Threshold expansion of massive coloured particle cross sections

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Loops and Legs in Quantum Field Theory
Wörlitz, Germany, April 25 - 30, 2010

Outline

- Introduction: heavy particle pair production and resummation near threshold
- SCET and PNRQCD factorization; soft and Coulomb gluon resummation
  Current results
- Squark anti-squark production
  Top anti-top production

MB, P. Falgari, C. Schwinn, 0907.1443 [hep-ph] and in preparation (factorization, colour, anomalous dimension, squark production);
MB, M. Czakon, P. Falgari, A. Mitov, C. Schwinn, arXiv:0911.5166 [hep-ph] (α_s^2 log terms for $t\bar{t}$);
MB, P. Falgari, S. Klein, C. Schwinn, work in progress (top quark production)

Pair production and resummation

$$ij\,(q\bar{q}, qg, gg, e^+e^-) \to HH' + X$$

$$\sigma = \sum_{i,j} \int_{4\bar{m}_H^2}^{s} d\hat{s}\, \mathcal{L}_{ij}(\hat{s}/s, \mu_f)\, \hat{\sigma}_{ij}(\hat{s}, m_H, m_H'; \mu_f)$$

$$\beta = \sqrt{1 - 4\bar{m}_H^2/\hat{s}}$$

“Non-perturbative” despite small couplings:

- Soft gluon (photon) resummation, Sudakov logarithms: $g^2 \ln^2 \beta \sim 1$.
- Strong Coulomb force: $g^2/\beta \sim 1$
- Sizeable decay widths of H, H': $\Gamma_H/m_H \sim g^2$
  Physical final states, “Dyson resummation”, non-resonant backgrounds

(Perturbative) Resummations in the frameworks of (P)NRQCD, SCET, unstable particle EFT.

Total partonic cross sections and invariant mass distributions

Partonic cross sections [here: $\hat{\sigma}$ and invariant mass distribution $d\hat{\sigma}/dM_{HH'}$] $q\bar{q}, qg, gg \to HH' + X$ contain

$$\left[\alpha_s \ln^2(1-z)\right]^n, \qquad z = M_{HH'}^2/\hat{s}$$

which should be resummed, if the total hadronic cross section is dominated by the partonic threshold. (DY: Sterman, 1997; Catani, Trentadue, 1989; Two particles/jets: Kidonakis, Sterman, 1997; Bonciani et al, 1998)

- Invariant mass distribution with $M_{HH'} \geq$ few $\times (m_H + m_{H'})$, such that H, H' are relativistic when $z \to 1$.

  Colour exchange and s, t-dependent anomalous dimensions.
  Coulomb gluons not relevant

  NNLL resummation recently done (Ahrens, Ferroglia, Neubert, Pecjak, Li, 2010/2009)

- Total cross section with $M_{HH'} = m_H + m_{H'}$, such that the threshold $\beta \to 0$ is dominated by non-relativistic H, H' $(1 - z = \beta^2)$.

  Colour exchange and anomalous dimensions simple.
  Constant terms in invariant mass distribution develop Coulomb singularities $(\alpha_s/\beta)^n \times$ logs in addition to soft-gluon threshold logs $(\alpha_s \ln^2 \beta)^n$.

  Requires factorization analysis and resummation formalism for soft and Coulomb terms (MB, Falgari, Schwinn, 2009/2010) (sth like DY resum $\times$ $e^+e^- \to t\bar{t}$ near threshold)

Other recent work

Other recent work on improving heavy particle pair production cross sections beyond NLO by resummation or approximate higher-order terms:

- $t\bar{t}$, total cross section

  Moch, Uwer, 2008; Cacciari, Frixione, Mangano, Nason, Ridolfi, 2008; Kidonakis, Vogt, 2008; Langenfeld, Moch, Uwer, 2009; Ahrens, Ferroglia, Neubert, Pecjak, Li, 2010

- sparticle pairs, total cross section

  Kulesza, Motyka, 2008/2009; Langenfeld, Moch, 2009; Beenakker, Brensing, Laenen, Krämer, Kulesza, Niessen, 2009

- Invariant mass distributions
  - Hagiwara, Sumino, Yokoya, 2008; Kiyo, Kühn, Moch, Steinhauser, Uwer, 2008 ($t\bar{t}$ threshold)
  - Hagiwara, Yokoya, 2009 (sparticle pair threshold)
  - Ahrens, Ferroglia, Neubert, Pecjak, Li, 2010 ($t\bar{t}$, resummation for large $z \longrightarrow 1$)

Present talk: towards joint NNLL resummation of soft and Coulomb gluons.

Systematics of soft-gluon and Coulomb resummation

Expansion of the partonic cross section

$$\hat{\sigma}(\beta) = \hat{\sigma}^{(0)} \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \ldots\Big]$$
$$\times \Big\{1 \text{ (LL,NLL)};\ \alpha_s,\ \beta \text{ (NNLL)};\ \alpha_s^2,\ \alpha_s\beta,\ \beta^2 \text{ (NNNLL)};\ \ldots\Big\}$$

Counting: $\alpha_s/\beta, \alpha_s \ln\beta \sim 1$

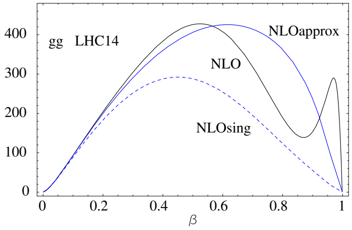
[Note: Non-relativistic logs can also be cast into the above form.]

In fixed order [Note: NNLL can be α_s/β[Coulomb] $\times$ β[sub-leading soft] $\times$ $\alpha_s \ln^2\beta$ – beyond the standard soft gluon approximation!]

LL	$\alpha_s \left\{\frac{1}{\beta}, \ln^2\beta\right\};\ \alpha_s^2 \left\{\frac{1}{\beta^2}, \frac{\ln^2\beta}{\beta}, \ln^4\beta\right\}; \ldots,$
NLL	$\alpha_s \ln\beta;\ \alpha_s^2 \left\{\frac{\ln\beta}{\beta}, \ln^3\beta\right\}; \ldots,$
NNLL	$\alpha_s \left\{1, \beta \times \ln^{2,1}\beta\right\};\ \alpha_s^2 \left\{\frac{1}{\beta}, \ln^{2,1}\beta, \beta \times \ln^{4,3}\beta\right\}; \ldots,$

Why threshold expansion?

- Strictly valid for high masses $2\bar{m}_H \rightarrow s_{\rm had}$ (heavy sparticles).
- Certainly not for tops at LHC7. Invariant mass distribution peaks at 380 GeV, corresponding to $\beta \approx 0.4$, but the average β is larger. No large logs.
- Assume that threshold expansion provides a good approximation for the integral over all β. Works reasonable well for gg at LO and NLO, less well for $q\bar{q}$, and probably better at NNLO, because the average is dominated by smaller β as the order increases.
  [Note: Mutliplying the expanded relative NLO correction by the exact tree improves the approximation.]

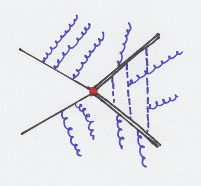


	Tevatron	LHC7	LHC14
$\langle\beta\rangle_{gg,\rm NLO}$	0.41	0.49	0.53
LO	5.25	101.9	563.3
NLO	6.49	149.9	842.7
$\rm NLO_{sing}$	6.76	138.8	751.2
$\rm NLO_{approx}$	7.45	159.0	867.6

Cross section in pb, MSTW2008nnlo PDFs.

Joint soft-gluon and Coulomb resummation

Factorization of soft and Coulomb gluons is non-trivial, since soft gluons attach to and between Coulomb ladders and the energy of Coulomb gluons is also soft.
Also Coulomb exchange carries colour structure $T_R^A \otimes T_{R'}^A$.



Still factorizes!

Factorization formula [sum over a includes sub-leading powers in β.]

$$\hat{\sigma}(\beta,\mu) = \sum_a \sum_{i,i'} H^a_{ii'}(M,\mu) \int d\omega \sum_{R_\alpha} J^a_{R_\alpha}(E-\frac{\omega}{2})\, W^{a,R_\alpha}_{ii'}(\omega,\mu).$$

At NNLL there is factorization dependence on μ also in the Coulomb function.

Soft-gluon decoupling

From the initial state:

$$\mathcal{L}_c = \bar{\xi}_c \left(in \cdot D + i \not{D}_{\perp c} \frac{1}{i\bar{n} \cdot D_c} i\not{D}_{\perp c} \right) \frac{\not{\bar{n}}}{2} \, \xi_c - \frac{1}{2} \, \mathrm{tr} \left(F_c^{\mu\nu} F^c_{\mu\nu} \right)$$

by the SCET field redefinitions (Bauer, Pirjol, Stewart, 2001) $\xi_c(x) = S_n^{(3)}(x_-)\, \xi_c^{(0)}(x)$, $A^A_{c\mu}(x) = S_n^{(8)}(x_-)\, A^{A(0)}_{c\mu}(x)$, such that $n \cdot D \to n \cdot D_c$.

From the final state:

$$\mathcal{L}_{\mathrm{PNRQCD}} = \psi^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^{\,\dagger} \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi'$$
$$+ \int d^3\vec{r} \left[\psi^\dagger \mathbf{T}^{(R)a} \psi \right](\vec{r}) \left(\frac{\alpha_s}{r} \right) \left[\psi'^{\,\dagger} \mathbf{T}^{(R')a} \psi' \right](0)$$

by the PNRQCD field redefinition $\psi_a(x) = S_v^{(R)}(x^0)_{ab}\, \psi_b^{(0)}(x)$, such that $D_s^0 \to \partial^0$.

[S_v drops out from the Coulomb interaction, since $S_v^{(R)\dagger} \mathbf{T}^{(R)a} S_v^{(R)} = [S_{\mathrm{ad}}]^{ab}\, \mathbf{T}^{(R)b}$ in any rep R; S_{ad} is real and independent of $\vec{r}$.]

Soft-gluon decoupling (II)

Proves decoupling of soft gluon and Coulomb resummation, since soft gluons disappear from the leading-order Lagrangians for the other fields. Sub-leading interactions can be treated as perturbations in β.

Soft gluons are collected in a soft function (vacuum correlator) of eight Wilson lines:

$$\hat{W}^{\{k\}}_{\{ab\}}(z,\mu) = \langle 0|\overline{\mathrm{T}}[S_{v,b_4k_2} S_{v,b_3k_1} S^{\dagger}_{\bar{n},jb_2} S^{\dagger}_{n,ib_1}](z)\mathrm{T}[S_{n,a_1i} S_{\bar{n},a_2j} S^{\dagger}_{v,k_3a_3} S^{\dagger}_{v,k_4a_4}](0)|0\rangle\,,$$

Everything can be made colour-diagonal by going to a colour basis constructed from the Clebsh-Gordan coefficients of the irreducible representations R_α of the HH' pair (MB, Falgari, Schwinn, 2009), which behaves like a single particle in rep α.
Anomalous dimensions satisfy Casimir scaling (up to two loops required for NNLL).

At this point can use a combination of known methods

- Diagrammtic threshold expansion (MB, Smrinov, 1997)
- SCET soft-gluon resummation formalism like for Drell-Yan production (Becher, Neubert, Xu, 2007)
- Non-relativistic resummation like for $e^+e^- \longrightarrow t\bar{t}$ (MB, Signer, Smirnov, 1999)

Leading term, hard matching coefficients

$$\hat{\sigma}(\beta,\mu) = \sum_i H_i(M,\mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2})\, W_i^{R_\alpha}(\omega,\mu).$$

- Prodcution described by local operators

$$\mathcal{O}^{(\ell)}_{\{a;\alpha\}}(\mu) = \left[\phi_{c;a_1}\phi_{\bar{c};a_2}\psi^\dagger_{a_3}\psi'^\dagger_{a_4}\right](\mu)$$

  of collinear (initial state) and non-relativistic (final state) fields.

- $H_i = C_i C_i^*$ – colour-unaveraged partonic (hard) cross section directly at threshold.
  C_i = matching coeffficient of production operator, contains hard quantum corrections to production.
  [No spin-separation needed at NNLL, for top quarks.]

- NLL needs H_i at tree level
  NNLL needs H_i at $\mathcal{O}(\alpha_s)$. Known for top anti-top production from (Czakon, Mitov, 2008).

Leading term, Coulomb function

$$\hat{\sigma}(\beta,\mu) = \sum_i H_i(M,\mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2})\, W_i^{R_\alpha}(\omega,\mu).$$

- J_{R_α} – sums Coulomb-exchange to all orders for HH' in irrep R_α.
  Related to a correlation function of non-relativistic fields in PNRQCD.

- At LL and NLL

$$J_{R_\alpha}(E) \propto \text{Im}\left[-\frac{(2m_{\text{red}})^2}{4\pi}\left\{\sqrt{-\frac{E}{2m_{\text{red}}}} + \alpha_s(-D_{R_\alpha})\left[\frac{1}{2}\ln\left(-\frac{8\,m_{\text{red}}E}{\mu^2}\right) + \psi\left(1 - \frac{\alpha_s(-D_{R_\alpha})}{2\sqrt{-E/(2m_{\text{red}})}}\right)\right]\right\}\right]$$

- At NNLL need resummation of insertion of one-loop correction to Coulomb potential.
  Known analytically from (MB, Signer, Smirnov, 1999).
  Plus log-enhanced higher-order terms (see below).

Leading term, soft function

$$\hat{\sigma}(\beta,\mu) = \sum_i H_i(M,\mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2})\, W_i^{R_\alpha}(\omega,\mu).$$

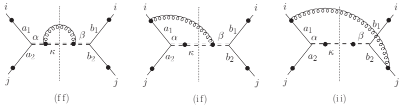
- Soft function for the production of a *single* particle in irrep R_α

$$\hat{W}^{R_\alpha}_{\{a\alpha,b\beta\}}(z,\mu) \equiv \langle 0|\overline{\mathrm{T}}[S^{R_\alpha}_{v,\beta\kappa} S^{\dagger}_{\bar{n},jb_2} S^{\dagger}_{n,ib_1}](z)\mathrm{T}[S_{n,a_1 i} S_{\bar{n},a_2 j} S^{R_\alpha\,\dagger}_{v,\kappa\alpha}](0)|0\rangle$$

- NLL requires $W_i^{R_\alpha}$ at tree level, NNLL at $\mathcal{O}(\alpha_s)$.
  Fourier transform ($L = 2\ln(iz\mu e_E^{\gamma}/2)$)

$$\hat{W}_i^{(1)R_\alpha}(L,\mu) = (C_r + C_{r'})\left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon}L + L^2 + \frac{\pi^2}{6}\right) + 2C_{R_\alpha}\left(\frac{1}{\epsilon} + L + 2\right).$$

(MB, Falgari, Schwinn, 2009;
for octet see also Idilbi, Kim, Mehen, 2009)



Resummation

- NNLL (NLL) soft-gluon resummation needs the 3-loop (2-loop) cusp anomalous dimension and 2-loop (1-loop) soft anomalous dimension in

$$\frac{d}{d\ln\mu}\hat{W}_i^{R_\alpha}(L) = \left((\Gamma^r_{\rm cusp} + \Gamma^{r'}_{\rm cusp})L - 2\gamma^{R_\alpha}_{W,i}\right)\hat{W}_i^{R_\alpha}(L),$$

- Two-loop soft anomalous dimension at threshold (MB, Falgari, Schwinn, 2009)

$$\gamma^{R_\alpha}_{W,i} = \gamma^{R_\alpha}_{H,s} + \gamma^r_s + \gamma^{r'}_s$$

is a sum over single particle terms that satisfy Casimir scaling (at least up to two loops) $\gamma^{R_\alpha}_{H,s} = C_{R_\alpha}\gamma_{H,s}$ with

$$\gamma^{(1)}_{H,s} = -C_A\left(\frac{98}{9} - \frac{2\pi^2}{3} + 4\zeta_3\right) + \frac{40}{9}T_F n_f,$$

- In Mellin space resummation formalism (see also Czakon, Mitov, Sterman, 2009):

$$D^{(1)R_\alpha}_{HH'} = 2\gamma^{(1)R_\alpha}_{H,s} - 8\beta_0 C_{R_\alpha} = -C_{R_\alpha}C_A\left(\frac{460}{9} - \frac{4\pi^2}{3} + 8\zeta_3\right) + \frac{176}{9}C_{R_\alpha}T_F n_f.$$

Resummation (II)

Resummation of soft gluon threshold logs via renormalization group equations.
Since the soft functions are diagonal (to all orders) in the chosen colour basis, can use the formalism developed for Drell-Yan (and Higgs) production by (Becher, Neubert, Xu. 2007)

$$\hat{\sigma}^{\text{res}}_{pp'}(\hat{s}, \mu_f) = \frac{\hat{\sigma}^{(0)}_{pp'}(\hat{s}, \mu_f)}{\beta} \sum_{i,R_\alpha} h_i(M, \mu_h) \; U_i^{R_\alpha}(M, \mu_h, \mu_s, \mu_f) \left(\frac{2M}{\mu_s}\right)^{-2\eta} \times \tilde{s}_i^{R_\alpha}(\partial_\eta, \mu_s) \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \int_0^\infty d\omega \, \frac{J_{R_\alpha}(E - \frac{\omega}{2})}{\omega} \left(\frac{\omega}{\mu_s}\right)^{2\eta} \quad (1)$$

$$U_i^{R_\alpha} = \exp[4S(\mu_h, \mu_s) - 2a_i^V(\mu_h, \mu_s) + 2a^{\phi,r}(\mu_s, \mu_f) + 2a^{\phi,r'}(\mu_s, \mu_f)] \left(\frac{4M^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h, \mu_s)}$$

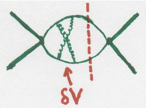
$$\tilde{s}_i^{R_\alpha}(\rho, \mu) = \int_{0_-}^\infty d\omega e^{-s\omega} \, \overline{W}_i^{R_\alpha}(\omega, \mu)$$

[$a(\mu_1, \mu_2)$ denote integrated anomalous dimensions]

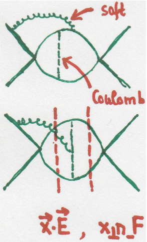
Non-relativistic log summation must be added separately – relevant from NNLL.

NNLL at $\mathcal{O}(\alpha_s^2)$

2-loop anomalous dimension not enough for NNLL resummation.
Consider $\alpha_s^2 \ln\beta$ terms: (MB, Czakon, Falgari, Mitov, Schwinn)



- $\hat{\sigma}^{(0)} \times \alpha_s^2 \ln\beta$ from singular heavy-quark potentials ($1/r^2$ etc.)
  Can be obtained from $e^+e^- \to t\bar{t}X$ calculation [MB, Signer, Smirnov, 1999] (+ colour, spin adjustment)
  Diagonal in the singlet-octet basis.



- $\hat{\sigma}^{(0)} \times \alpha_s^2 \ln^{2,1}\beta$ from sub-leading (non-eikonal) β-suppressed soft interactions in SCET and NRQCD. Implies new soft functions with operator insertions between Wilson lines.

  Off-diagonal in the singlet-octet basis. Corresponds to three-particle correlations.

  Vanish for the total cross section to all orders in α_s (Lorentz-invariance + scaling), but not for the amplitude.

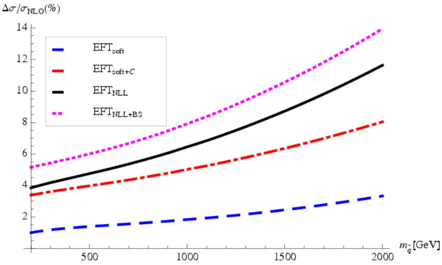
- Above + diagonal leading soft function consistent with (Ferroglia et al., 2009). No extra terms for $t\bar{t}$ total cross section, though.

Squark-antisquark production (MB, Falgari, Schwinn)

Results at NLL for $pp \to$ squark+antisquark $+ X$ at $\sqrt{s} = 14$ TeV

NLL = Tree C and W, 1-loop anomalous dim. + LO Coulomb Green function + matching to NLO fixed order ((Beenakker et al, 1997; Prospino code), fitting functions from (Langenfeld, Moch, 2009))

Size of corrections beyond NLO



($\sqrt{s} = 14$ TeV; $m_{\tilde g}/m_{\tilde q} = 1.25$)

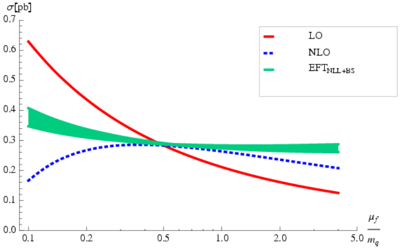
Resummation correction significantly larger than in other implementations (Kulesza, Motyka; Beenakker et al.), because:

- Coulomb scale is lower
- Interference of soft and Coulomb correction included (to black)
- Bound state contributions included (to purple)

Resummation is a few to 10% effect at the natural scale.

Squark-antisquark production (II)

Results at NLL for $pp \rightarrow$ squark+antisquark $+ X$ at $\sqrt{s} = 14$ TeV
Scale dependence at LO, NLO, NLL



($\sqrt{s} = 14$ TeV; $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$; $m_{\tilde{q}} = 1$ TeV; Green band: variation of the soft and hard scale.)

Scale dependence reduced (but only slightly).

Top anti-top cross section results

Total hadronic $t\bar{t}$ cross section with NLL resummation and singular terms at NNLO.
Differences to previous results:

- NLL resummation with SCET (rather than Mellin N-space) formalism
- Correct singular terms in β at $\mathcal{O}(\alpha_s^2)$ (MB, Czakon,Falgari, Mitov, Schwinn)

Numerically, the differences are at the few percent level.
Complete NNLL in progress.

Example: $gg \to [t\bar{t}]_8 X$

$$\hat{\sigma}(\beta)^{(2)}_{gg\to[t\bar{t}]_8 X} = \hat{\sigma}(\beta)^{(0)} \left(\frac{\alpha_s}{4\pi}\right)^2 \left[\frac{\pi^4}{27\beta^2} + \frac{\pi^2}{\beta}\left\{-32\ln^2\beta + \left(\frac{118}{9} - 32\ln 2\right)\ln\beta + \frac{46}{9}\ln 2 - \frac{5\pi^2}{18} - \frac{127}{27}\right\}\right.$$
$$+ 4608\log^4\beta + \left(27648\ln 2 - \frac{65152}{3}\right)\log^3\beta + \left(59904\ln^2 2 - 96576\ln 2 - 3088\pi^2 + \frac{204944}{3}\right)\log^2\beta$$
$$\left. + \left(55296\ln^3 2 - 137952\ln^2 2 - 9264\pi^2\ln 2 + 202352\ln 2 + 33120\zeta(3) + \frac{65908\pi^2}{9} - \frac{1244776}{9}\right)\log\beta\right]$$

Top anti-top cross section results (II)

All results preliminary.

		Tevatron	LHC7	LHC10	LHC14
NLO	MSTW08	$6.50\,^{+0.32+0.33}_{-0.70-0.24}$	$150\,^{+18+9}_{-19-9}$	$380\,^{+44+17}_{-47-18}$	$842\,^{+97+30}_{-97-32}$
	ABKM09	$6.43\,^{+0.23+0.15}_{-0.61-0.15}$	$122\,^{+13+7}_{-15-7}$	$323\,^{+36+15}_{-38-15}$	$738\,^{+81+27}_{-83-27}$
NLL+NLO	MSTW08	$6.54\,^{+0.98+0.33}_{-0.38-0.24}$	$151\,^{+24+8}_{-14-8}$	$381\,^{+60+17}_{-36-18}$	$845\,^{+131+30}_{-081-32}$
	ABKM09	$6.46\,^{+0.89+0.15}_{-0.35-0.15}$	$122\,^{+19+7}_{-11-7}$	$323\,^{+49+15}_{-30-15}$	$741\,^{+110+27}_{-069-27}$
NNLO_{app}	MSTW08	$7.13\,^{+0.00+0.36}_{-0.33-0.26}$	$162\,^{+3+9}_{-3-9}$	$380\,^{+11+17}_{-05-18}$	$895\,^{+29+31}_{-07-33}$
	ABKM09	$7.01\,^{+0.06+0.18}_{-0.36-0.18}$	$132\,^{+2+8}_{-2-8}$	$345\,^{+8+16}_{-4-16}$	$785\,^{+23+29}_{-06-29}$
NNLO_{app} + NLL	MSTW08	$7.13\,^{+0.08+0.36}_{-0.41-0.26}$	$162\,^{+2+9}_{-1-9}$	$380\,^{+9+18}_{-2-18}$	$895\,^{+23+31}_{-04-33}$
	ABKM09	$7.00\,^{+0.13+0.18}_{-0.44-0.18}$	$132\,^{+1+8}_{-1-8}$	$345\,^{+6+16}_{-1-16}$	$784\,^{+17+29}_{-03-29}$
NNLL			work in progress		

Cross section in pb.
Error includes scale variation $\mu_i/2\ldots 2\mu_i$ for all μ_i (first number) and PDF error (second error).
(MB, Falgari, Klein, Schwinn, in progress)

Conclusion

1) Joint resummation possible due to factorization of soft and Coulomb gluon effects (in SCET $\times$ NRQCD)
   Leading soft function diagonal to all orders in a simple colour basis. 2-loop anomalous dimension at threshold determined.

2) Top pair cross section at threshold known at $\mathcal{O}(\alpha_s^2)$ at threshold up to the constant term.
   For any coloured heavy particles, given the one-loop hard matching coefficients.

3) Soft gluon resummation at NNLL for total cross section possible.
   For complete NNLL combine with non-relativstic log resummation.
   Phenomenological analysis forth-coming.