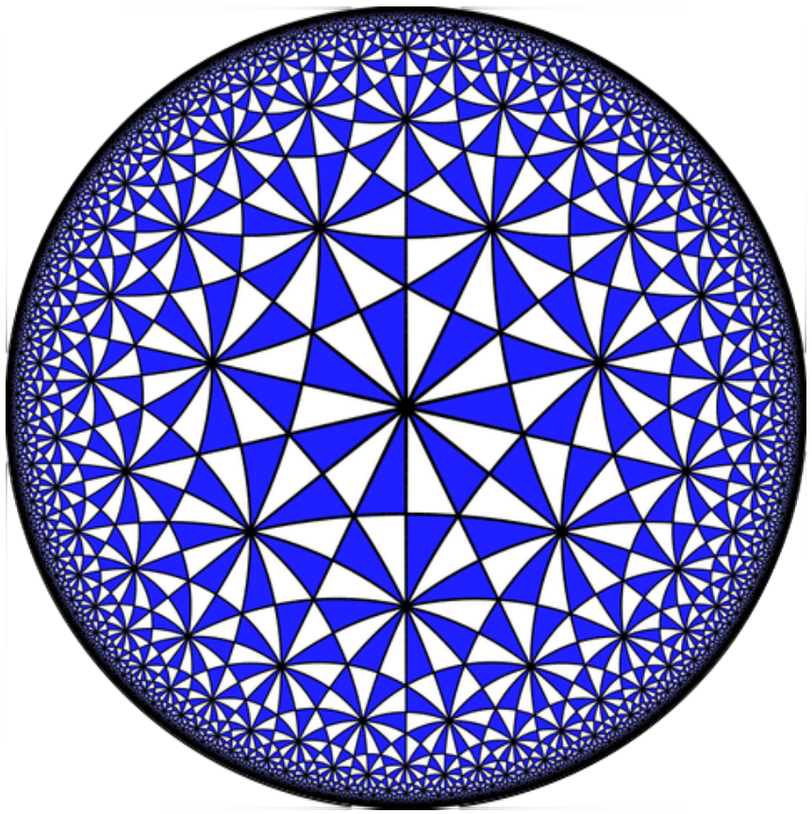
HOLOGRAPHY AND CRITICALITY IN MATCHGATE TENSOR NETWORKS

and other recent results on tensor networks



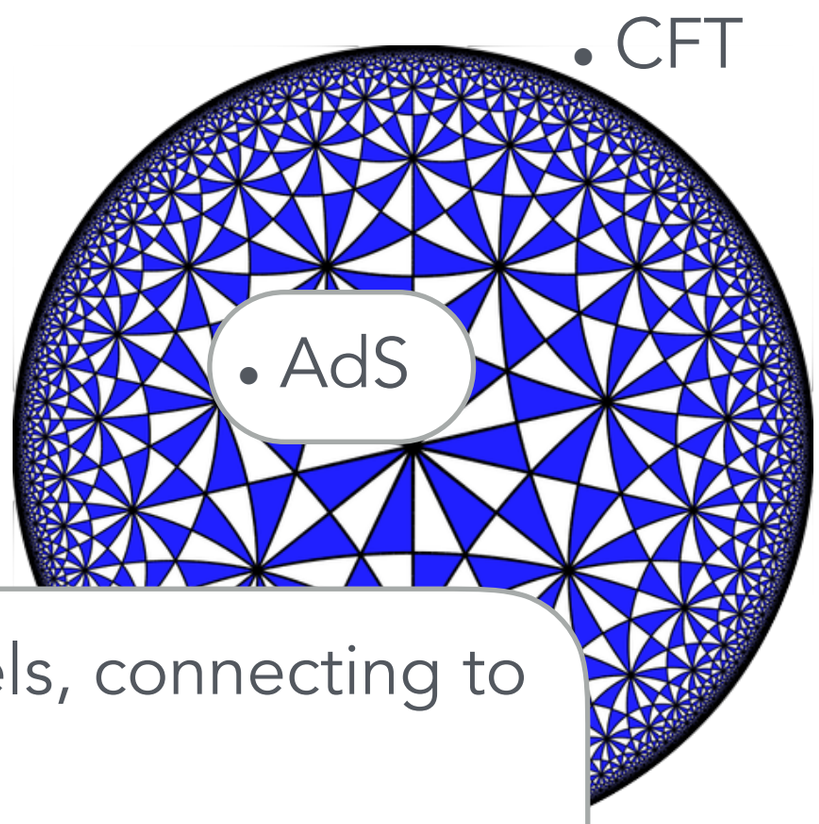
Jens Eisert, Freie Universität Berlin

Joint work with Alexander Jahn, Marek Gluza, Fernando Pastawski

arXiv:1711.03109 and arXiv:1903.xxxxx

ADS-CFT CORRESPONDENCE

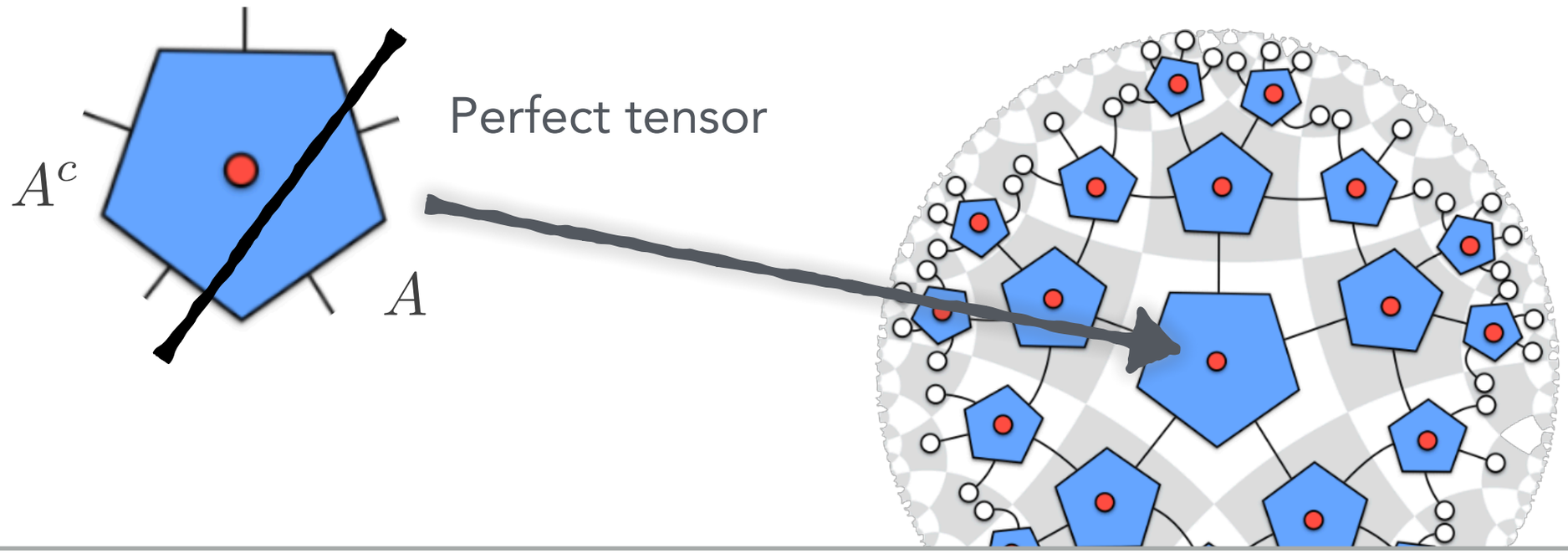
- Duality between Einstein gravity in $D+2$ Anti de Sitter spacetime and
- conformal field theory in $D+1$ dimensions



- **Tensor-network** based toy models, connecting to
  - condensed matter
  - quantum information

Maldacena, Avd Th Math Phys 2, 231 (1998)
Witten, Adv Theor Math Phys 2, 253 (1998)
van Raamsdonk, Gen Rel Grav 42, 2323 (2010)

- **Quantum error correction:** Holographic pentagon code



- Perfect tensor: Any bi-partite cut with $|A| \leq |A^c|$ is proportional to isometry

- $[[2n-1, 1, n]]$ quantum error correcting code
- Here, $2n-1=5$, "Pentagon code"

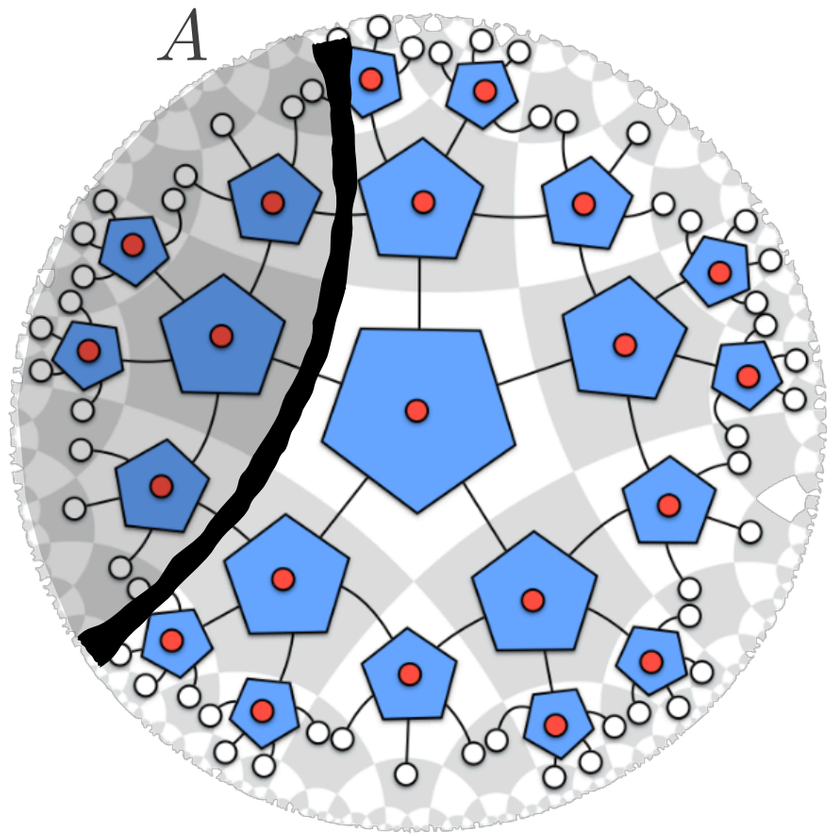
- Holographic state: Product state fed into bulk

Pastawski, Yoshida, Harlow, Preskill, JHEP 2015, 149 (2015)
Helwig, Cui, Latorre, Riera, Lo, Phys Rev A 86, 052335 (2012)

- Entanglement entropy of a connected region of a boundary satisfies

$$S_A = |\gamma_A|$$

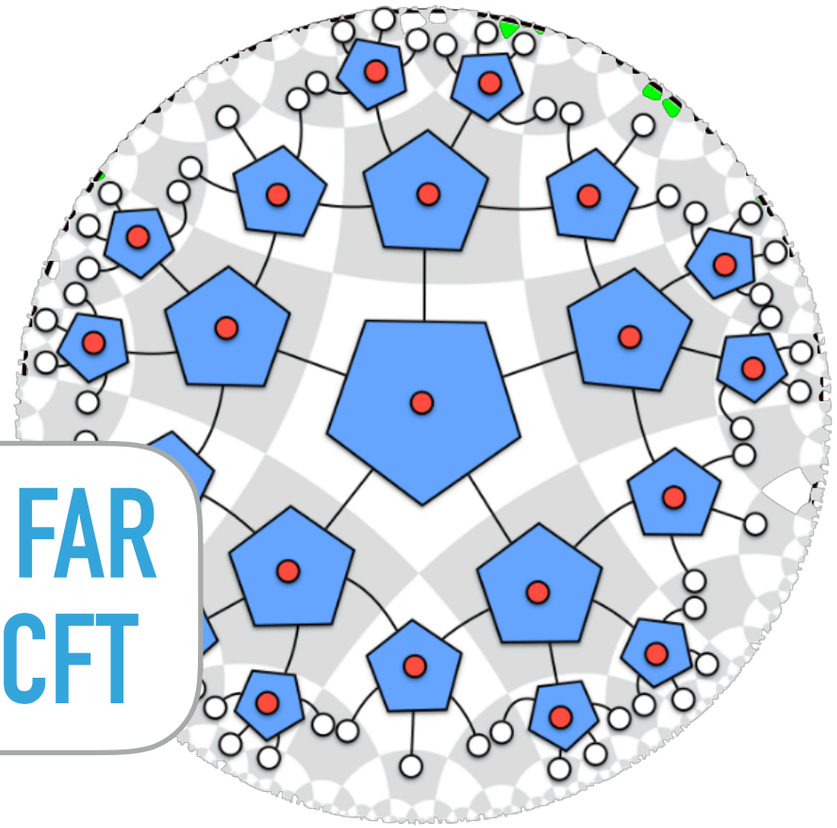
- γ_A minimal bulk geodesic
- Lattice version of Ryu-Takayanagi formula



Pastawski, Yoshida, Harlow, Preskill, JHEP 2015, 149 (2015)
Ryu, Takayanagi, Phys Rev Lett 96, 181602 (2006)

“MODEL 1”: PENTAGON CODES

- Connection of **AdS-cft** to **holographic quantum error correction**

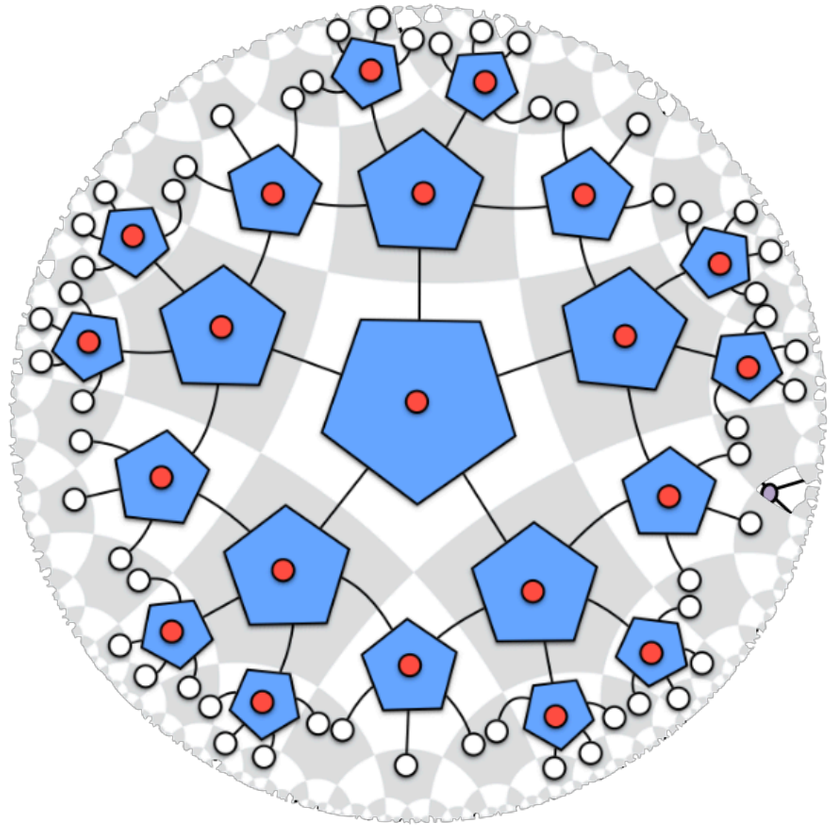


BUT, BOUNDARY STATE FAR FROM A REASONABLE CFT

Almheiri, Dong, Harlow, JHEP 1504, 163 (2015)
Harris, McMahon, Brennen, Stace, Phys Rev A 98, 052301 (2018)

“MODEL 2”: MULTISCALE ENTANGLEMENT RENORMALIZATION (MERA)

- Tensor network consisting of **isometries** and **disentanglers**
- Approximates critical quantum states

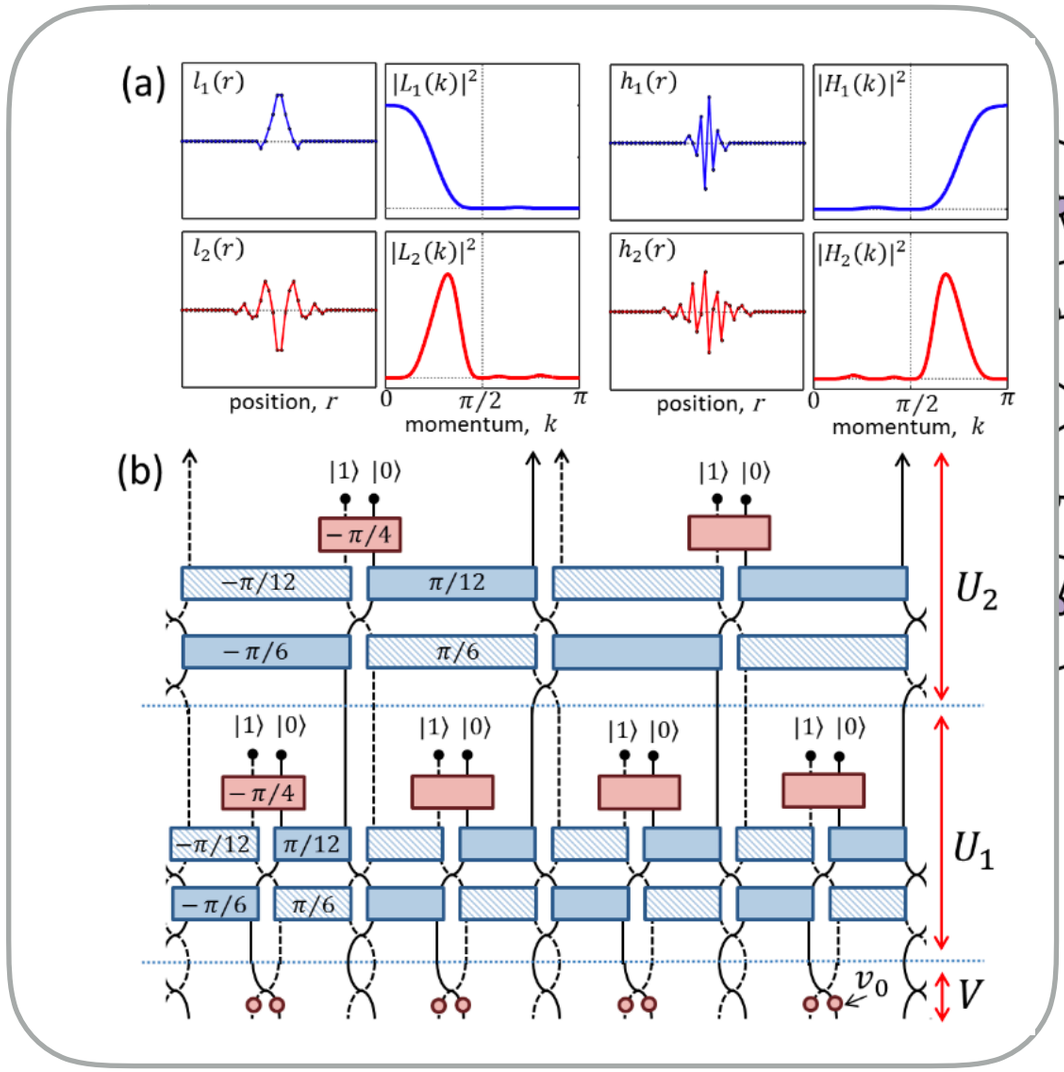


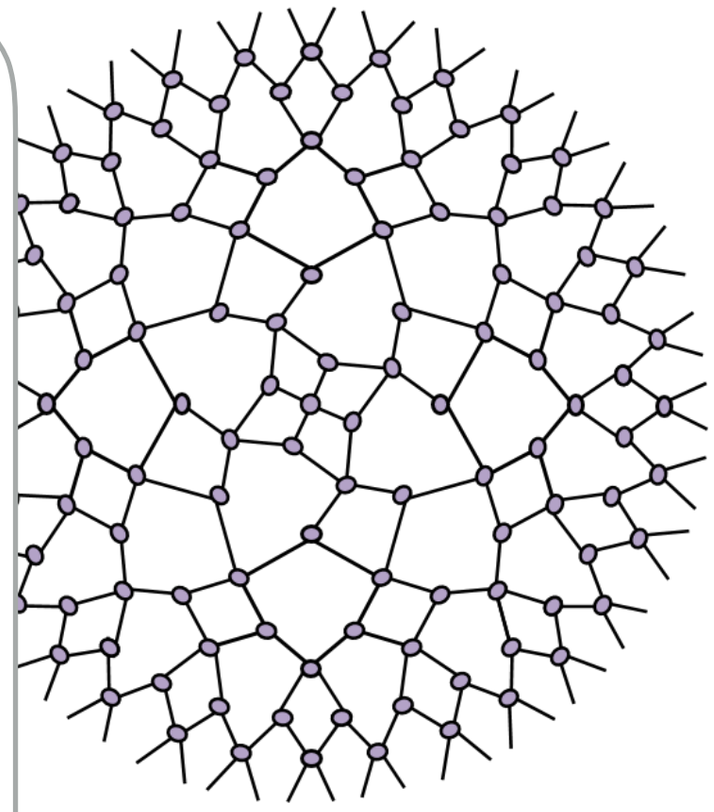
Vidal, Phys Rev Lett 101, 110501 (2008)
Evenbly, Vidal, Phys Rev B, 79, 144108 (2009)
Dawson, Eisert, Osborne, Phys Rev Lett 100, 130501 (2008)
Swingle, Phys Rev D 86, 065007 (2012)

“MODEL 2”: MULTISCALE ENTANGLEMENT RENORMALIZATION (MERA)

- Free fermionic MERA based on wavelets approximates Ising critical theory

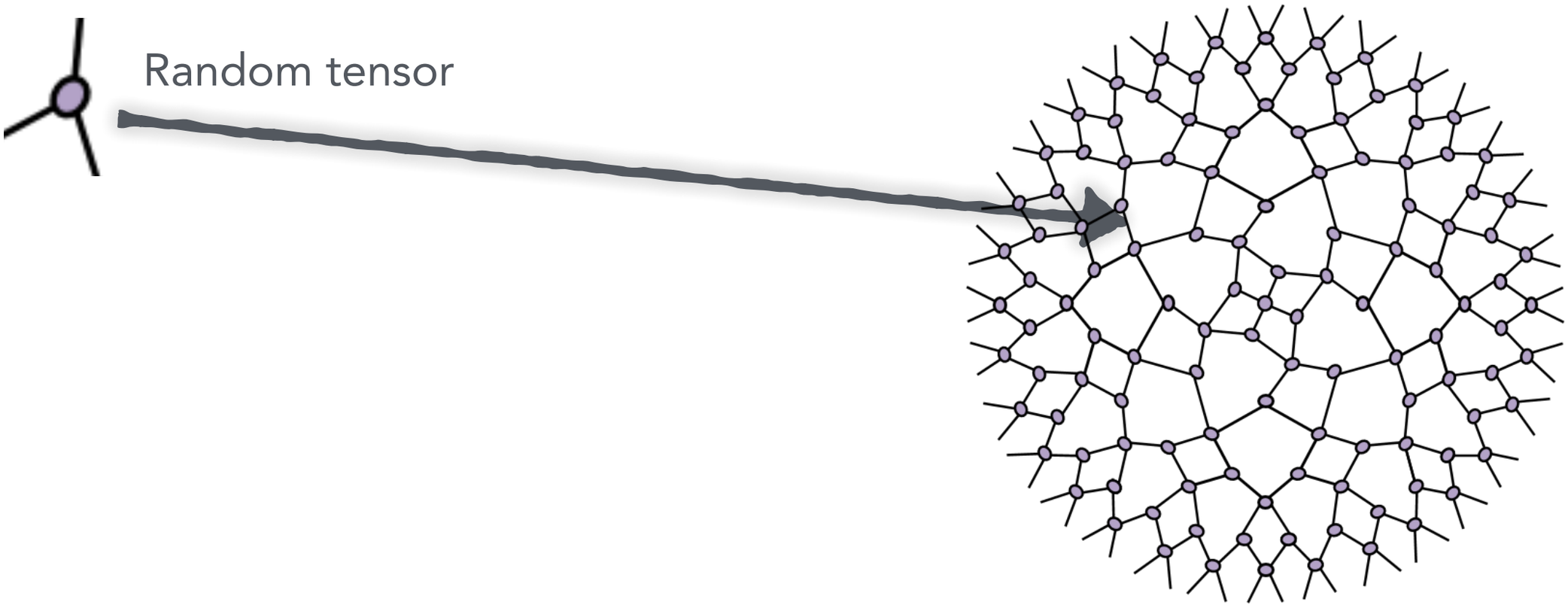
$$\hat{c}_j = \sum_{j,k} A_{j,k} c_k$$



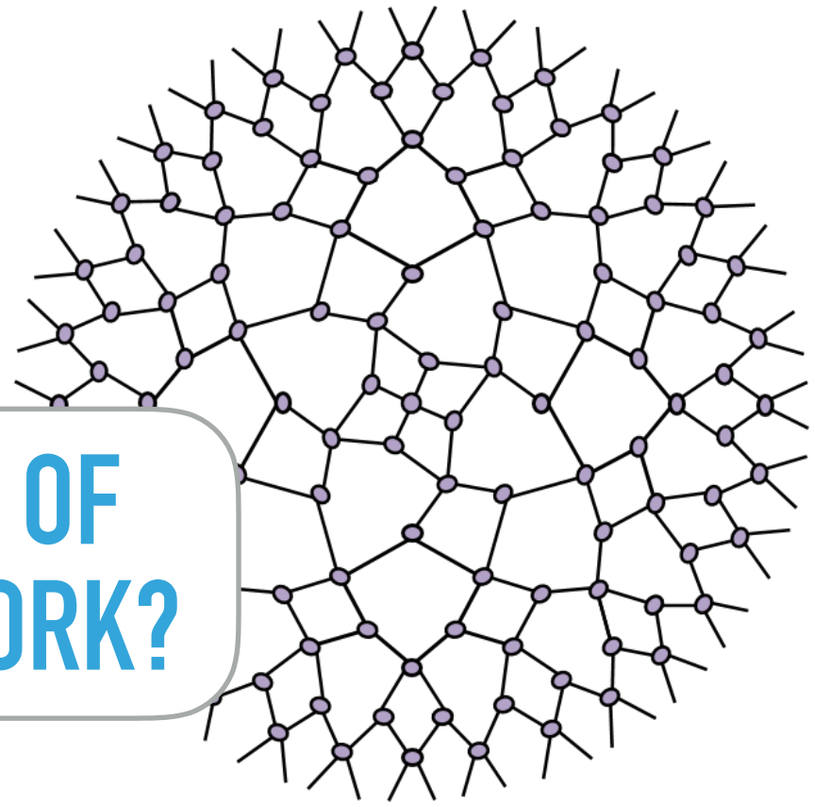


Evenbly, White, Phys Rev Lett 116, 140403 (2016)
Haegeman, Swingle, Walter, Cotler, Evenbly, Scholz, Phys Rev X 8, 011003 (2018)

- Random isometric tensors are with high probability close to being perfect



Hayden, Nezami, Qi, Thomas, Walter, Yang, JHEP 2016, 9 (2016)



CAN ONE EMBODY (ASPECTS) OF THEM IN A LARGER FRAMEWORK?

Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109
Jahn, Gluza, Pastawski, Eisert, in preparation (2019)

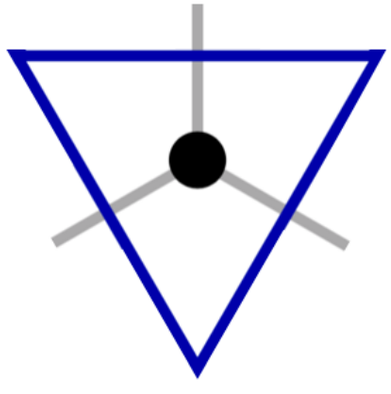
GETTING TO WORK: MATCHGATE TENSOR NETWORKS

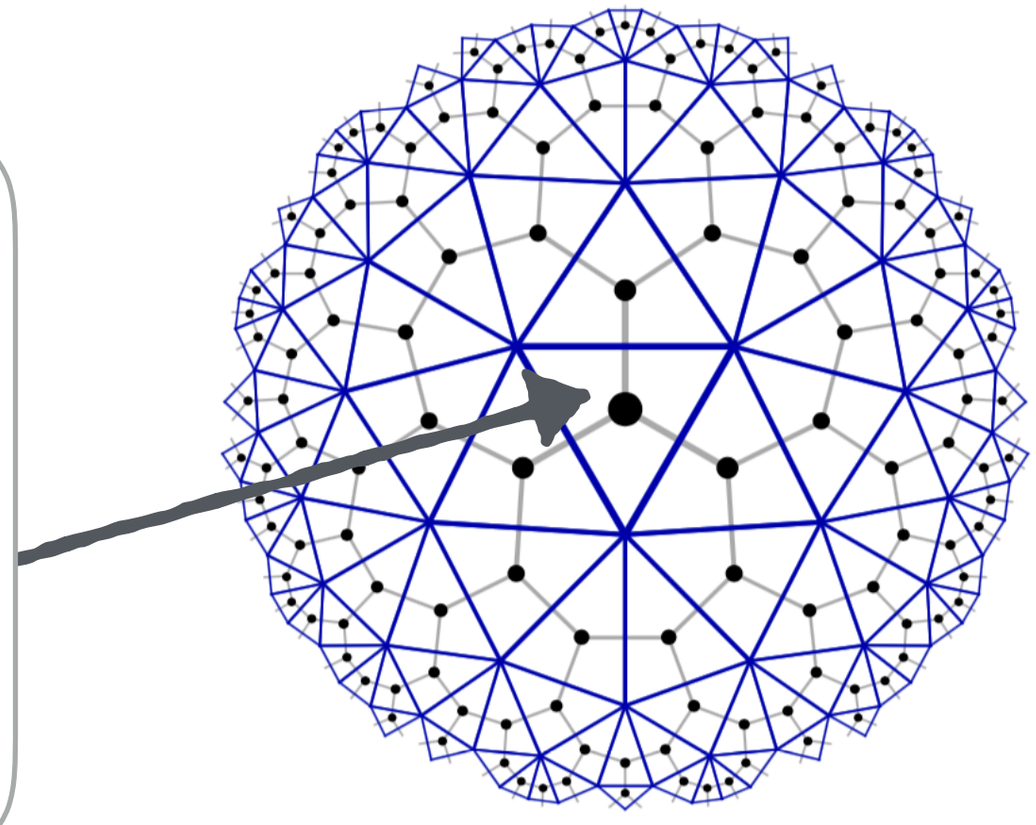
- Choose some some tiling of the plane

- Matchgate tensor

$$T_v : \{0,1\}^{\times r} \to \mathbb{C}$$

per vertex $v \in V$





- Boundary state obtained by tensor contraction $|\psi\rangle = \sum_{j \in \{0,1\}^{\times L}} \mathcal{T}(j)\ |j\rangle$

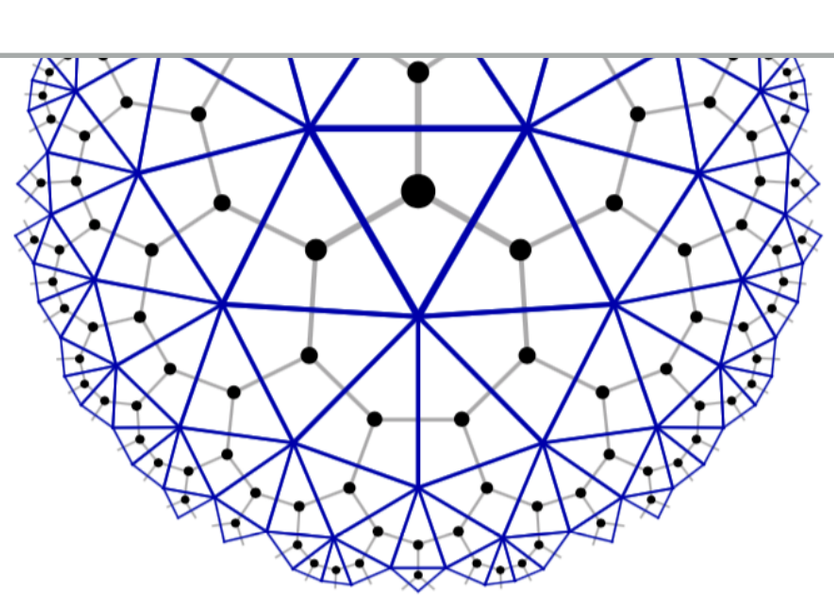
Bravyi, Cont Math 482, 179 (2009)
Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

- **Matchgate tensors:** Consider a rank-r tensor $T(x)$ with inputs $x \in \{0,1\}^{\times,r}$, $T(x)$ is a matchgate if there exists an antisymmetric matrix $A \in \mathbb{C}^{r\times r}$ and a reference index $z \in \{0,1\}^r$ such that

$$T(x) = \mathrm{Pf}(A_{|x\mathrm{XOR}z})T(z)$$

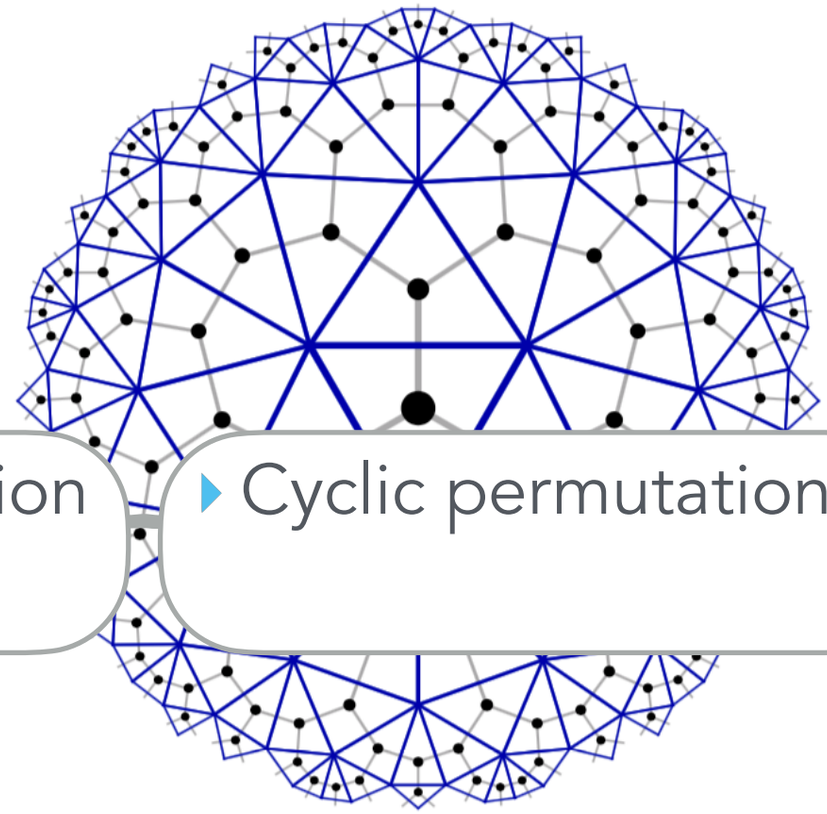
where $\mathrm{Pf}(A)$ is the Pfaffian of A and $A_{|x}$ is the submatrix of A acting on the subspace supported by x

Cai, Choudhary, Lu, CCC07, IEEE Conference (2007)



Bravyi, Cont Math 482, 179 (2009)
Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

- **Observation:** The contraction requires $O(L^2 N)$ steps for L boundary sites and N contracted tensors



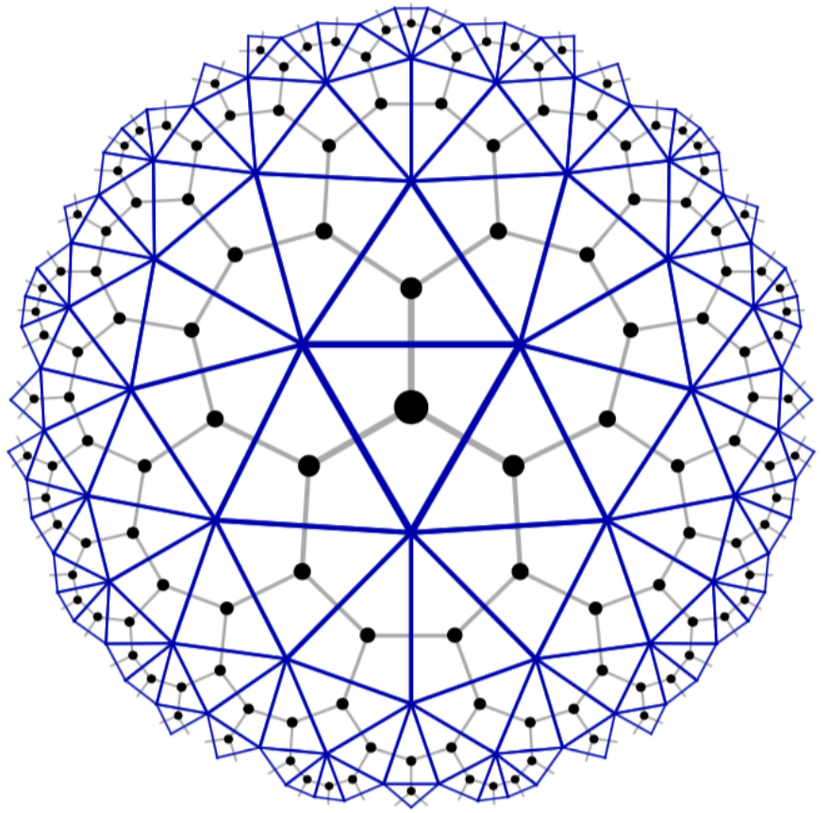
- Contraction of tensors with generating matrices A and B
- Self-contraction (tedious)
- Cyclic permutations
- Generic even matchgate with $\bar{z} = 0$ has

$$\Phi_T(\theta) = T(\bar{0}) \exp\left(\frac{1}{2} \sum_{j,k=1}^{r} A_{j,k}\theta_j\theta_k\right)$$

with generating matrix A

Bravyi, Cont Math 482, 179 (2009)
Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

Place generating matrices on some tiling



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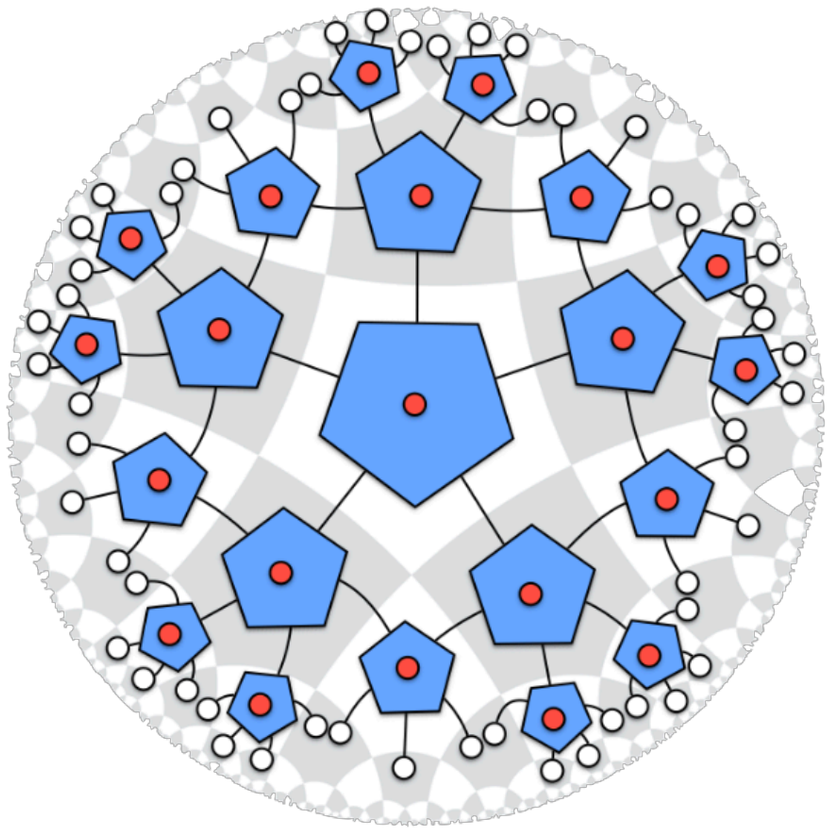
with generating matrix A

Bravyi, Cont Math 482, 179 (2009)
Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

LET US PLAY

RELATIONSHIP TO PENTAGON CODES

- **Observation:** The holographic pentagon code with computational basis input in the bulk yields a matchgate tensor network



- Gives rise to stabilizer code $\langle S_j \rangle_{j=1}^{5}$, e.g.,

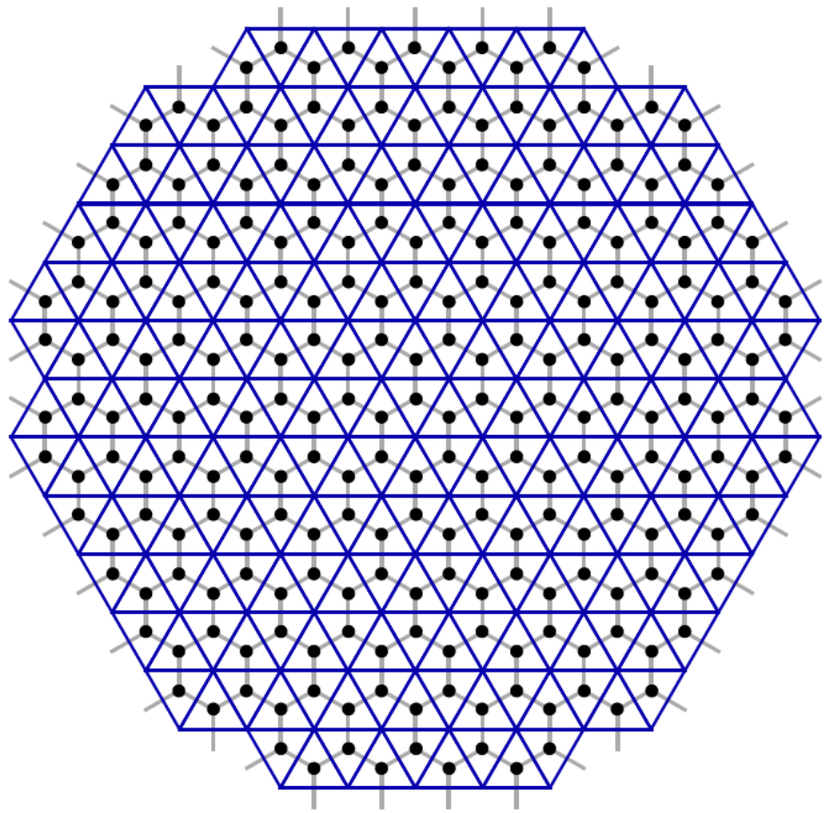
$$S_1 = \sigma^x \otimes \sigma^z \otimes \sigma^z \otimes \sigma^x \otimes \mathbb{1}_2 = \mathrm{i} m_7 m_2$$
$$S_2 = \mathbb{1}_2 \otimes \sigma^x \otimes \sigma^z \otimes \sigma^z \otimes \sigma^x = \mathrm{i} m_9 m_4$$

expressed in Majoranas

Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

- $\{3, 6\}$ flat tiling

Schläfli symbol

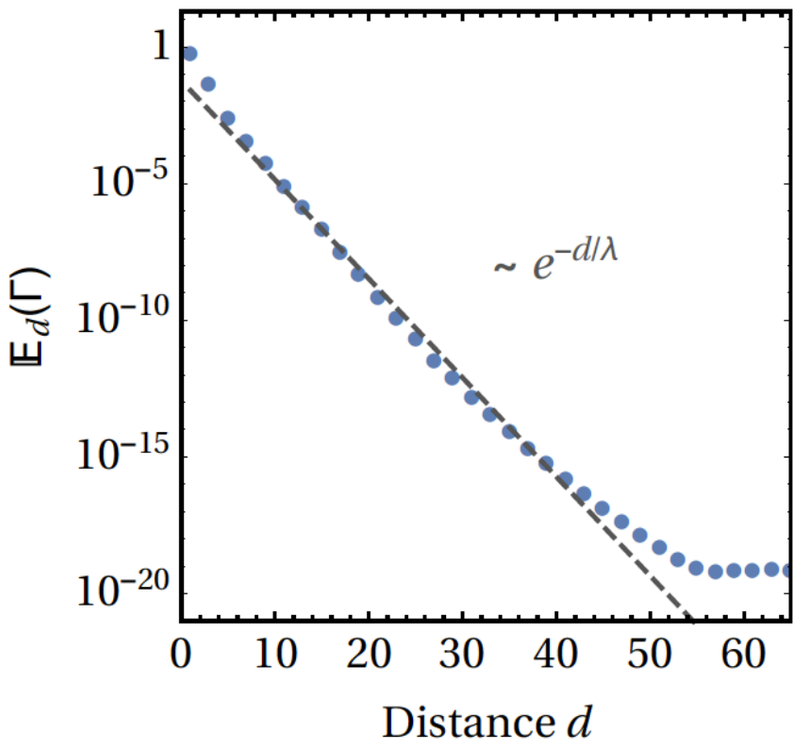


- Anti-symmetric matrix A, single parameter a

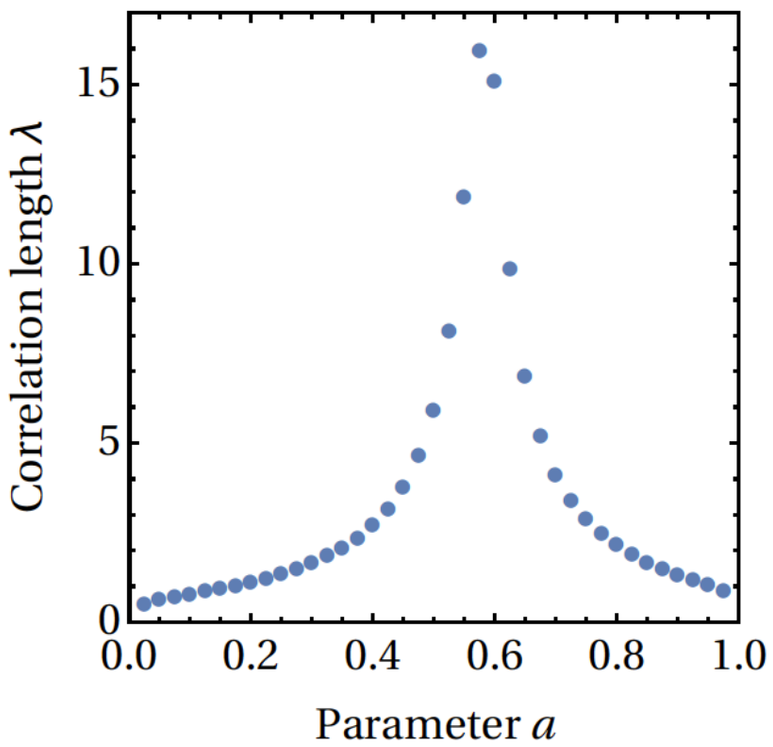
$$A = \begin{pmatrix} 0 & a & a \\ -a & 0 & a \\ -a & -a & 0 \end{pmatrix}$$

Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

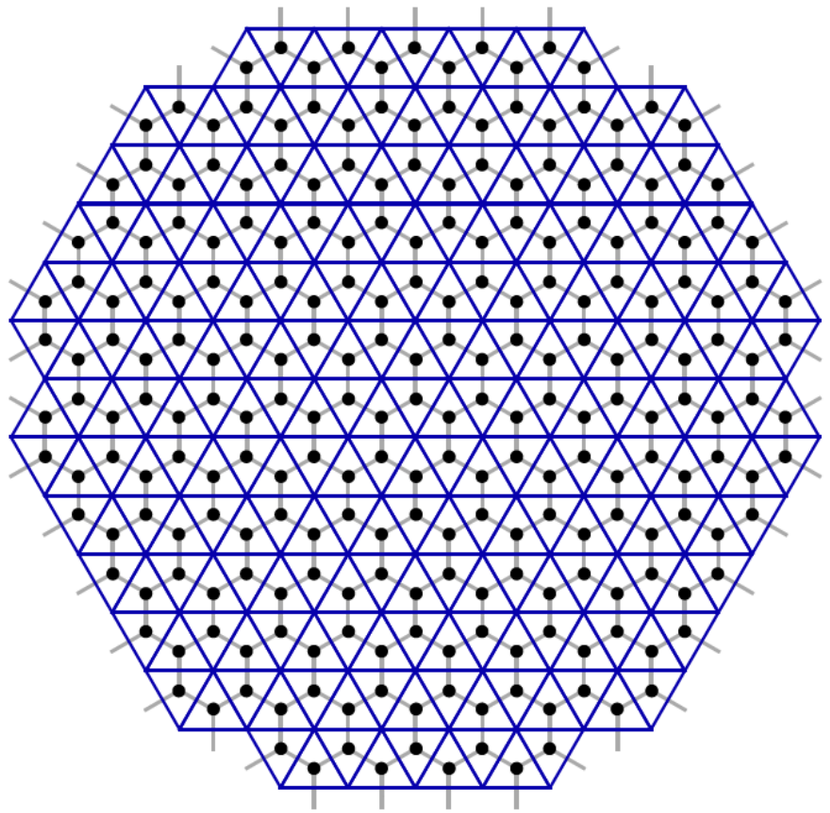
- Gapped system



- A ameter a



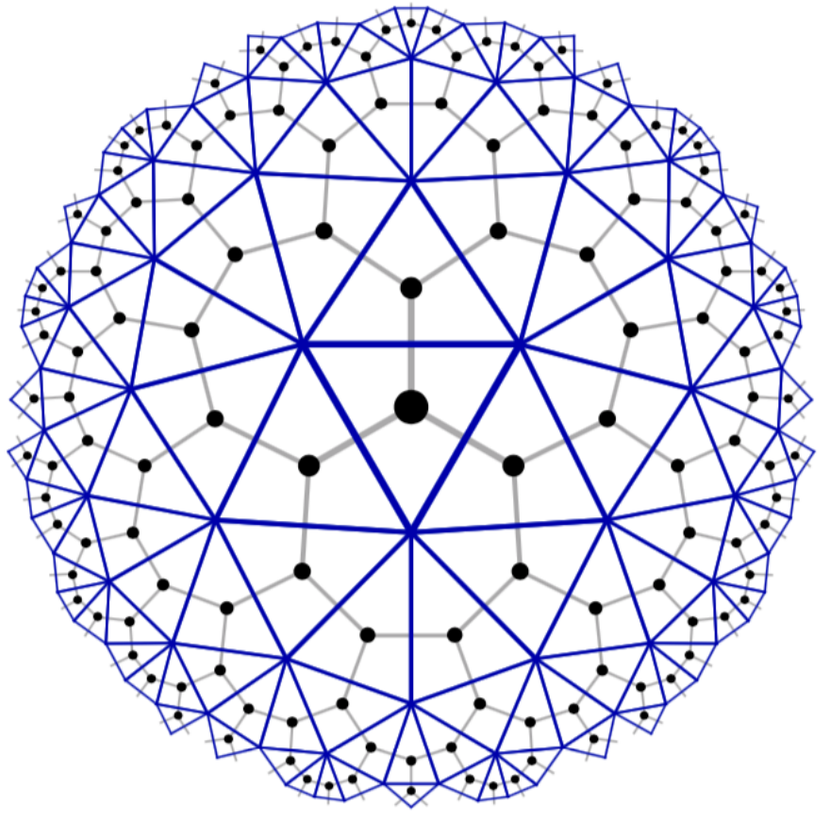
- $\{3, 6\}$ flat tiling



Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

REGULAR TILINGS AND BULK CURVATURE

- $\{3, k\},\ k > 6$, hyperbolic tiling

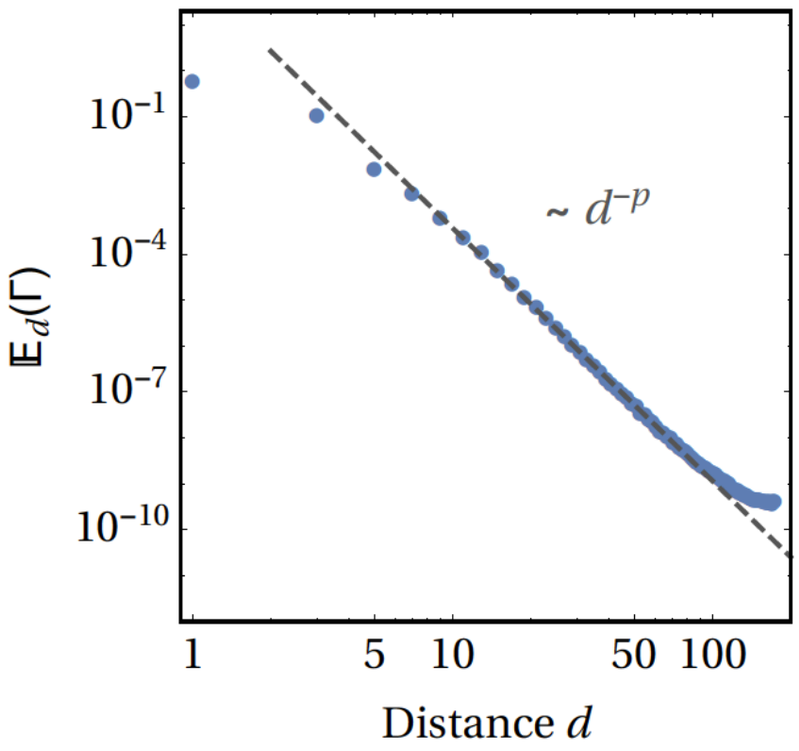


- Anti-symmetric matrix A, single parameter a

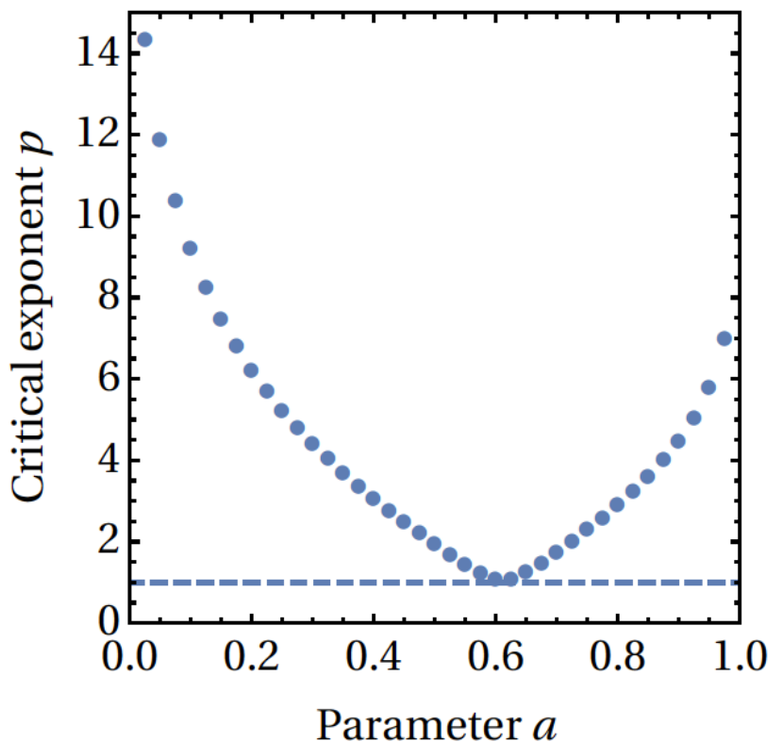
$$A = \begin{pmatrix} 0 & a & a \\ -a & 0 & a \\ -a & -a & 0 \end{pmatrix}$$

Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

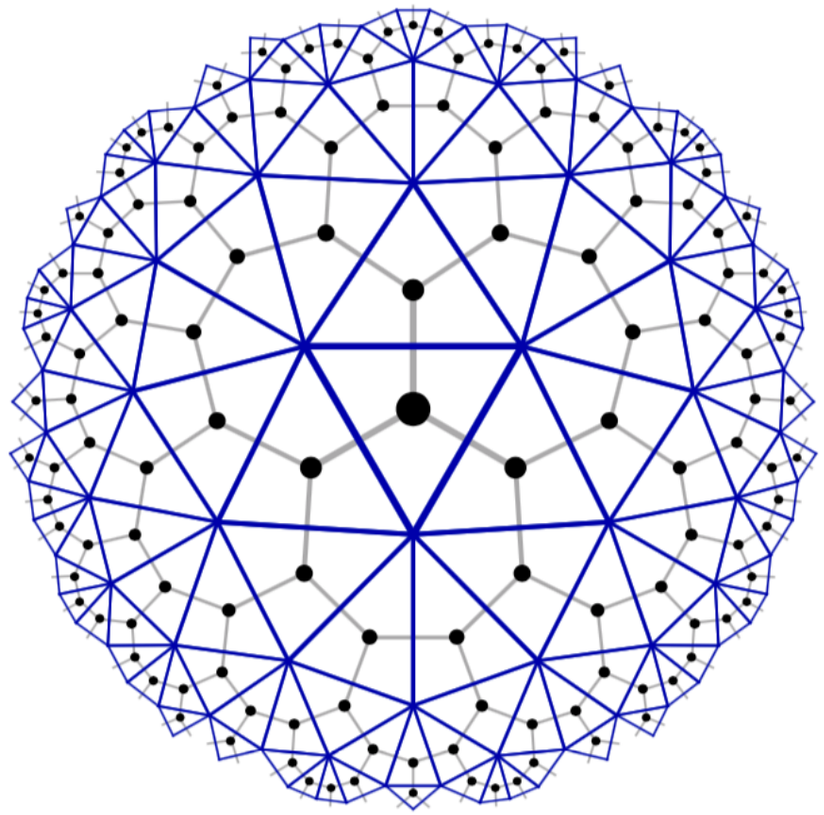
- $\{3, 7\}$ critical system



- A ameter a



- $\{3, k\},\ k > 6$, hyperbolic tiling

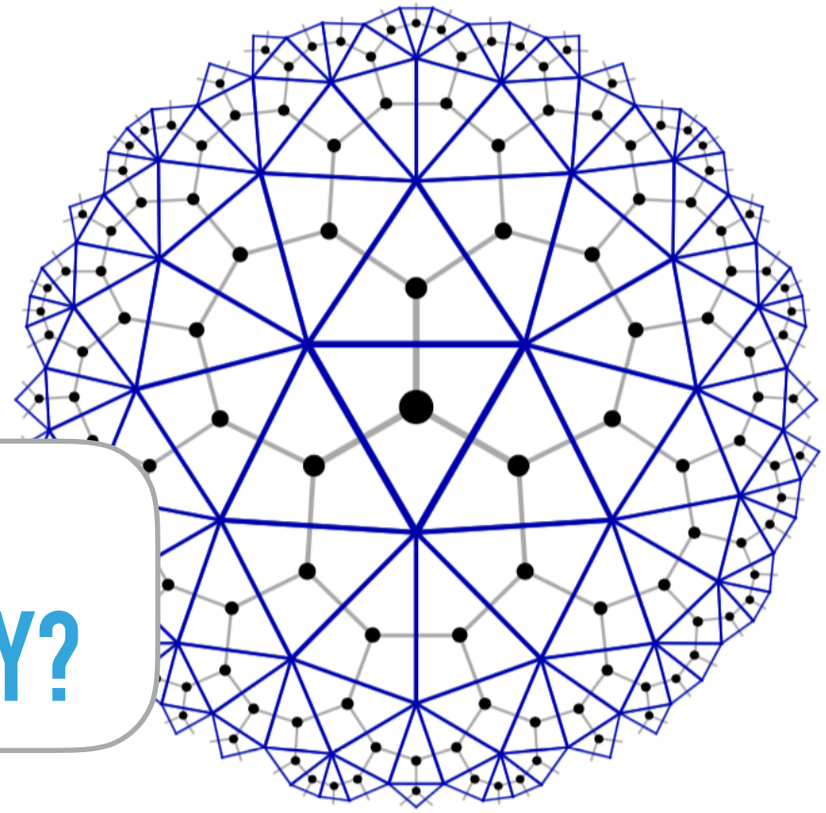


- For $a \approx 0.61$, critical Ising theory

$$H = \mathrm{i} \left(\sum_{i=1}^{N-1} m_i m_{i+1} + m_1 m_N \right)$$

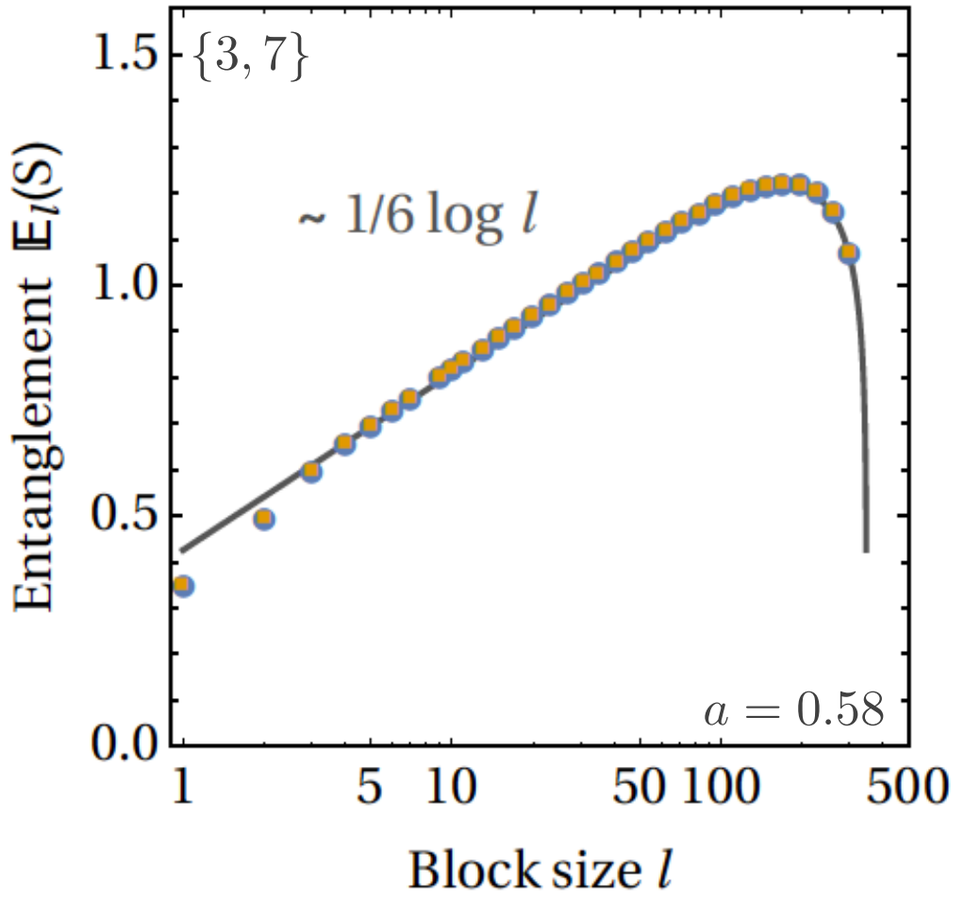
Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

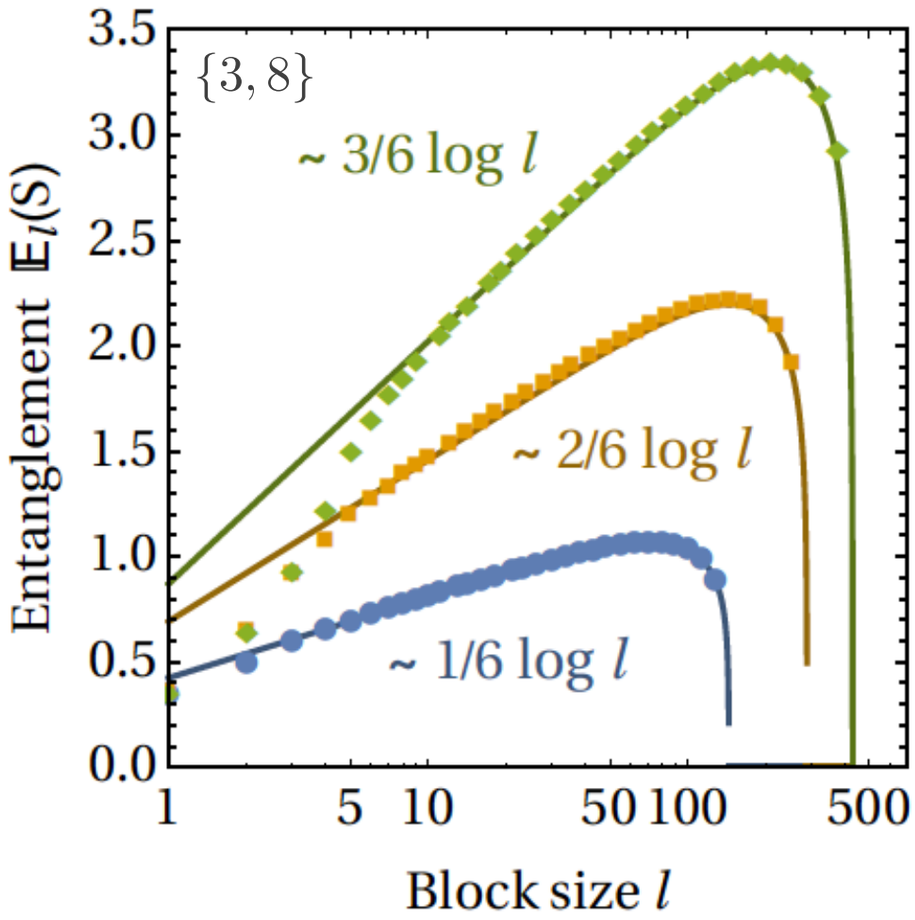
- $\{3, k\},\ k > 6$, hyperbolic tiling



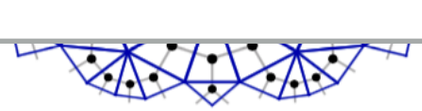
HOW ABOUT THE ENTANGLEMENT ENTROPY?

Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109





Tiling with higher bond dimensions χ (144, 288, 432 Majorana fermions for χ = 2, 4, 8, respectively)

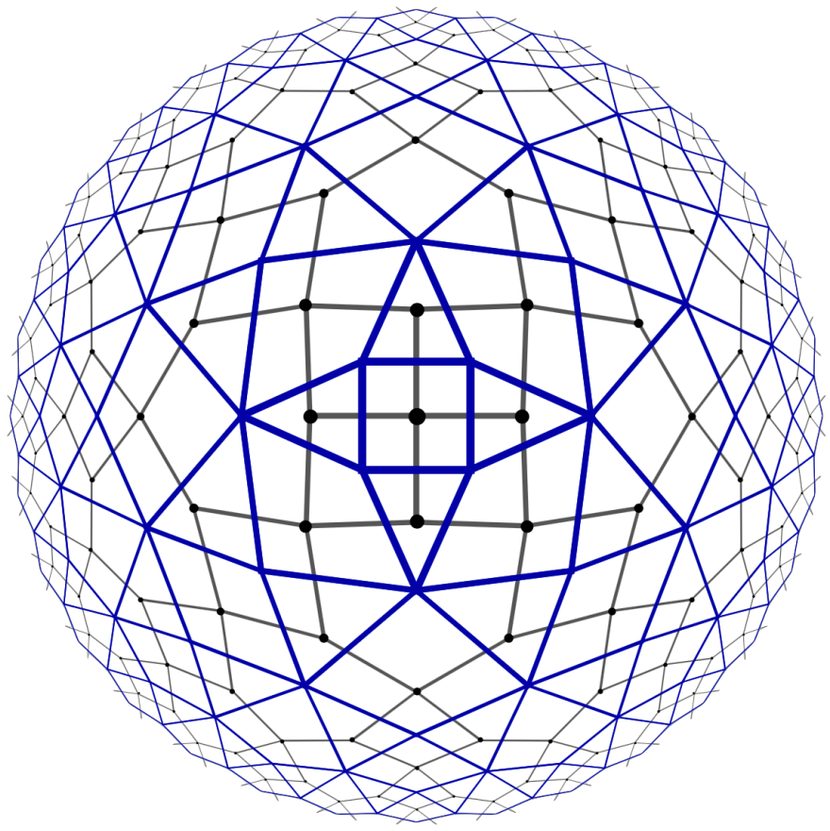


- CFT entanglement entropy of a block

$$S_\ell = \frac{c}{3} \ln \left(\frac{L}{\pi\epsilon} \sin \frac{\pi\ell}{L} \right) \simeq \frac{c}{3} \ln \frac{\ell}{\epsilon} + O\left((\ell/L)^2\right)$$

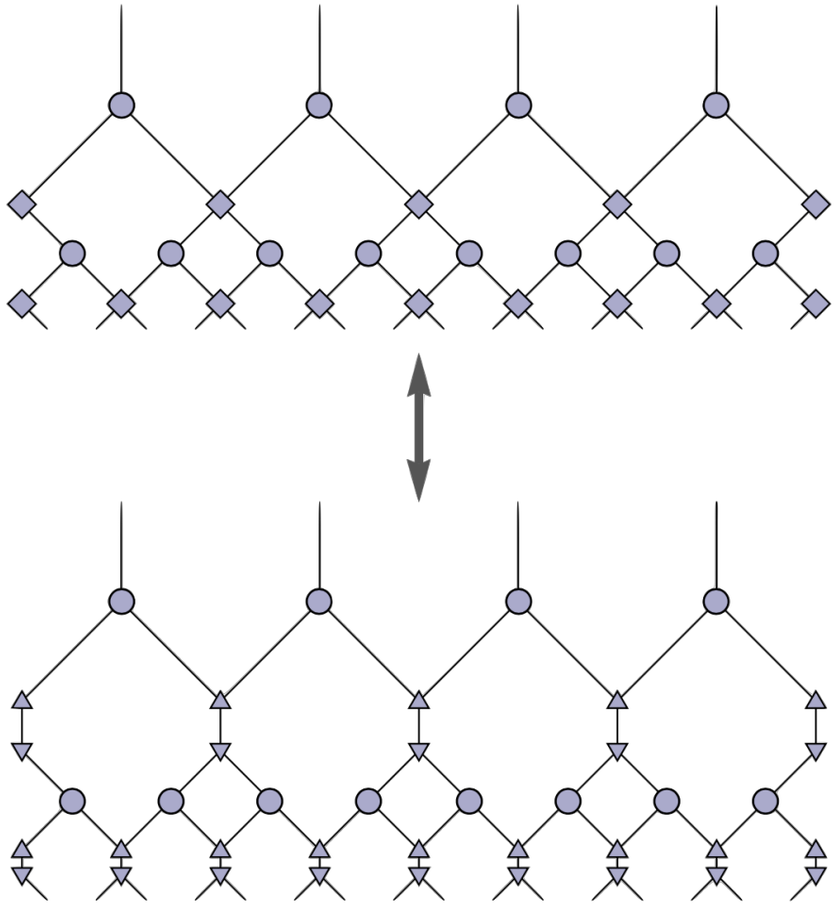
Holzhey, Larsen, Wilczek, Nucl Phys B 424, 443 (1994)
Calabrese, Cardy, J Stat Mech 0406, 06002 (2004)
Eisert, Cramer, Plenio, Rev Mod Phys 82, 277 (2010)

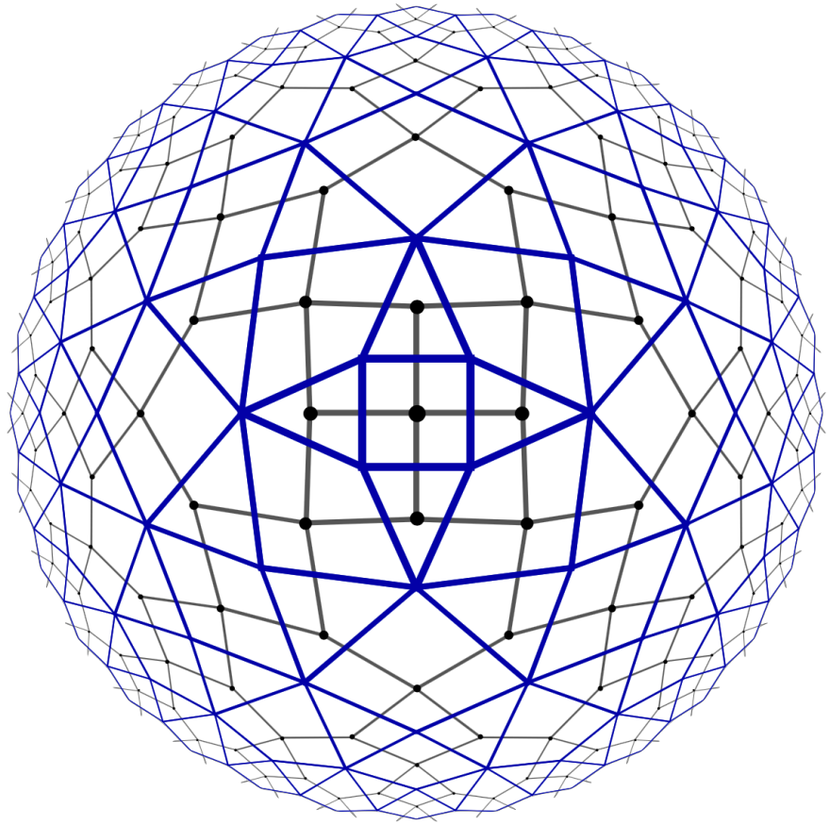
MERA?



Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

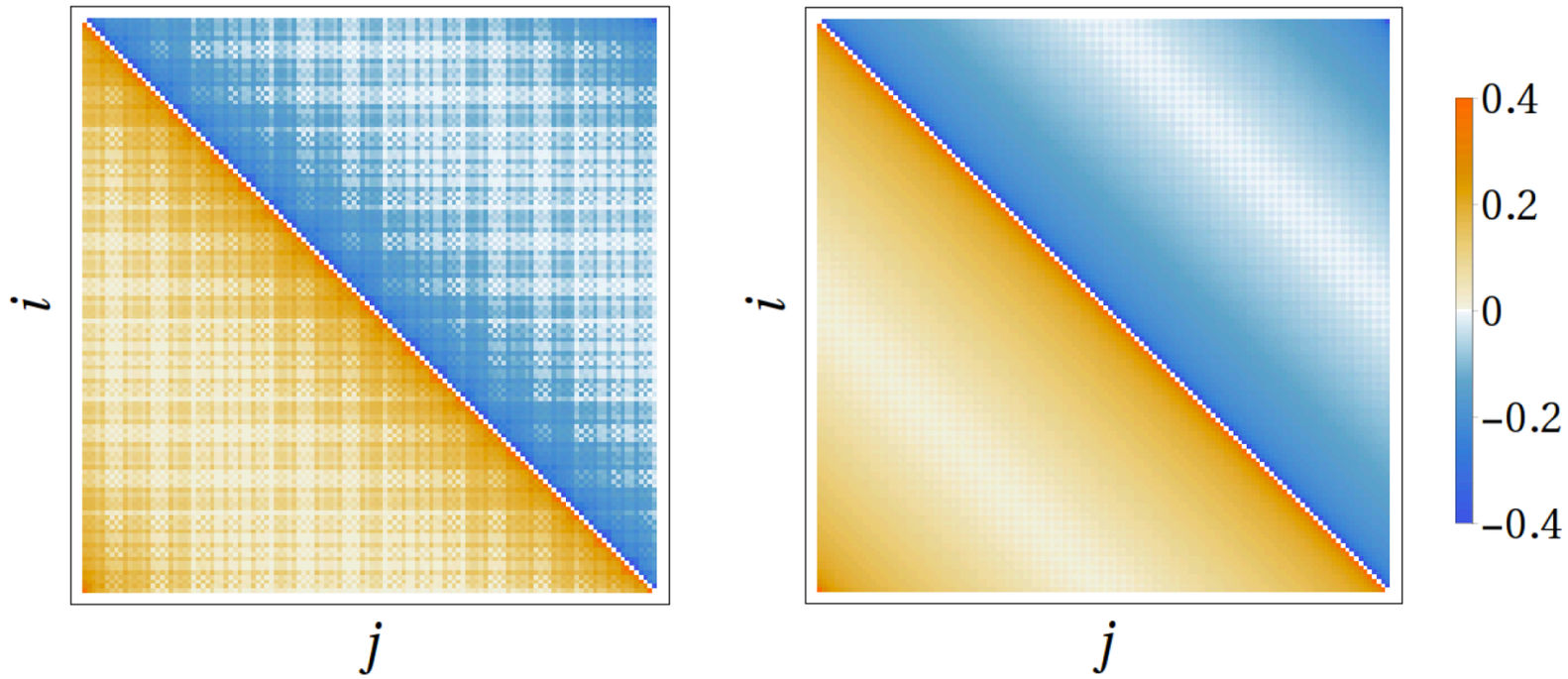
- Tiling with 3-and 4-leg **MERA tensor network**
- At $L = 256$, for $a = 0.566$, $b = 0.443$, $c = 0.363$, relative energy density error to continuum solution is about $\epsilon = 0.0002$



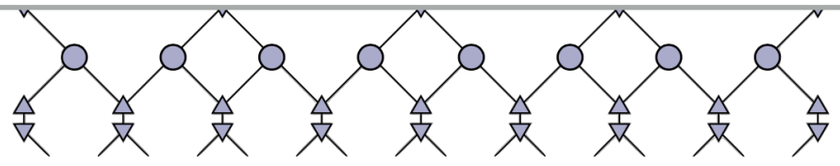


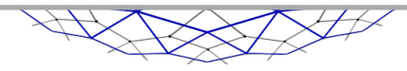
Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

- Tiling with 3-and 4-leg **MERA** tensor network



Entries of the fermionic correlation matrix as a function of distance, left for$\{3, 7\}$ bulk tiling with 252 Majorana fermions, right for MERA with 256





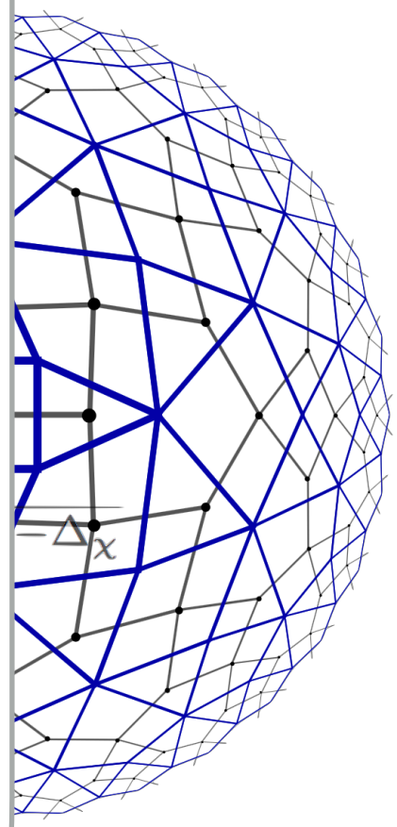
Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

EXTRACTING CRITICAL DATA

- Ising theory at criticality described by a 1+1-dimensional CFT
- T

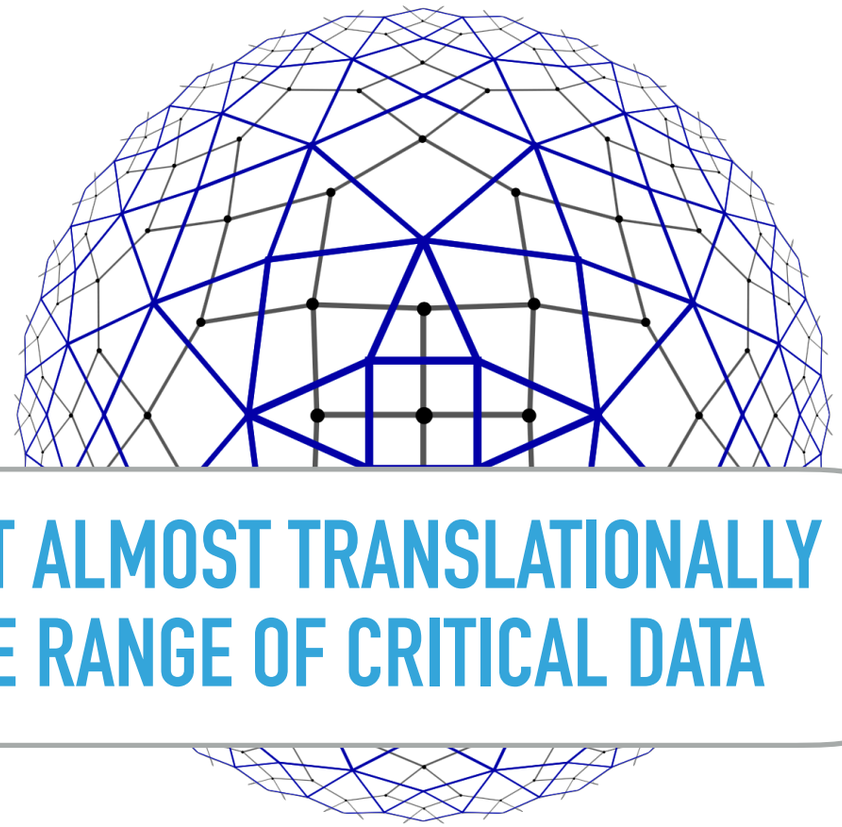
Parameter	Exact	$\{3,6\}$ bulk	$\{3,7\}$ bulk	mMERA	Wavelets
ϵ_0	-0.6366	-0.6139	-0.5617	-0.6365	-0.6211
c	0.5000	0.5006	0.5018	0.4958	0.4957
$\Delta_\psi, \Delta_{\bar{\psi}}$	0.5000	0.4948	0.4951	0.5023	0.5000
Δ_ϵ	1.0000	0.9856	1.0121	1.0027	1.0000
Δ_σ	0.1250	0.1403	0.1368	0.1417	0.1402
$C_{\sigma,\sigma,\epsilon}$	0.5000	0.5470	0.5336	0.5156	0.4584

TABLE I. Table of *conformal data* for the regular $\{3,6\}$ and $\{3,7\}$ bulk tilings as well as the mMERA, compared to the exact results and the wavelet MERA [16]. Listed are the ground-state energy density ϵ_0, central charge c, scaling dimensions Δ_ϕ of the fields $\phi = \psi, \bar{\psi}, \epsilon, \sigma$, and the structure constant $C_{\sigma,\sigma,\epsilon}$



Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

EXTRACTING CRITICAL DATA



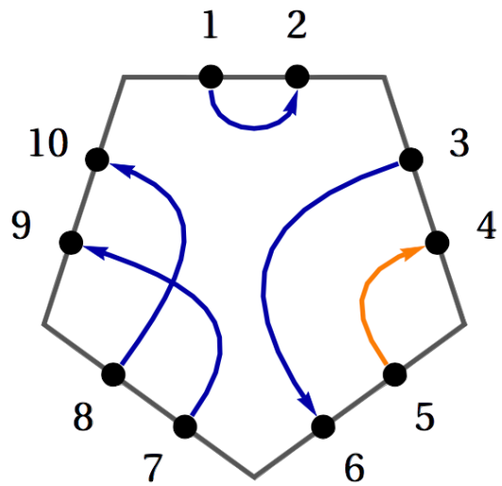
SO WITH FEW PARAMETERS, ONE ARRIVES AT ALMOST TRANSLATIONALLY INVARIANT STATES AND CAN EXTRACT A WIDE RANGE OF CRITICAL DATA

Jahn, Gluza, Pastawski, Eisert, arXiv:1711.03109

GETTING MORE SERIOUS ON QUANTUM ERROR CORRECTION

HOLOGRAPHIC MAJORANA DIMER MODELS OF QUANTUM ERROR CORRECTION

- For holographic stabilizer codes such as pentagon code develop picture of **paired Majorana dimers**



$$\Gamma_{i,j} = \begin{cases} -1 & \text{for an arrow } i \to j \\ 1 & \text{for an arrow } j \to i \\ 0 & \text{if no arrow connects } i \text{ and } j \end{cases}$$

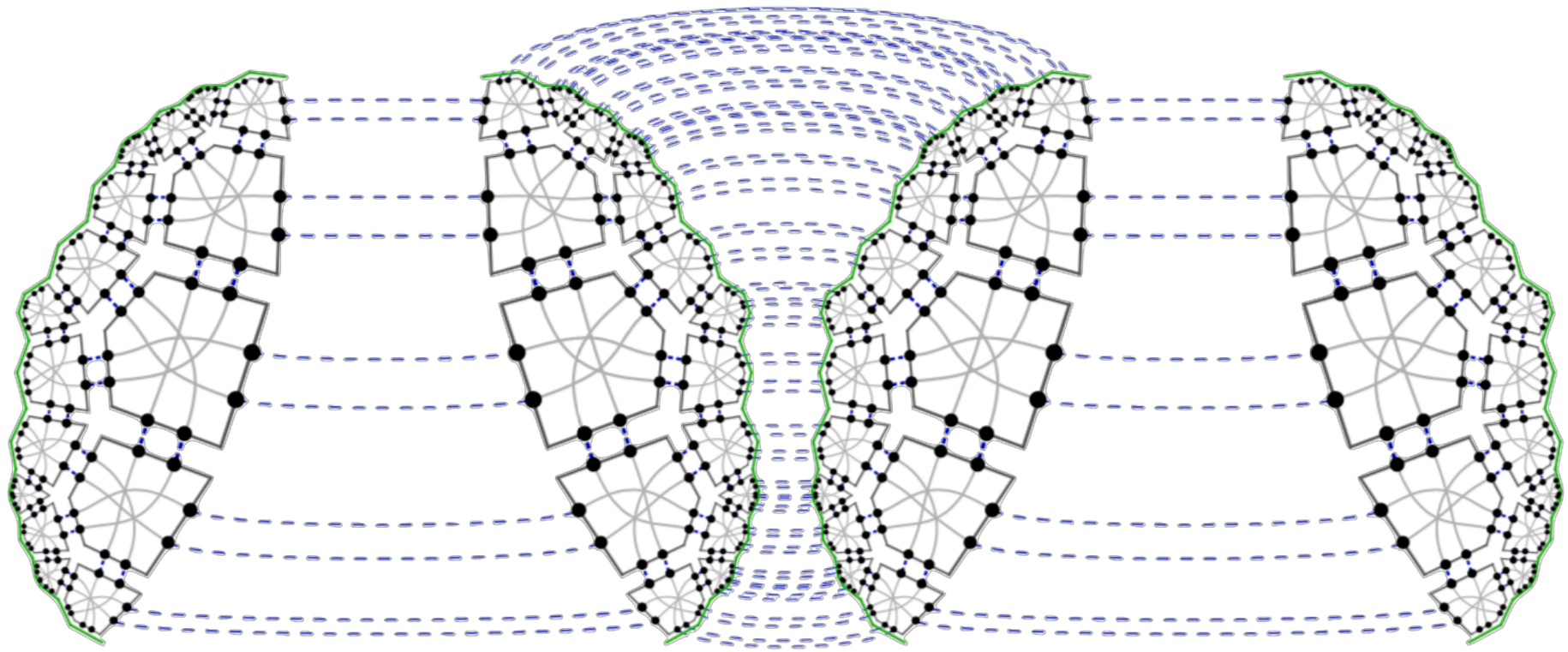
- Diagrammatic contraction rules amenable to analytical analysis

Jahn, Gluza, Pastawski, Eisert, in preparation (2019)

HOLOGRAPHIC MAJORANA DIMER MODELS OF QUANTUM ERROR CORRECTION

- New picture of **holographic QEC** in paired Majoranas

$$\rho_A^2 = 2^{2N_{C,W}}$$



- **Theorem:** Computational basis state vectors of the bulk are dual to Majorana dimer states on the boundary
  - Can compute **Renyi entropies**
  - Can compute second moments of **non-Gaussian states** arising in **quantum error correcting codes** etc

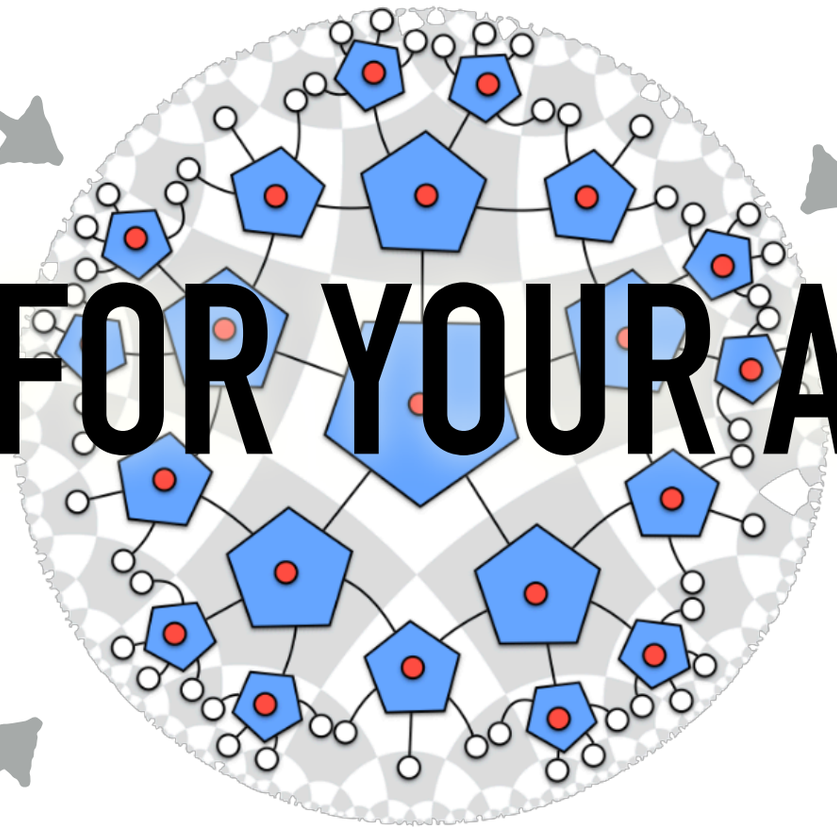
Jahn, Gluza, Pastawski, Eisert, in preparation (2019)

MATCHGATE TENSOR NETWORKS PROVIDE VERSATILE FRAMEWORK, ALLOWING FOR NEW INSIGHTS INTO HOLOGRAPHIC CODES

OUTLOOK AND SUMMARY

Interesting endeavor to think of tensor network models capturing holographic aspects

Fun tool provided by matchgate tensor networks



THANKS FOR YOUR ATTENTION!

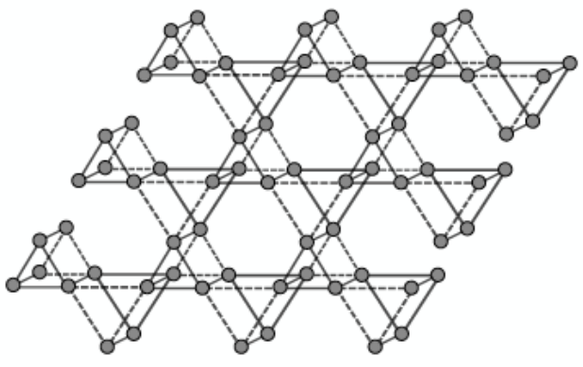
Tunable correlations, curvature, entanglement, MERA

More on error correction

- Random matchgate tensors?
  Hayden, Nezami, Qi, Thomas, Walter, Yang, JHEP 2016, 9 (2016)
- Interacting theories, connection to string nets?
  Wille, Buerschaper, Eisert, Phys Rev B 95, 245127 (2017)
  Bultinck, Williamson, Haegeman, Verstraete, Phys Rev B 95, 075108 (2017)
- Steps towards parametrizing physical CFTs?
- Non-unitary MERA, further perspectives?
- Quasiperiodic tilings?
  Boyle, Dickens, Flicker, arXiv:1812.02365

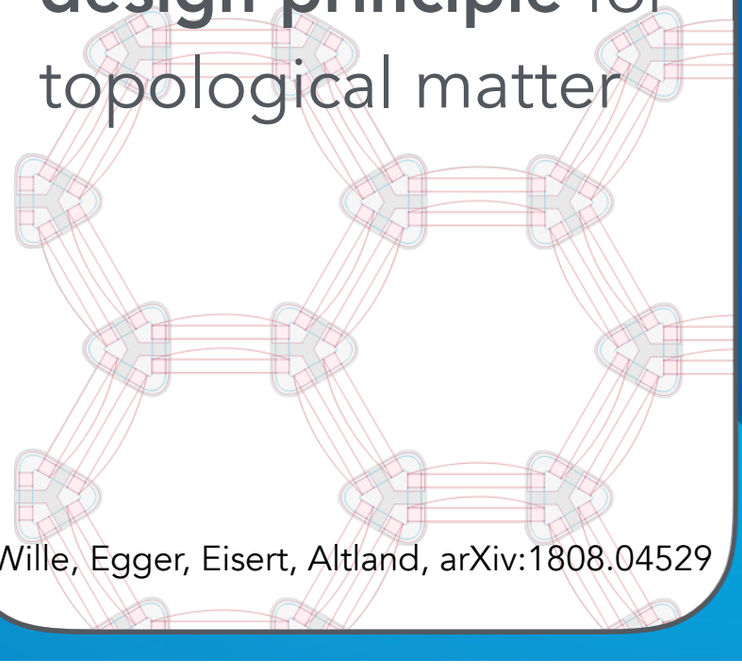
THANKS FOR YOUR ATTENTION!

- Tensor networks for **quantum materials**



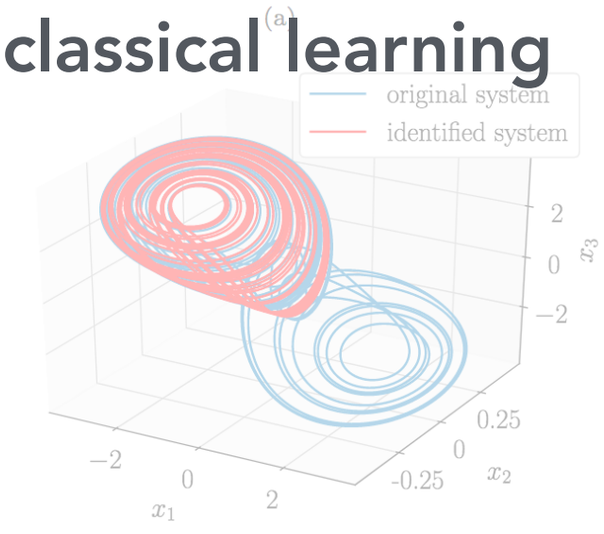
Kshetrimayum, Rizzi, Eisert, Orus, PRL 122, 122, 070502 (2019). Kshetrimayum, Nietner, Lake, Eisert, in preparation

- Tensor networks as **design principle** for topological matter



Wille, Egger, Eisert, Altland, arXiv:1808.04529

- Tensor networks in **classical learning**



Gelss, Klus, Eisert, Schuette, arXiv:1809.02448