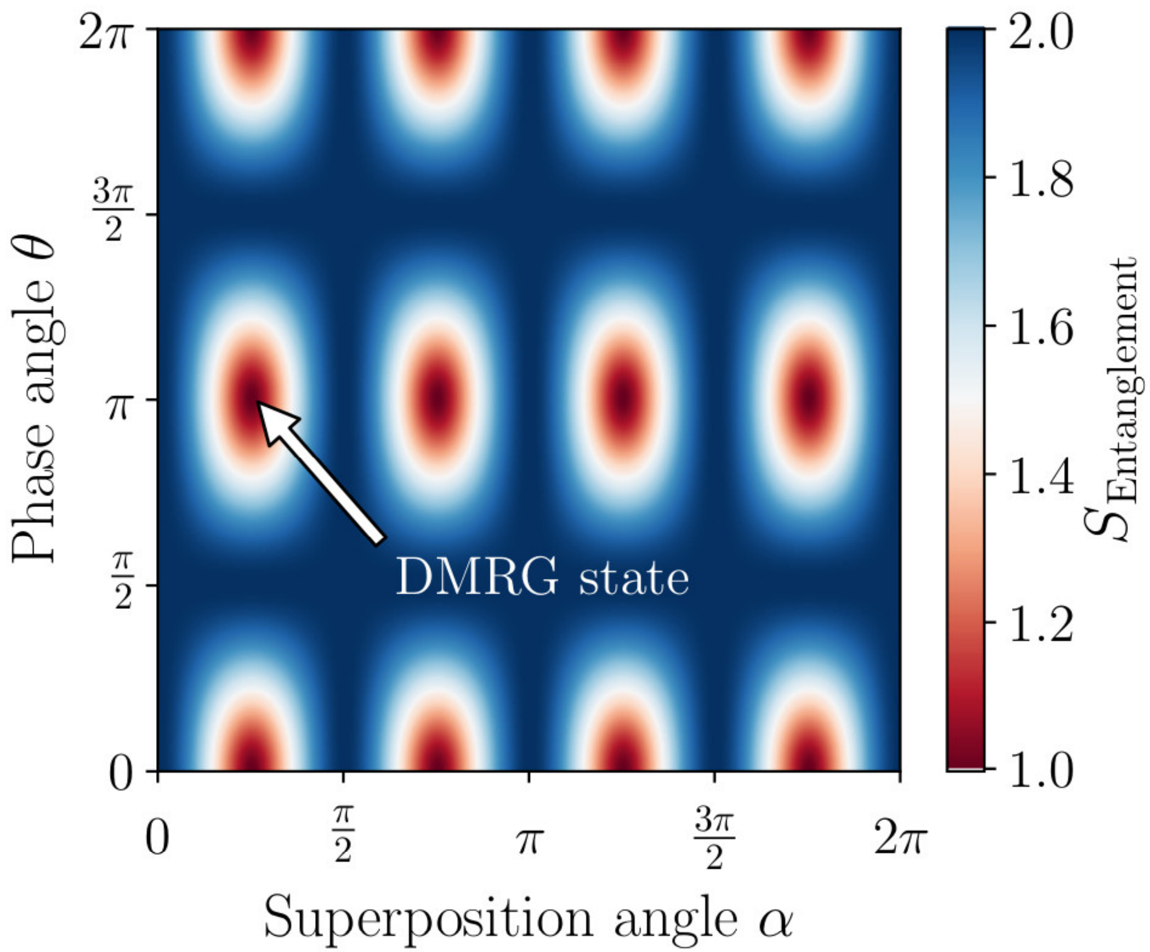




$\sigma_h = 0.05, \sigma_V = 0$

The Method: DMRG-X

- compute overlap of DMRG state has with all ED states
- equal overlap with two ED states of almost same energy: edges.
- DMRG minimizes entanglement. duh.

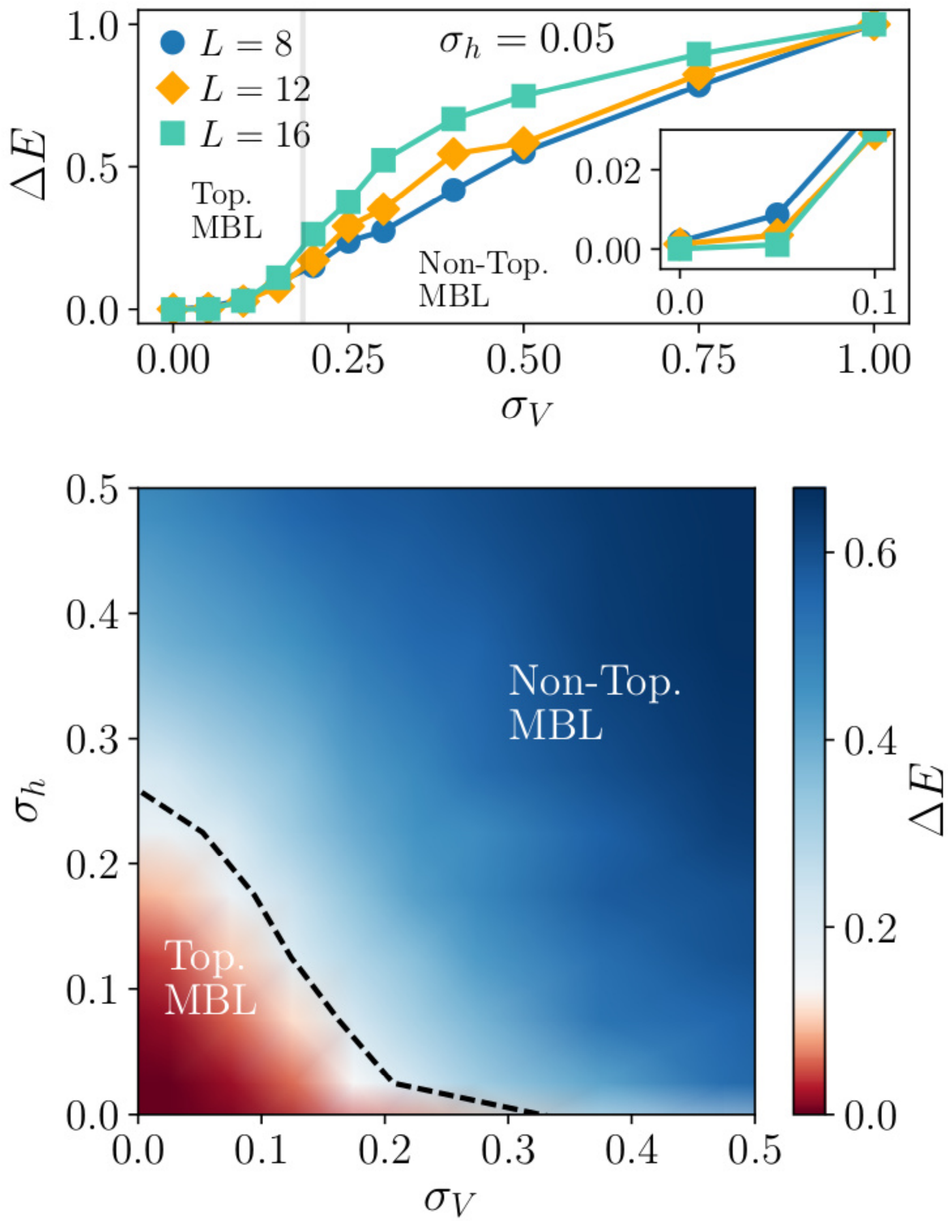


DMRG-X not suited. study problem with ED.

Detecting Topology

- use ED to compute spectrum
- OBC: each eigenstate four-fold degenerate in TD limit
- introduce measure ΔE
- mid-spectrum states
- trivial insulator for $h_i, V_i \to \infty$ (classical Ising chain)

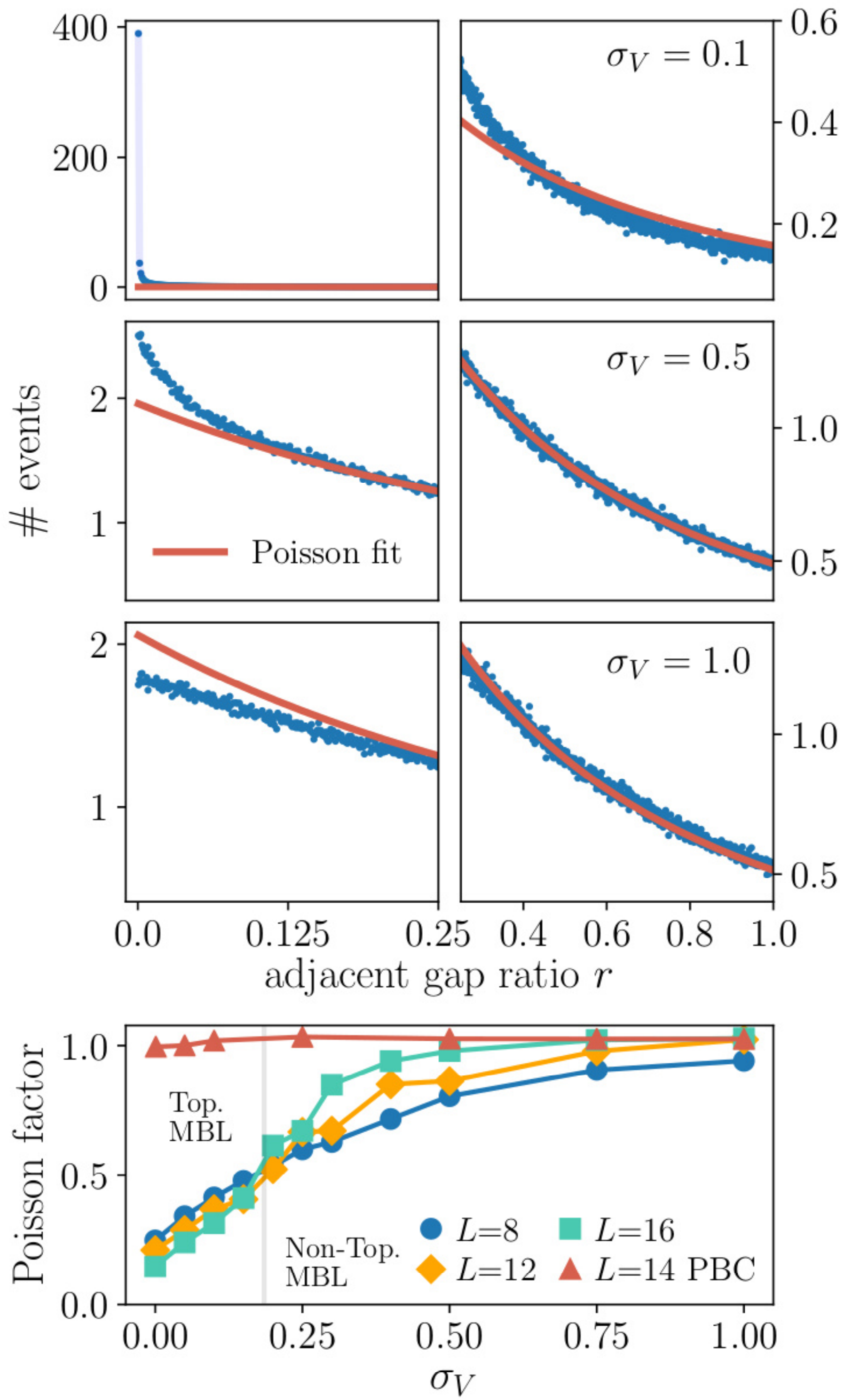
$\Rightarrow$ topological phase stable



Detecting MBL

- compute adjacent gap ratio
- localized regime: Poissonian form
- true for periodic BC
- open BC:
  zero-energy peak + Poissonian

⇒ always localized?!



Localization Length

- scaling of entanglement entropy

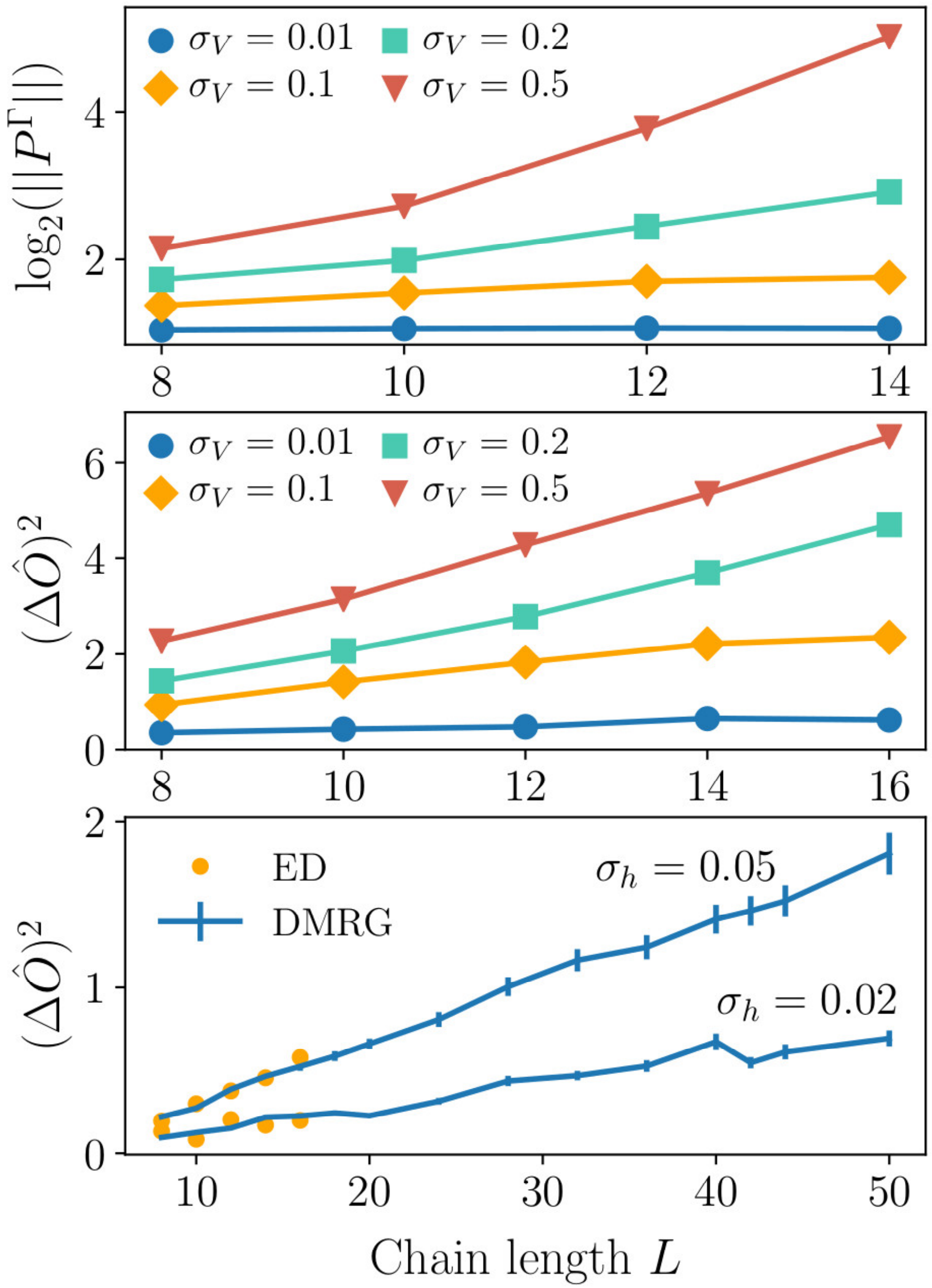
$$S \sim \begin{cases} \text{vol} & \text{ergodic} \\ \text{area} & \text{localized}, L > L_{\text{loc}} \end{cases}$$

- same: bi-partite spin fluctuations

- problem: degenerate spectrum!

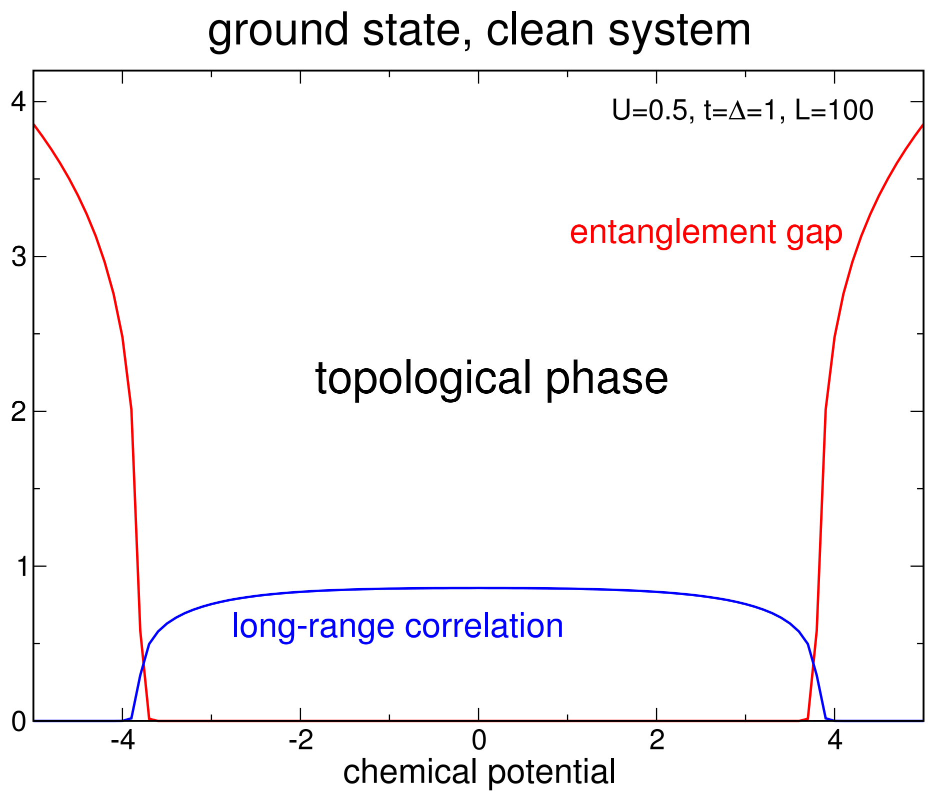
  $\Rightarrow$ use entanglement negativity

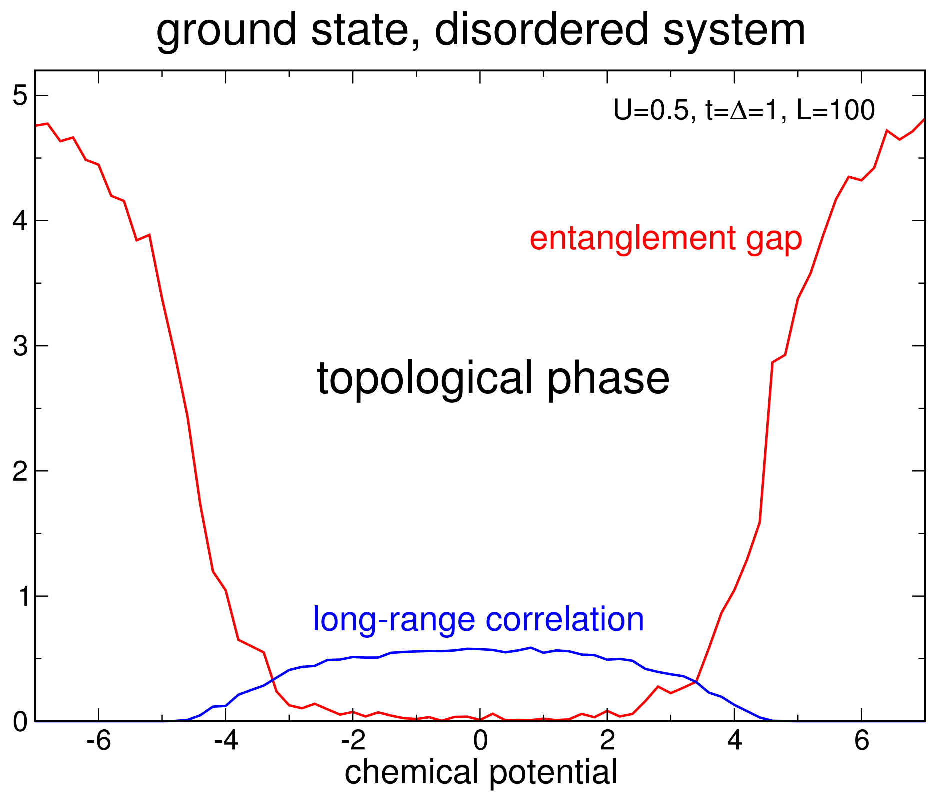
data inconclusive



Kitaev: Ground State

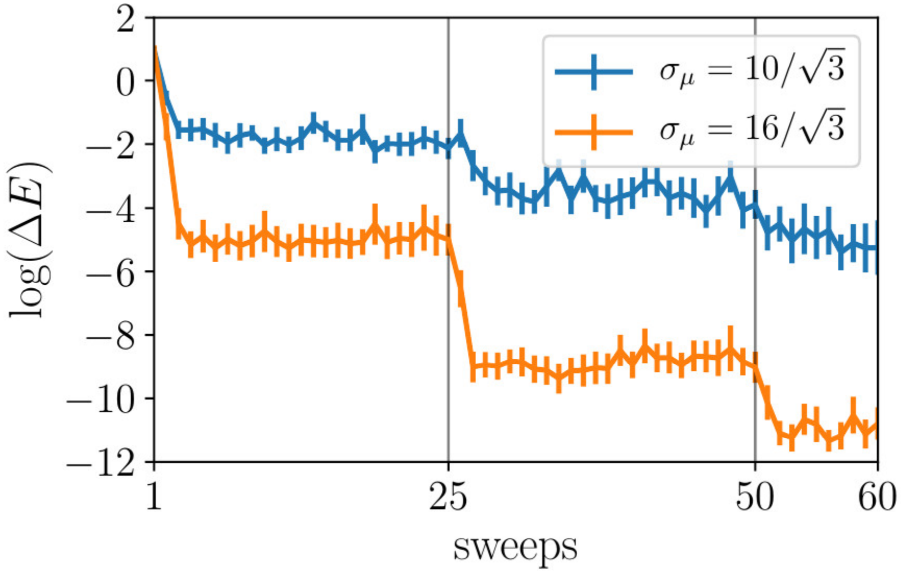
- Kitaev chain $H = \sum_i -t\sigma_i^x\sigma_{i+1}^x + U\sigma_i^z\sigma_{i+1}^z - \frac{1}{2}\mu_i\sigma_i^z$
- topological if $|\mu| < 2t$ for $U = 0$ without disorder *(Gergs et al.'16)*
- use variational DMRG to find phase diagram; top. stable for moderate disorder



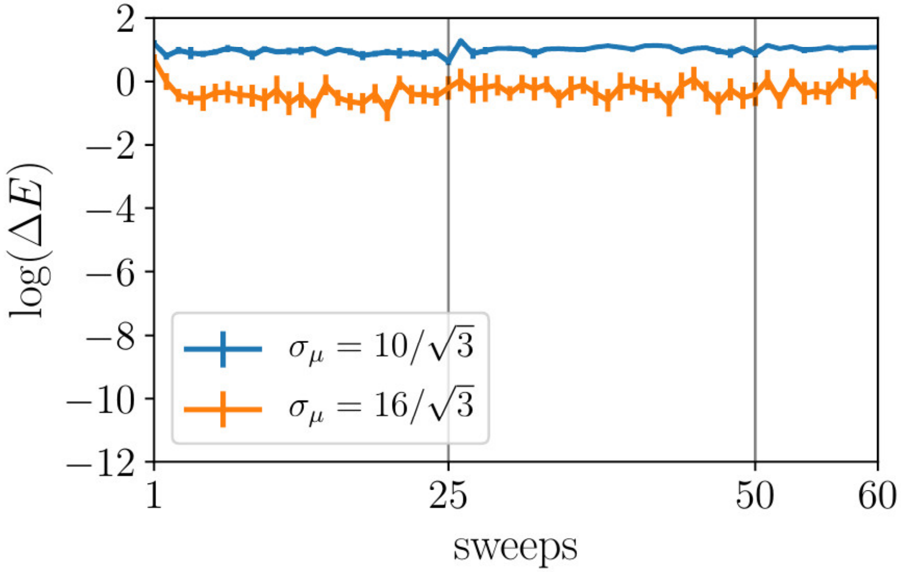


MBL + topology: what about excited states?

Kitaev: DMRG-X



(a) $\Delta = t$, $U = 0$



(c) $\Delta = t$, $U = 0.5t$

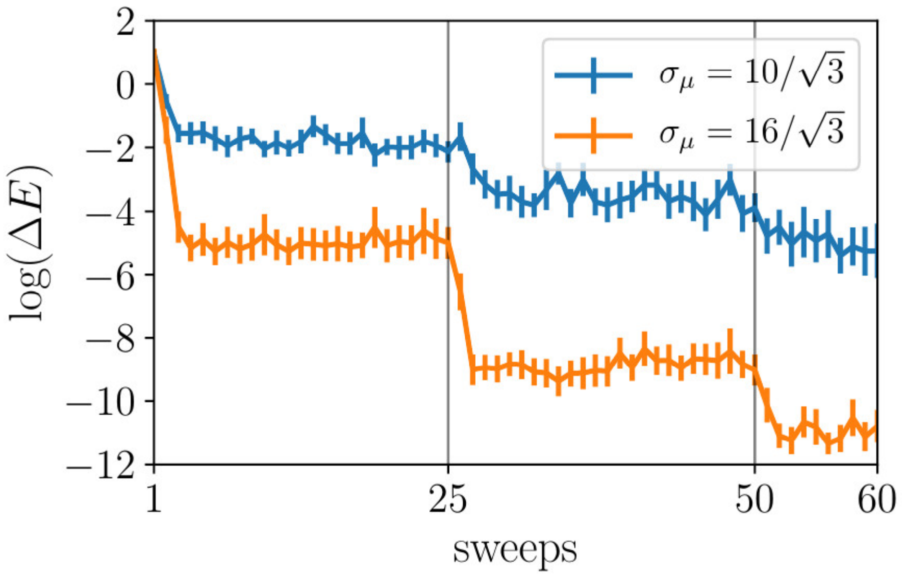
disorder:

small enough for topology

large enough for MBL

$L = 24$

Kitaev: DMRG-X   no intermediate regime!



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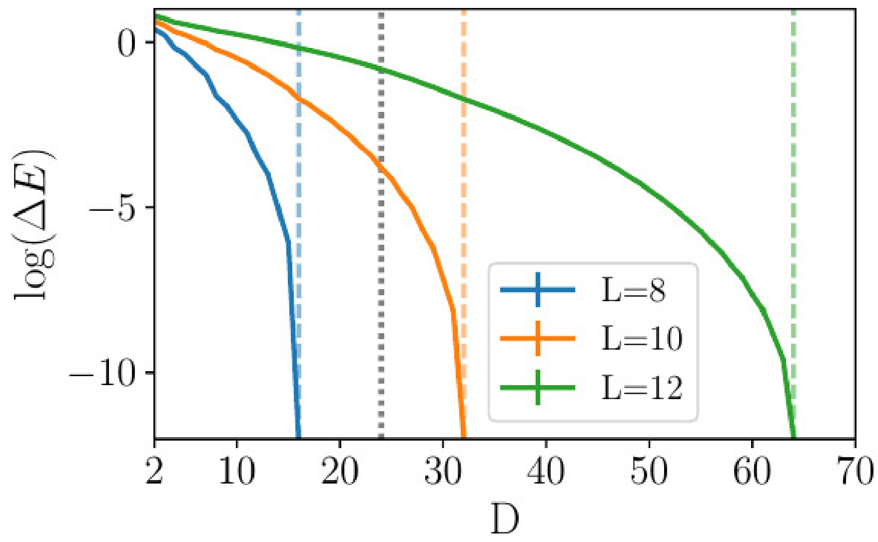
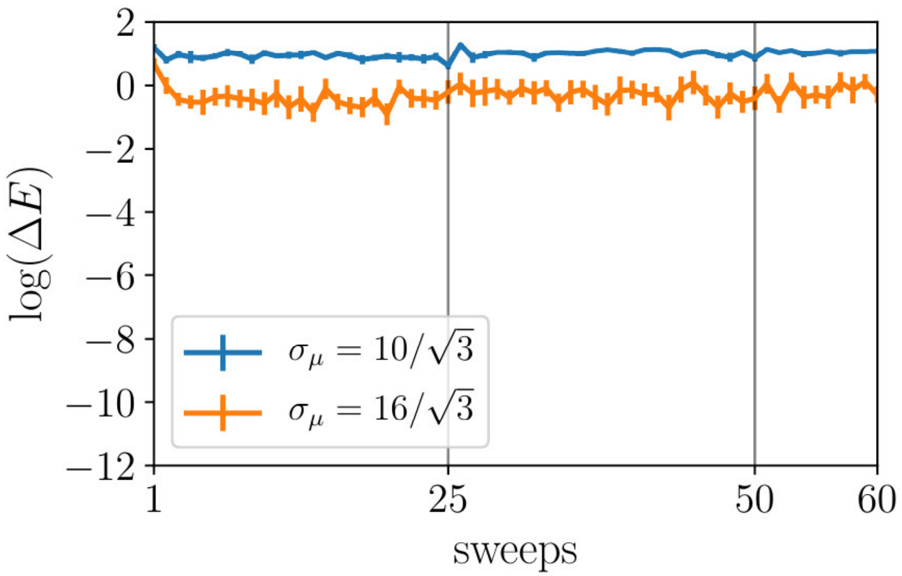
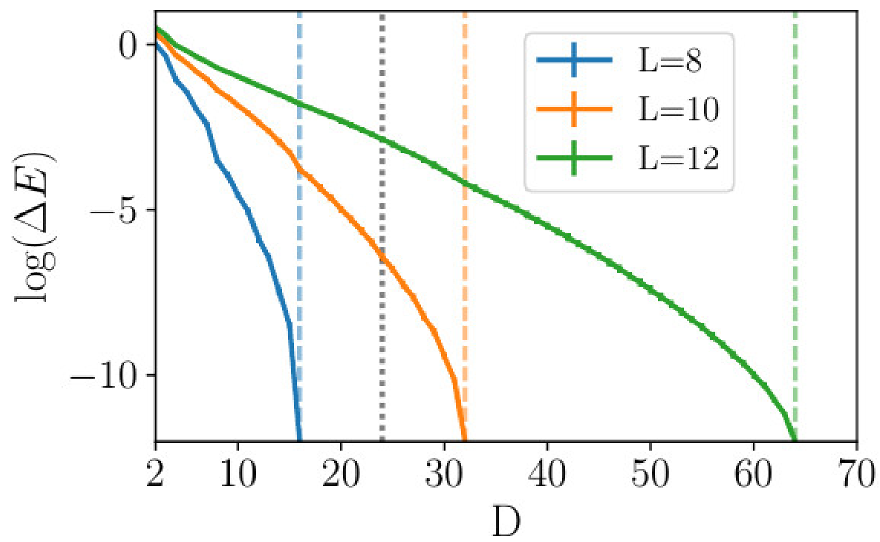


(a) $U = 0.5t$, $W = \frac{3}{\sqrt{3}}$

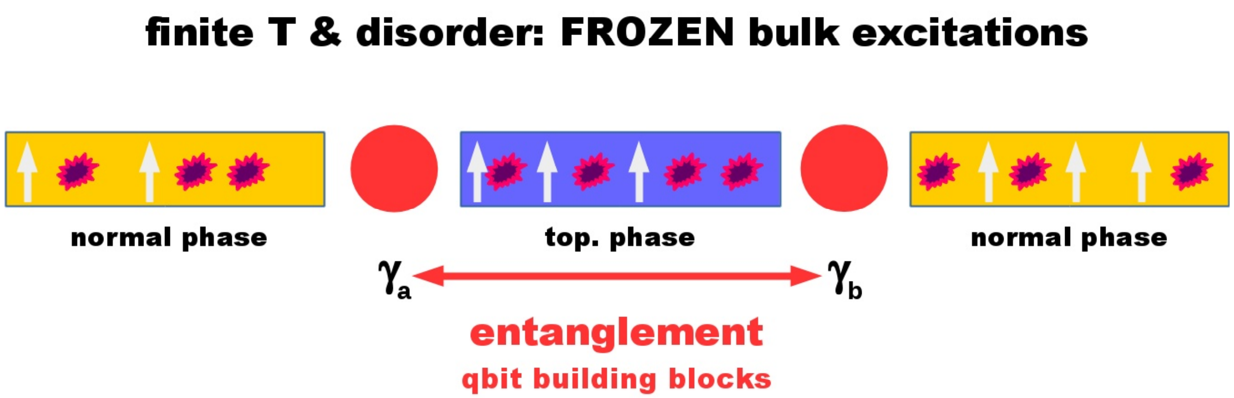


(c) $\Delta = t$, $U = 0.5t$

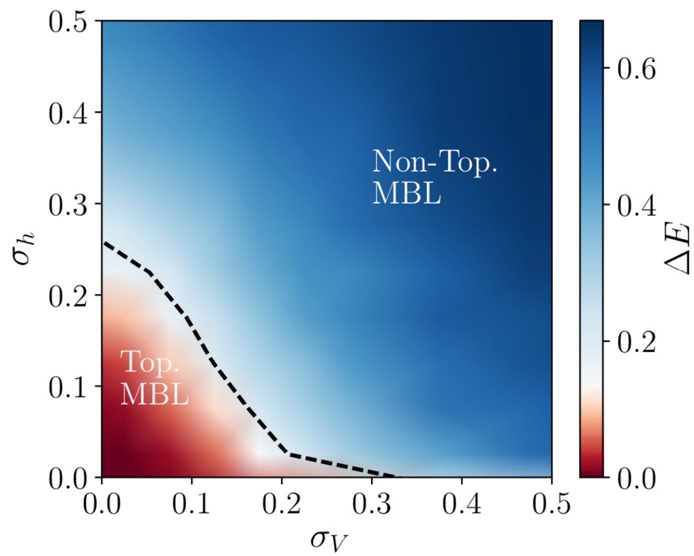


(b) $U = 0.5t$, $W = \frac{12}{\sqrt{3}}$

Topology & Disorder



- Kitaev: no "intermediate regime" found
- study toy model
- DMRG-X not suited for degenerate spectra. symmetries?!
- ED phase diagram



DMRG

(CK+..., PRB'16)

- MBL: ex. states have low entanglement
- How to find their MPS representation?

  (Pollmann et al.'16, Yu et al.'17, Lim/Sheng'16)
- most NAIVE approach: GS of $(H - E)^2$