Finite top quark mass effects in NNLO Higgs boson production at LHC

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work done in collaboration with
Matthias Steinhauser and Mikhail Rogal

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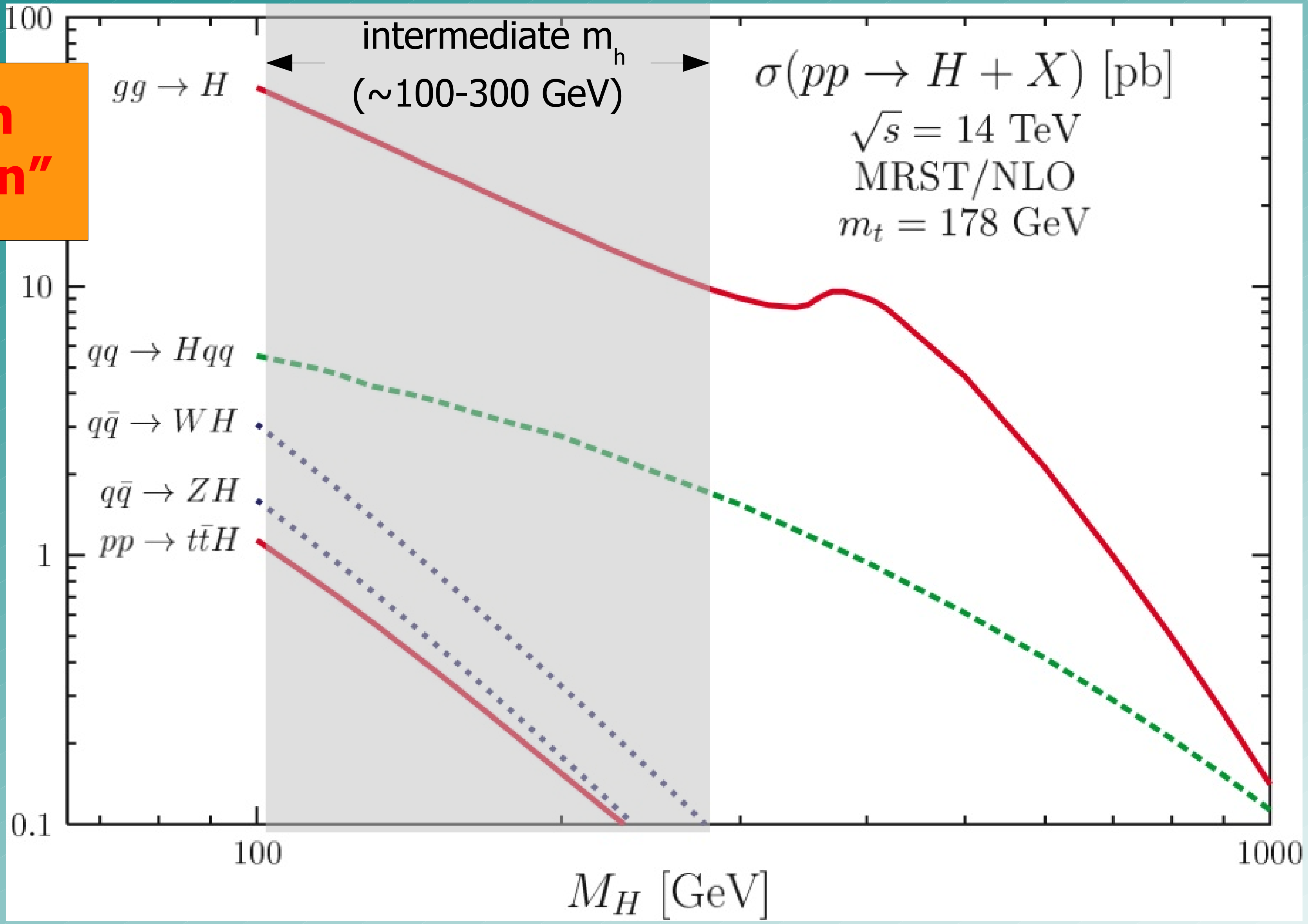
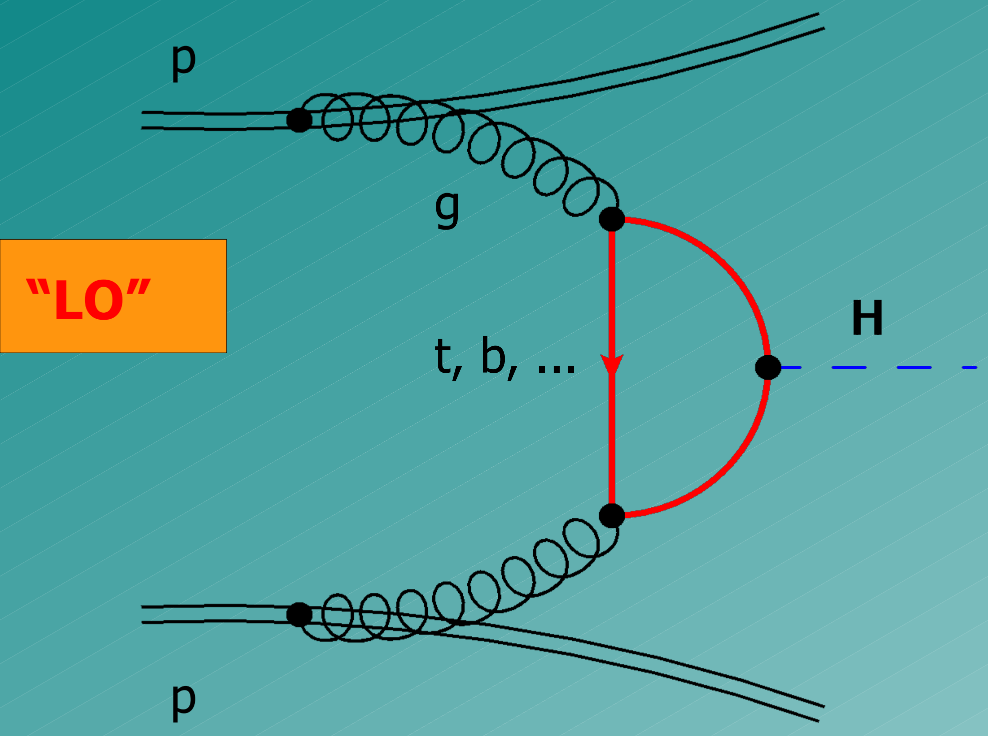
Higgs boson production at the LHC: pp → H+X

"Gluon fusion"



(plot borrowed from arXiv:0911.4409 by Bustamante, Cieri, Ellis)

Higgs boson production at the LHC: pp → H+X



Dominant mode: **gg → H** via a top-quark loop

Very well studied process!

Characteristic scales:

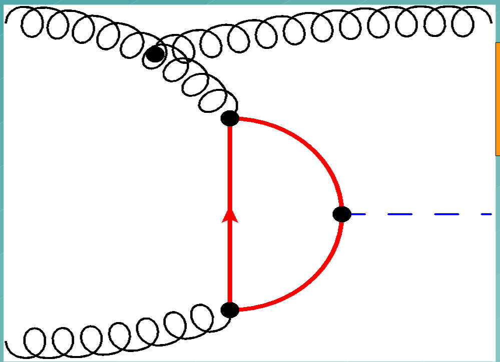
$$\sqrt{S} \sim 14\,\mathrm{TeV}\,(\,protons\,)$$
$$\sqrt{s} \sim 100 - 14000\,\mathrm{GeV}\,(\,partons\,)$$
$$m_h \sim 100 - 300\,\mathrm{GeV}$$
$$m_t \sim 170\,\mathrm{GeV}$$

Leading order: **[Geordi, Glashow, Machacek, Nanopoulos '78]**
(full dependence on m_h , m_t)

Theoretical predictions (until recently)

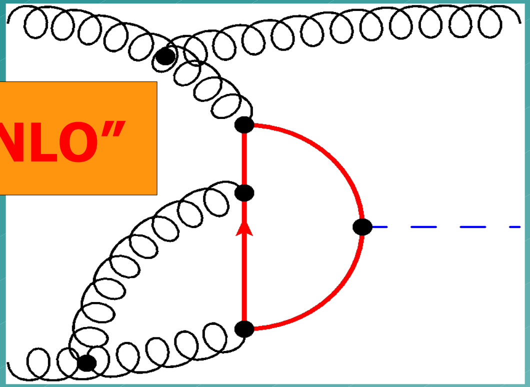
Inclusive cross-section @ $O(\alpha_s)$: ~O(70%)

[Dawson; Djouadi, Spira, Zerwas '91]
[Spira et al '95] (exact)



"NLO"

"NNLO"



Cross-section @ $O(\alpha_s^2)$: ~O(10%), scale dep. O(%)

[Harlander, Kilgore '02] (soft expansion)
[Anastasiou, Melnikov '02],
[Ravindran, Smith, van Neerven '03]

Beyond fixed order PT (improve scale dep.):
NNLO + NNLL
N^3LO threshold-enhanced
π^2-resummation, ...

Fully differential:
NLO (exact), **NNLO**

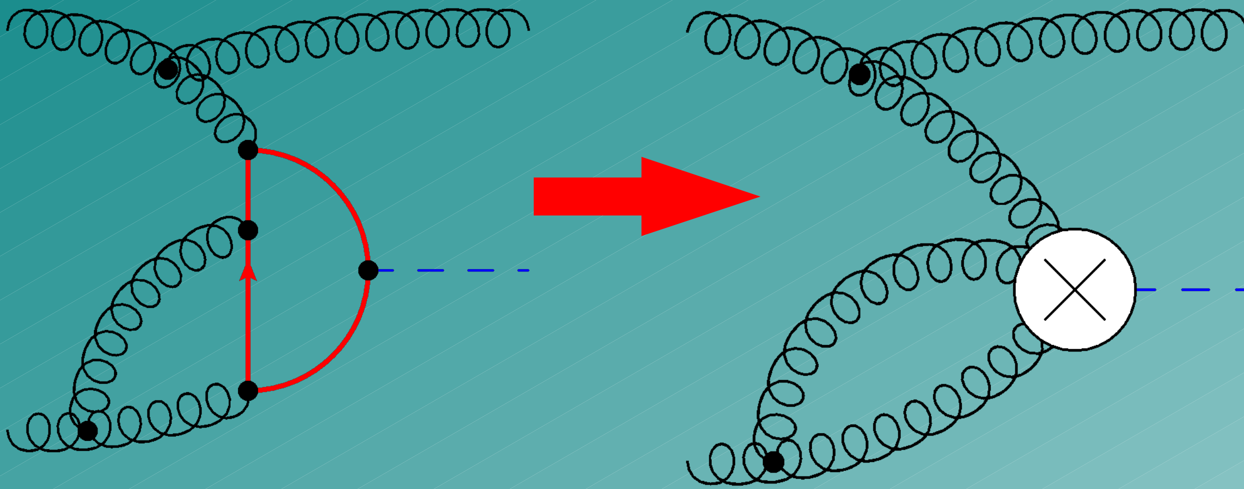
Also: EW, QCD-EW, ...

Catani, de Florian, Grazzini, Nason; Ahrens, Becher, Neubert, Yang; Actis, Passarino, Sturm, Uccirati; Anastasiou, Boughezal, Petriello; Moch, Vogt; ...

Results in green: $m_t \to \infty$ as heavy top effective field theory (EFT)

Heavy top limit: effective theory

Formally integrate top quark out => effective ggH, gggH, ... vertices



$$L_{eff} = C \cdot H\, G_{\mu\nu} G^{\mu\nu}$$

[Shroeder, Steinhauser; Chetyrkin, Kuehn, Sturm; Spira et al]

Assumptions:

$$\frac{m_h^2}{4\, m_t^2} \ll 1,$$

$$s = (p_1 + p_2)^2 \ll m_t^2$$

← **i.e. CM energy assumed much less than top mass**

Heavy top limit: effective theory

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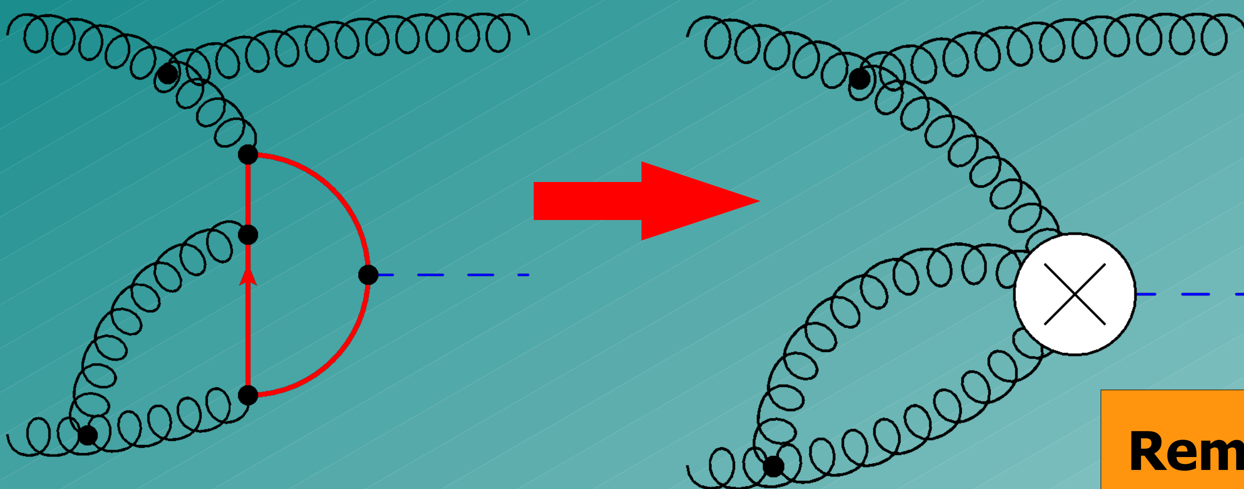

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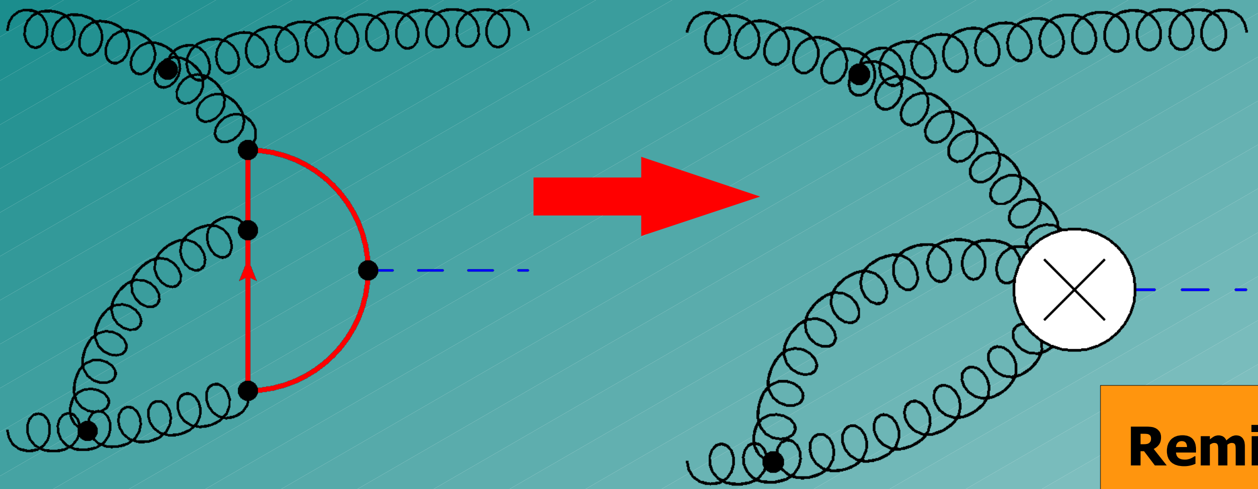

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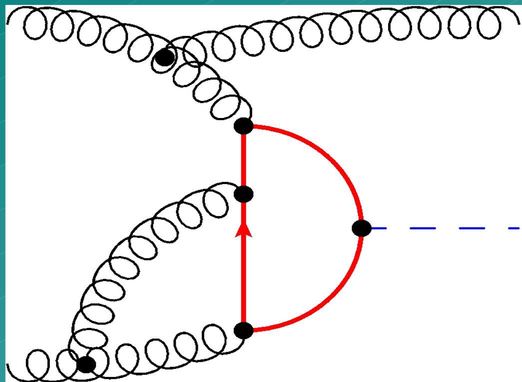
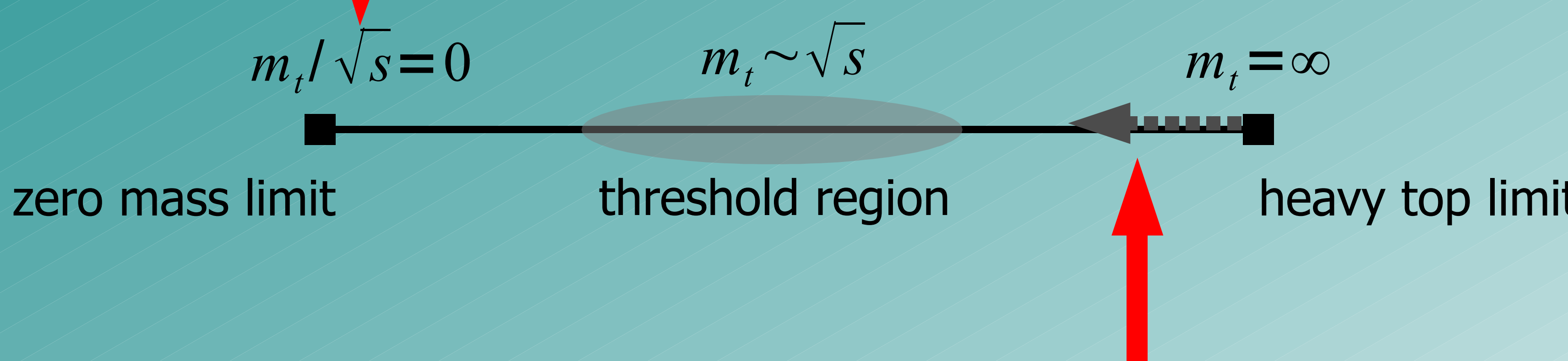
Nevertheless, agrees perfectly with exact NLO (and very convenient). What about NNLO?

Beyond the $m_t \to \infty$ limit at the NNLO



NNLO large s leading logs (m_t/s, $m_h/s \to 0$):

[Marzani et al '08]: gg channel (also quark channels in [Harlander,Mantler,Marzani,Ozeren '09])

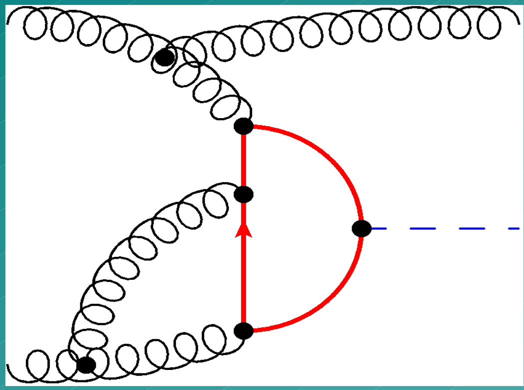


1/m_t expansion near heavy top limit (in s/m_t, $m_h/m_t \to 0$):

[Harlander, Ozeren; Pak, Steinhauser, Rogal '09]: virtual corrections

[Harlander, Ozeren '09]: full NNLO cross-section (soft expansion)

Beyond the $m_t \to \infty$ limit at the NNLO



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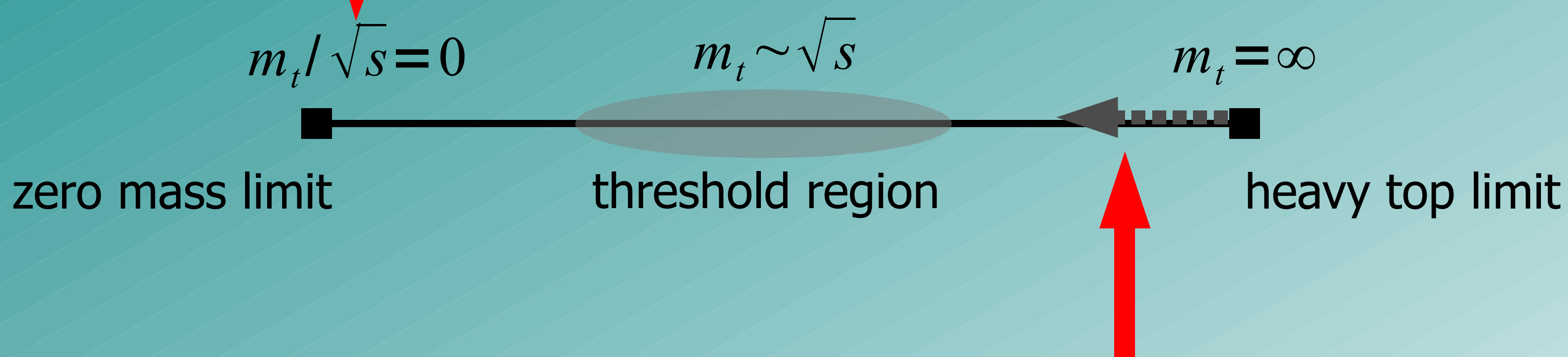

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[Harlander, Ozeren; Pak, Steinhauser, Rogal '09]: virtual corrections

[Harlander, Ozeren '09]: full NNLO cross-section (soft expansion)

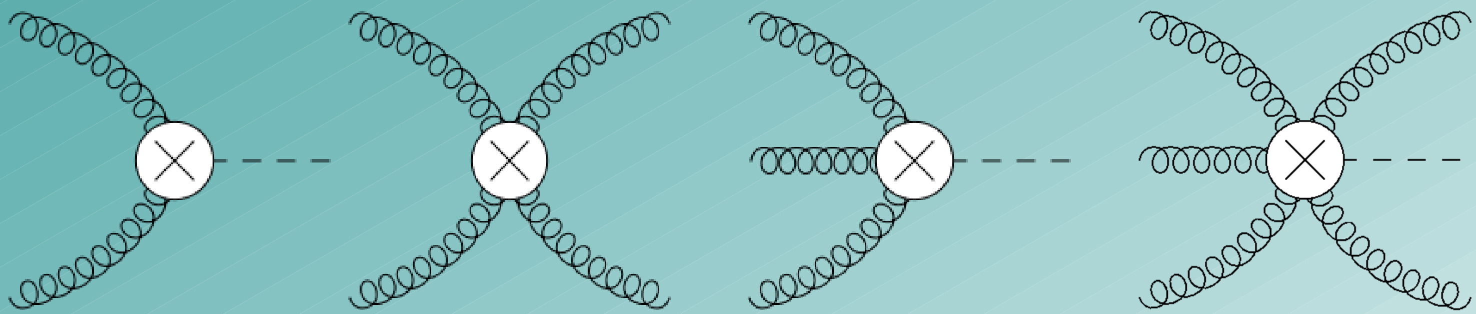
This work: same expansion, full NNLO cross-section, independent confirmation by a different method

Asymptotic expansion as alternative to EFT

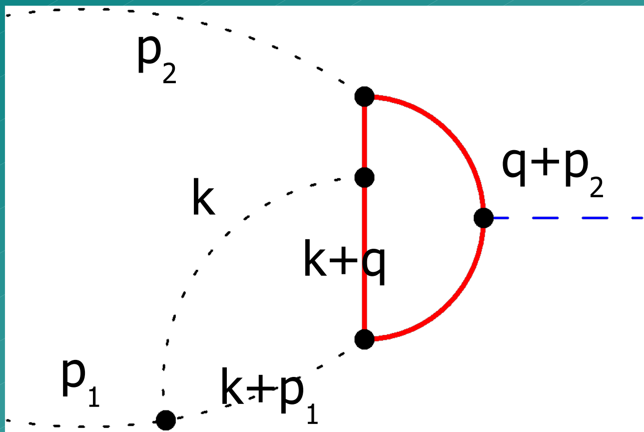
Can use EFT to obtain $O(1/m_t^n)$ terms (e.g. [Neill, 09]):

- complicated power counting
- operators of higher dimensions
- Wilson coefficients
- Feynman rules
- renormalization

complexity grows with expansion order



Alternative to EFT: asymptotic expansion

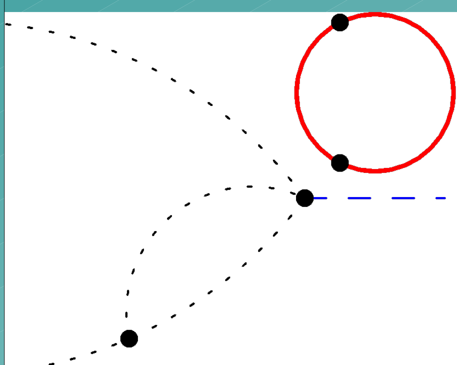


$$\int \frac{d^D k\, d^D q}{((q+k)^2+m_t^2)(k+p_1)^2 \ldots}$$

loop momenta *k*, *q* can be "large" or "small", => two non-zero expansion "regions":

$m_t, |q| \gg |k|, |p_1|, |p_2|$

$$\frac{1}{(q+k)^2+m_t^2} = \frac{1}{q^2+m_t^2} - \frac{2qk+k^2}{(q^2+m_t^2)^2} + \ldots$$

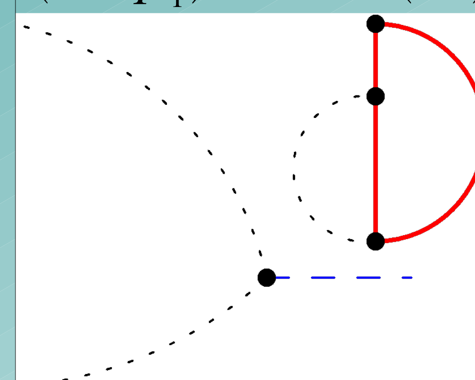


$$\times \theta(L-|k|)$$

$+$

$m_t, |q|, |k| \gg |p_1|, |p_2|$

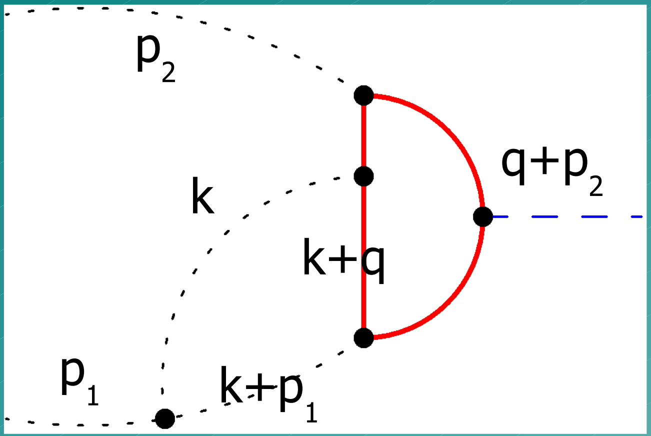
$$\frac{1}{(k+p_1)^2} = \frac{1}{k^2} - \frac{2k\,p_1}{(k^2)^2} + \ldots$$



$$\times \theta(|k|-L)$$

Cutoff L: $m_t \gg L \gg |p_1|, |p_2|$ **- not real loop integrals, difficult to compute**

Alternative to EFT: asymptotic expansion



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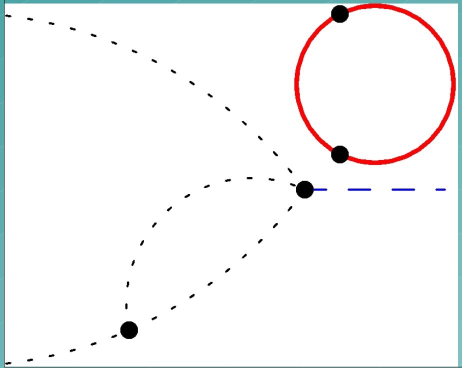

+

$m_t, |q|, |k| \gg |p_1|, |p_2|$

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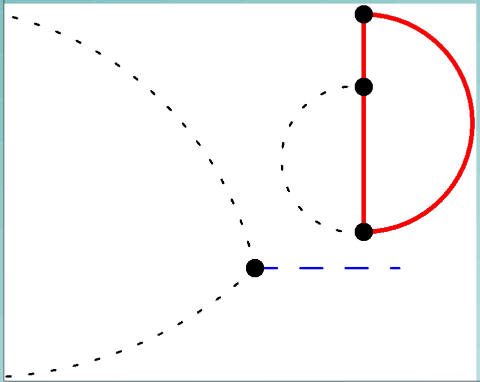
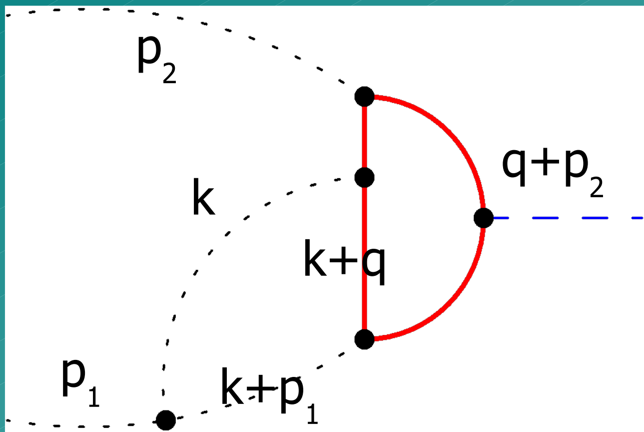


Non-trivial step: drop constraints, no double-counting occurs!
[Chetyrkin, Smirnov, Tkachov, ...]

Alternative to EFT: asymptotic expansion



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loop momenta *k*, *q* can be "large" or "small",
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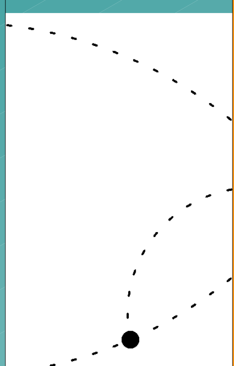
$m_t, |q| \gg |k|, |p_1|, |p_2|$

$\frac{1}{(q+k)^2+ \ldots} = \frac{1}{\ldots} - \frac{2qk+k^2}{\ldots} + \ldots$



$m_t, |q|, |k| \gg |p_1|, |p_2|$

$\frac{1}{\ldots} = \frac{1}{\ldots} - \frac{2k\,p_1}{\ldots} + \ldots$

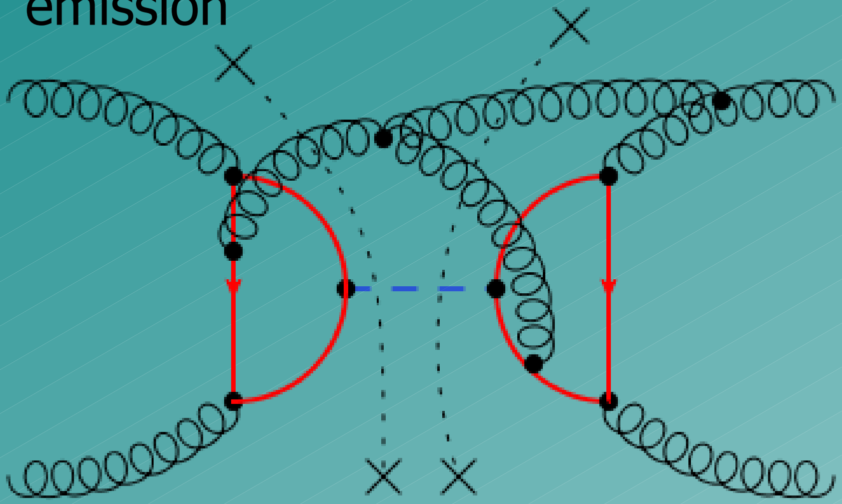
Advantage of asymptotic expansion over EFT:
- need only program the expansion and integrals once, additional orders just require more CPU time

Disadvantages:
- larger number of more complex diagrams, heavy top limit harder to reproduce

Still, exactly the same limitations and predictive power! (identical analytic results)

Technical details

Optical theorem:
avoid summation over final states and integration over phase space, uniformly treat virtual and real emission



4-loop, imaginary part only, forward scattering diagrams

Many diagrams: ~ 20000

Can apply multi-loop methods to phase space integration!

Use multi-loop toolchain:

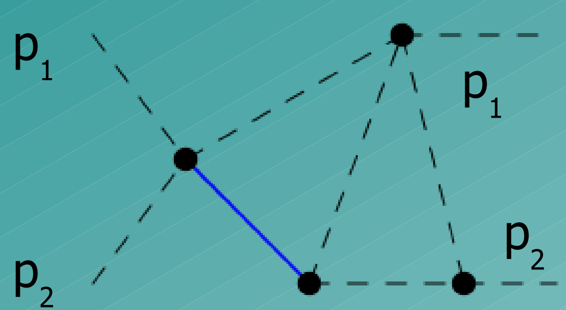
- Automated diagrams generation (**QGRAF** + custom filters)
- Automated asymptotic expansion (**EXP/Q2E** and custom program)
- Symbolic calculations: **FORM** (**MATAD** and custom program), **Mathematica**, custom Laporta implementation (Perl, C++, uses **FERMAT**)
- Most steps have independent cross-checks, analytic and numerical

Rather demanding:
~ month @ TTP cluster,
~ 100s GB of intermediate data

Technical details

Master integrals: published but required cross-checks (many errors).
Method of differential equations (DE):

$$U(x, D) =$$



$$x = \frac{m_h^2}{s}, \quad s = (p_1 + p_2)^2$$

With IBP's: $$\frac{d}{dx} U(x, D) = A(x, D) \cdot U(x, D) + (\text{simpler integrals})$$

Solve order by order in ε, use soft expansion (x = 1 limit) to fix integration constants

Results in terms of Harmonic Polylogarithms - special functions, very convenient for automated computations, valid for any values of x

$$\int_0^y x^a (1-x)^b (1+x)^c H(1,0,-1,\ldots,x) = y^d (1-y)^e (1+y)^f H(\ldots,y) + \ldots$$

Technical details

- DE solution to O(ε^n): $$U = x^n (1-x)^m (1+x)^k H(\ldots, x) + \ldots$$
- Can easily be divergent at x=1, need to restore delta- and plus-pieces
- Soft expansion to O(ε^{n+1}), leading term only: $$C(\epsilon)(1-x)^{k-a\epsilon}$$
- Ansatz:

$$U \to U + C(\epsilon)(1-x)^{k+1}\left[(1-x)^{-1-a\epsilon} - (1-x)^{-1-a\epsilon}\right]$$

Expand in distributions:

$$\frac{1}{y^{1+a\epsilon}} = \frac{\delta(y)}{a\epsilon} + \left[\frac{1}{y}\right]_+ + \ldots$$

Expand "naively":

$$\frac{1}{y^{1+a\epsilon}} = \frac{1}{y} - \frac{a \ln y}{y} + \ldots$$

- cancels singularities in HPLs

Structure of partonic cross-sections

After adding all contributions, renormalization, cancellation of collinear singularities, etc:

$$\sigma_{gg} \sim A^{(0)}\delta(1-x) + \left(\frac{\alpha_s}{\pi}\right)\left(A^{(1)}\delta(1-x) + B_{+}^{(1)}(x) + C^{(1)}(x)\right)$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^2\left(A^{(2)}\delta(1-x) + B_{+}^{(2)}(x) + C^{(2)}(x)\right)$$

$$x = \frac{m_h^2}{s}$$
$$s = (p_1 + p_2)^2$$
$$\rho = \frac{m_h^2}{m_t^2}$$

Coefficients A, B, C – series in ρ (only even powers)

We have managed to obtain O($1/m_t^4$) terms for gg, and O($1/m_t^6$) terms for the quark channels.

Expanding our results in powers of (1-x) ("soft expansion"), we find complete agreement with Harlander and Ozeren!

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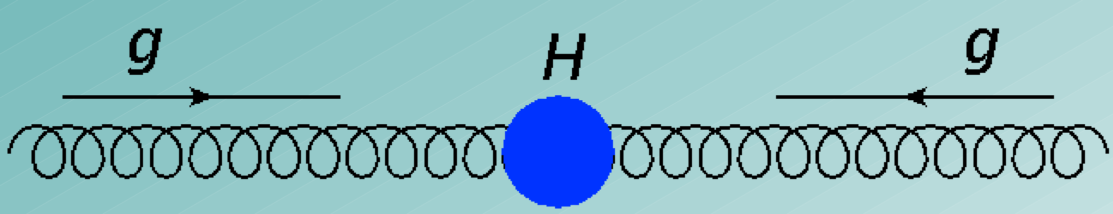
$$x = \frac{m_h^2}{s}$$
$$s = (p_1 + p_2)^2$$
$$\rho = \frac{m_h^2}{m_t^2}$$

Purely virtual contributions, no + ultra-soft real radiation, $1/m_t$ expansion is completely OK



$$\sqrt{s} = m_h$$
$$x = 1$$

Structure of partonic cross-sections

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$$x = \frac{m_h^2}{s}$$

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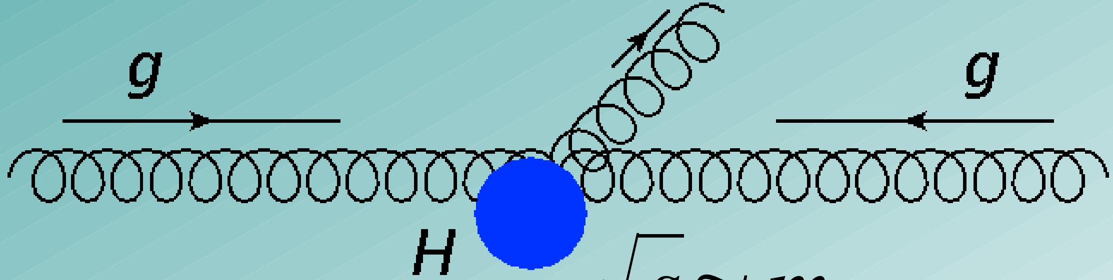
$$\rho = \frac{m_h^2}{m_t^2}$$

Plus-distributions (enhanced near x=1), radiation of very soft gluons, $1/m_t$ expansion is OK



$$\sqrt{s} \approx m_h$$

$$(1-x) \ll 1$$

Structure of partonic cross-sections

After adding all contributions, renormalization, cancellation of collinear singularities, etc:

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$$+\left(\frac{\alpha_s}{\pi}\right)^2\left(A^{(2)}\delta(1-x) + B_+^{(2)}(x) + C^{(2)}(x)\right)$$

$$x = \frac{m_h^2}{s}$$
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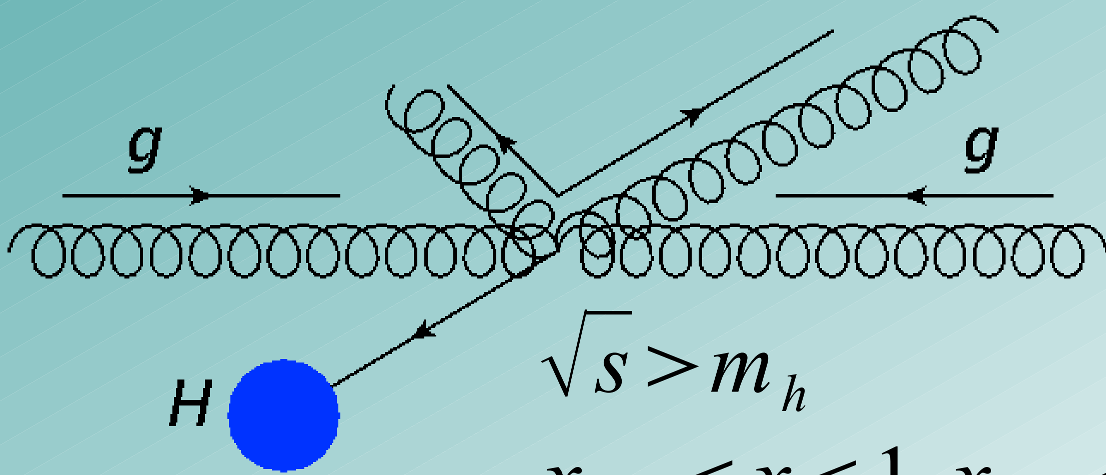
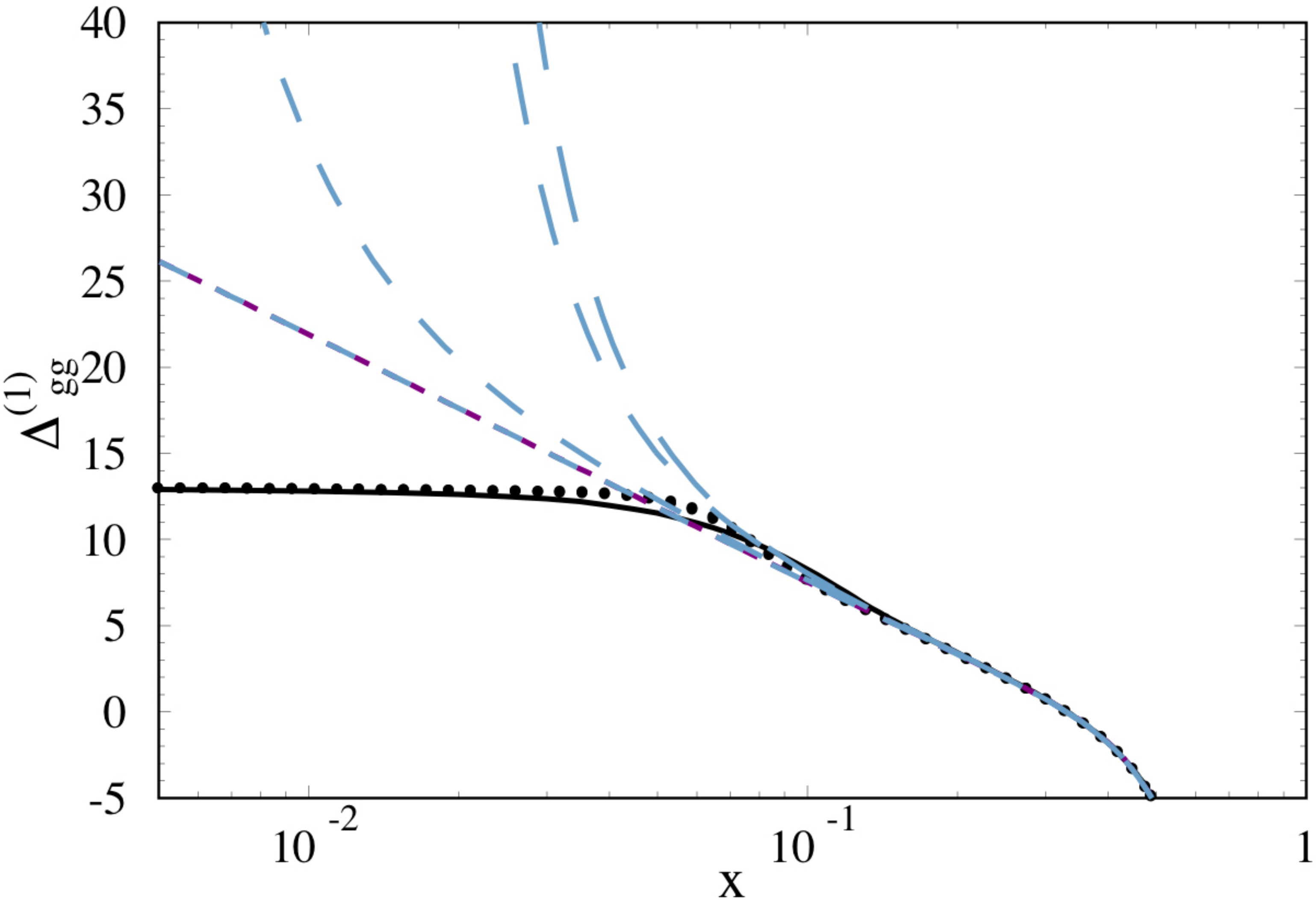
Hard real radiation, functions depending on x.
$1/m_t$ expansion OK below the top production threshold



$$\sqrt{s} > m_h$$
$$x_{min} < x < 1, x_{min} \ll 1$$

NLO top mass effects, gg channel (non-singular)



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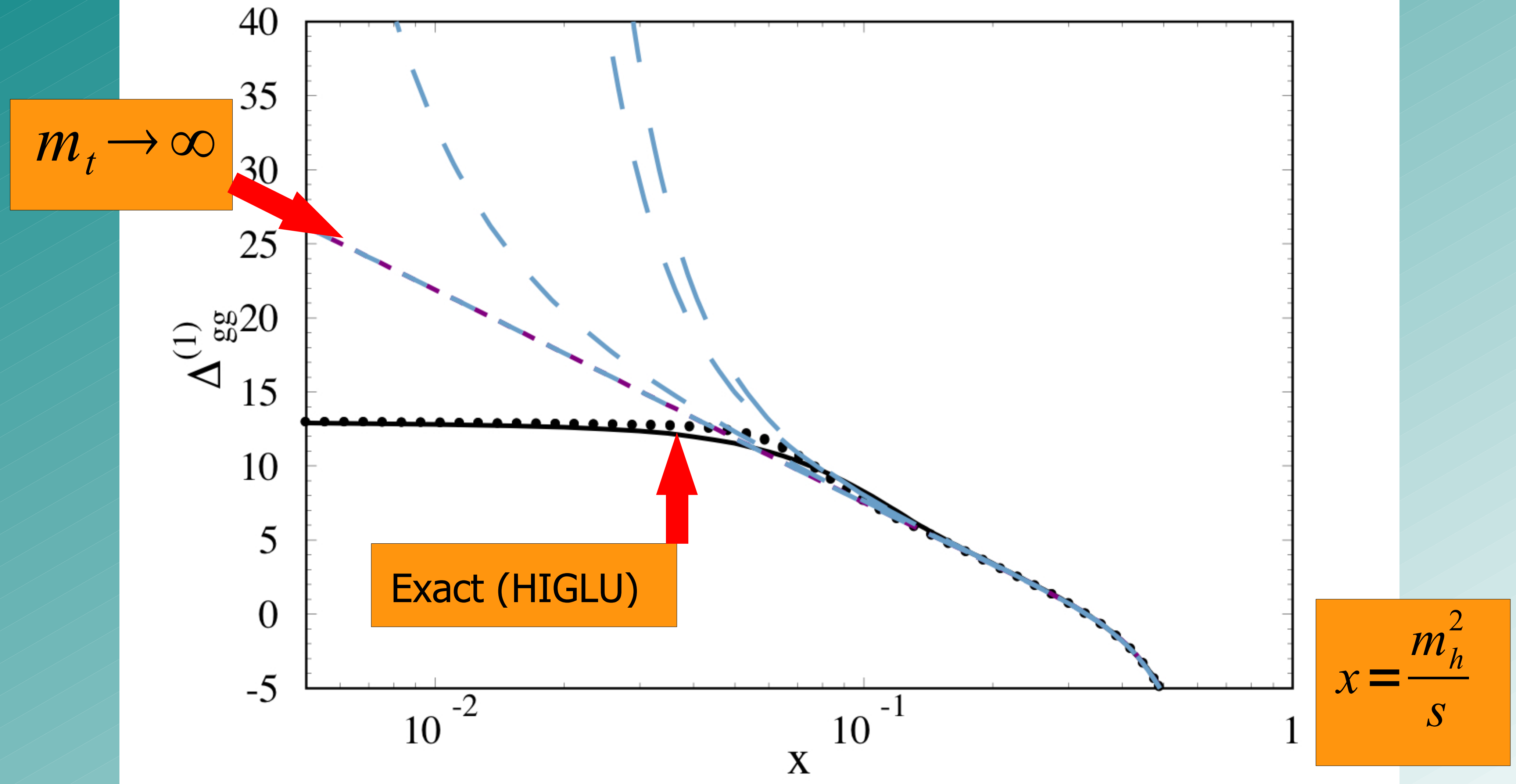


NLO top mass effects, gg channel (non-singular)

Singularities due to:

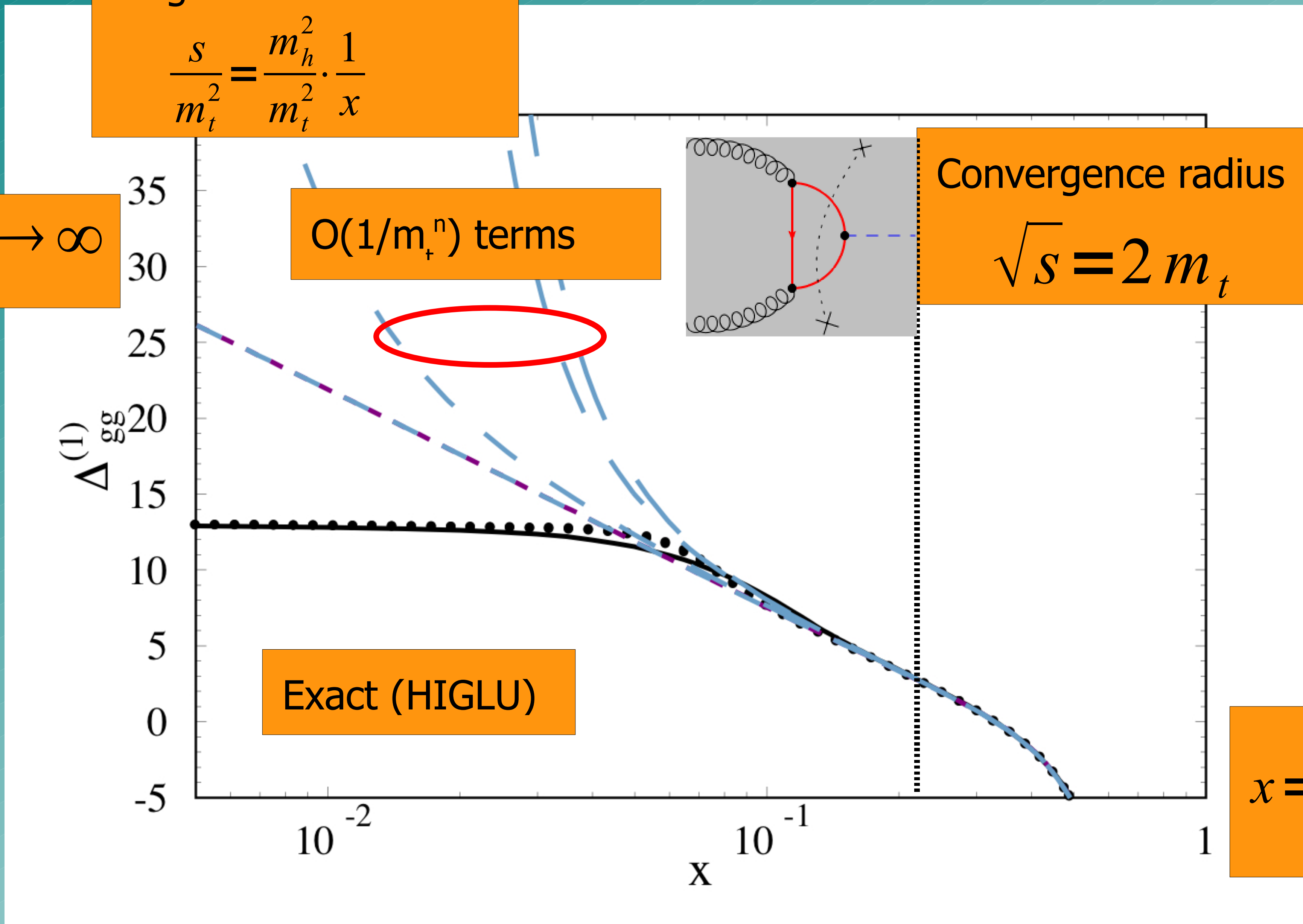
$$\frac{s}{m_t^2} = \frac{m_h^2}{m_t^2} \cdot \frac{1}{x}$$

$m_t \to \infty$

O(1/m_t^n) terms

Convergence radius

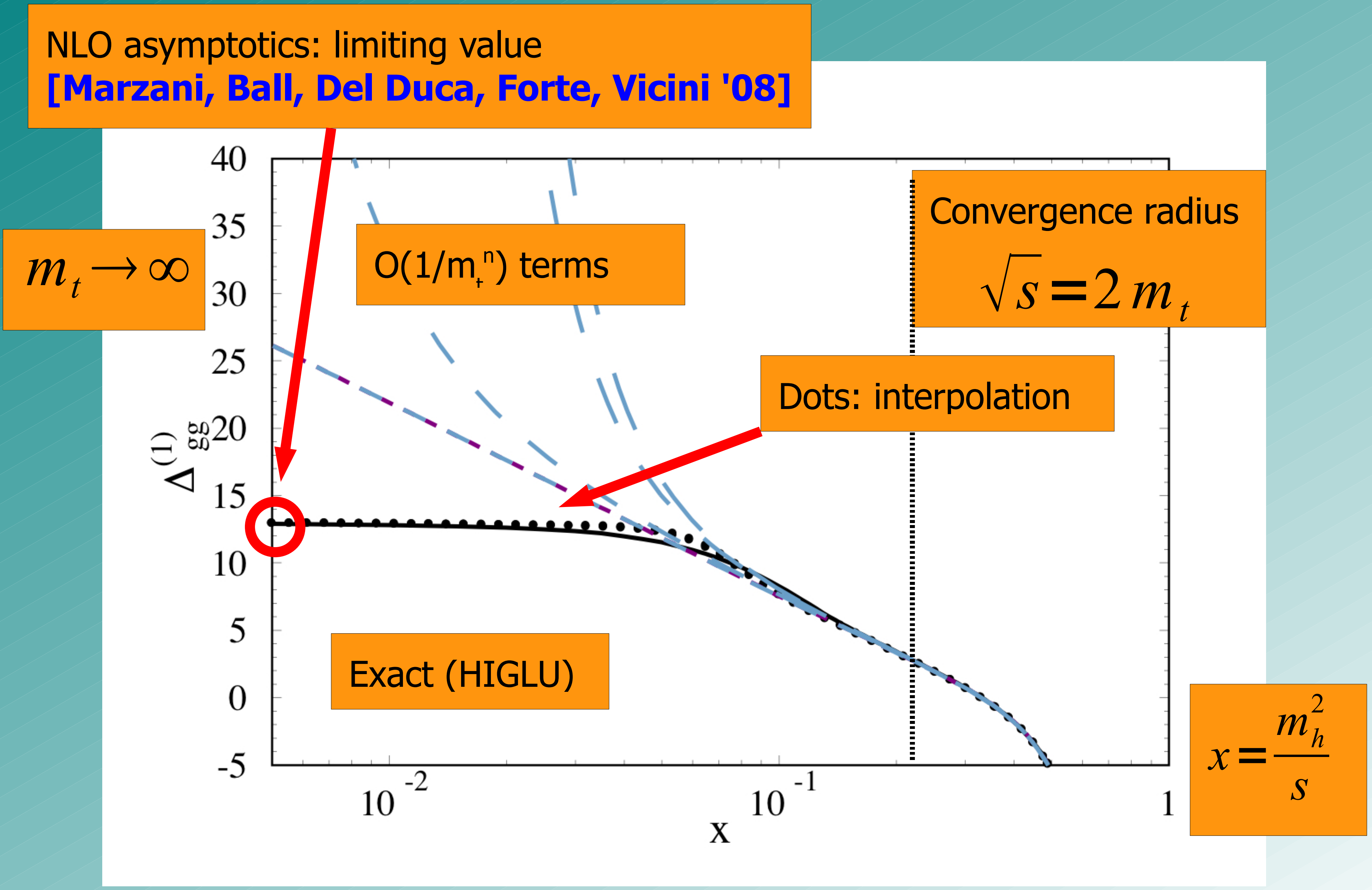
$$\sqrt{s} = 2\, m_t$$



Exact (HIGLU)

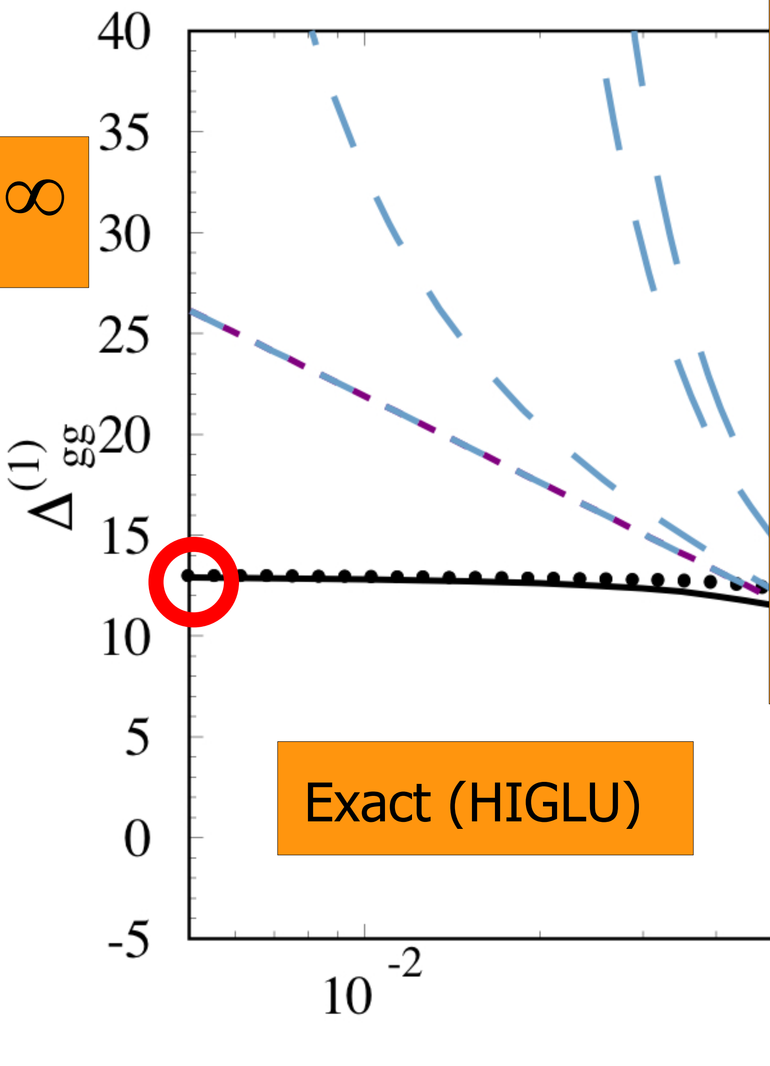
$$x = \frac{m_h^2}{s}$$

NLO top mass effects, gg channel (non-singular)



NLO top mass effects, gg channel (non-singular)

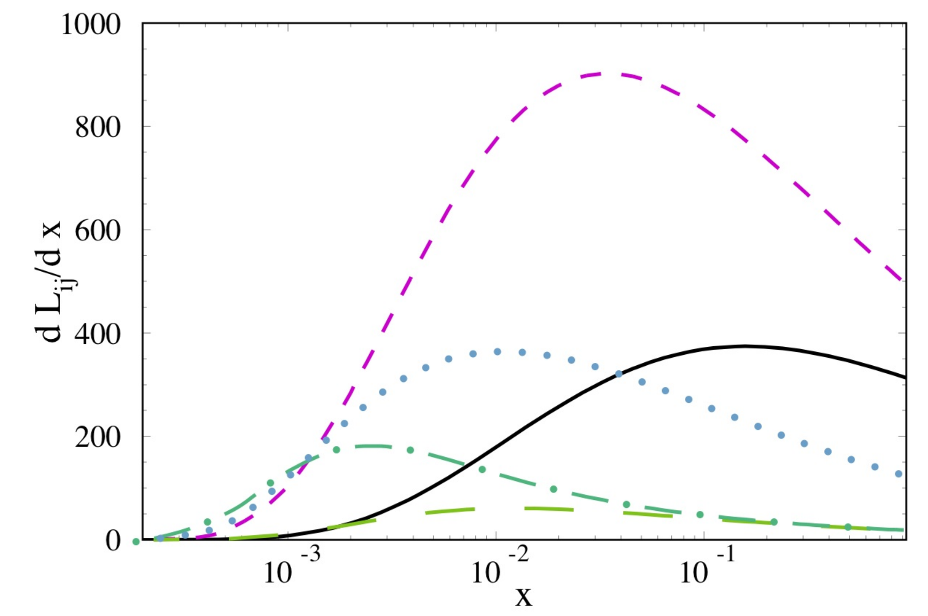
$m_t \to \infty$



Exact (HIGLU)

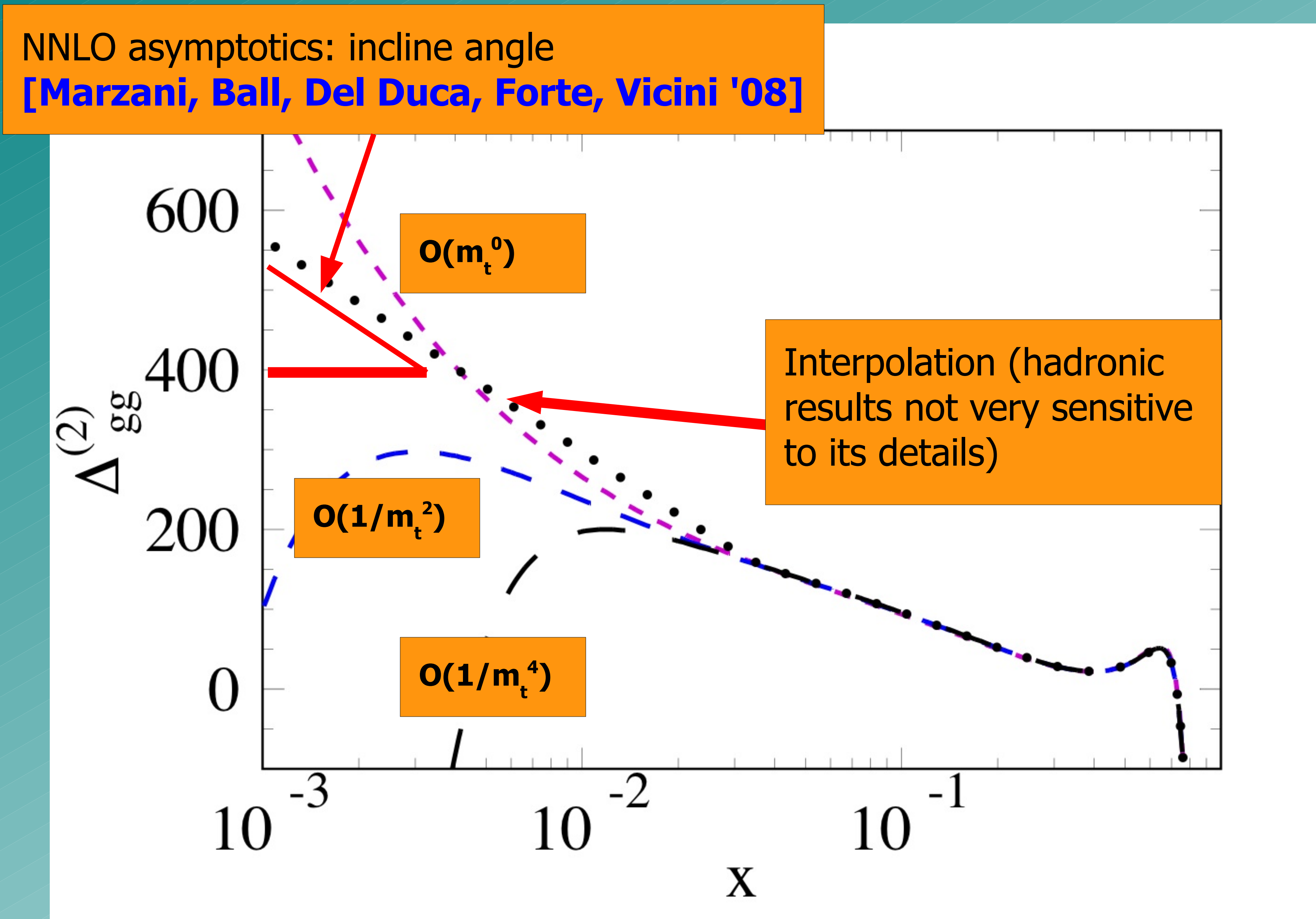
Luminosity functions: gg, qg, qq, ...

All suppressed near x=0

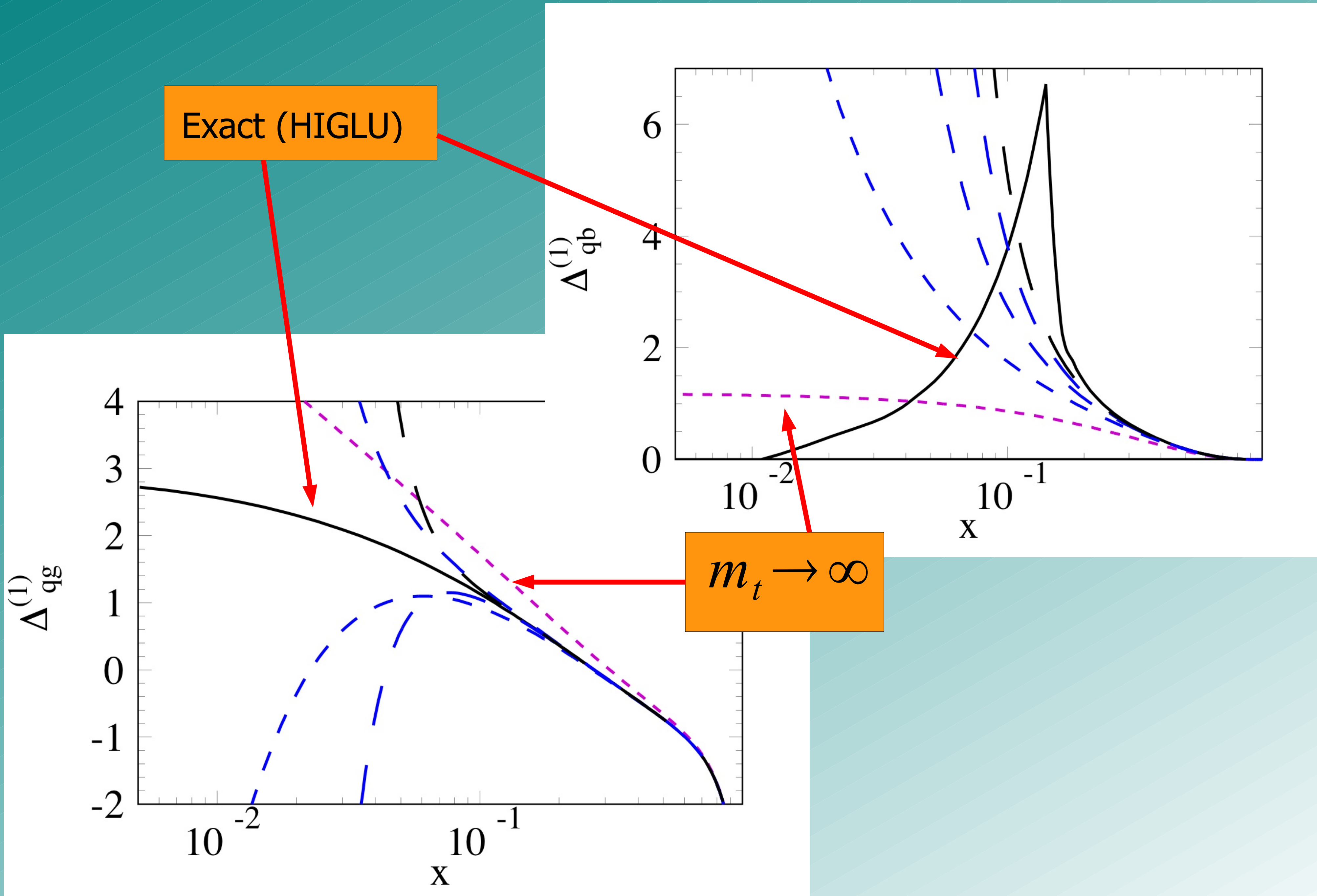


$$x = \frac{m_h^2}{s}$$

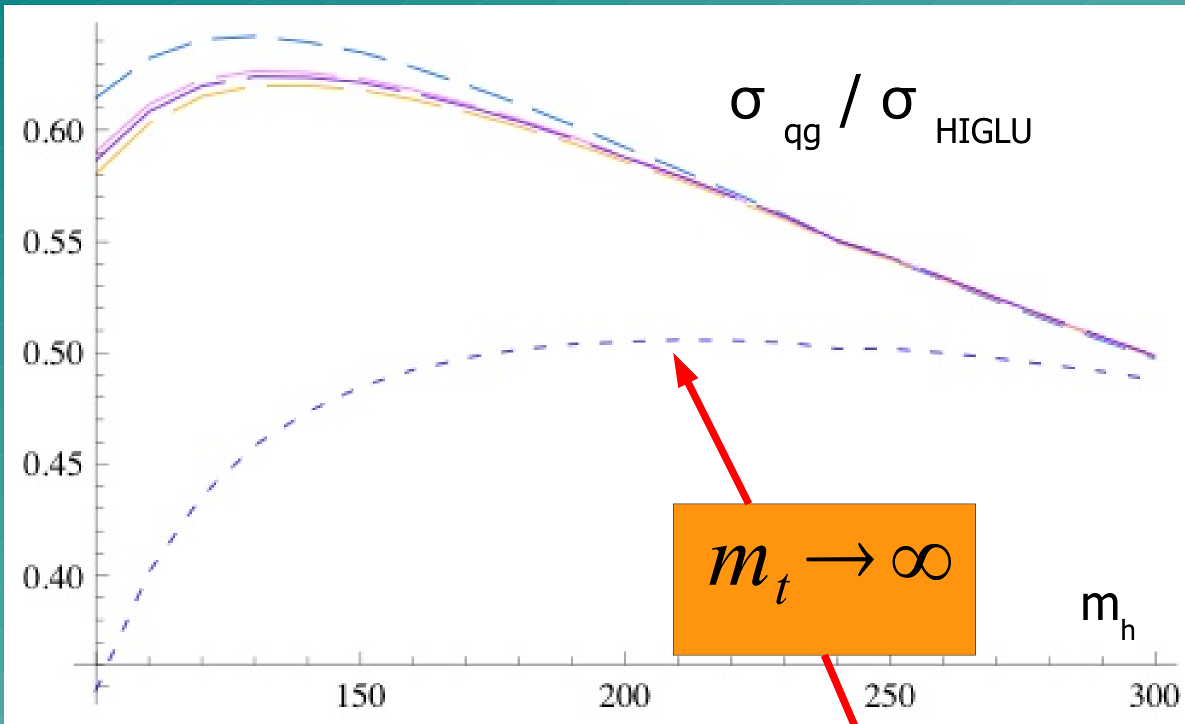
NNLO top mass effects, gg channel



NLO top mass effects, qg and qq channels

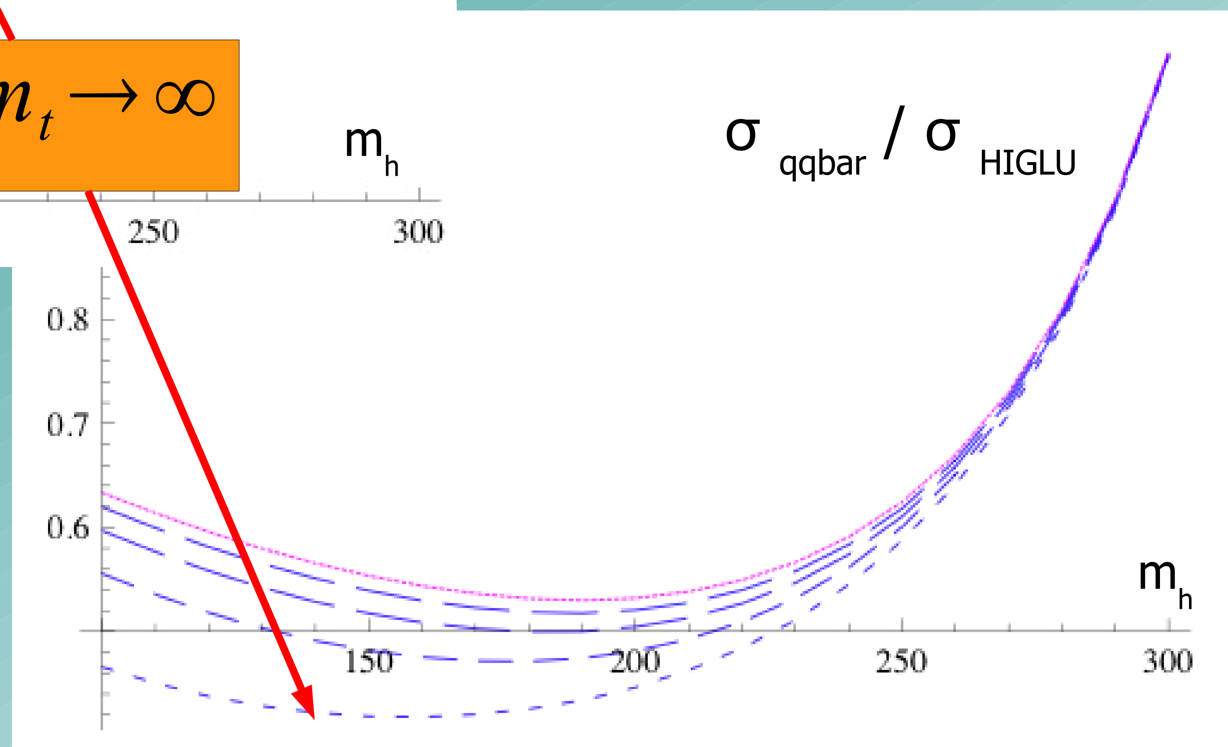


NLO qg and qqbar: hadronic study



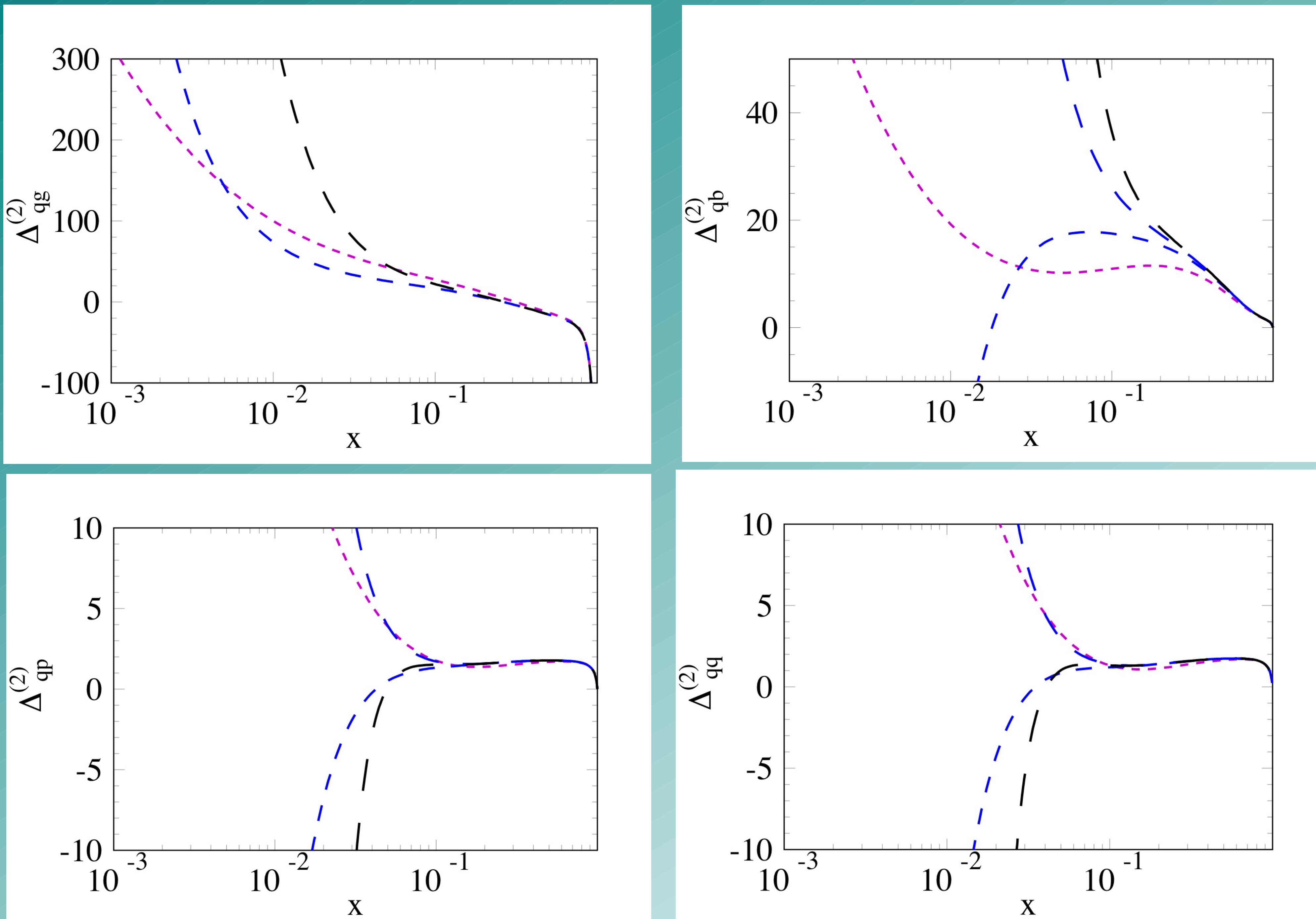
Poor-man's recipe:
use $1/m_t$ expansion below threshold, and heavy top limit above

$m_t \to \infty$

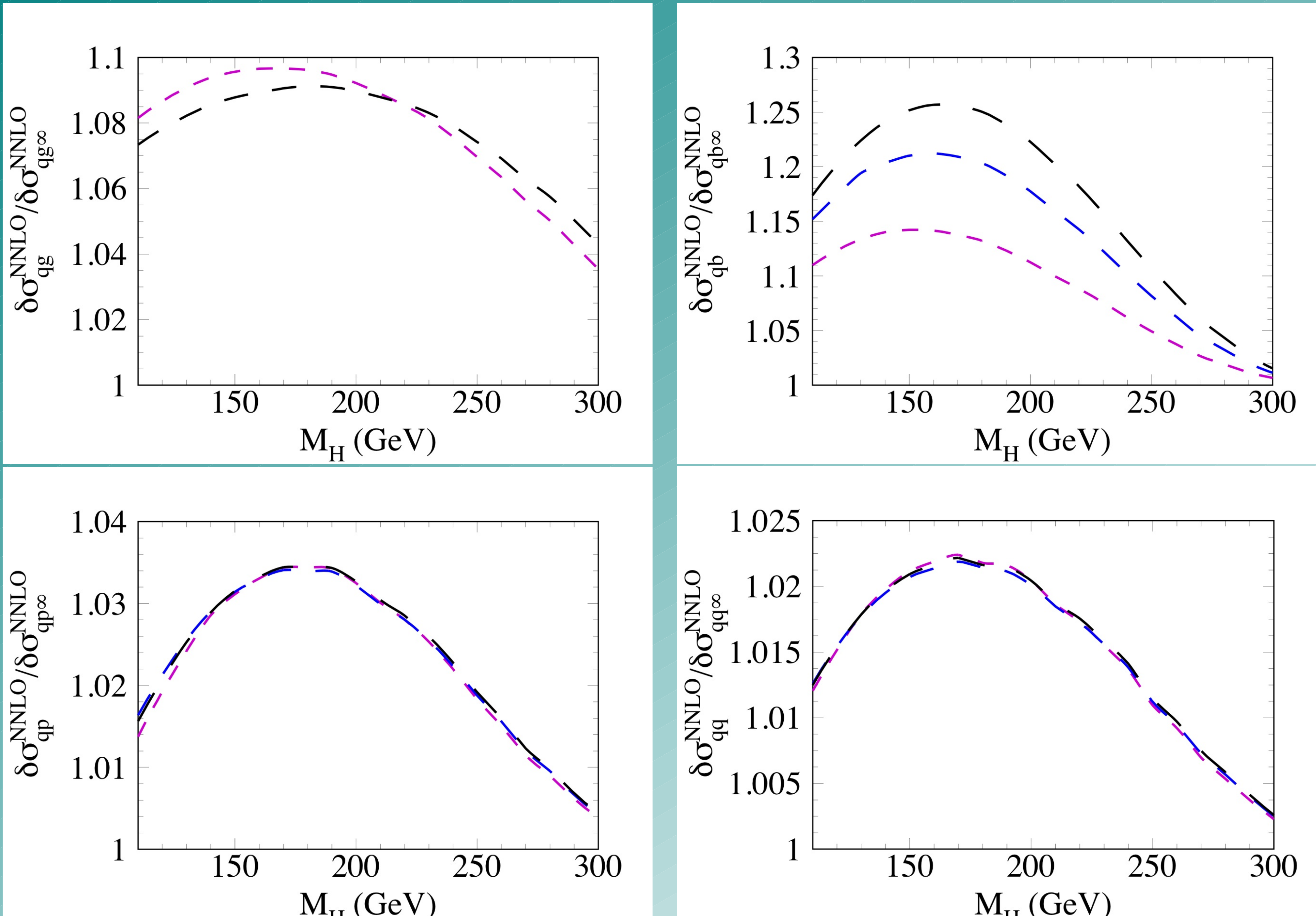


Not particularly bad: O(50%) difference for subleading terms

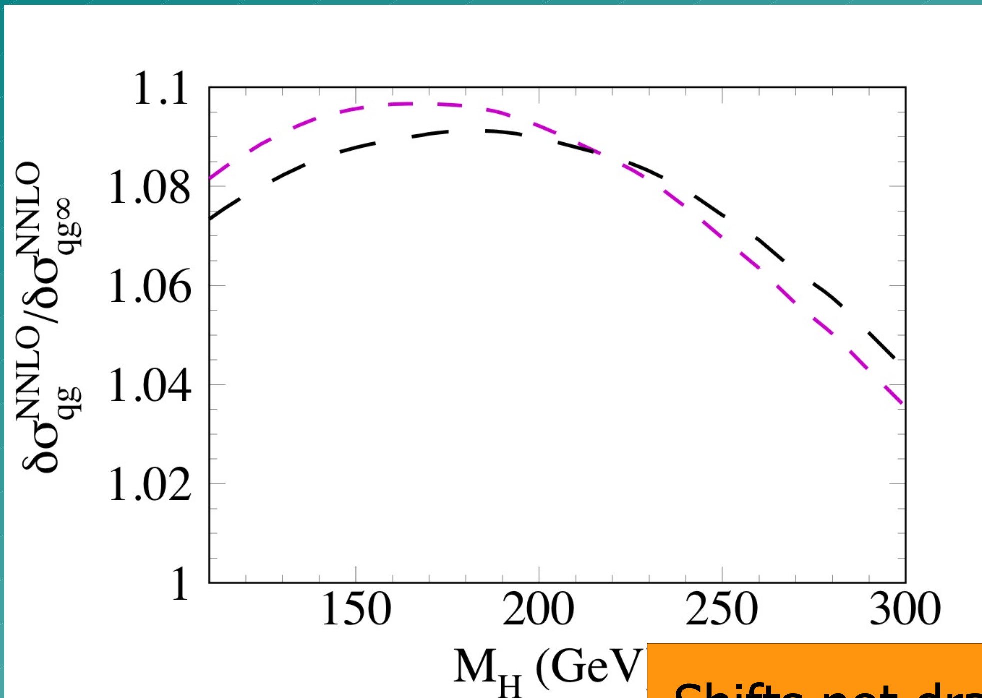
NNLO top mass effects, quark channels

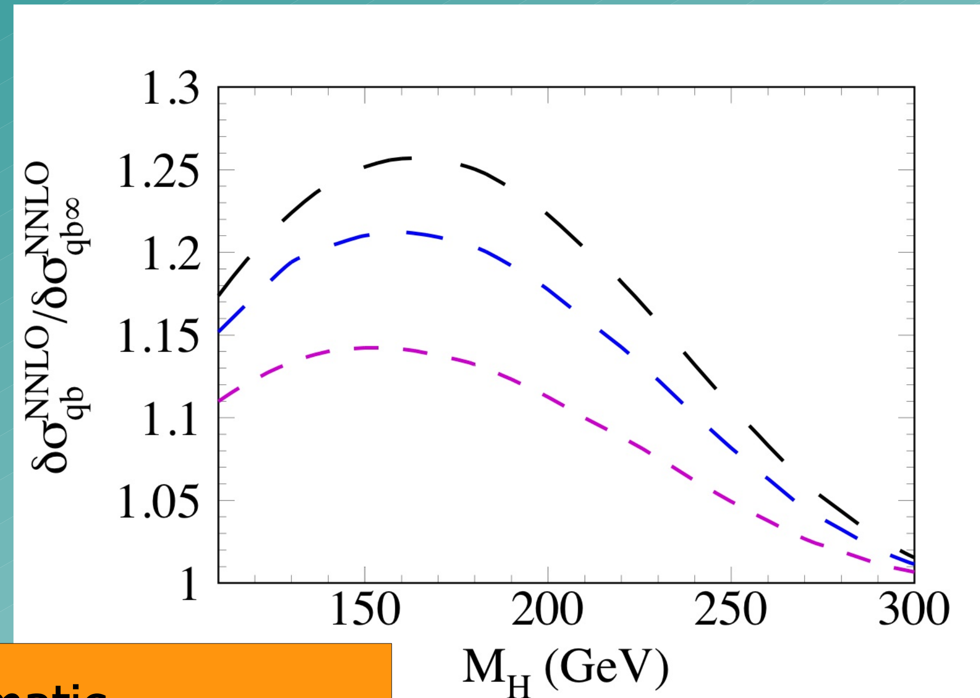


NNLO hadronic results, quark channels

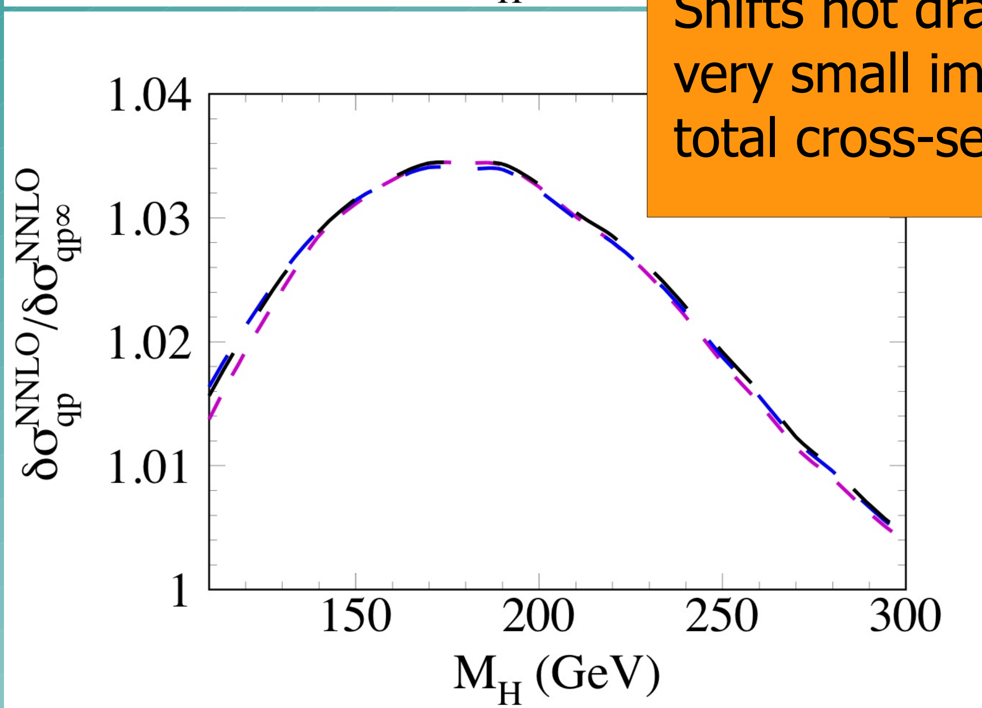


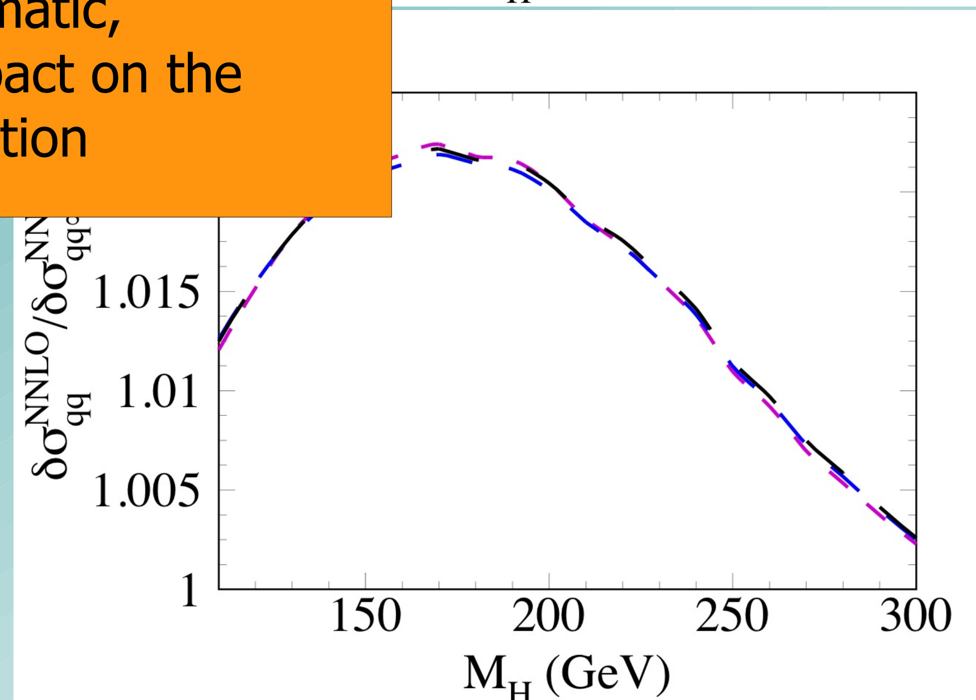
NNLO hadronic results, quark channels



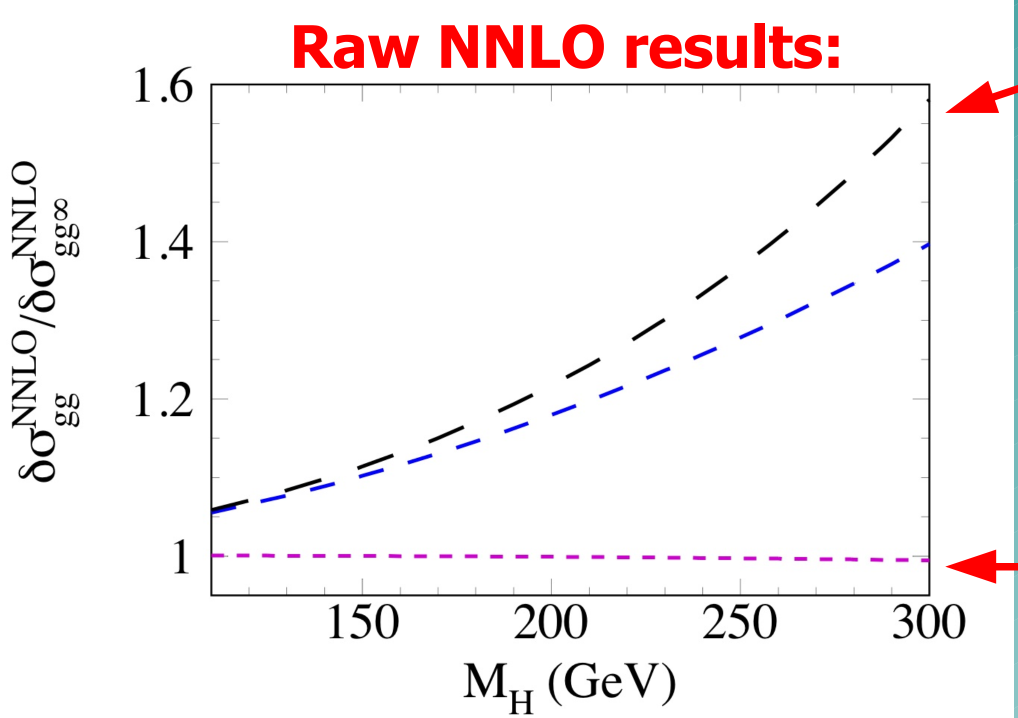


Shifts not dramatic,
very small impact on the
total cross-section





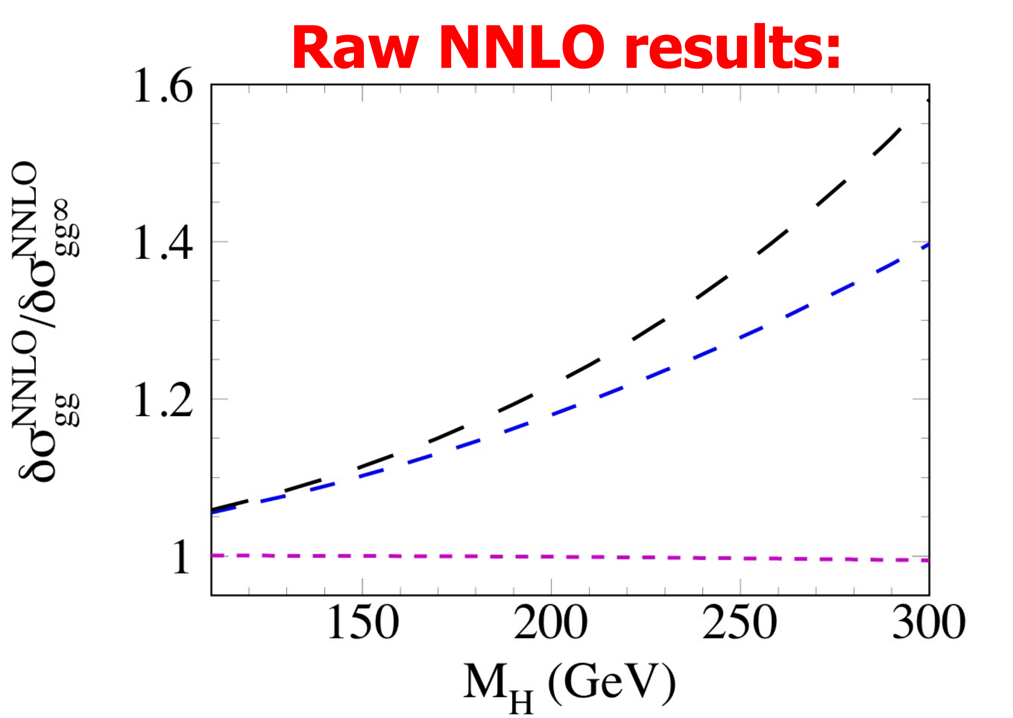
NNLO hadronic results, gg channel

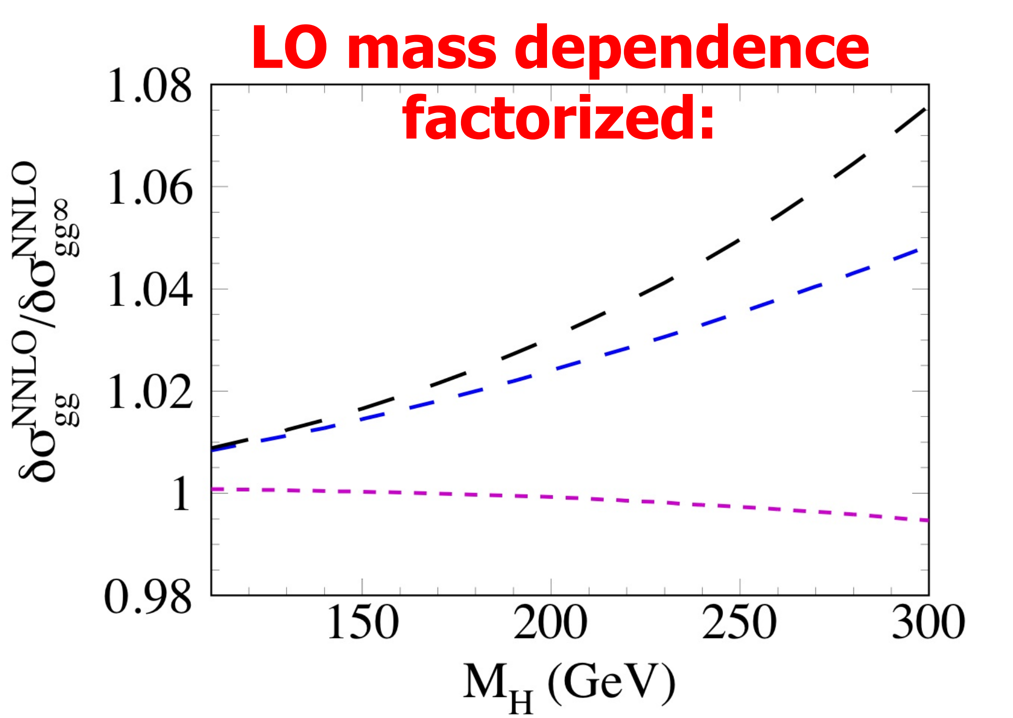


60% - huge effect!

not exactly constant (effects of interpolation)

NNLO hadronic results, gg channel

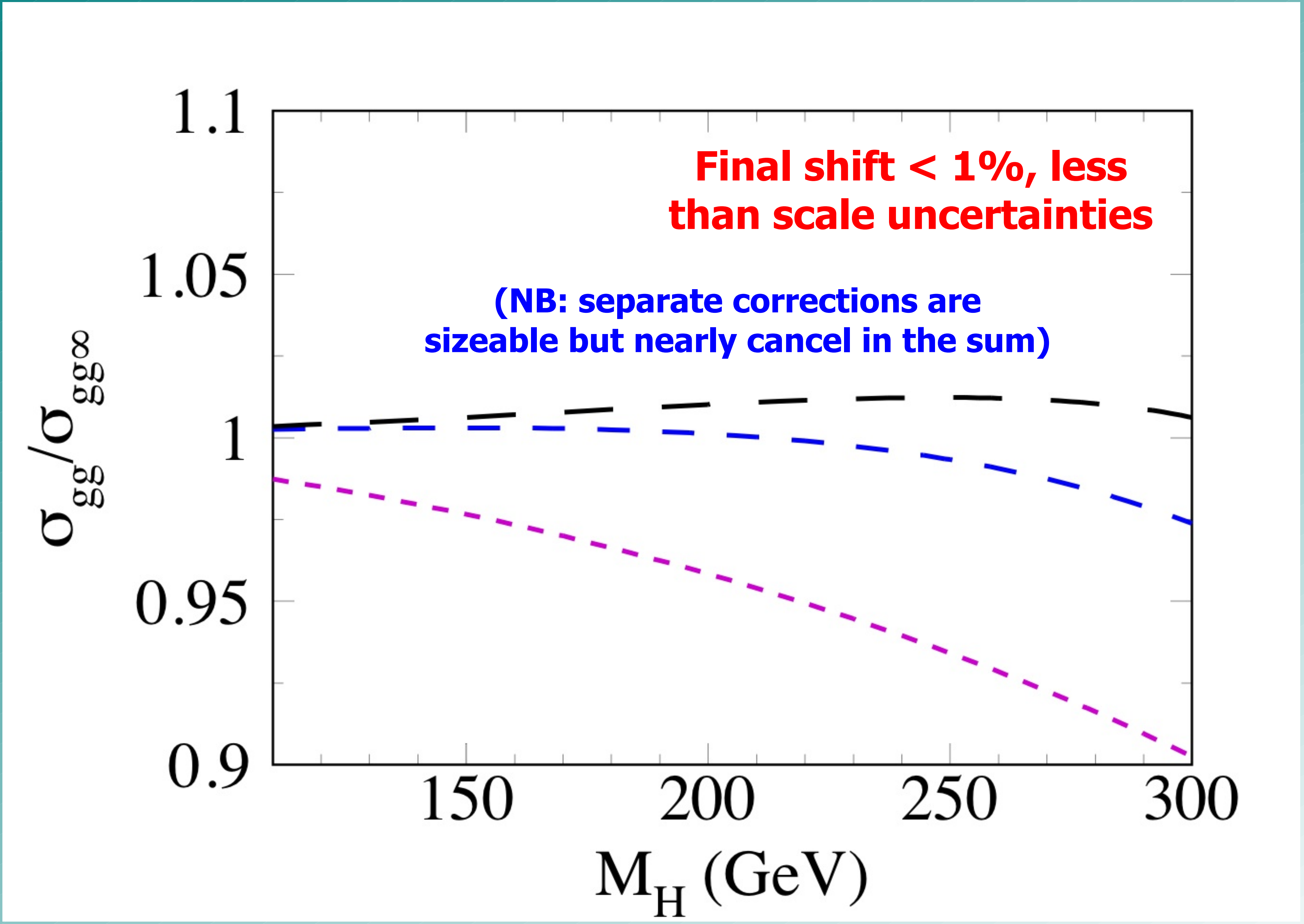




Common recipe:

$$\sigma^{NNLO}_{factorized} = \sigma^{LO}_{exact}(m_t)\left(\frac{\sigma^{NNLO}}{\sigma^{LO}}\right)_{O(1/m_t^n)}$$

NNLO hadronic results, total gg cross-section



Summary

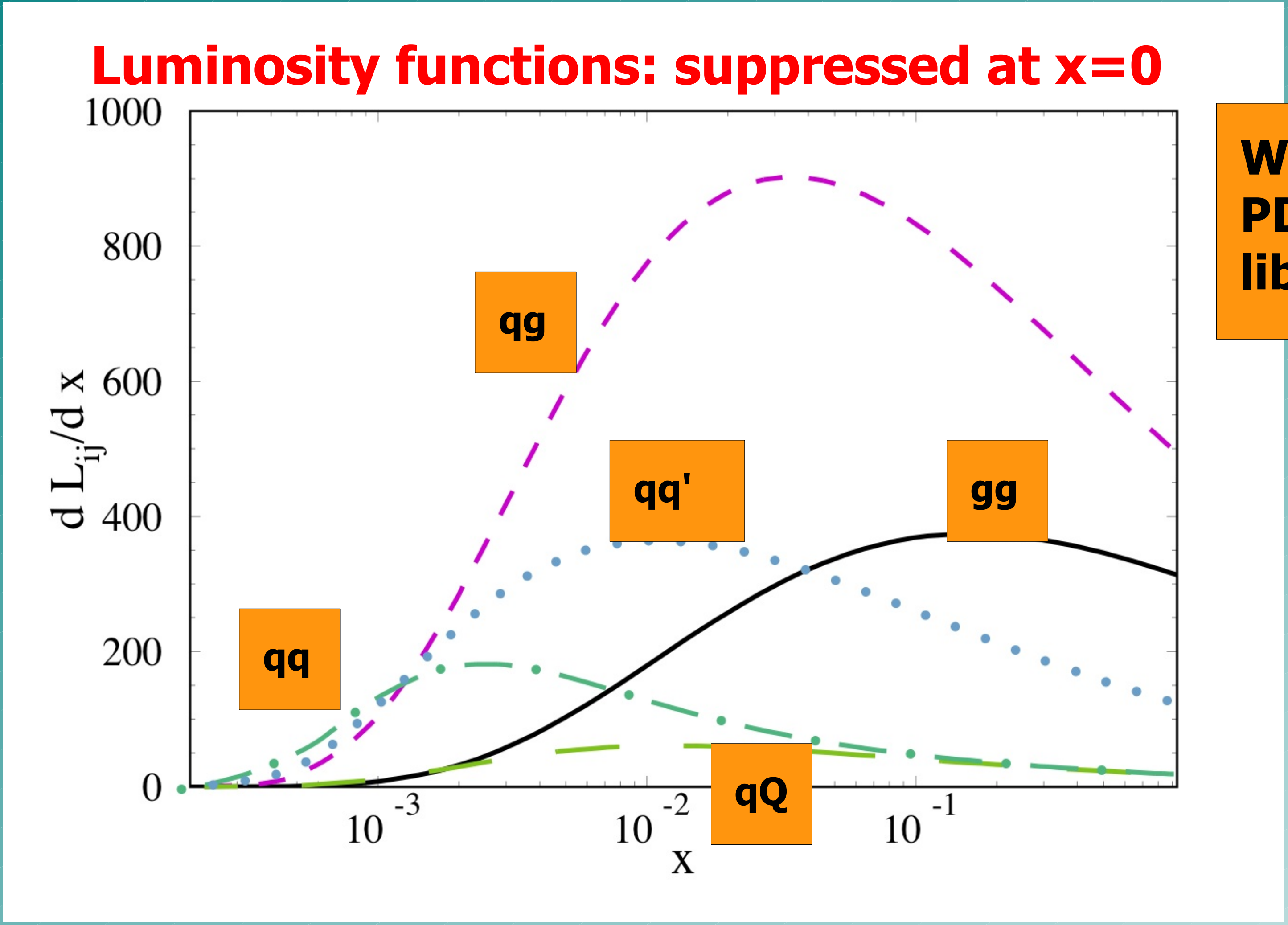
- Top mass corrections (expanded in $1/m_t$) to Higgs production have been found exactly in x, results by Harlander and Ozeren confirmed

- Shift of hadronic results smaller than scale uncertainties (a non-trivial result!)

- **Verdict: heavy top approximation is justified at NNLO**

Thank you for your attention!

From partonic to hadronic cross-sections



We use MSTW2008 PDFs from LHAPDF library

$$\sigma_{pp\to H+X} = \sum_{ij=gg,\dots} \int_{m_h^2/s}^{1} dx \left[\frac{d L_{ij}}{dx} \right](x)\, \sigma_{ij\to H+X}(x)$$

Workflow of the calculation

- Diagrams generation: **QGRAF** (~10^6 4-loop diagrams) + Perl scripts that sort out zeros
- Asymptotic expansion, mapping on pre-defined topologies: **Q2E/EXP** and custom Perl/C++ program (more general expansion algorithm)
- Calculation: **FORM** programs: **MATAD** setup and independent program
- IBP reduction: Laporta algorithm, C++ program **rows** (internally uses **FERMAT**)
- Convolutions with splitting functions: done in Mellin space with **FORM**
- Master integrals: by differential equations with a **Mathematica** program, use **HPL.m** and by soft expansion
- Many steps have independent cross-checks, analytic and numerical