Multi-Jet Processes for the LHC

Radiative Corrections to (H/W/Z+)Multi-Jets

Jeppe R. Andersen

Loops and Legs
April 28, 2010

Multiple ($\geq$ 2) hard jets...

Smaller number of jets solved satisfactory (?) already...(POWHEG, MC@NLO, NNLO,...)

Special radiation pattern from **current-current** scattering

Look into **higher order corrections beyond** "inclusive K-factor"

Concentrate on the **hard, perturbative corrections** relevant for a description of the final state **in terms of jets**?

Goal

Build framework for **all-order summation** (virtual+real emissions). Exact in another limit than the usual soft&collinear. Better suited for describing **radiation relevant for multi-jet** production.

Insight

Can use the insight gained from studying the relevant limit to **guide and improve** analyses: CP-properties of the Higgs-boson couplings

The Challenge, The Solution, Status

The Challenge (fka Problem), (in trivial statements)

Hard emission is less suppressed at **increasing collider energies**.
New problem for the LHC-era (W+jets, H+jets, ...)
NLO gets the **one** hard emission right, but **one may not be sufficient**.
Parton shower does **many emissions**, but **not the hard ones**.
PS+matching is **good at Tevatron**, but **sufficient at LHC?**

The Solution

High Energy Jets (HEJ): What it is; what it is **not**

Status

What **HEJ** can do for you
What $1fb^{-1}$@7TeV can tell us about our perturbative tools

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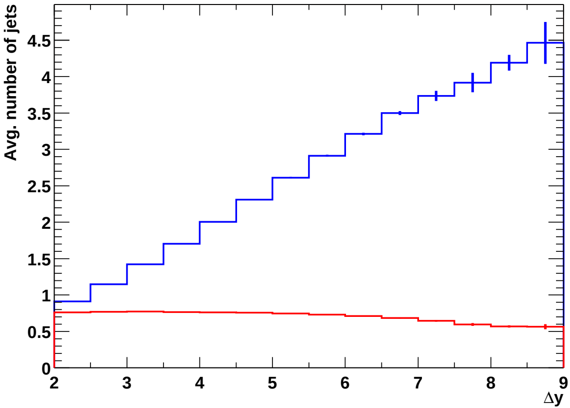
Prerequisites for the discussion

Observables: Focus is on the final state in terms of jet count and configuration (but not jet substructure).
The obtained description is fully exclusive. Will however concentrate the discussion on a few of the many possible observables, which capture the relationship between the **increasing phase space** (for increasing Δy between most forward and most backward hard jet) and the **amount of hard radiation**:

$\frac{\sigma_{N+1}}{\sigma_N}$, $\langle \#\text{jets} \rangle$,... vs. Δy.

Δy “large” can arise as a **result of specific phase space cuts** (H+jets), **or naturally** (W+jets) as a result of a dominance from qg initial state.

The Challenge: $\langle \#\text{jets} \rangle$ vs. Δy



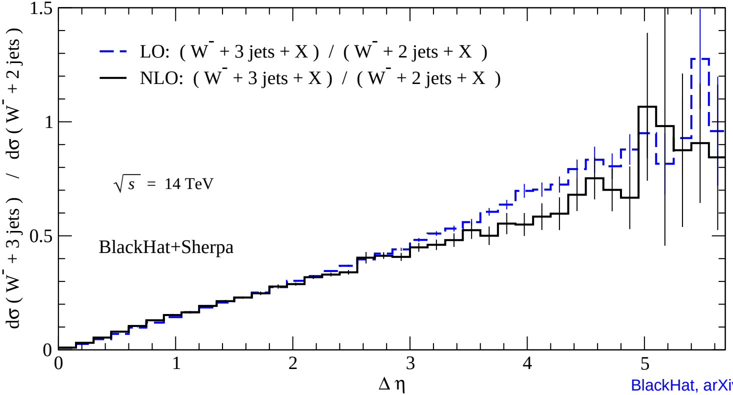
Red: Average number of central ($|y| < 1$) jets.

JRA, V. Del Duca, F. Maltoni, W.J. Stirling, hep-ph/0105146

Basic observation of increasing phase space for hard emissions with increasing Δy is the motivation for e.g. BFKL resummation.

However, don't just take *my* word for it...

W+Multiple Jets @ NLO

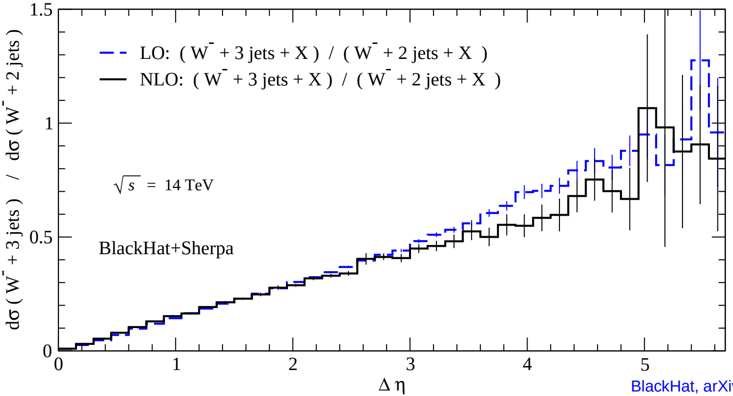


BlackHat, arXiv:0912.4927

The inclusive 3-jet rate is large compared to the inclusive 2-jet rate, even for normal rapidity spans obviously, the inclusive 3-jet rate "ought to" be smaller than the inclusive 2-jet rate.

The large contribution from real radiative corrections to W+dijets is not revealed by the inclusive K-factor (actually less than one)

W+Multiple Jets @ NLO



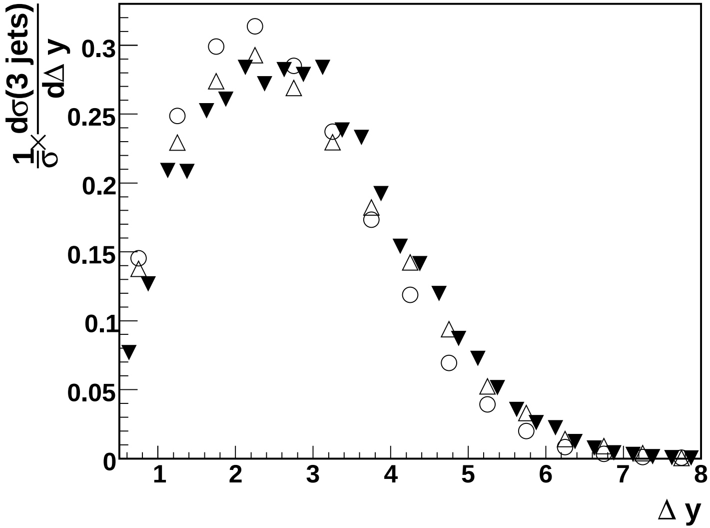
BlackHat, arXiv:0912.4927

All calculational methods and processes will agree on the opening of phase space as Δy increases

The mechanism for emission differ between processes (WBF vs. GF) and calculational methods (full NLO, shower, ...). Can be tested against data!

$1/\sigma\ d\sigma/d\Delta y$

JRA, M. Campanelli, J. Campbell, V. Ciulli, J. Huston, P. Lenzi, R. Mackeprang, arXiv:1003.1241



$\Delta y \approx 2 - 3$ (where σ_{3j}/σ_{2j} is already very large) is not “tail of distribution”!

HEJ (High Energy Jets)

- What is this HEJ?
- What is it **not**

HEJ (High Energy Jets)

Goal (inspired by the great Fadin & Lipatov)

Sufficiently **simple** model for hard radiative corrections that the all-order sum can be evaluated explicitly (completely exclusive)

but. . .

Sufficiently **accurate** that the description is relevant

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Soft limit $\longrightarrow$ **eikonal approximation** $\longrightarrow$ enters all parton shower (and much else) resummation.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider…

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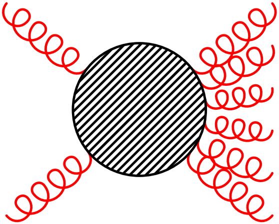
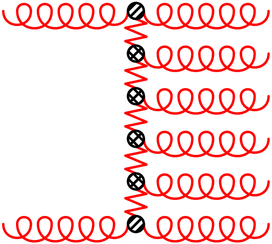


High Energy Limit

$$|\hat{t}| \quad \text{fixed}, \quad \hat{s} \to \infty$$



$$\mathcal{A}^R_{2\to 2+n} = \frac{\Gamma_{A'A}}{q_0^2}\left(\prod_{i=1}^{n} e^{\omega(q_i)(y_{i-1}-y_i)}\frac{V^{J_i}(q_i,q_{i+1})}{q_i^2 q_{i+1}^2}\right) e^{\omega(q_{n+1})(y_n-y_{n+1})}\frac{\Gamma_{B'B}}{q_{n+1}^2}$$

$q_i = \mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l$

LL: Fadin, Kuraev, Lipatov; NLL: Fadin, Fiore, Kozlov, Reznichenko

Maintain (at LL) terms of the form

$$\left(\alpha_s \ln \frac{\hat{s}_{ij}}{|\hat{t}_i|}\right)$$

to all orders in α_s.

At LL only gluon production; at NLL also quark–anti-quark pairs produced. Approximation of any-jet rate possible.

Comparison of 3-jet scattering amplitudes

Universal behaviour of scattering amplitudes in the HE limit:

$$\forall i \in \{2,\ldots,n-1\} : y_{i-1} \gg y_i \gg y_{i+1}$$
$$\forall i,j : |p_{i\perp}| \approx |p_{j\perp}|$$

$$\left|\overline{\mathcal{M}}^{MRK}_{gg\to g\cdots g}\right|^2 = \frac{4\,s^2}{N_C^2-1}\,\frac{g^2\,C_A}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_A}{|p_{n\perp}|^2}.$$

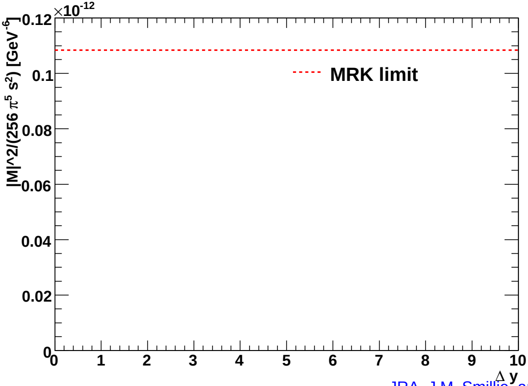
$$\left|\overline{\mathcal{M}}^{MRK}_{qg\to qg\cdots g}\right|^2 = \frac{4\,s^2}{N_C^2-1}\,\frac{g^2\,C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_A}{|p_{n\perp}|^2},$$

$$\left|\overline{\mathcal{M}}^{MRK}_{qQ\to qg\cdots Q}\right|^2 = \frac{4\,s^2}{N_C^2-1}\,\frac{g^2\,C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_F}{|p_{n\perp}|^2},$$

Allow for analytic resummation (BFKL equation).
However, how well does this actually approximate the amplitude?

Study just a slice in phase space:

40GeV jets in Mercedes star (transverse) configuration. Rapidities at $-\Delta y, 0, \Delta y$.



JRA, J.M. Smillie, arXiv:0908.2786

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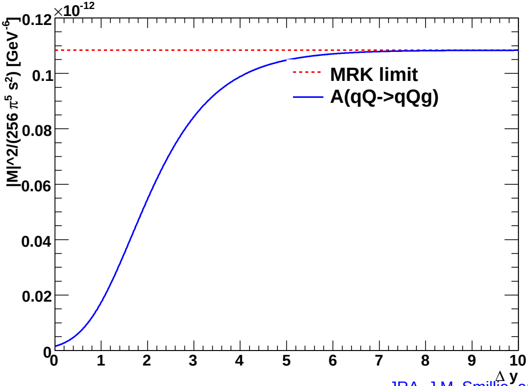

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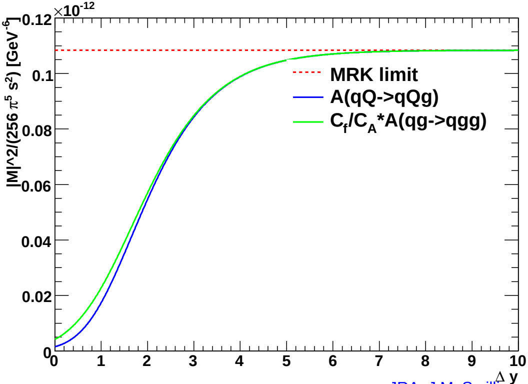

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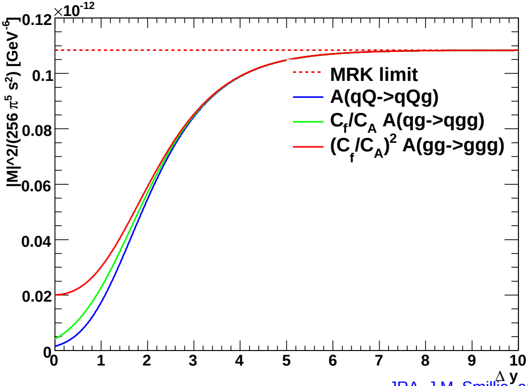

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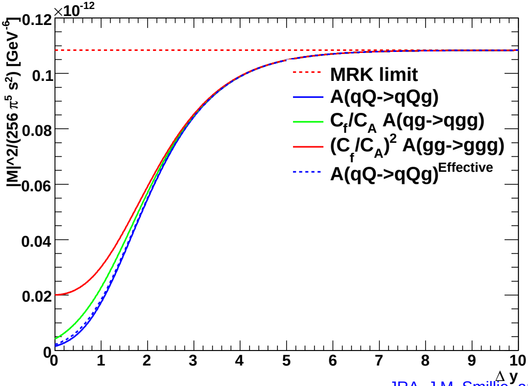

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1) Inspiration from Fadin&Lipatov: dominance by t-channel
2) No kinematic approximations in the position of these poles (denominator)
3) Accurate definition of currents (coupling through t-channel exchange)
4) Gauge invariance. Not just asymptotically.

Scattering of qQ-Helicity States

Start by describing quark scattering. Simple matrix element for $q(a)Q(b) \to q(1)Q(2)$:

$$M_{q^-Q^-\to q^-Q^-} = \langle 1|\mu|a\rangle \frac{g^{\mu\nu}}{t} \langle 2|\nu|b\rangle$$

t-**channel factorised**: Contraction of (local) currents across *t*-channel pole

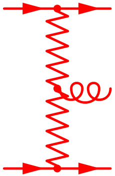
$$\begin{aligned}\left|\overline{\mathcal{M}}^t_{qQ\to qQ}\right|^2 = &\frac{1}{4\left(N_C^2-1\right)} \left\|S_{qQ\to qQ}\right\|^2\\ &\cdot \left(g^2\, C_F\, \frac{1}{t_1}\right)\\ &\cdot \left(g^2\, C_F\, \frac{1}{t_2}\right).\end{aligned}$$

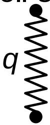
Extend to $2 \to n$ ...

J.M.Smillie and JRA: arXiv:0908.2786

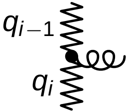
Building Blocks for an Amplitude

Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles



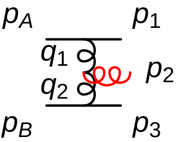


$$\frac{1}{q^2}\exp\left(\hat{\alpha}(q)\Delta y\right)$$

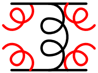


$$V^{\mu}(q_{i-1}, q_i)$$

$$j^{\nu} = \overline{\psi}\gamma^{\nu}\psi$$



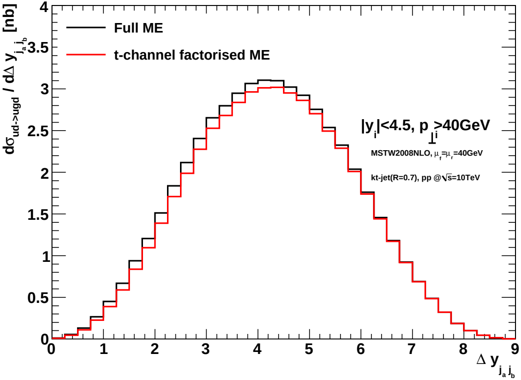
$$\begin{aligned} V^{\rho}(q_1, q_2) = &- (q_1 + q_2)^{\rho} \\ &+ \frac{p_A^{\rho}}{2}\left(\frac{q_1^2}{p_2 \cdot p_A} + \frac{p_2 \cdot p_B}{p_A \cdot p_B} + \frac{p_2 \cdot p_n}{p_A \cdot p_n}\right) + p_A \leftrightarrow p_1 \\ &- \frac{p_B^{\rho}}{2}\left(\frac{q_2^2}{p_2 \cdot p_B} + \frac{p_2 \cdot p_A}{p_B \cdot p_A} + \frac{p_2 \cdot p_1}{p_A \cdot p_1}\right) - p_B \leftrightarrow p_3. \end{aligned}$$



Building Blocks for an Amplitude

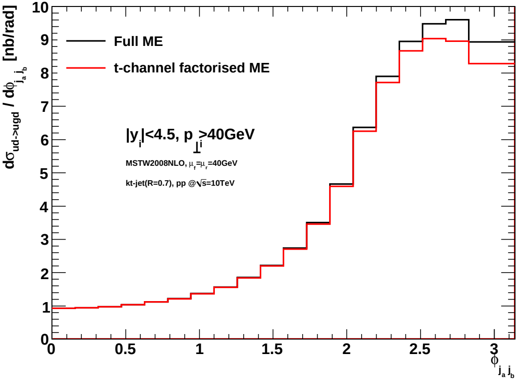
$p_g \cdot V = 0$ can easily be checked (gauge invariance)
The approximation for $qQ \to qgQ$ is given by

$$\left|\overline{\mathcal{M}}^t_{qQ\to qgQ}\right|^2 = \frac{1}{4\,(N_C^2-1)}\ \left\|S_{qQ\to qQ}\right\|^2$$
$$\cdot\ \left(g^2\,C_F\,\frac{1}{t_1}\right)\cdot\ \left(g^2\,C_F\,\frac{1}{t_2}\right)$$
$$\cdot\left(\frac{-g^2 C_A}{t_1 t_2}\,V^\mu(q_1,q_2)V_\mu(q_1,q_2)\right).$$



J.M.Smillie and JRA: arXiv:0908.2786

3 Jets @ 10 TeV



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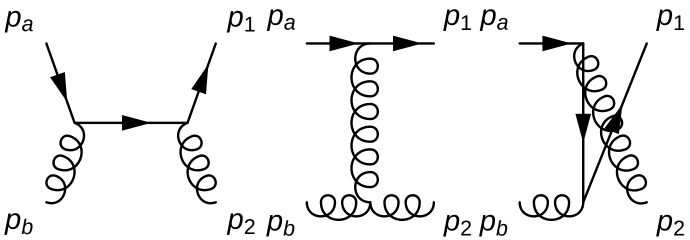
Quark-Gluon Scattering

"What happens in $2 \to 2$-processes with gluons? Surely the t-channel factorisation is spoiled!"



Direct calculation ($q^- g^- \to q^- g^-$):

$$M = \frac{g^2}{\hat{t}} \times \frac{p_{2\perp}^*}{|p_{2\perp}|} \left(t^2_{ae} t^b_{e1} \sqrt{\frac{p_b^-}{p_2^-}} - t^b_{ae} t^2_{e1} \sqrt{\frac{p_2^-}{p_b^-}} \right) \langle b|\sigma|2\rangle \ \times \langle 1|\sigma|a\rangle.$$

Complete t-channel factorisation!

J.M.Smillie and JRA

Quark-Gluon Scattering

For the helicity choices where a qQ-channel exists, the t-channel current generated by a gluon in qg scattering is that of a quark, but with a colour factor

$$\frac{1}{2}\left(C_A - \frac{1}{C_A}\right)\left(\frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-}\right) + \frac{1}{C_A}$$

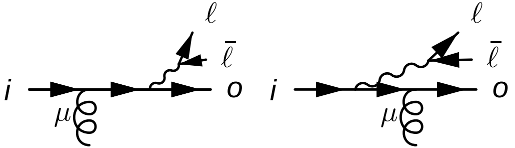
instead of C_F. Tends to C_A in MRK limit.

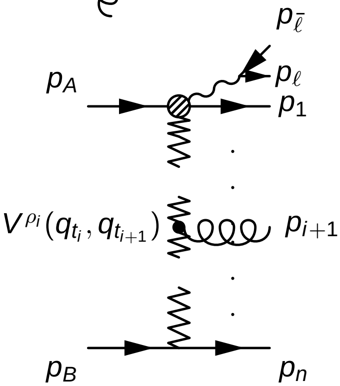
Similar results for e.g. $g^+g^- \to g^+g^-$. **Exact, complete** t-channel **factorisation**.

By using the formalism of **current-current scattering**, we get a better description of the t-channel pole than by using just the kinematic limit.

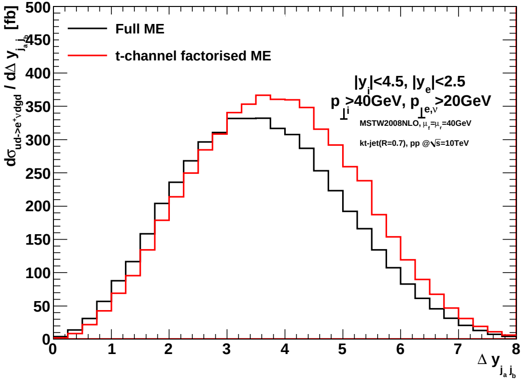
W+Jets

Two currents to calculate for $W + jets$:

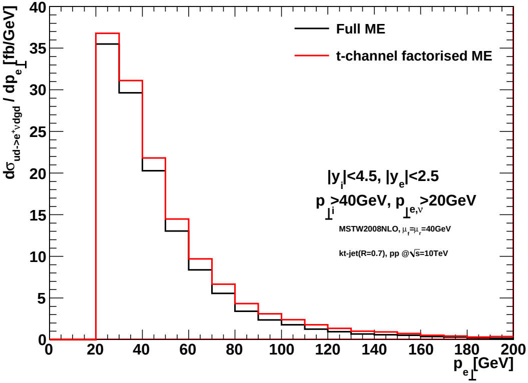




W+ 3 Jets @ LHC



J.M.Smillie and JRA: arXiv:0908.2786



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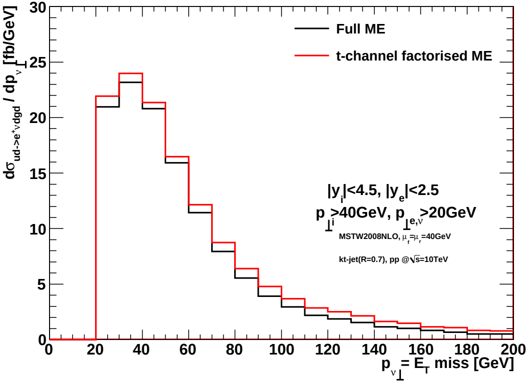


J.M.Smillie and JRA: arXiv:0908.2786

All-Orders and Regularisation

- Have prescription for $2 \to n$ matrix element, including virtual corrections: Lipatov Ansatz $1/t \to 1/t\,\exp(-\omega(t)\Delta y_{ij})$
- Organisation of cancellation of IR (soft) divergences is easy
- Can calculate the sum over the n-particle phase space explicitly ($n \sim 30$) to get the all-order corrections (just as if one had provided all the $N^{30}LO$ matrix elements and a regularisation procedure)
- **Match** to n-jet tree-level where known

J.M. Smillie, JRA arXiv:0908.2786, arXiv: 0910:5113

What HEJ is NOT

- **Small-x evolution of pdfs**. x isn't even small. And we are using standard collinear factorisation - which allows for a stringent comparison with standard PT!
- **BFKL**
  - We have no approximation of kinematic invariants. $q_\perp^2 \neq -t$ at LHC energies. Try for yourself. It is orders of magnitude off!
  - No evolution equation
  - No kernel
  - No impact factors
  - . . . but we do have gauge invariance. Everywhere in phase space. Not just asymptotically.

What HEJ can do for you

Describe the hard multi-jet environment for several processes (all matched):

NOW

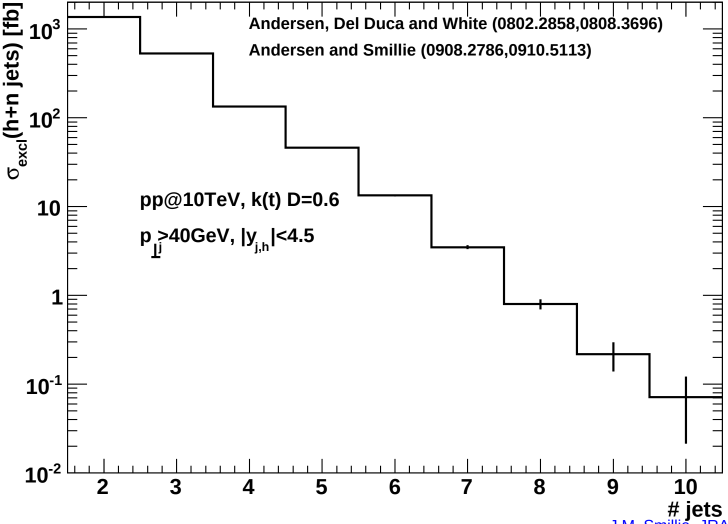
Released code: H+jets

root n-tuples: W+jets (or ask nicely and you will get the code)

soon

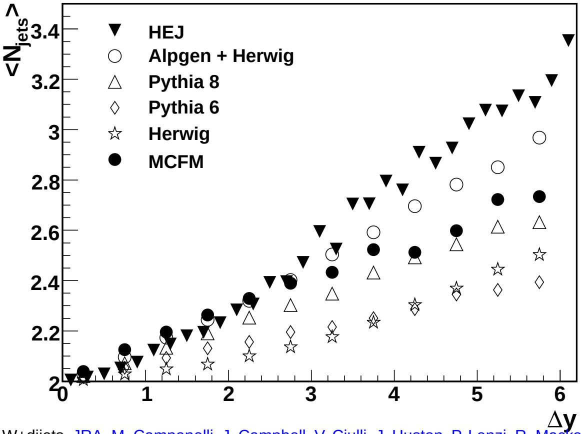
Z+jets, jets...

What HEJ can do for you



J.M. Smillie, JRA arXiv:1001.4463

What can $1fb^{-1}$ tell us about our perturbative tools



W+dijets, JRA, M. Campanelli, J. Campbell, V. Ciulli, J. Huston, P. Lenzi, R. Mackeprang, arXiv:1003.1241

$1fb^{-1}$@7TeV could be enough to tell the predictions apart!

Obviously, similar results for pure dijets with much less data

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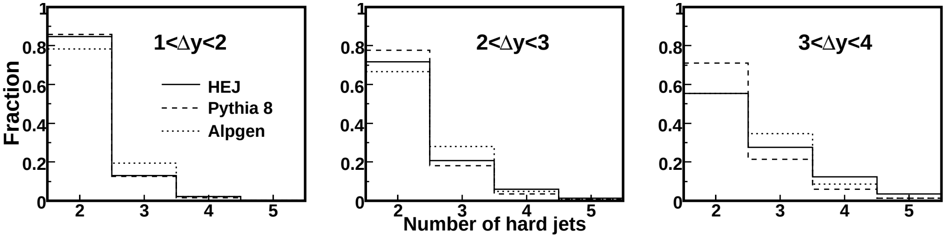

Many handles to distinguish the predictions from various perturbative approaches using early data

W+dijets, JRA, M. Campanelli, J. Campbell, V. Ciulli, J. Huston, P. Lenzi, R. Mackeprang, arXiv:1003.1241

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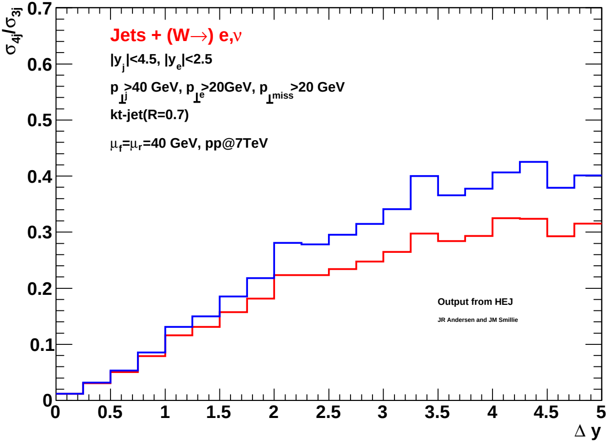
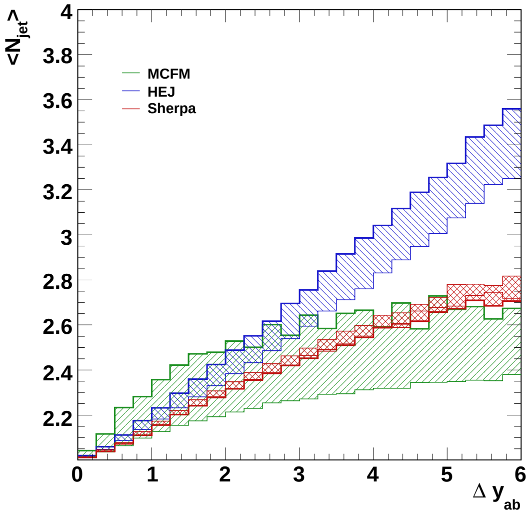
Predictions for W+4 jets



(to be compared to future results from e.g. BlackHat)

Why is ⟨#jets⟩ in W+jets interesting?

Similarities to H+dijets

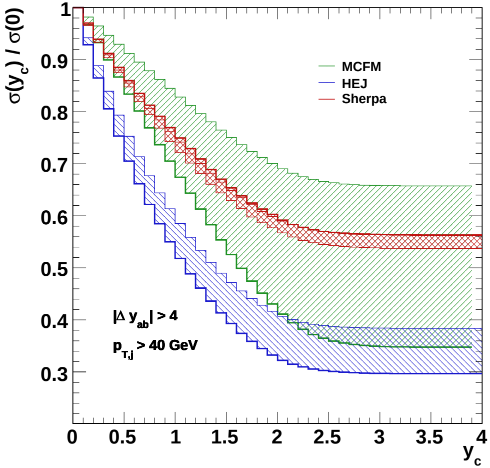


W+jets can help in investigating jet veto efficiencies, . . .

JRA, J. Campbell, S. Höche,

arXiv:1003.1241

Effect of Central Jet Veto



J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.1241

$$\forall j \in \{\text{jets with } p_{j\perp} > 40\text{GeV}\} \setminus \{a, b\} \;:\; \left| y_j - \frac{y_a + y_b}{2} \right| > y_c$$

CP Properties of Higgs-Boson Couplings from Hjj through Gluon Fusion

Stabilising the Extraction against Higher Order Corrections

Why Hjj, The Problem, The Solution

Why study Higgs Boson production in Association with Dijets?

The distribution in the **azimuthal angle** between the **two** jets in *Hjj* allows for a **clean extraction** of CP properties

The Problem

. . . in a region of phase space where the **perturbative corrections are large**.

How do we deal with events with **three or more** jets?

The Solution

By constructing an azimuthal observable, which takes into account the **information from all the jets** of the event!

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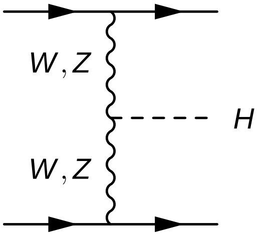
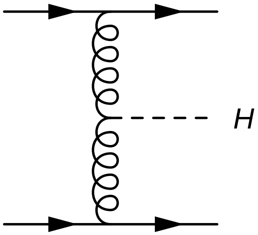


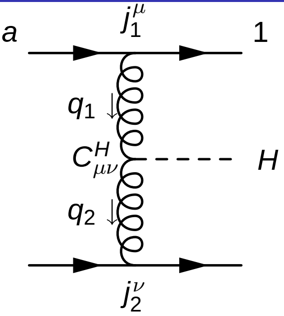
Considerations for Weak Boson Fusion

Higgs Couplings through Azimuthal Correllations



. . . and gluon fusion (Higgs coupling to gluons through top loop)

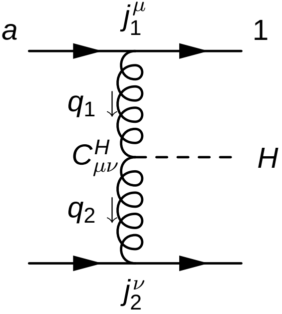
Higgs Couplings through Azimuthal Correllations



$$\mathcal{M} \propto \frac{j_1^\mu \, C^H_{\mu\nu} \, j_2^\nu}{t_1 \, t_2}, \qquad j_1^\mu = \overline{\psi}_1 \gamma^\mu \psi_a$$

$$C_H^{\mu\nu} = a_2 \, (q_1 q_2 g^{\mu\nu} \; - \; q_1^\nu q_2^\mu) \\ + a_3 \, \varepsilon^{\mu\nu\rho\sigma} \, q_{1\rho} \, q_{2\sigma}.$$

Higgs Couplings through Azimuthal Correllations



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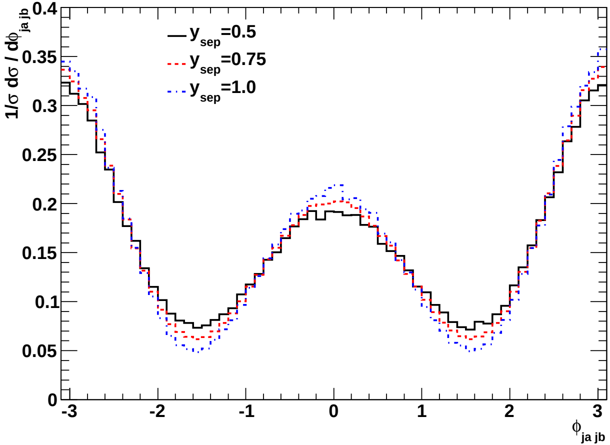
Take e.g. the term $\varepsilon^{\mu\nu\rho\sigma}\, q_{1\rho}\, q_{2\sigma}$: for $|p_{1,z}| \gg |p_{1,x,y}|$ and for small energy loss (i.e. $p_{a,e} \sim p_{1,e}$):

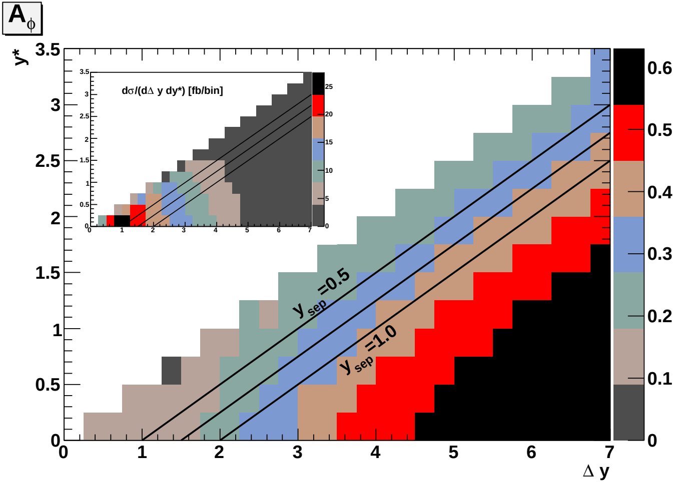
$$\left[j_1^0\, j_2^3 - j_1^3\, j_2^0\right] (\mathbf{q}_{1\perp} \times \mathbf{q}_{2\perp}).$$

In this limit, the azimuthal dependence of the propagators is also suppressed: $|\mathcal{M}|^2$:$\sin^2(\phi)$ **(CP-odd)**, $\cos^2(\phi)$ **(CP-even)**.



JRA, K. Arnold, D. Zeppenfeld, arXiv:1001.3822

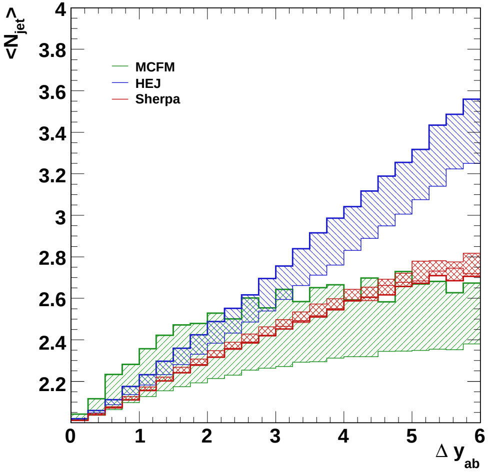
$$CP\text{-even}, \; p_{j\perp} > 40 \text{ GeV}, \quad y_{ja} < y_h < y_{jb},$$
$$|y_{j_a,j_b}| < 4.5, \min\left(|y_h - y_{j_a}|, |y_h - y_{j_b}|\right) > y_{\text{sep}}.$$



$$\Delta y = |y_{j_a} - y_{j_b}|, \quad y^* = y_h - \frac{y_{j_a}+y_{j_b}}{2}.$$

JRA, K. Arnold, D. Zeppenfeld

Rapidity separation between the jets and the Higgs Boson **enhance the azimuthal correlation**.



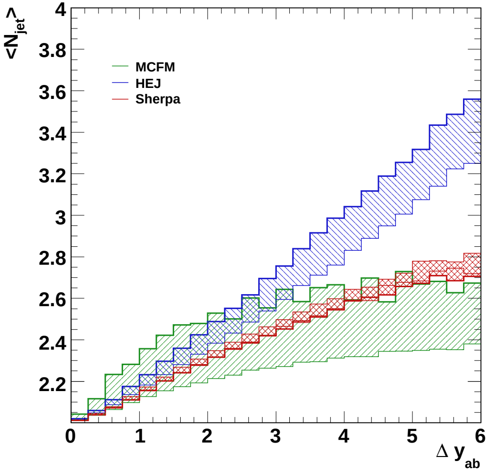
All models show a clear increase in the number of hard jets as the rapidity span increases.

How to extract the *CP*-structure of the Higgs boson coupling from events with **three or more** jets?

2 hardest jets?

J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.1241

Increasing Rapidity Span → Increasing Number of Jets



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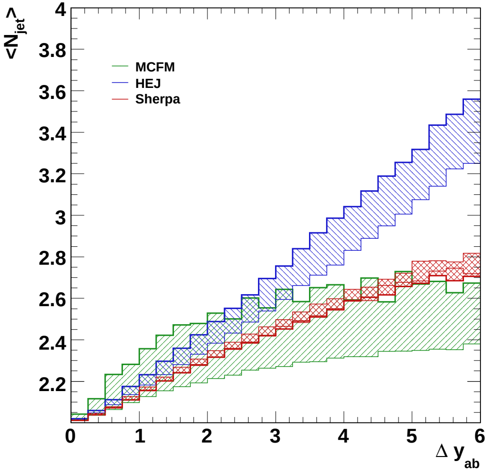
How to extract the *CP*-structure of the Higgs boson coupling from events with **three or more** jets?

2 hard jets furthest apart in rapidity?

J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.1241



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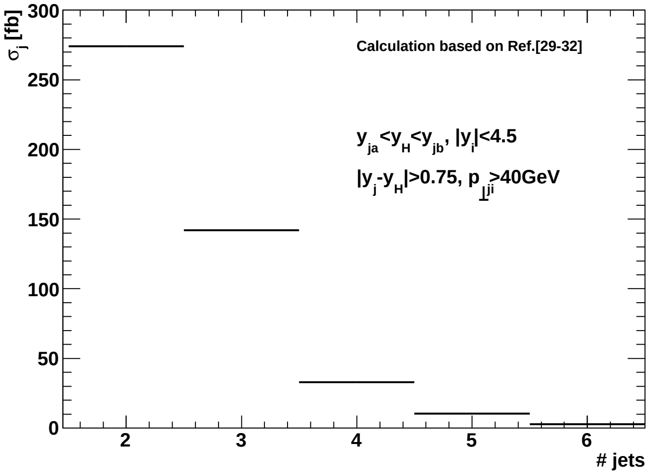
How to extract the *CP*-structure of the Higgs boson coupling from events with **three or more** jets?

Significant washing out of the azimuthal correlation observed at tree-level *hjj*

J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.1241

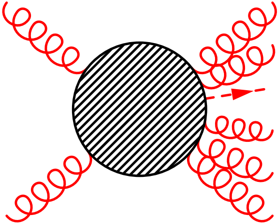
Many Jets!



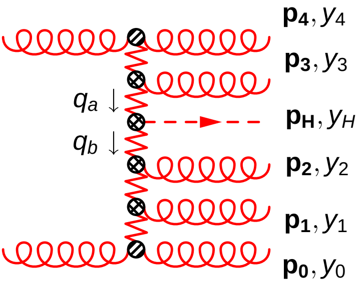
Calculation based on all-order approximant to the *n*-particle matrix element, which reproduces the exact result in the limit of large invariant mass between all particles.

JRA&C.D. White, JRA&J.M. Smillie

Develop Insight Into the Perturbative Corrections



High Energy Limit
$\longrightarrow$
$|p_{\perp,i}|$ fixed, $\hat{s}_{ij} \to \infty$

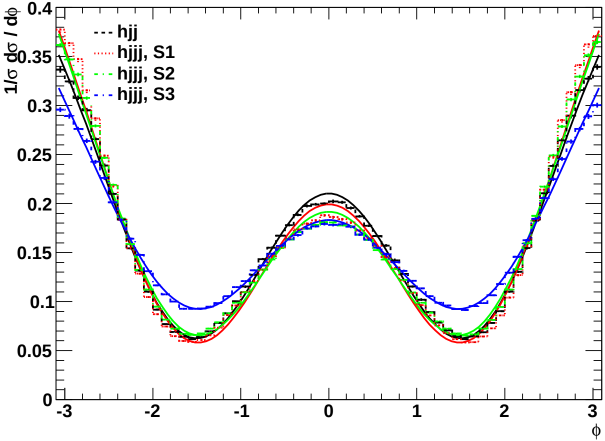


$$\left|\mathcal{M}_{gg\to g\cdots ghg\cdots g}\right|^2 \to \frac{4\hat{s}^2}{N_C^2-1}\left(\prod_{i=1}^{j}\frac{C_A\, g_s^2}{\mathbf{p}_{i\perp}^2}\right)\frac{|C^H(\mathbf{q}_{a\perp},\mathbf{q}_{b\perp})|^2}{\mathbf{q}_{a\perp}^2\ \mathbf{q}_{b\perp}^2}\left(\prod_{i=j+1}^{n}\frac{C_A\, g_s^2}{\mathbf{p}_{i\perp}^2}\right)$$

$$C^H(\mathbf{q}_{a\perp},\mathbf{q}_{b\perp}) \;=\; -i\,\frac{\alpha_s}{3\pi v}\,\mathbf{q}_{a\perp}\cdot\mathbf{q}_{b\perp}, \quad y_0 < \cdots < y_j < y_H < y_{j+1} < y_n$$

The **High Energy Limit** tells us to investigate the **azimuthal angle** between the **sum of the jet vectors** either side in rapidity of the Higgs Boson!

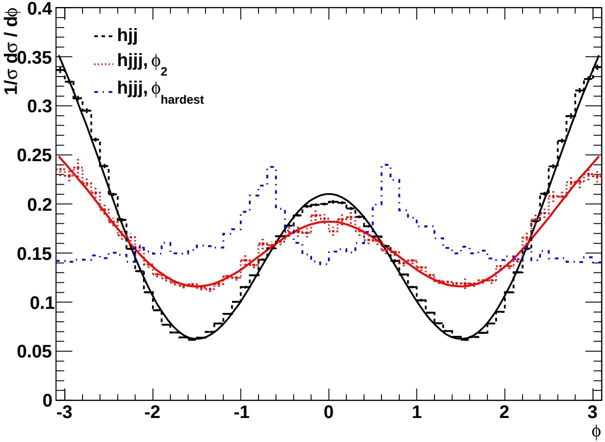
And It Even Works!



JRA, K. Arnold, D. Zeppenfeld, arXiv:1001.3822

Three subsamples of tree-level three-jet events: two jets on same side of the Higgs boson parallel (S1), perpendicular (S2) or anti-parallel (S3). Azimuthal correlation almost unchanged from hjj.

...Much Better Than Any Alternative



JRA, K. Arnold, D. Zeppenfeld, arXiv:1001.3822

Two hardest jets on one side, and the softest on the other (all above 40GeV - $1/3$ of inclusive 3-jet cross section). Using **just the two hardest** jets gives **unsatisfactory** result.

Summary of H+Dijet Study

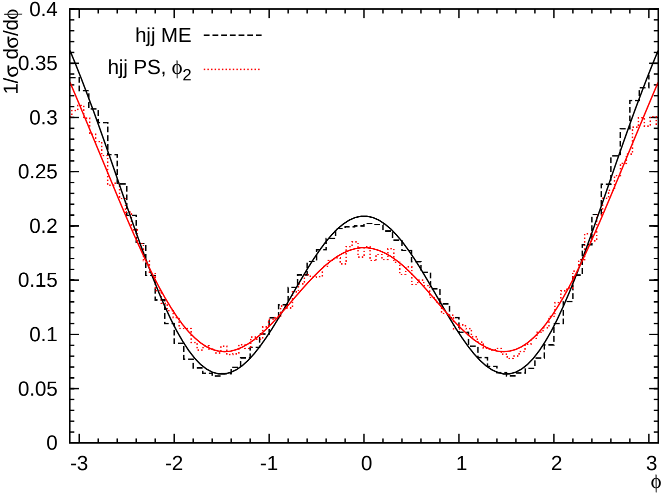
1. Full hjjj tree-level confirms expectations from High Energy Limit
2. Observable stable when shower+hadronisation effects are added (LO+HERWIG++)
    1. However, the parton shower delivers a very poor description of the multi-jet configurations, when compared to e.g. hjjj tree-level
3. Observable stable when additional hard perturbative corrections are summed to all orders (HEJ)

See arXiv:1001.3822 for all the details

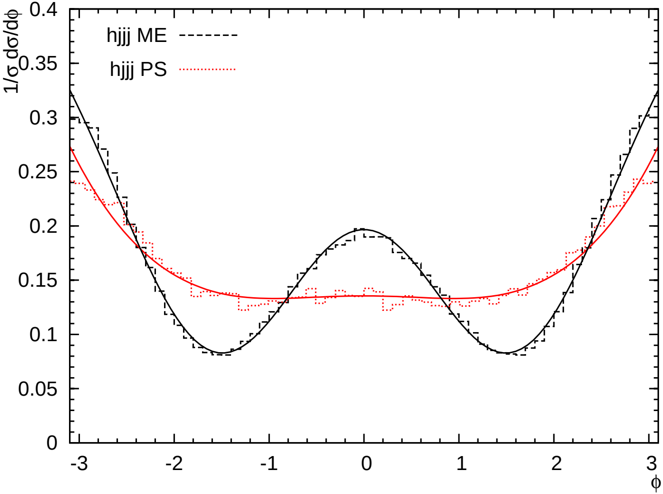
Summary

1. HEJ is a new perturbative tool for the description of multi-jet events at high energy colliders
   1. Simplify pert. corrections by concentrating on widely separated emissions
   2. Filling in the details of each jet (soft, collinear) is a job left for a parton shower
2. The insight gained from the study has already improved analysis for the LHC
3. Even the 1st $fb^{-}1$@7TeV will shed light on the multi-jet environment in the new high energy domain.

Stability Against Corrections Implemented in a Parton Shower



. . . however, shower does not describe three-jet sample accurately



Stability Against Hard, Higher Order Corrections