Infrared singularities of one-loop amplitudes

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Multileg NLO calculations

What one aims for:

- NLO calculations for multi-parton processes at the LHC.
  Multi-parton processes: 3, 4, 5, ... partons in the final state.

- For a given process the program should be usable for any infrared-safe observable.

- Need to compute the virtual corrections and the real corrections.

The subtraction method for the real emission

The subtraction methods subtracts out a simple term, which approximates the real emission in all singular limits:

$$\int_{n+1} d\sigma^R + \int_n d\sigma^V \quad = \quad \underbrace{\int_{n+1} \left(d\sigma^R - d\sigma^A\right)}_{\text{convergent}} + \underbrace{\int_n \mathbf{I} \otimes d\sigma^B}_{\text{singular}} + \int_n d\sigma^V$$

- Residue subtraction: S. Frixione, Z. Kunszt and A. Signer, '95,
  V. Del Duca, G. Somogyi, Z. Trócsányi, '05
- Dipole subtraction: Catani and Seymour '96, Phaf and S.W. '01, Catani, Dittmaier, Seymour and Trócsányi '02.
- Antenna subtraction: A. Gehrmann-De Ridder, Th. Gehrmann, N. Glover, '05

Computational costs

- Efficient methods like recursion relations known for Born and real contribution.
- Integration over momenta of the final state particles is done by Monte Carlo.
- Real emission (minus the subtraction terms) can be automated.

  S.W., '05, T. Gleisberg and F. Krauss, '07, M. Seymour and C. Tevlin, '08, K. Hasegawa, S. Moch and P. Uwer, '08, R. Frederix, T. Gehrmann and N. Greiner, '08, M. Czakon, C. Papadopoulos and M. Worek, '09.
- Insertion term $\mathbf{I} \otimes d\sigma^B$ is cheap.
- Virtual corrections usually reduced to a set of master integrals, which are then calculated analytically.

  Popular methods: Traditional Feynman graph approach and cut-based techniques.
- CPU-time for real emission sets time scale.

Never change a winning team

Use subtraction also for the virtual part:

$$\int_{n+1} d\sigma^R + \int_n d\sigma^V = \underbrace{\int_{n+1} \left(d\sigma^R - d\sigma^A\right)}_{\text{convergent}} + \underbrace{\int_n (\mathbf{I}+\mathbf{L}) \otimes d\sigma^B}_{\text{finite}} + \underbrace{\int_n \left(d\sigma^V - d\sigma^{A'}\right)}_{\text{convergent}}$$

- In the last term $d\sigma^V - d\sigma^{A'}$ the Monte Carlo integration is over a phase space integral of n final state particles plus a 4-dimensional loop integral.
- All explicit poles cancel in the combination $\mathbf{I}+\mathbf{L}$.
- Divergences of one-loop amplitudes related to IR-divergences (soft and collinear) and to UV-divergences.

Numerical NLO QCD calculations

Proceed through the following steps:

1. Local subtraction terms for soft, collinear and ultraviolet singular part of the integrand of one-loop amplitudes

2. Contour deformation for the 4-dimensional loop integral.

3. Numerical Monte Carlo integration over phase space and loop momentum.

Not a new idea: Nagy and Soper proposed in ’03 this method, working graph by graph.

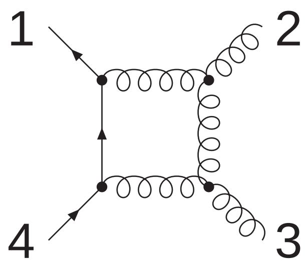
What is new: The IR-subtraction terms can be formulated at the level of amplitudes, no need to work graph by graph.

The IR-subtraction terms are universal and amasingly simple.

Primitive amplitudes

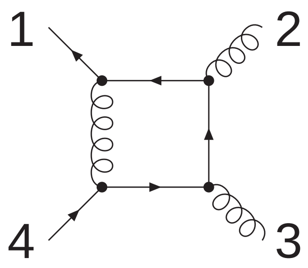
Colour-decomposition of one-loop amplitudes:

$$\mathcal{A}^{(1)} = \sum_j C_j A_j^{(1)}.$$



Primitive amplitudes distinguished by:

- fixed cyclic ordering
- definite routing of the fermion lines
- particle content circulating in the loop



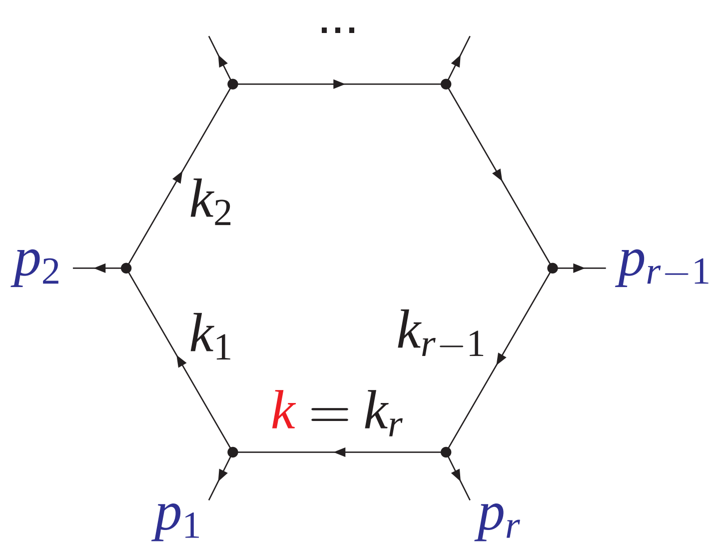
Z. Bern, L. Dixon, D. Kosower, ’95

Notation and kinematics

All momenta specified by p_1, ..., p_n and k:

$$k_i = k - (p_1 + ... + p_i)$$

For cyclic ordered amplitudes we have only n different propagators.



Write primitive one-loop amplitude as

$$A^{(1)}_{\text{bare}} = \int \frac{d^Dk}{(2\pi)^D} G^{(1)}_{\text{bare}},$$

$$G^{(1)}_{\text{bare}} = P(k) \prod_{i=1}^{n} \frac{1}{k_i^2 - m_i^2 + i\delta}.$$

Integrand can be calculated efficiently using recursion relations.

The infrared subtraction terms for the virtual corrections

Local unintegrated form:

$$G^{(1)}_{\text{soft+coll}} = -4\pi\alpha_s i \sum_{i\in I_g} \left(\frac{4p_i p_{i+1}}{k_{i-1}^2 k_i^2 k_{i+1}^2} - 2\frac{S_i g^{UV}_{i-1,i}}{k_{i-1}^2 k_i^2} - 2\frac{S_{i+1} g^{UV}_{i,i+1}}{k_i^2 k_{i+1}^2} \right) A_i^{(0)}.$$

with $S_q = 1$, $S_g = 1/2$. The function $g^{UV}_{i,j}$ provides damping in the UV-region:

$$\lim_{k\to\infty} g^{UV}_{i,j} = O\left(k^{-2}\right), \qquad \lim_{k_i || k_j} g^{UV}_{i,j} = 1.$$

Integrated form:

$$S_\varepsilon^{-1}\mu^{2\varepsilon}\int \frac{d^Dk}{(2\pi)^D} G^{(1)}_{\text{soft+coll}} = \frac{\alpha_s}{4\pi}\frac{e^{\varepsilon\gamma_E}}{\Gamma(1-\varepsilon)}\sum_{i\in I_g}\left[\frac{2}{\varepsilon^2}\left(\frac{-2p_i\cdot p_{i+1}}{\mu^2}\right)^{-\varepsilon} + \left(\frac{2}{\varepsilon}+2\right)(S_i+S_{i+1})\left(\frac{\mu_c^2}{\mu^2}\right)^{-\varepsilon}\right] A_i^{(0)}$$
$$+O(\varepsilon),$$

M. Assadsolimani, S. Becker, S.W., ’09

Remarks

- Contrary to the subtraction terms for the real emission, there are no spin correlations.

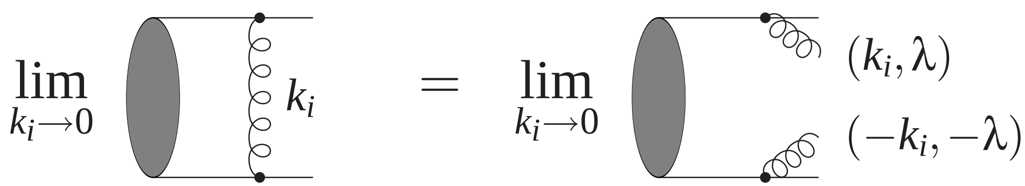
- Contrary to the subtraction terms for the real emission, there is no dependence on the variant of dimensional regularization (conventional dimensional regularisation, 't Hooft-Veltman scheme, four-dimensional scheme, ...).

$$\begin{aligned}
\mathcal{A}_{ren}(p_1,...,p_n,\alpha_s) &= \left(Z_2^{1/2}\right)^{n_q+n_{\bar{q}}} \left(Z_3^{1/2}\right)^{n_g} \mathcal{A}_{bare}\left(p_1,...,p_n,Z_g^2 S_\varepsilon^{-1}\mu^{2\varepsilon}\alpha_s\right), \\
Z_2 &= 1+\frac{\alpha_s}{4\pi}C_F\left(\frac{1}{\varepsilon_{IR}}-\frac{1}{\varepsilon_{UV}}\right)+O(\alpha_s^2), \\
Z_3 &= 1+\frac{\alpha_s}{4\pi}(2C_A-\beta_0)\left(\frac{1}{\varepsilon_{IR}}-\frac{1}{\varepsilon_{UV}}\right)+O(\alpha_s^2).
\end{aligned}$$

- The integrated form reproduces in the pole terms the well-known result.

Proof of the formula: Soft singularities

In the soft limit the three propagators $(i-1)$, i and $(i+1)$ become singular.



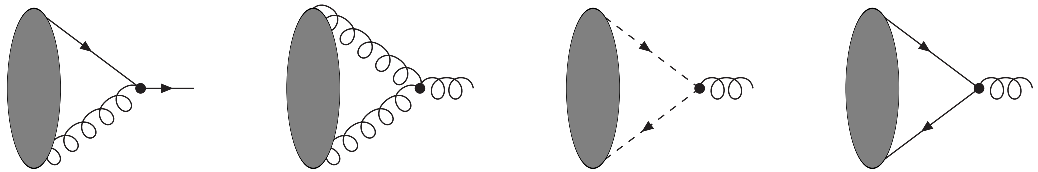
Self-energy corrections on external lines not singular enough in the soft limit.

Can replace soft gluons with eikonal factors.

The sum of all diagrams forms a gauge-invariant set.

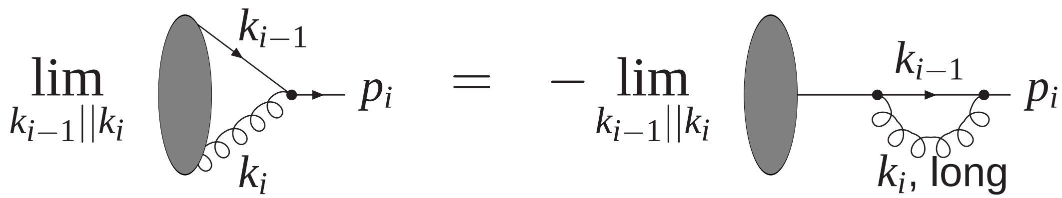
Proof of the formula: Collinear singularities

In the collinear limit two propagators $(i-1)$ and i become singular.



The splittings $g \longrightarrow c\bar{c}$ and $g \longrightarrow q\bar{q}$ are not singular enough.

In the splittings $q \longrightarrow qg$ and $g \longrightarrow gg$ one gluon is longitudinal polarised.



Outlook: UV-subtraction terms

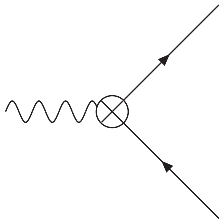
In a fixed direction in loop momentum space the amplitude has up to quadratic UV-divergences.

Only the integration over the angles reduces this to a logarithmic divergence.

For a local subtraction term we have to match the quadratic, linear and logarithmic divergence.

The subtraction terms have the form of counter-terms for propagators and vertices.

Example:



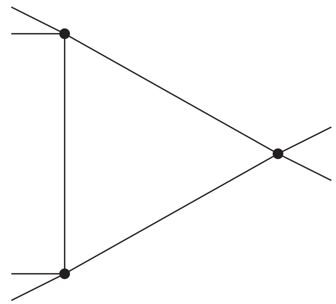
$$= \; i g^3 S_\varepsilon^{-1} \mu^{4-D} \int \frac{d^D k}{(2\pi)^D i} \frac{2\,(1-\varepsilon)\,\bar{\not{k}}\,\gamma^\mu \bar{\not{k}} + 4\mu_{UV}^2 \gamma^\mu}{\left(\bar{k}^2 - \mu_{UV}^2\right)^3}$$

S. Becker, Ch. Reuschle, S.W., to appear

Outlook: Contour deformation

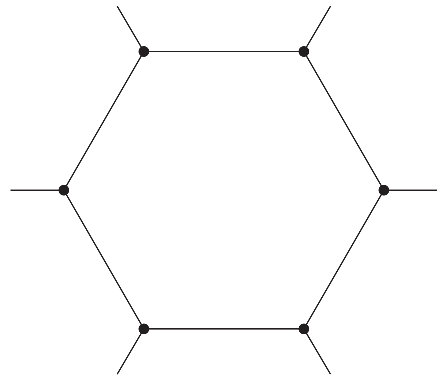
With the subtraction terms for UV- and IR-singularities one removes

- UV divergences
- Pinch singularities due to soft or collinear partons



Still remains:

- Singularities in the integrand, where a deformation into the complex plane of the contour is possible.
- Pinch singularities for exceptional configurations of the external momenta (thresholds, anomalous thresholds ...)



Summary

The main new result: A simple formula, which approximates a one-loop amplitude in all singular soft and collinear limits.

Application: Computation of multi-leg NLO processes.