Radiative corrections to quarkonium HFS and η_b mass puzzle

Alexander Penin

University of Alberta & INR Moscow

Loops and Legs in Quantum Field Theory

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Topics covered

✔ QCD theory of quarkonium HFS

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- ✔ Discovery of pseudoscalar bottomonium state η_b

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- ✔ Discovery of pseudoscalar bottomonium state η_b
- ✔ $M^{\rm exp}(\eta_b) - M^{\rm QCD}(\eta_b) \sim 3\sigma$ ⇨ "η_b mass puzzle"

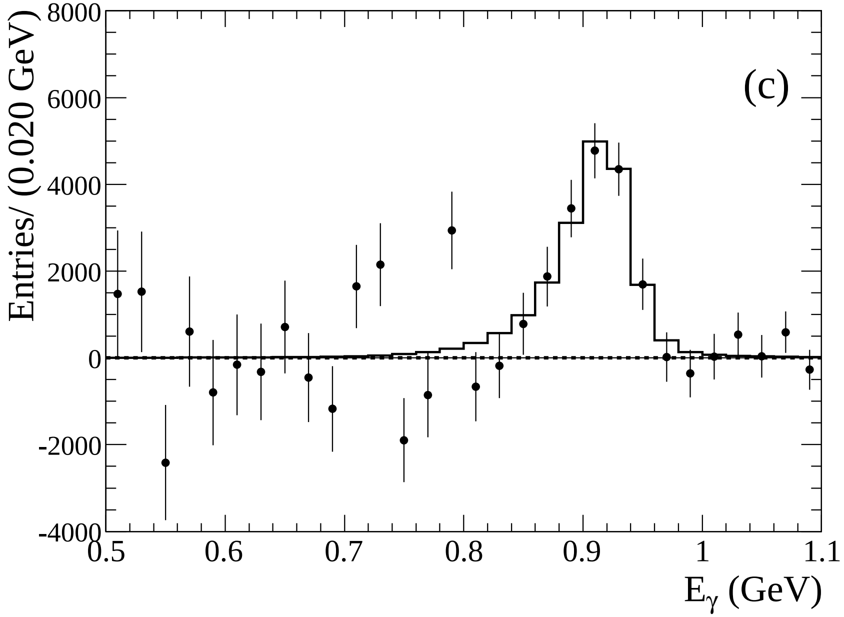
Heavy quarkonium experiment 2003-2008

Charmonium	Bottomonium
$X(3872)$, $X(3940)$	
$Y(3940)$, $Y(4260)$	*nothing*
$Z(3930)$	

Descivery of η_b - June 2008

BABAR Collaboration, Phys.Rev.Lett. 101, 071801 (2008)

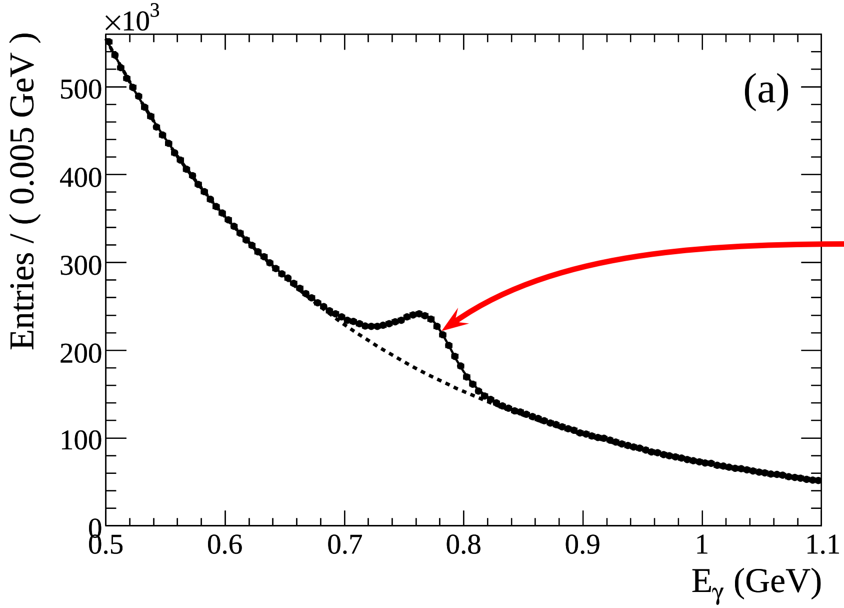
Single photon spectrum at $s = M(\Upsilon(3S))$



Peak at $E_\gamma = 921.2^{+2.1}_{-2.8}\text{stat} \pm 2.4_{\text{syst}}\ MeV$ ⇨ $\Upsilon(3S) \to \gamma\eta_b$

Descivery of η_b - June 2008

BABAR Collaboration, Phys.Rev.Lett. 101, 071801 (2008)

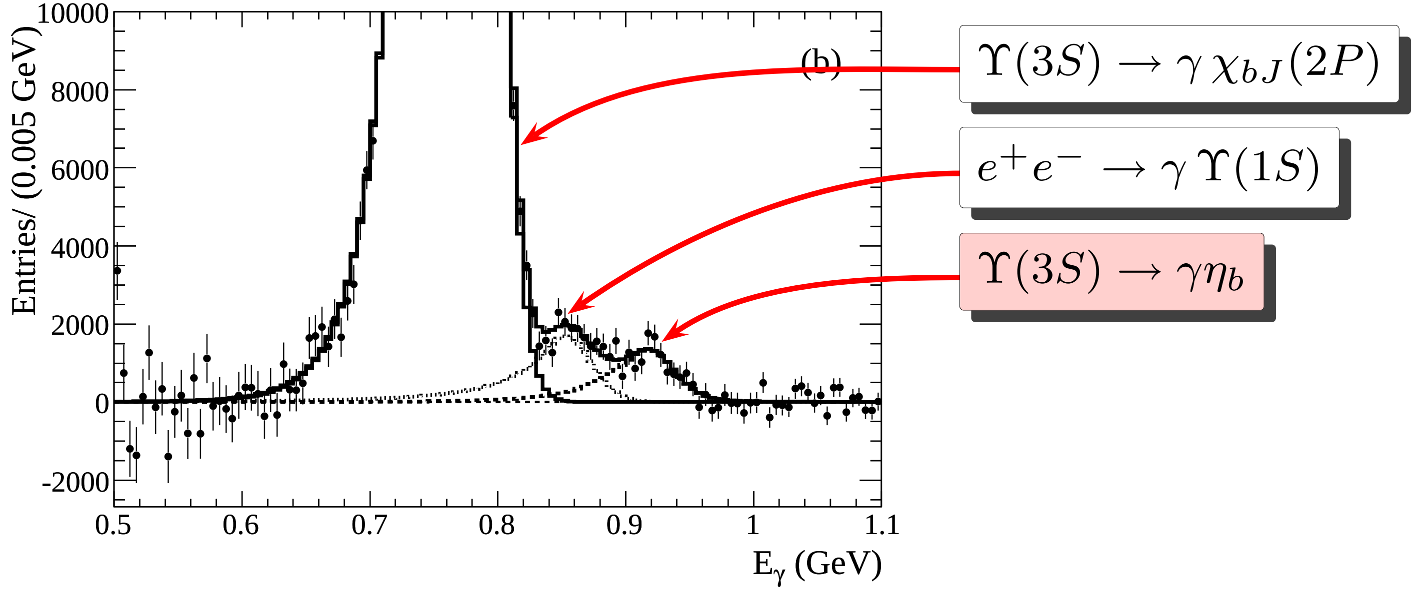


$\Upsilon(3S) \to \gamma\, \chi_{bJ}(2P)$

Inclusive spectrum

Descivery of η_b - June 2008

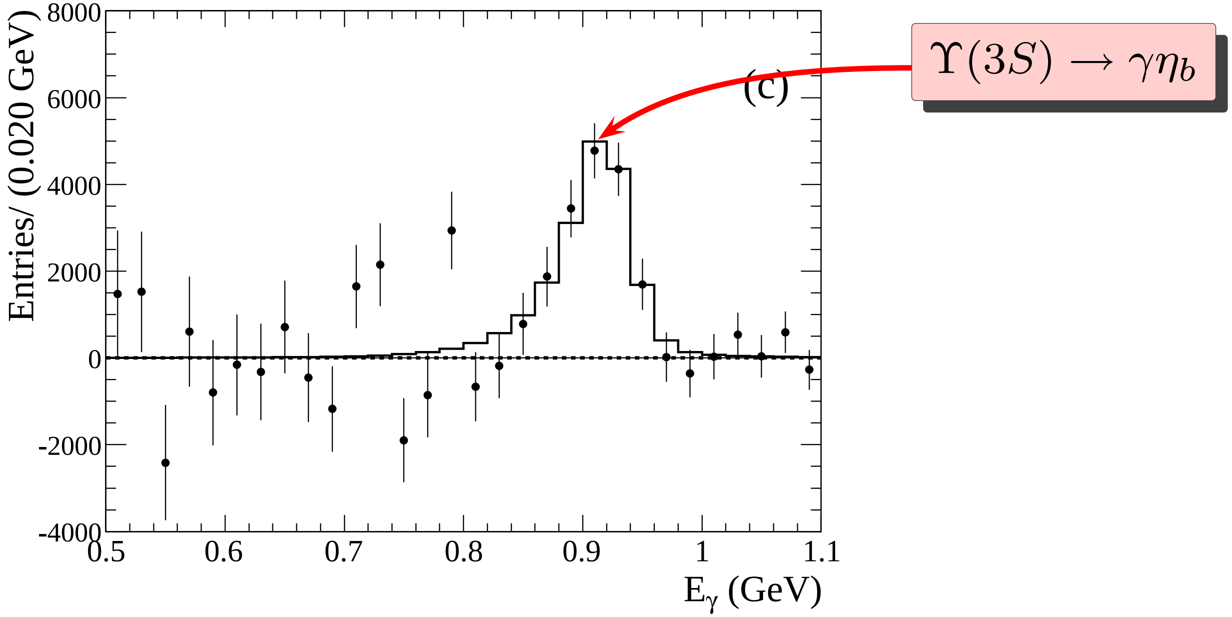
BABAR Collaboration, Phys.Rev.Lett. 101, 071801 (2008)



Non-peak background subtracted

Descivery of η_b - June 2008

BABAR Collaboration, Phys.Rev.Lett. 101, 071801 (2008)



All background subtracted

Hyperfine Splitting $E_{\rm hfs} = M(\Upsilon_{1S}) - M(\eta_b)$

$$E^{\rm exp}_{\rm hfs} = 71.4 \pm 2.7\,({\rm syst})\,{}^{+2.3}_{-3.1}\,({\rm stat})\ {\rm MeV}$$

$\Upsilon(3S) \to \gamma\eta_b$

$$E^{\rm exp}_{\rm hfs} = 67.4 \pm 2.0\,({\rm syst})\,{}^{+4.8}_{-4.6}\,({\rm stat})\ {\rm MeV}$$

$\Upsilon(2S) \to \gamma\eta_b$

Theory of Hyperfine Splitting

✔ Perturbative QCD

✔ Lattice QCD

✘ Potential models

Perturbative QCD

- NRQCD, pNRQCD ⇨ systematic expansion in α_s

  (Caswell, Lepage; Pineda, Soto; ...)

- Dimensional regularization
  Threshold expansion ⇨ loops in pNRQCD

  (Pineda, Soto; Beneke, Smirnov; ...)

- Log resummation ⇨ reduced scale dependence

  (Luke, Manohar, Rothstein; Pineda; ...)

- Nonperturbative contribution $\mathcal{O}(v^2)$ ⇨ NNLO

  (Voloshin; Leutwiller)

NLO corrections to HFS

Spin-flip potential:

$$V_S(\boldsymbol{q}^2) \quad = \quad \frac{4\pi C_F \alpha_s \boldsymbol{S}^2}{3m_q^2}$$

NLO corrections to HFS

Spin-flip potential:

$$V_S(\boldsymbol{q}^2) = \frac{4\pi C_F \alpha_s \boldsymbol{S}^2}{3m_q^2}\left\{1 + \frac{\alpha}{\pi}\left[\left(1+\frac{7}{8}\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{m_q^2}\right)\right)C_A - \frac{1}{2}C_F + \frac{6-6\ln 2+i3\pi}{4}T_F\right]\right.$$

hard contribution

QCD on-shell on-threshold amplitude

NLO corrections to HFS

Spin-flip potential:

$$V_S(\boldsymbol{q}^2) = \frac{4\pi C_F \alpha_s \boldsymbol{S}^2}{3m_q^2}\Bigg\{1 + \frac{\alpha}{\pi}\left[\left(1+\frac{7}{8}\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{m_q^2}\right)\right)C_A - \frac{1}{2}C_F + \frac{6-6\ln 2+i3\pi}{4}T_F\right]$$
$$+\frac{\alpha}{\pi}\left[\left(-\frac{7}{18}-\frac{7}{8}\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{\boldsymbol{q}^2}\right)\right)C_A - \frac{5}{9}T_F n_l\right]\Bigg\}$$

hard contribution

QCD on-shell on-threshold amplitude

soft contribution

NQCD, static heavy quark propagator

Coulomb potential:

$$V_C(\boldsymbol{q}^2) = -\frac{4\pi C_F \alpha_s(\boldsymbol{q}^2)}{\boldsymbol{q}^2}\left[1 + \frac{\alpha_s}{\pi}\left(\frac{31}{36}C_A - \frac{5}{9}T_F n_l\right)\right]$$

NLO corrections to HFS

Quantum Mechanical PT *(potential contribution)*

$$\delta^{NLO}E_{\rm hfs} = \langle\psi^{\rm Coulomb}|V_S^{\rm 1-loop}|\psi^{\rm Coulomb}\rangle + 2\langle\psi^{\rm 1-loop}|V_S^{\rm tree}|\psi^{\rm Coulomb}$$

NLO corrections to HFS

Quantum Mechanical PT *(potential contribution)*

$$\delta^{NLO}E_{\rm hfs} = \langle\psi^{\rm Coulomb}|V_S^{1-{\rm loop}}|\psi^{\rm Coulomb}\rangle + 2\langle\psi^{1-{\rm loop}}|V_S^{\rm tree}|\psi^{\rm Coulomb}$$

Result for general n

$$E_{\rm hfs}^{NLO}(n) = \frac{1}{3}\frac{C_F^4\alpha_s^4}{n^3}\left\{1+\frac{\alpha_s}{\pi}\left[\frac{7\,C_A}{4}\ln\left(\frac{C_F\alpha_s}{n}\right)-\frac{C_F}{2}+\frac{-15-11\,n+12\,n^2\,\Psi_2(n)}{9\,n}\right.\right.$$

$$\left.\left.\times n_f\,T_F+\frac{393+150\,n+126\,\gamma_E\,n+126\,n\,\Psi_1(n)-264\,n^2\,\Psi_2(n)}{72\,n}C_A\right]\right\}$$

Penin, Steinhauser, Phys.Lett. B538, 335 (2002)

Nonrelativistic Renormalization Group

- Several scales: m_q, $m_q v$, $m_q v^2$
- Logarithmic integrals between the scales ⇨ $\ln v$ ⇨ $\ln \alpha_s$

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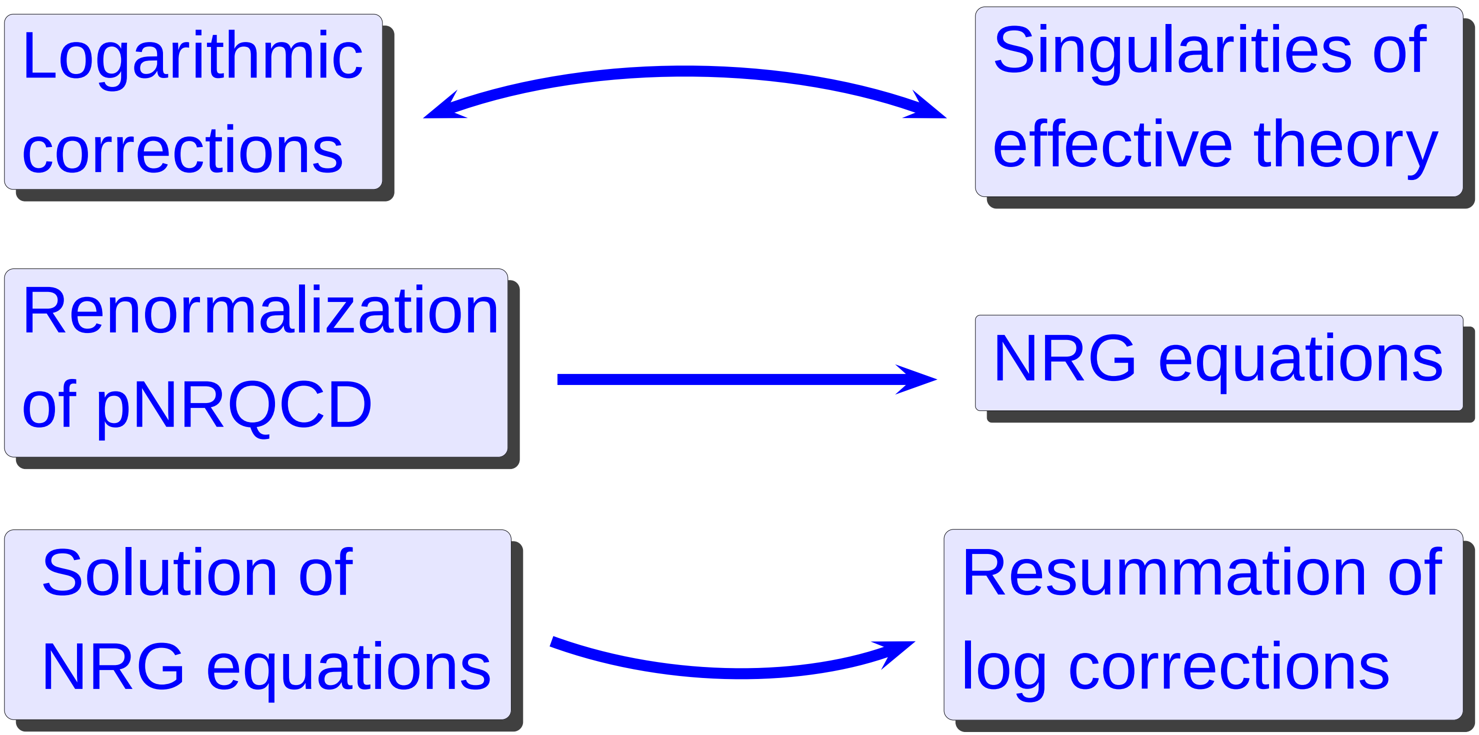
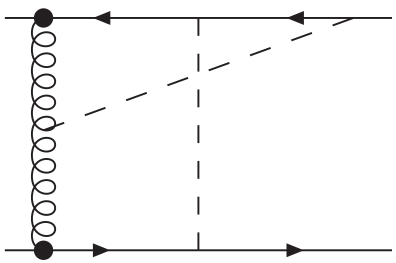
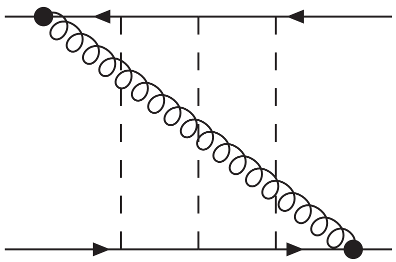


NRG running of spin-flip potential

- NLL running ⇨ $\mathcal{O}(\alpha_s^m \ln^{m-1} \alpha_s)$
  - Two-loop soft running:



  - Three-loop ultrasoft-potential running



NRG running of spin-flip potential

- NLL soft running
  - *NRG equation*

$$\mu_s \frac{d}{d\mu_s} D^{(2)}_{S^2,s} = \alpha_s(\mu_s) c_F^2(\mu_s) \gamma_s(\alpha_s)$$

  - *LL Fermi coupling* $c_F(\mu) = (\alpha_s(\mu)/\alpha_s(m_q))^{-C_A/\beta_0}$
  - *Two-loop anomalous dimension* $\gamma_s = \gamma_s^{(1)} \frac{\alpha_s}{\pi} + \cdots$

$$\gamma_s^{(2)} = \frac{1}{216}\left[{C_A}^2\,\left(5 - 36\,\pi^2\right) + 88\,C_A\,n_l\,T_F + 4\,n_l\,T_F\,\left(27\,C_F - 40\,n_l\,T_F\right)\right]$$

Penin, Pineda, Smirnov, Steinhauser, Phys.Lett. B593, 124 (2004)

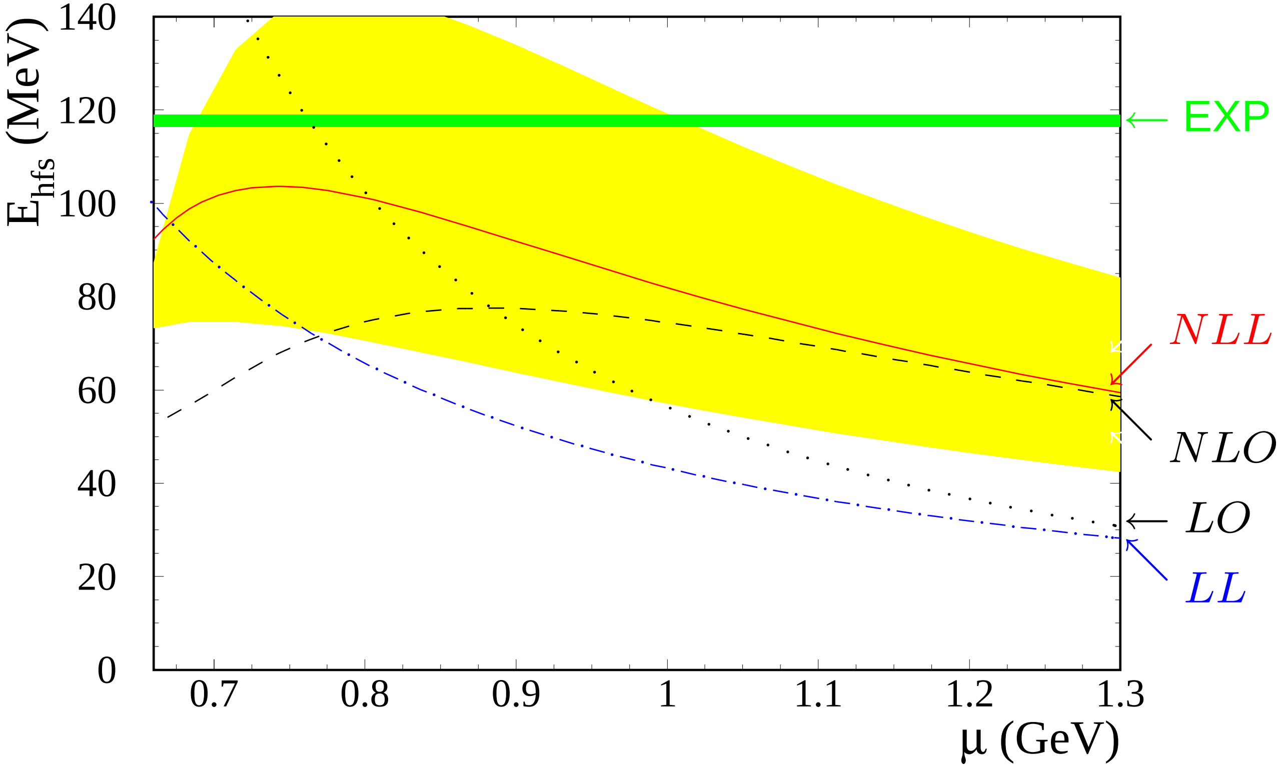
Bottomonium HFS in NLL approximation

Kniehl, Penin, Pineda, Smirnov, Steinhauser, Phys.Rev.Lett. 92, 242001 (2004)

$$
\begin{aligned}
M_{\Upsilon(1S)} - M_{\eta_b} = \frac{C_F^4 \alpha_s^4(\nu) m_b}{3} \Bigg\{ \frac{27}{14} y^{-1} - \frac{13}{14} y^{-\frac{18}{25}} + \frac{\alpha_s(m_b)}{\pi} \Bigg[\Bigg(\frac{1037}{224} + \frac{405086361761\,\pi^2}{25617160800} \\
- \frac{3}{4} \ln 2 \Bigg) \times y^{-1} - \frac{1024\,\pi^2}{143} y^{-\frac{39}{50}} - \left(\frac{102973}{26250} + \frac{184336\,\pi^2}{25725} \right) y^{-\frac{18}{25}} + \frac{1024\,\pi^2}{675} y^{-\frac{1}{2}} + \frac{671\,\pi^2}{1029} y^{-} \\
- \frac{3\,\pi^2}{23} y^{-\frac{2}{25}} + \left(- \frac{13427921}{1260000} + \frac{88057\,\pi^2}{151200} \right) y^{\frac{7}{25}} + \frac{4\,\pi^2}{41} y^{\frac{16}{25}} + \frac{1377}{56} - \frac{1253587\,\pi^2}{227500} - \frac{629\,\pi^2}{7500} y \\
- \frac{2873\,\pi^2}{7182} y^{\frac{32}{25}} \, {}_2F_1 \left(\frac{57}{25}, 1; \frac{82}{25}; \frac{y}{2} \right) + \frac{2873\,\pi^2}{3591} y^{-1} \, {}_2F_1 \left(1, 1; \frac{82}{25}; -1 \right) + \left(\frac{675}{28} - \frac{533}{42} y^{\frac{7}{25}} \right) \\
\times \ln \left(\frac{\mu}{C_F \alpha_s(\mu) m_b} \right) + \frac{85248\,\pi^2}{30625} y^{-1} \ln y + \left(- \frac{45834}{4375} y^{-1} + \frac{21216}{4375} - \frac{2873}{1575} y^{\frac{7}{25}} + \frac{243}{1250} y \right) \\
\times \pi^2 \ln(2 - y) \Bigg] \Bigg\} ,
\end{aligned}
$$

$$
y = \frac{\alpha_s(\mu)}{\alpha_s(m_b)}
$$

HFS in charmonium



$E_{\rm hfs} = M(J/\Psi) - M(\eta_c)$

$$n = 1$$

- QCD NLL approximation:

$$E^{\rm th}_{\rm hfs} = 112\ {\rm MeV}$$

- Experiment:

$$E^{\rm exp}_{\rm hfs} = 117.7 \pm 1.3\ {\rm MeV}$$

$E_{\rm hfs} = M(J/\Psi) - M(\eta_c)$

$\boldsymbol{n} = 2$

- QCD NLL approximation:

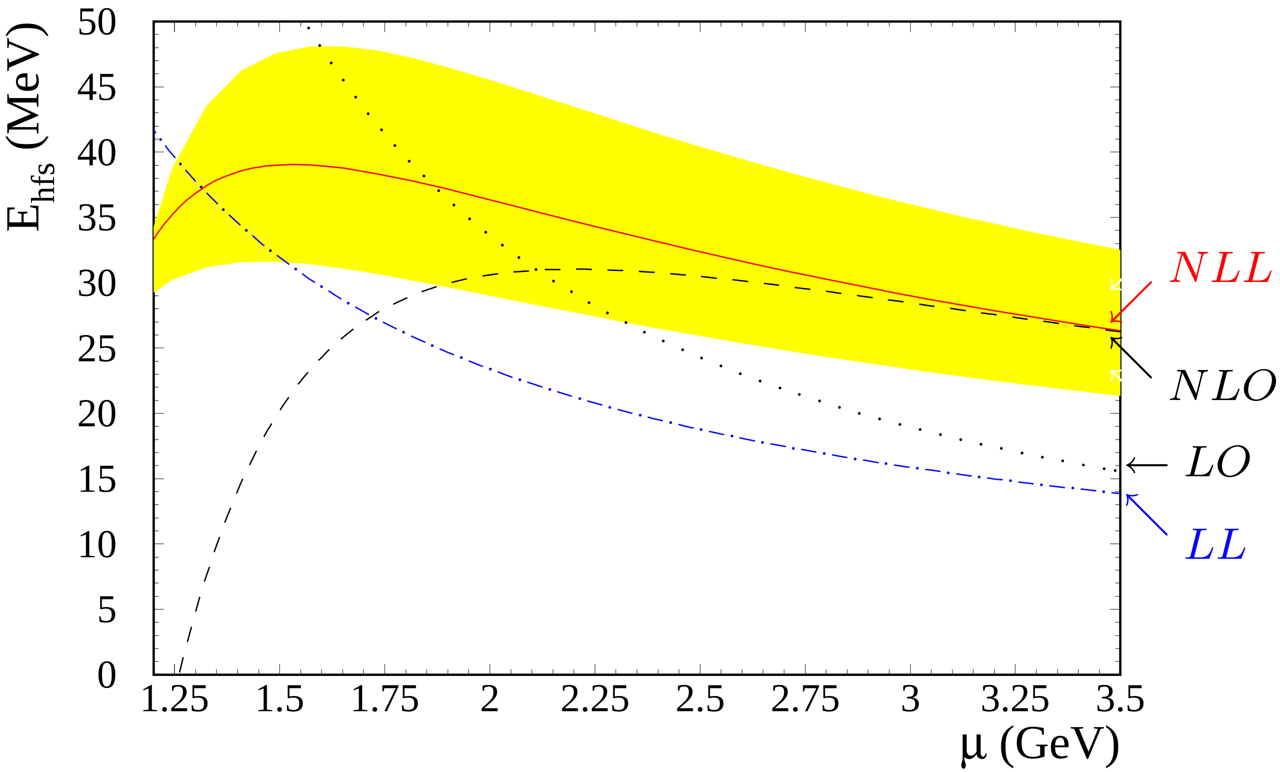
$$E_{\rm hfs}^{\rm th} = 41 \ \mathrm{MeV}$$

- Experiment:

$$E_{\rm hfs}^{\rm exp} = 41 \pm 3 \ \mathrm{MeV}$$

HFS in bottomonium

(Kniehl, Penin, Pineda, Smirnov, Steinhauser)



$E_{\rm hfs} = M(\Upsilon_{1S}) - M(\eta_b)$

- QCD NLL approximation:

$$E^{\rm th}_{\rm hfs} = 41 \pm 11\,({\rm th})\,{}^{+9}_{-8}\,(\delta\alpha_s)\ {\rm MeV}$$

- Experiment:

$$E^{\rm exp}_{\rm hfs} = 71.4 \pm 2.7\,({\rm syst})\,{}^{+2.3}_{-3.1}\,({\rm stat})\ {\rm MeV}$$

Why perturbative QCD *works* for charmonium but *fails* for bottomonium HFS?

- *Nonperturbative correction scales as* $1/m_q^4$
  Perturbative correction scales as $\alpha_s(\alpha_s m_q)$

➥ *Cancellation of huge perturbative and nonperturbative corrections for charmonium, no cancellation for bottomonium?*

- *Hard to belive since leading* $\mathcal{O}(v^2)$ *nonperturbative contribution is positive* ⇨ *bottomonium HFS would be even smaller!*

Lattice QCD

- “Full” lattice NRQCD simulations:

$$E_{\rm hfs}^{\rm lat} = 61 \pm 14 \ {\rm MeV}$$

A. Gray *et al.*, Phys.Rev. D72, 094507 (2005)

- Experiment:

$$E_{\rm hfs}^{\rm exp} = 71.4 \pm 2.7\,({\rm syst})\,{}^{+2.3}_{-3.1}\,({\rm stat})\ {\rm MeV}$$

Lattice QCD

- Hard cutoff $1/a \sim 2\ \mathrm{GeV}$ from spin-avarage spectrum
- Logarithmic contribution of the hard modes:

$$\delta^{\mathrm{hard}} E_{\mathrm{hfs}} = -\frac{\alpha_s}{\pi} \frac{7\, C_A}{4} \ln\left(a m_b\right) E_{\mathrm{hfs}} \approx -20\ \mathrm{MeV}$$

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$$E^{\mathrm{lat}}_{\mathrm{hfs}} + \delta^{\mathrm{hard}} E_{\mathrm{hfs}} \approx 40\ \mathrm{MeV}$$

➥ *in perfect agreement with perturbative QCD!*

η_b mass puzzle

- Origin of discrepancy
  - *Underestimate of theoretical uncertainty*
  - *Underestimate of experimental uncertainty*
  - *Exotic* e.g. *light CP-odd Higgs Boson*

Domingo *et al.*, Phys.Rev.Lett. 103, 111802 (2009)

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- BaBar guys are very angry
  - ➥ better to solve the problem now!