Perturbative calculations of radiative B decay

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1. Motivation

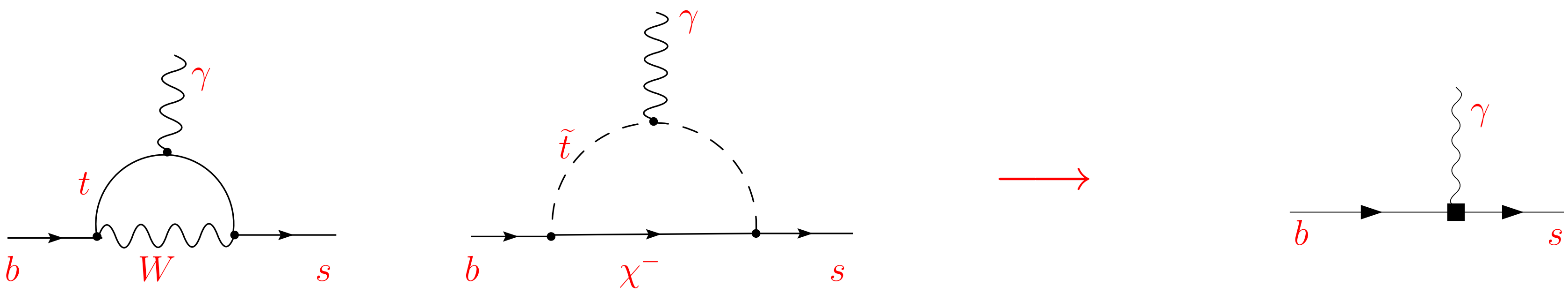
2. Completed and ongoing calculations of $b \to X_s^p \gamma$ at $\mathcal{O}(\alpha_s^2)$

3. Example: G_{27} from imaginary parts of 4-loop diagrams
   [M. Czakon, T. Schutzmeier, R. Boughezal, T. Huber]

4. Overview of non-perturbative effects in $\mathcal{B}(\bar{B} \to X_s \gamma)$

5. Summary

Information on electroweak-scale physics in the $b \to s\gamma$ transition is encoded in an effective low-energy local interaction:



Basic properties:

- **Sensitivity to new physics at scales (a few)$\times\mathcal{O}(100\,\mathrm{GeV})$ with the present th/exp accuracy, even in models with Minimal Flavour Violation (MFV).**

- **Perturbative calculability of the inclusive rate**

$$\Gamma(\bar{B} \to X_s\,\gamma)_{E_\gamma > E_0} \;=\; \Gamma(b \to X_s^p\,\gamma)_{E_\gamma > E_0} \;+\; \left(\begin{array}{c} \textbf{non-perturbative effects} \\ \sim\;(2 \pm 5)\% \end{array} \right)$$

provided E_0 is large $\left(E_0 \sim m_b/2\right)$ but not too close to the endpoint $\left(m_b - 2E_0 \gg \Lambda_{\rm QCD}\right)$.

- **The known/estimated NNLO $\mathcal{O}(\alpha_s^2)$ contributions to the partonic rate are $\sim \mathcal{O}(10\%)$. An uncertainty of $\pm 3\%$ is assumed for the unknown part.**

A very recent and detailed analysis of dominant non-perturbative effects: M. Benzke, S. J. Lee, M. Neubert and G. Paz, arXiv:1003.5012.

Results of the SM calculations:

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm NNLO}_{E_\gamma > 1.6\ {\rm GeV}} = \begin{cases} (3.15 \pm 0.23) \times 10^{-4}, & \text{hep-ph/0609232, using the 1S scheme,} \\ (3.26 \pm 0.24) \times 10^{-4}, & \text{following the kin scheme analysis of arXiv:0805.0271, but } \overline{m}_c(\overline{m}_c)^{\rm 2loop} \text{ rather than } \overline{m}_c(\overline{m}_c)^{\rm 1loop}. \end{cases}$$

Experimental world averages:

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm EXP}_{E_\gamma > 1.6\ {\rm GeV}} = \begin{cases} (3.55 \pm 0.26) \times 10^{-4}, & \text{[HFAG, winter 2010],} \\ (3.50 \pm 0.17) \times 10^{-4}, & \text{[Artuso, Barberio, Stone, arXiv:0902.3743].} \end{cases}$$

$\Rightarrow$ **Clean signals of new physics — unlikely.**

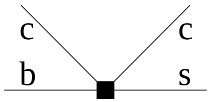
(even after reducing the uncertainties by factors of 2 on both sides)

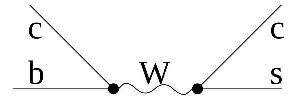
Constraints on new physics — certainly.

Decoupling of W, Z, t, H^0 $\Rightarrow$ effective weak interaction Lagrangian:

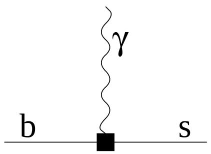
$$L_{\text{weak}} \sim \sum C_i(\mu_b)\, Q_i$$

where

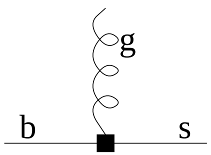
Q_2 = 

$= (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$, **from** 

, $\quad C_2(\mu_b) \simeq 1$

Q_7 = 

$\sim (\bar{s}_L \sigma^{\mu\nu} b_R)\, F_{\mu\nu}$, $\quad C_7(\mu_b) \simeq -0.3$

Q_8 = 

$\sim (\bar{s}_L \sigma^{\mu\nu} T^a b_R)\, G^a_{\mu\nu}$, $\quad C_8(\mu_b) \simeq -0.2$

Q_1 **differs from** Q_2 **only by color structure.**

Q_3**, . . . ,** Q_6 – **other 4-quark operators with small Wilson coefficients** $C_i(\mu)$**.**

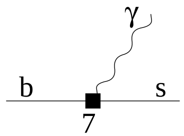
All the $C_i(\mu)$ are known up to $\mathcal{O}(\alpha_s^2)$ (NNLO) in the SM.

[Bobeth, MM, Urban, 2000], [MM, Steinhauser, 2004], [Gorban, Haisch, 2005], [Gorban, Haisch, MM, 2005], [Czakon, Haisch, MM, 2007].

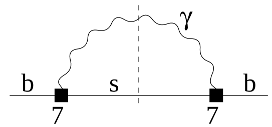
Evaluation of the NNLO matrix elements at $\mu_b \sim \frac{m_b}{2}$.

$$\Gamma(b \to X_s^{\rm p}\gamma)_{E_\gamma > E_0} = \frac{G_F^2 m_b^5 \alpha_{\rm em}}{32\pi^4} |V_{ts}^* V_{tb}|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b)$$

LO: $G_{ij} = \delta_{i7}\delta_{j7}$



$\Leftrightarrow$



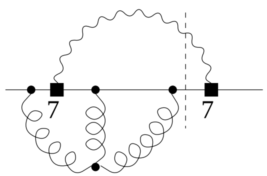
$|C_{1,2}(\mu_b)| \sim 1$, $|C_{3,4,5,6}(\mu_b)| < 0.07$,
$C_7(\mu_b) \sim -0.3$, $C_8(\mu_b) \sim -0.15$.

NLO: **The most important G_{ij} $(i,j = 1,2,7,8)$ are known since 1996.** { [Greub, Hurth, Wyler, 1996] [Ali, Greub, 1991-1995]

The remaining G_{ij} are known since 2002. { [Buras, Czarnecki, MM, Urban, 2002] [Pott, 1995]

NNLO: **Only $i,j = 1,2,7,8$ have been considered so far.**

Only G_{77} is fully known:



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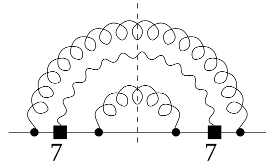
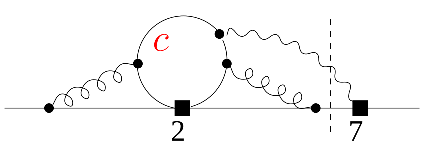


$+ \ldots$

{ [Blokland et al., 2005] [Melnikov, Mitov, 2005] [Asatrian et al., 2006-2007]

G_{27}**:**
(and analogous G_{17})
coming soon
for m_c=0



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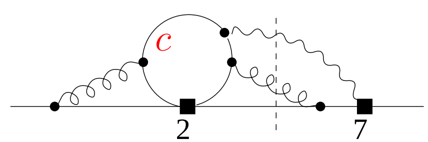

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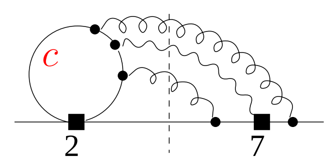


$+ \ldots$

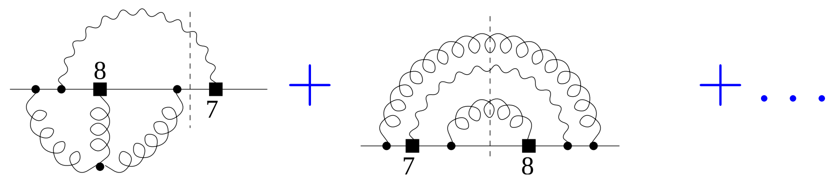
Boughezal, Czakon, Schutzmeier

Czakon, Huber, Schutzmeier

$\mathcal{O}(200)$ massive 4-loop on-shell master integrals...

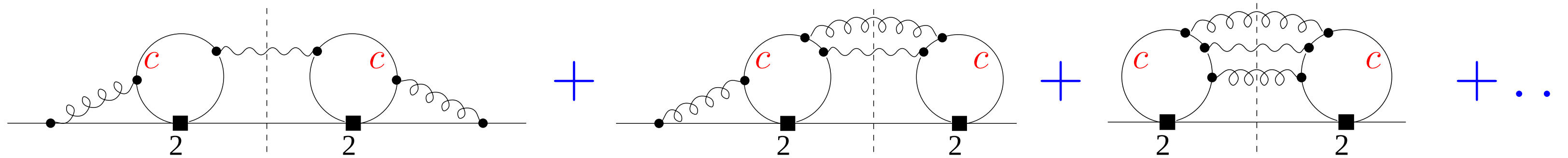
Medieval monk job? M.C.: "not really..."

G_{78}:



coming soon [H.M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub, G. Ossola]

G_{22}:
(and analogous G_{11} & G_{12})



Two-particle cuts are known (just $|\text{NLO}|^2$).

Three- and four-particle cuts vanish at the endpoint $E_\gamma = m_b/2$.
Analogous NLO corrections are not big (+3.6%).

The phenomenological estimate at the NNLO in 2006 relied on using the BLM approximation together with the large-m_c asymptotics of the non-BLM correction. The latter correction has been interpolated in m_c under the assumption that it vanishes at $m_c = 0$.

Large-m_c asymptotics of G_{ij}^{NNLO} ($m_c \gg m_b/2$):

1	2	7	8	
+	+	+	+	**1**
	+	+	+	**2**
		+	×	**7**
			−	**8**

[MM, Steinhauser, 2006]

The BLM approximation for G_{ij}^{NNLO} (arbitrary m_c):

1	2	7	8	
+	+	+	×	**1**
	+	+	×	**2**
		+	+	**7**
			+	**8**

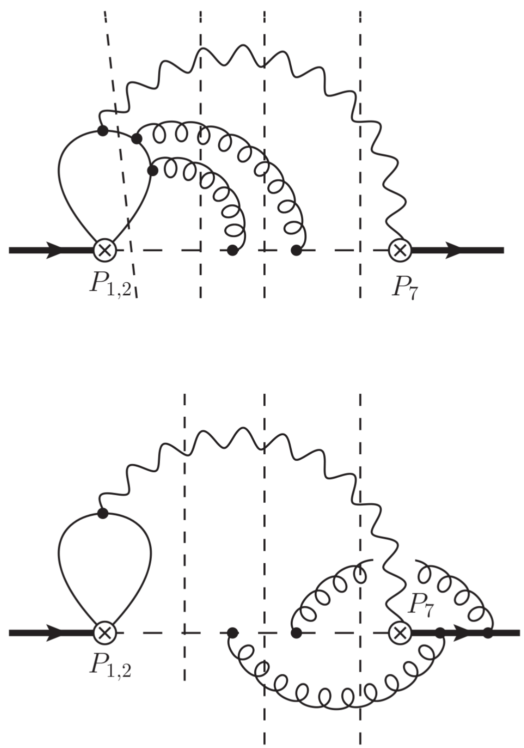
The BLM corrections to G_{78}, G_{88} are small.

G_{18} and G_{28} are small at the NLO.

[Bieri, Greub, Steinhauser, 2003]
[Ligeti, Luke, Manohar, Wise, 1999]
[Ferroglia, Haisch, unpublished] (G_{88})
[MM, Poradziński, coming soon]

Beyond BLM, diagrams with massive quark loops on gluon lines are now known for all the relevant G_{ij}.
[Boughezal, Czakon, Schutzmeier, 2007], [Asatrian, Ewerth, Gabrielyan, Greub, 2007], [Ewerth, 2008]

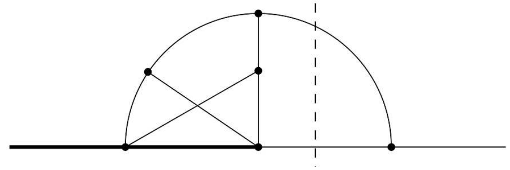
Other examples of diagrams contributing to G_{27}
(Figures here and in the next few slides are from the Ph.D. thesis of T. Schutzmeier):



However:

(i) **Diagrams with fermion loops on the gluon lines can be removed from the $m_c = 0$ calculation because they are known for arbitrary m_c.**
[Bieri, Greub, Steinhauser, 2003], [Boughezal, Czakon, Schutzmeier, 2007]

(ii) **In all the remaining diagrams, charm lines should not be cut. No open charm production occurs in $\bar{B} \to X_s\gamma$ by definition. Ignoring such cuts is possible even in the $m_c = 0$ case without introducing IR singularities.**

Evaluation of the master integrals I_k. (e.g.: )

(i) Generalization to the off-shell case $z \equiv \frac{p^2}{m_b^2} \neq 1$

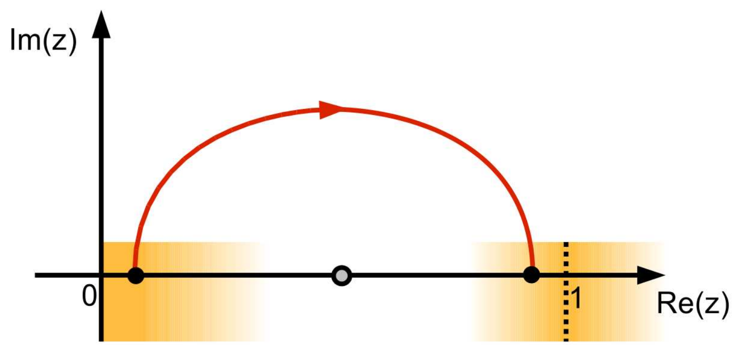
(ii) Automatic derivation (with the help of IBP) of differential equations of the form:

$$\frac{d}{dz} I_n = \sum_k w_{nk}(z, \epsilon) I_k$$

where w_{nk} are rational functions of their arguments.

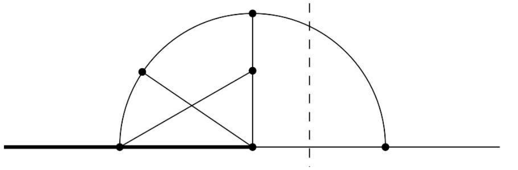
(iii) Establishing initial conditions from expansions around $z = 0$ that involve massless integrals only (apart from massive tadpoles).

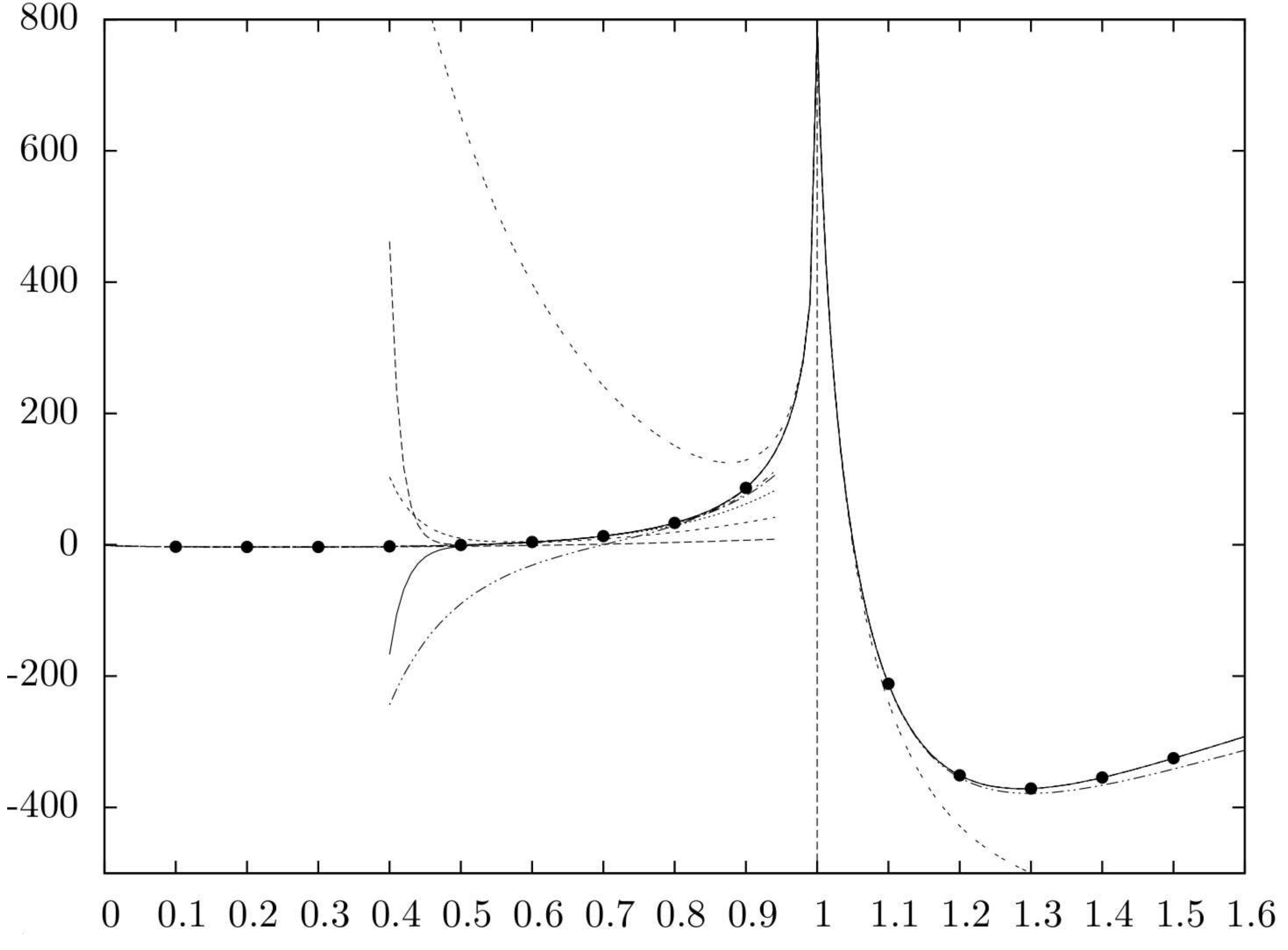
(iv) Evolving to the vicinity of $z = 1$ using precise numerical solutions to the differential equations.



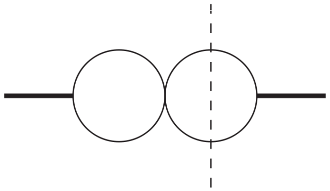
The evolution goes either in the upper or in the lower part of the complex z-plane to bypass spurious singularities of w_{nk} on the real axis. Path-independence of the final results serves as a test.

(v) Matching with expansions around $z = 1$, assuming their form $\sum c_{pq} (1-z)^p \ln^q(1-z)$ (with unknown coefficients c_{pq}). This is necessary only if numerical instabilities occur at $z = 1$.

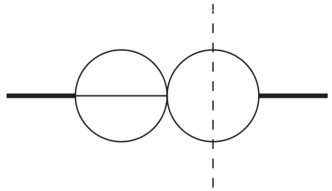
Example of z-dependence (the $\mathcal{O}(\epsilon^0)$ part of  **):**



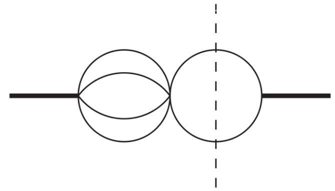
Massless integrals with two- and three-particle cuts for the initial conditions



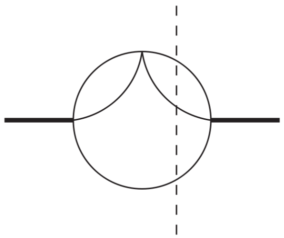
C_2^{2L}



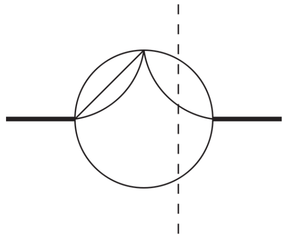
C_2^{3L}



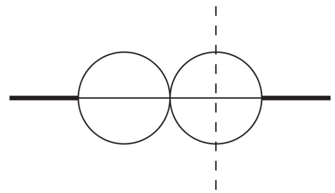
C_2^{4L}



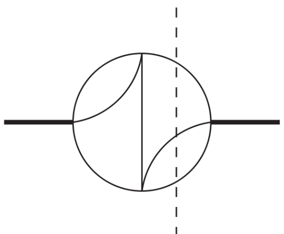
$C_{3,1}^{3L}$



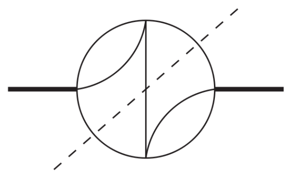
$C_{3,1}^{4L}$



$C_{3,2}^{4L}$

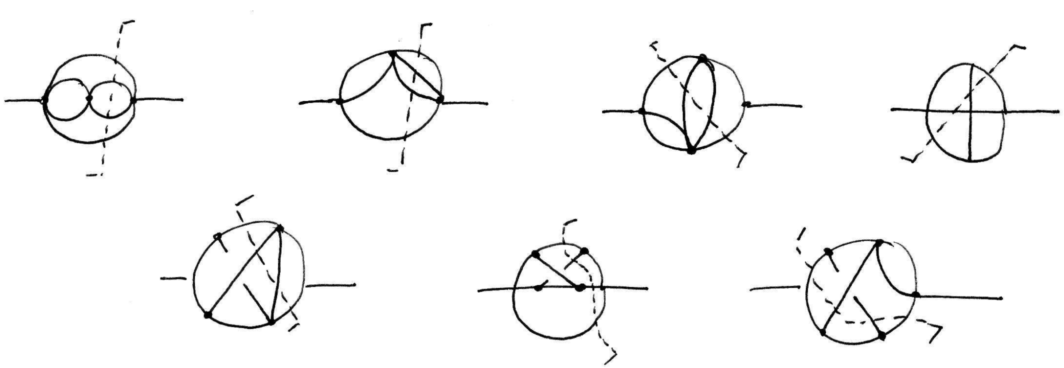


$C_{3,3}^{4L}$



$C_{3,4}^{4L}$

Massless integrals with four-particle cuts for the initial conditions (calculated by T. Huber using Mellin-Barnes techniques):

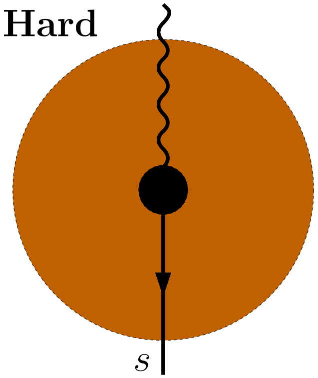


Energetic photon production in charmless decays of the $\bar{B}$-meson

$(E_\gamma \gtrsim \frac{m_b}{3} \simeq 1.6\,\mathrm{GeV})$ [see MM, arXiv:0911.1651]

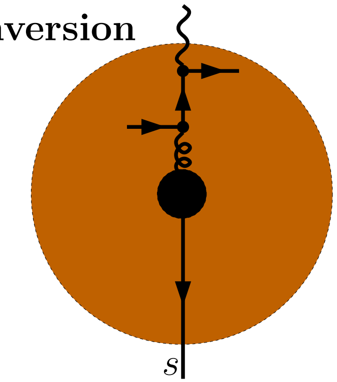
A. Without long-distance charm loops:

1. **Hard**



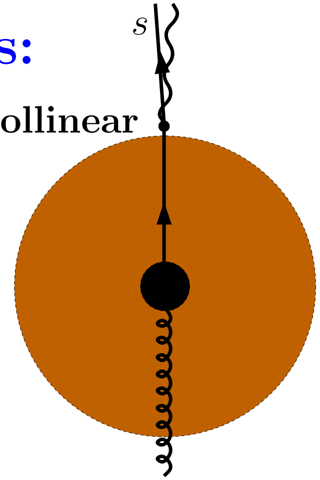
Dominant, well-controlled.

2. **Conversion**



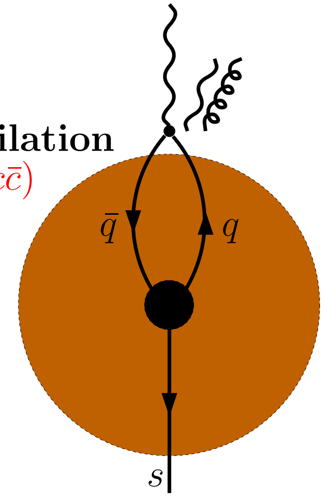
$\mathcal{O}(\alpha_s \Lambda / m_b)$, $(-1.5 \pm 1.5)\%$.
[Lee, Neubert, Paz, 2006]

3. **Collinear**



Pert. $< 1\%$, nonp. $\sim -0.2\%$.
[Kapustin,Ligeti,Politzer, 1995]

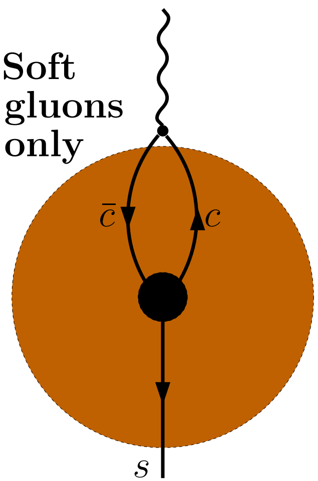
4. **Annihilation** $(q\bar{q} \neq c\bar{c})$



Exp. π^0, η, η', ω subtracted.
Perturbatively $\sim 0.1\%$.

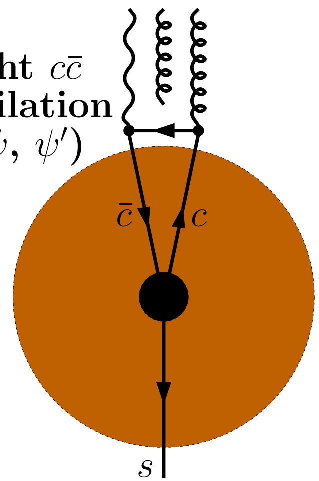
B. With long-distance charm loops:

5. **Soft gluons only**



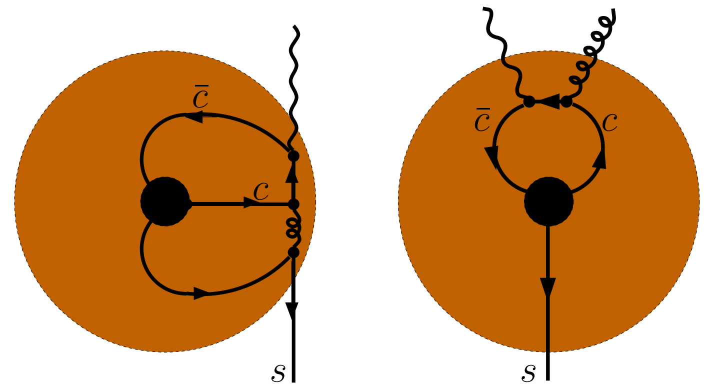
$\mathcal{O}(\Lambda^2/m_c^2)$, $\sim +3.1\%$.
[Voloshin, 1996], [...],
[Buchalla, Isidori, Rey, 1997]

6. **Boosted light $c\bar{c}$ state annihilation (e.g. η_c, J/ψ, ψ')**



Exp. J/ψ subtracted $(< 1\%)$.
Perturbatively (including hard): $\sim +3.6\%$.
$\phi_{ij}^{(1)}(\delta)$, $\phi_{ij}^{(2)\beta_0}(\delta)$, $i, j = 1, 2$

7. **Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state**



$\mathcal{O}(\alpha_s(\Lambda/M)^2)$ $\mathcal{O}(\alpha_s \Lambda/M)$
$M \sim 2m_c, 2E_\gamma, m_b$.

e.g. $\mathcal{B}[B^- \to D_{sJ}(2457)^- \; D^*(2007)^0 \;] \simeq 1.2\%$,
$\mathcal{B}[B^0 \to D^*(2010)^+ \; \bar{D}^*(2007)^0 K^-] \simeq 1.2\%$.

Summary

- **Given the present consistency of measurements and SM calculations, observing clean signals of new physics in $\bar{B} \to X_s\gamma$ is unlikely, even if the uncertainties were reduced by factors of 2 on both sides. However, achieving such a reduction is worth an effort, as it would lead to strengthening constraints on most popular beyond-SM theories (e.g. MSSM with MFV).**

- **New perturbative NNLO results are coming soon. This is going to improve the m_c-interpolation. No BLM approximation at $m_c = 0$ will be necessary any more.**

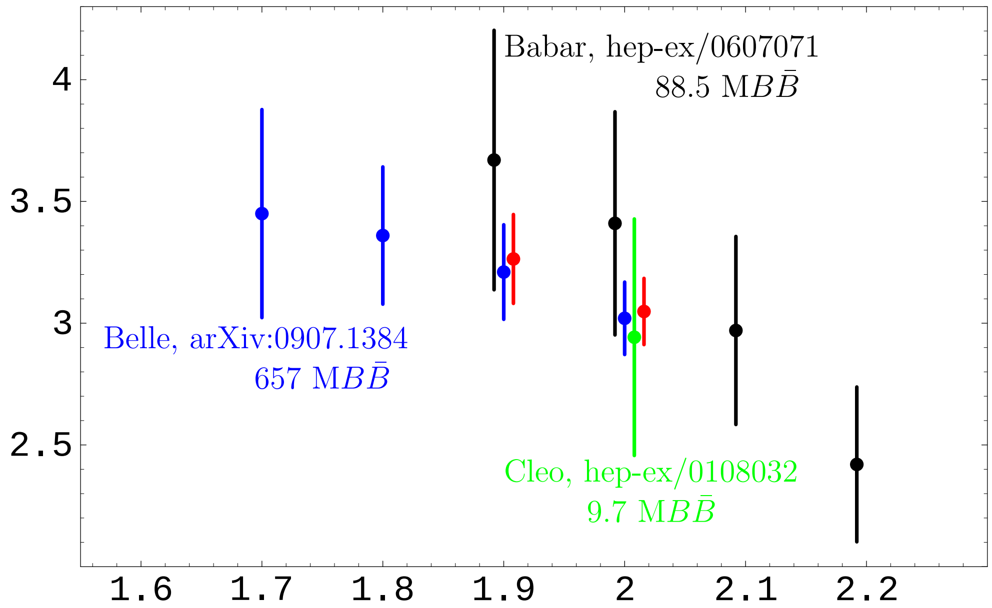
- **Non-perturbative uncertainties remain at the 5% level. However, their estimate has become more solid thanks to the recent analysis in arXiv:1003.5012.**
  [M. Benzke, S.J. Lee, M. Neubert, G. Paz].

BACKUP SLIDES

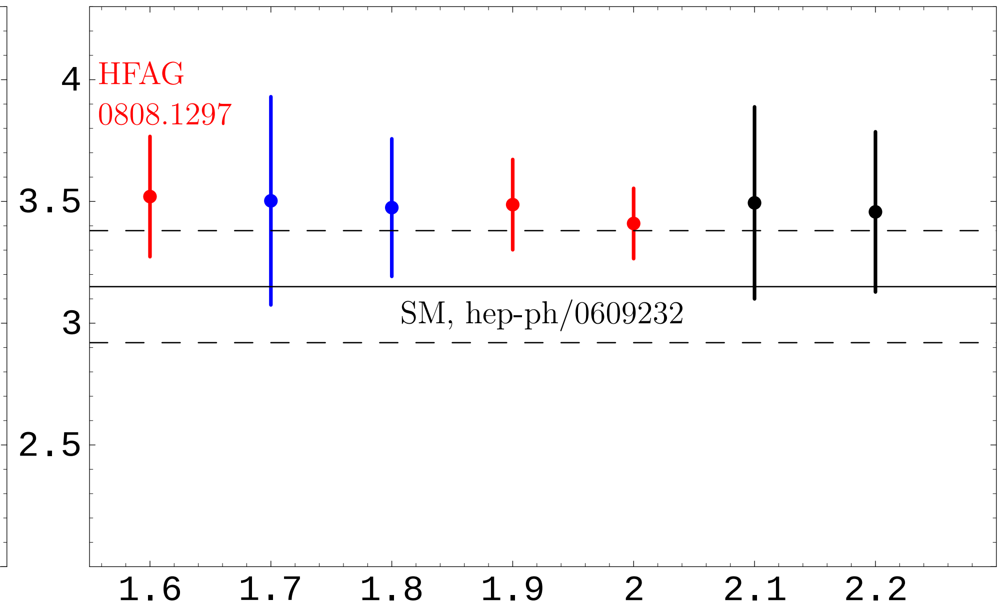
Updated $\mathcal{B}(B \to X_s\gamma)$ measurement by Belle.

A. Limosani *et al*, arXiv:0907.1384, PRL 103 (2009) 241801.

$\mathcal{B} \times 10^4$ **for each** $E_\gamma^{\rm min}$ **[GeV]**



Averages for each $E_\gamma^{\rm min}$ **rescaled to** $E_\gamma^{\rm min} = 1.6$ **GeV**



The displayed measurements are only the fully-inclusive, no-hadronic-tag ones.
Other methods (included in the HFAG average):

- **Semi-inclusive (systematics-limited),**
- **With hadronic tags of the recoiling B meson (not necessarily fully reconstructed).**
  Low systematic errors, but statistics-limited at present.

Interpolation in m_c

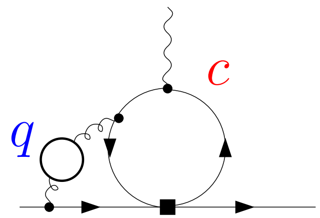
$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \underbrace{X}_{\text{normalization}} \left[\underbrace{P(E_0)}_{\text{perturbative}} + \underbrace{N(E_0)}_{\text{non-perturbative}} \right]$$

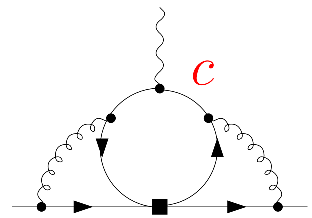
Expansion of $P(E_0)$:

$$P = \underbrace{P^{(0)} + \frac{\alpha_s(\mu_b)}{4\pi}\left(P_1^{(1)} + P_2^{(1)}(r)\right) + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 \left(P_1^{(2)}\right.}_{\textbf{known}} + P_2^{(2)}(r) + \left.\underbrace{P_3^{(2)}(r)}_{\textbf{known}}\right)$$

$$P_1^{(1)}, P_3^{(2)} \sim C_i^{(0)} C_j^{(1)}, \qquad P_2^{(1)}, P_2^{(2)} \sim C_i^{(0)} C_j^{(0)}, \qquad P_1^{(2)} \sim \left(C_i^{(0)} C_j^{(2)},\ C_i^{(1)} C_j^{(1)}\right)$$

Moreover: $P_2^{(2)} = A\, n_f + B = -\frac{3}{2}(11 - 2/3 n_f)A + \frac{33}{2}A + B = P_2^{(2)\beta_0} + P_2^{(2)\text{rem}}$



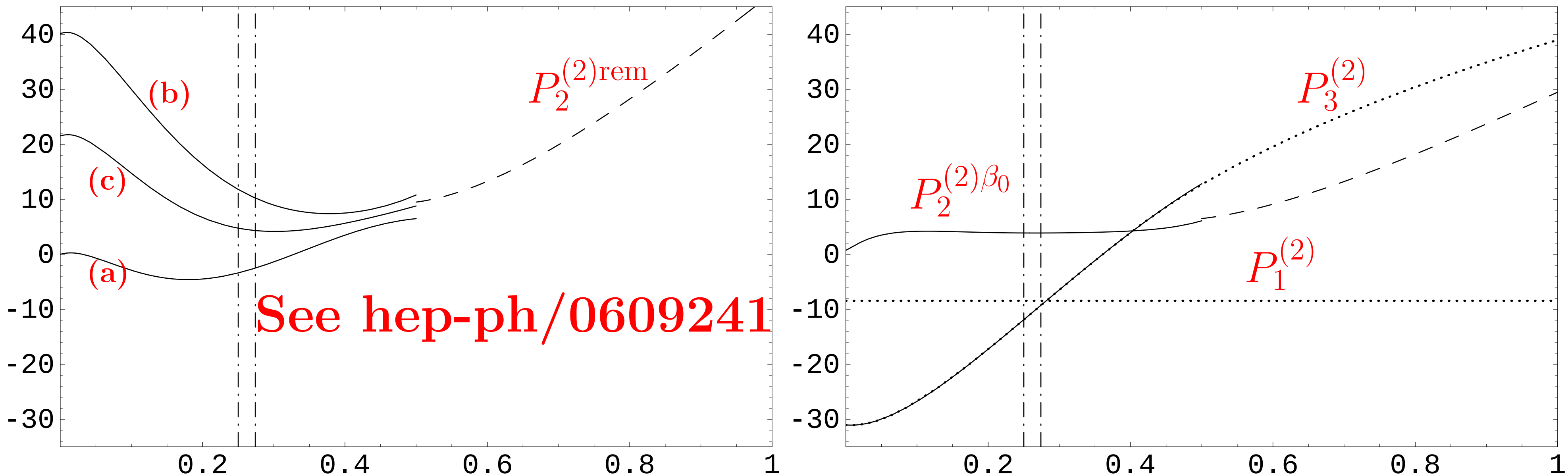


$$r = \frac{m_c(m_c)}{m_b^{1S}}$$

$P_2^{(2)\beta_0}$ **known for all r**

The complete $P_2^{(2)}$ has been calculated only for $r \gg \frac{1}{2}$.

The NNLO corrections $P_k^{(2)}$ as functions of $r = m_c(m_c)/m_b^{1S}$



Dotted: exact, **Solid: small-r expansions,** **Dashed: leading large-r asymptotics.**

Interpolation:

$$P_2^{(2)\mathrm{rem}}(r) = x_1 + x_2\, P_2^{(1)}(r) + x_3\, r\, \frac{d}{dr} P_2^{(1)}(r) + x_4\, P_2^{(2)\beta_0}(r) + x_5 |A_{\mathrm{NLO}}(r)|^2$$

The coefficients x_k are determined from the asymptotic behaviour at large r

and from the requirement that either **(a)** $P_2^{(2)\mathrm{rem}}(0) = 0$,

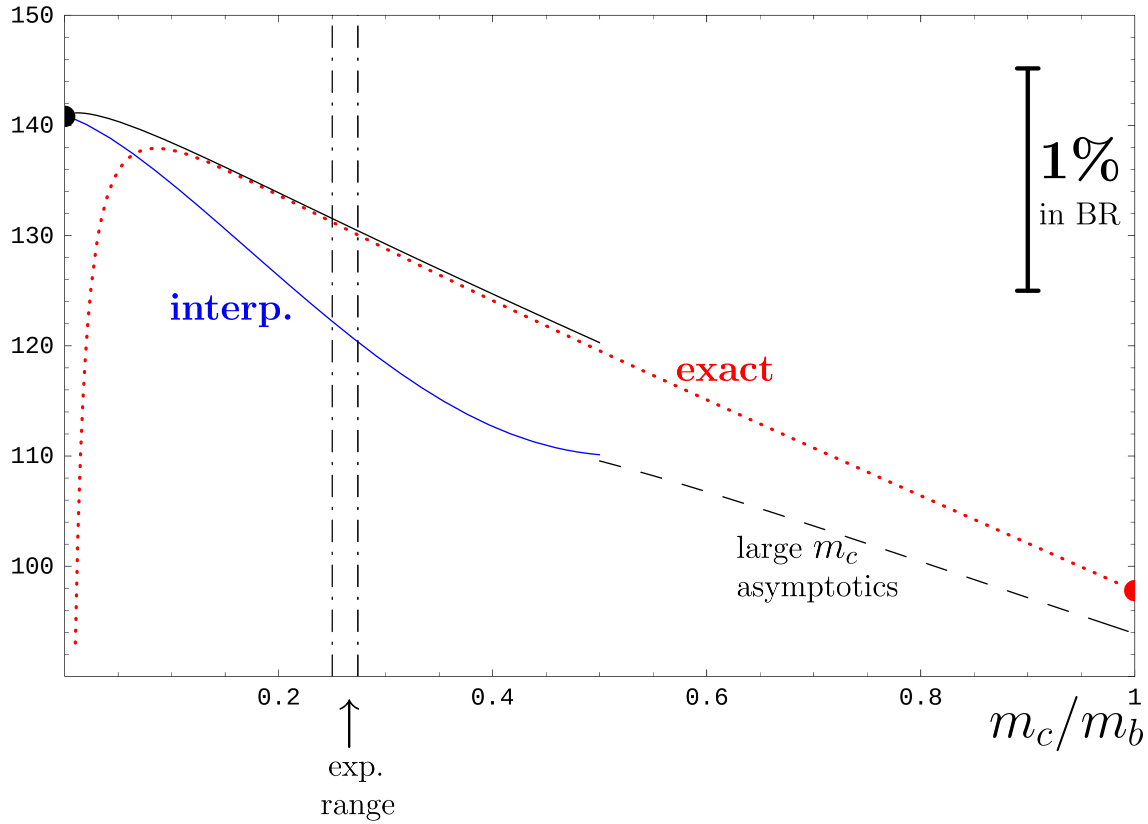
or **(b)** $P_1^{(2)} + P_2^{(2)\mathrm{rem}}(0) + P_3^{(2)}(0) = 0$,

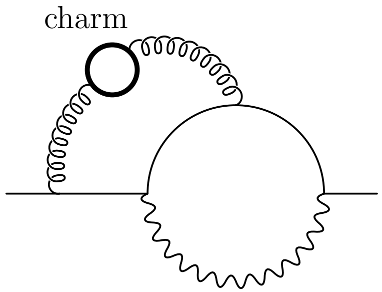
or **(c)** $P_2^{(2)\mathrm{rem}}(0) = \left[P_2^{(2)\mathrm{rem}}(0)\right]_{77}$.

The average of (a) and (b) is chosen to determine the central value of the NNLO branching ratio.
The difference between these two cases is used to estimate the interpolation ambiguity.

The m_c-dependence of $P_2^{(2)\rm rem} = C_i^{(0)}(\mu_b) C_j^{(0)}(\mu_b) K_{ij}^{(2)\rm rem}(\mu_b, E_0)$.

Example: $K_{77}^{(2)\rm rem}(2.5\ \text{GeV}, 1.6\ \text{GeV})$ **as a function of** m_c/m_b:





Value at $m_c = 0$: **Blokland et al., hep-ph/0506055** (**$c\bar{c}$ production included**).

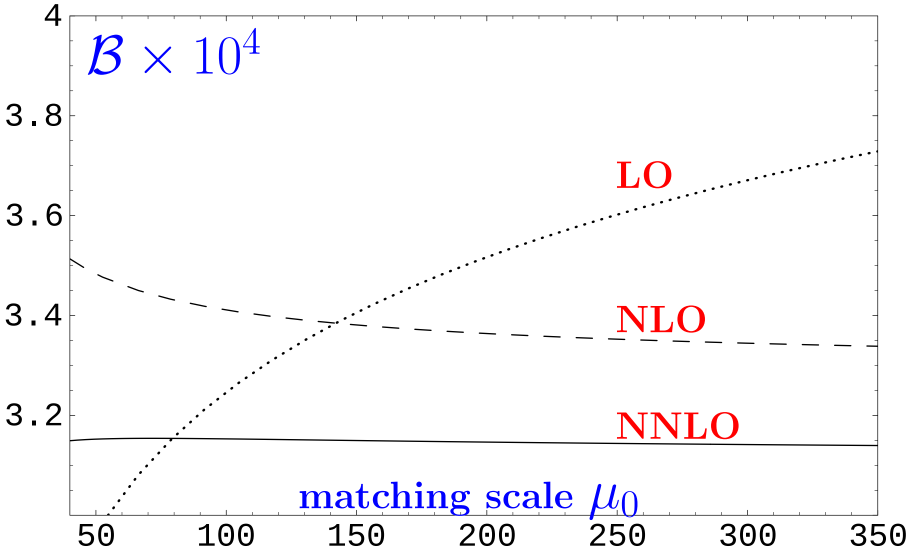
Large-m_c asymptotics: **Steinhauser, MM, hep-ph/0609241.**

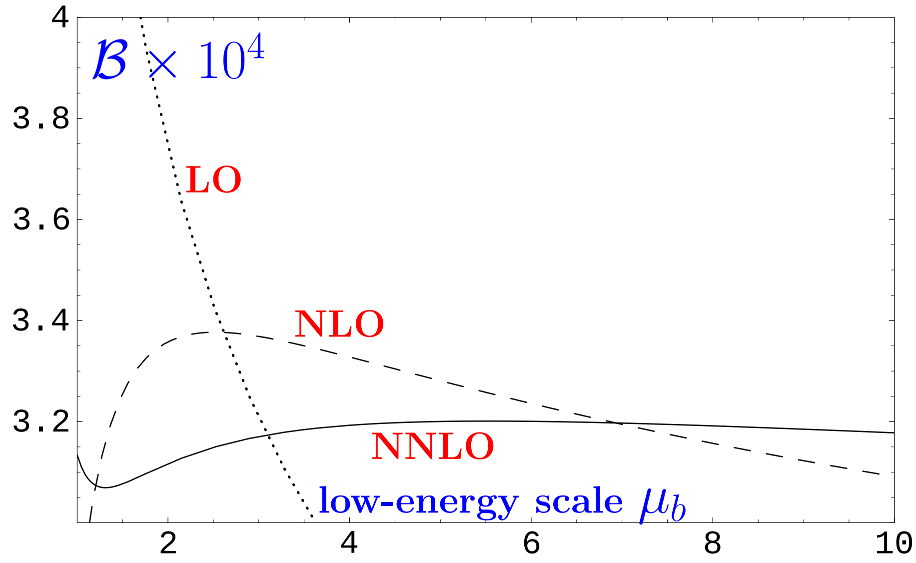
Interpolation: " " " (**$c\bar{c}$ production included**).

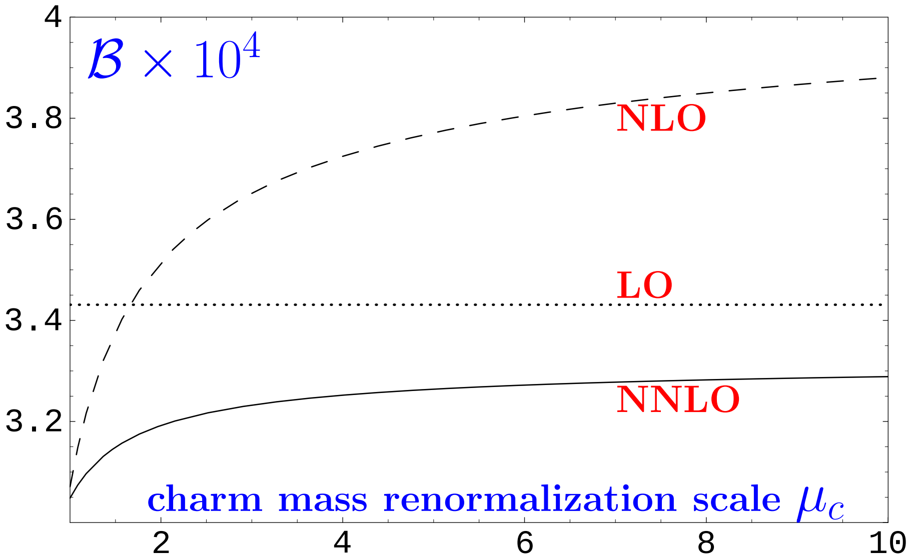
Exact $b \to X_s \gamma$: **Asatrian et al, hep-ph/0611123** (**$c\bar{c}$ production excluded**).

Exact $b \to X_u e \bar{\nu}$: **Pak, Czarnecki, arXiv:0803.0960** (**$c\bar{c}$ production included**).

Renormalization scale dependence of $\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > 1.6\ \mathrm{GeV}}$







"Central" values:

$$\mu_0 = 160\ \mathbf{GeV}$$

$$\mu_b = 2.5\ \mathbf{GeV}$$

$$\mu_c = 1.5\ \mathbf{GeV}$$

The inclusive branching ratio in the SM:

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm NNLO}_{E_\gamma > 1.6\ \rm GeV} = \begin{cases} (3.15 \pm 0.23) \times 10^{-4}, & \textbf{hep-ph/0609232, using the 1S scheme,} \\ (3.26 \pm 0.24) \times 10^{-4}, & \textbf{following the kin scheme analysis of arXiv:0805.0271, but } \overline{m}_c(\overline{m}_c)^{\rm 2loop} \textbf{ rather than } \overline{m}_c(\overline{m}_c)^{\rm 1loop}. \end{cases}$$

Contributions to the total uncertainty:

5% non-perturbative, mainly $\mathcal{O}\left(\alpha_s \frac{\Lambda}{m_b}\right)$ → **Improved measurements of Δ_{0-} should help.**

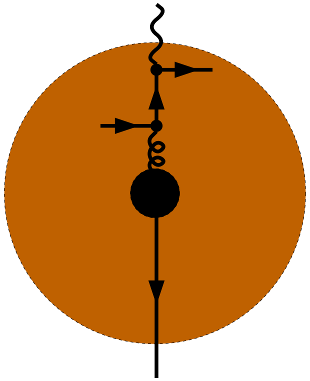
3% parametric $\left(\alpha_s(M_Z),\ \mathcal{B}^{\rm exp}_{\rm semileptonic},\ m_c\ \&\ C,\ \ldots\right)$

- $\alpha_s(M_Z)$: **2.0%**
- $\mathcal{B}^{\rm exp}_{\rm semileptonic}$: **1.6%**
- m_c & C: **1.1% (1S)**, **2.5% (kin)**

3% m_c-interpolation ambiguity → **The calculation of G_{17} and G_{27} for $m_c = 0$ should help a lot.**

3% higher order $\mathcal{O}(\alpha_s^3)$ → **This uncertainty will stay with us.**

Gluon-to-photon conversion in the QCD medium



This is hard gluon scattering on the valence quark or a "sea" quark that produces an energetic photon. The quark that undergoes this Compton-like scattering is assumed to remain soft in the $\bar{B}$-meson rest frame to ensure effective interference with the leading "hard" amplitude. Without interference the contribution would be negligible ($\mathcal{O}(\alpha_s^2\Lambda^2/m_b^2)$).

Suppression by Λ can be understood as originating from dilution of the target (size of the $\bar{B}$-meson $\sim \Lambda^{-1}$).

A rough estimate using vacuum insertion approximation gives

$$\Delta\Gamma/\Gamma \in [-3\%, -0.3\%] \qquad (\mathcal{O}(\alpha_s\Lambda/m_b)).$$

[Lee, Neubert, Paz, hep-ph/0609224]

However:

1. **Contribution to the interference from scattering on the "sea" quarks vanishes in the $SU(3)_{\text{flavour}}$ limit because $Q_u + Q_d + Q_s = 0$.**

2. **If the valence quark dominates, then the isospin-averaged $\Delta\Gamma/\Gamma$ is given by:**

$$\frac{\Delta\Gamma}{\Gamma} \simeq \frac{Q_d+Q_u}{Q_d-Q_u}\,\Delta_{0-} = -\frac{1}{3}\Delta_{0-} = \left(+0.2 \pm 1.9_{\text{stat}} \pm 0.3_{\text{sys}} \pm 0.8_{\text{ident}}\right)\%,$$

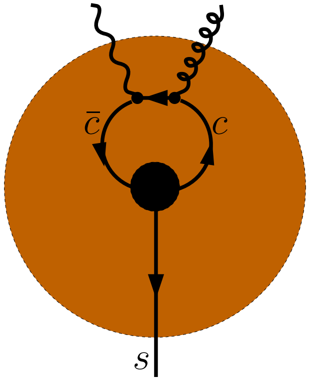
using the BABAR measurement (hep-ex/0508004) of the isospin asymmetry

$$\Delta_{0-} = [\Gamma(\bar{B}^0 \to X_s\gamma) - \Gamma(B^- \to X_s\gamma)]/[\Gamma(\bar{B}^0 \to X_s\gamma) + \Gamma(B^- \to X_s\gamma)],$$

for $E_\gamma > 1.9$ GeV.

Quark-to-photon conversion gives a soft s-quark and poorly interferes with the "hard" $b \to s\gamma g$ amplitude.

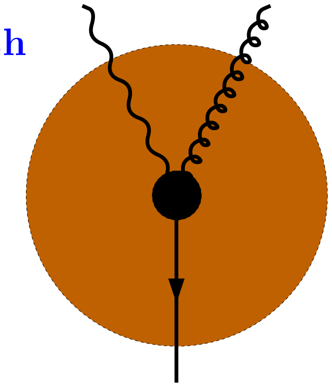
Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state



Heavy $\Leftrightarrow$ **Above the $D\bar{D}$ production threshold**

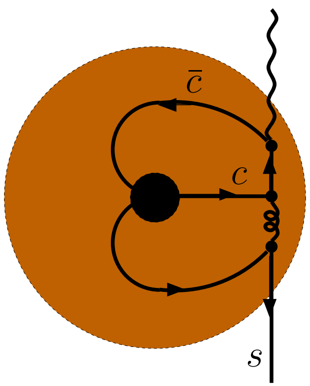
Long-distance $\Rightarrow$ **Annihilation amplitude is suppressed with respect to the open-charm decay due to the order Λ^{-1} distance between c and $\bar{c}$. By analogy to the B-meson decay constant $f_B \sim \Lambda(\Lambda/m_b)^{1/2}$, we may expect that the suppression factor scales like $(\Lambda/M)^{3/2}$, where $M \sim 2m_c, 2E_\gamma, m_b$.**

Hard gluon $\Leftrightarrow$ **Suppression by α_s of the interference with**
(non-soft)



Altogether: $\mathcal{O}\left(\alpha_s(\Lambda/M)^{3/2}\right)$.

To stay on the safe side, assume $\mathcal{O}(\alpha_s\Lambda/m_b)$ for numerical error estimates.



This type of amplitude interferes with the leading term but receives an additional Λ/M suppression (at least) due to participation of the s-quark in the hard annihilation.