THREE-LOOP EFFECTS IN GLUON-FUSION HIGGS PRODUCTION

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Loops and Legs in Quantum Field Theory
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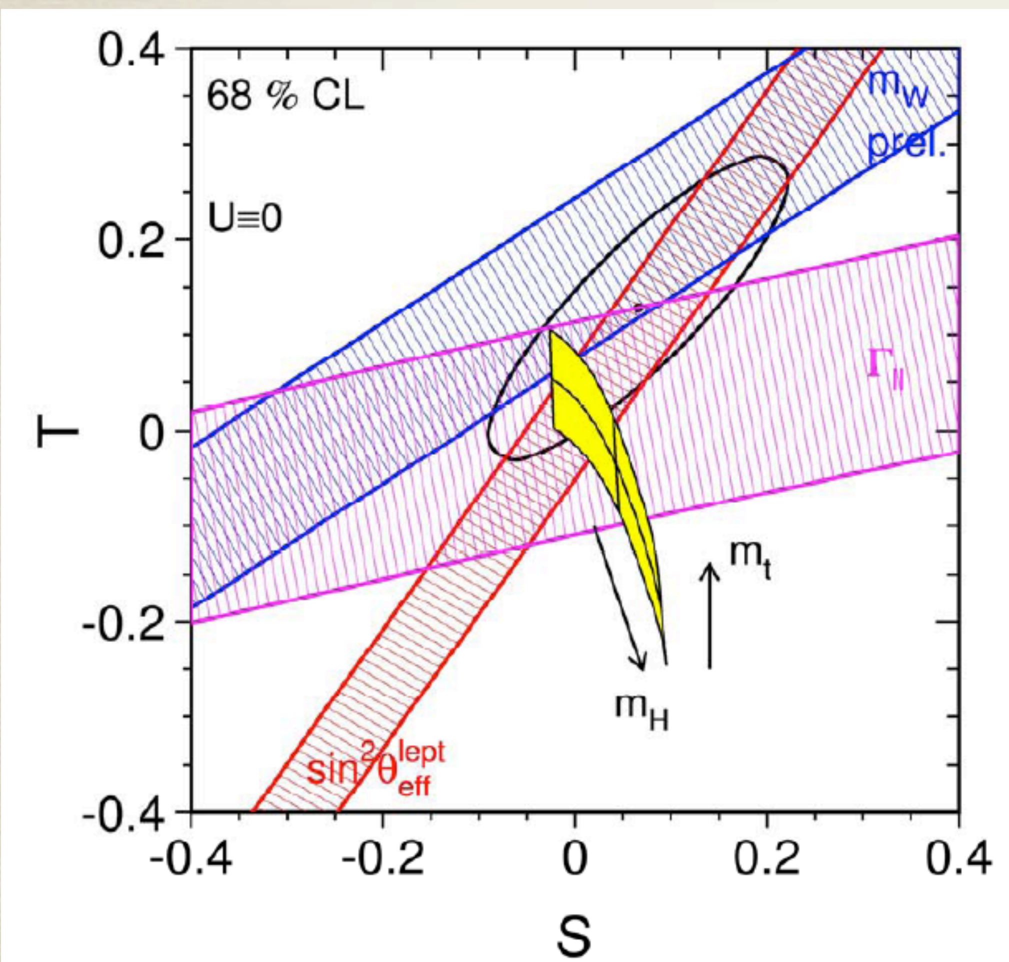
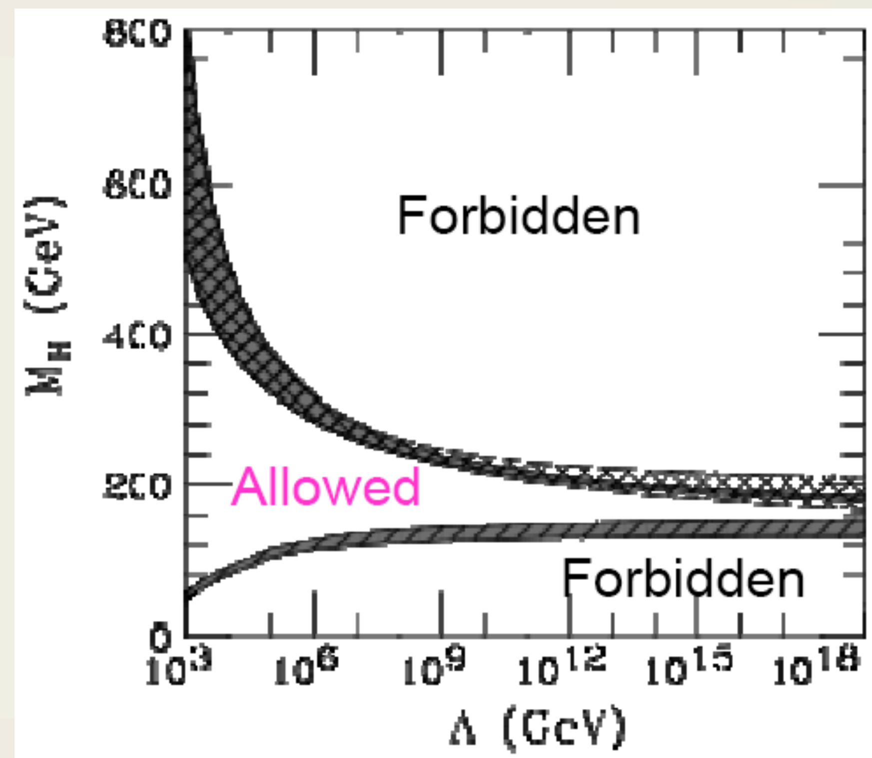
Outline

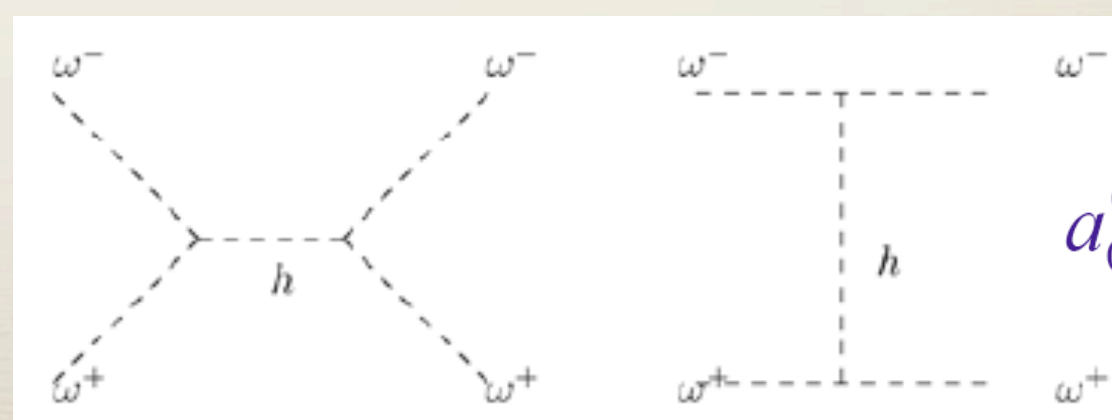
- Brief review of experiment, theory for SM Higgs
- The very effective Lagrangian for the Higgs
- Mixed 3-loop QCD-EW corrections to gg→h and Tevatron numerics (Anastasiou, Boughezal, FP JHEP 0904:003 (2009))
- Excluding new physics with the Higgs limit: color-octets at the Tevatron through NNLO (Boughezal, FP arXiv:1003.2046)

Why we expect a TeV scale Higgs

- Last undiscovered particle of the SM
- Many reasons to expect it (or something else) to be observed soon





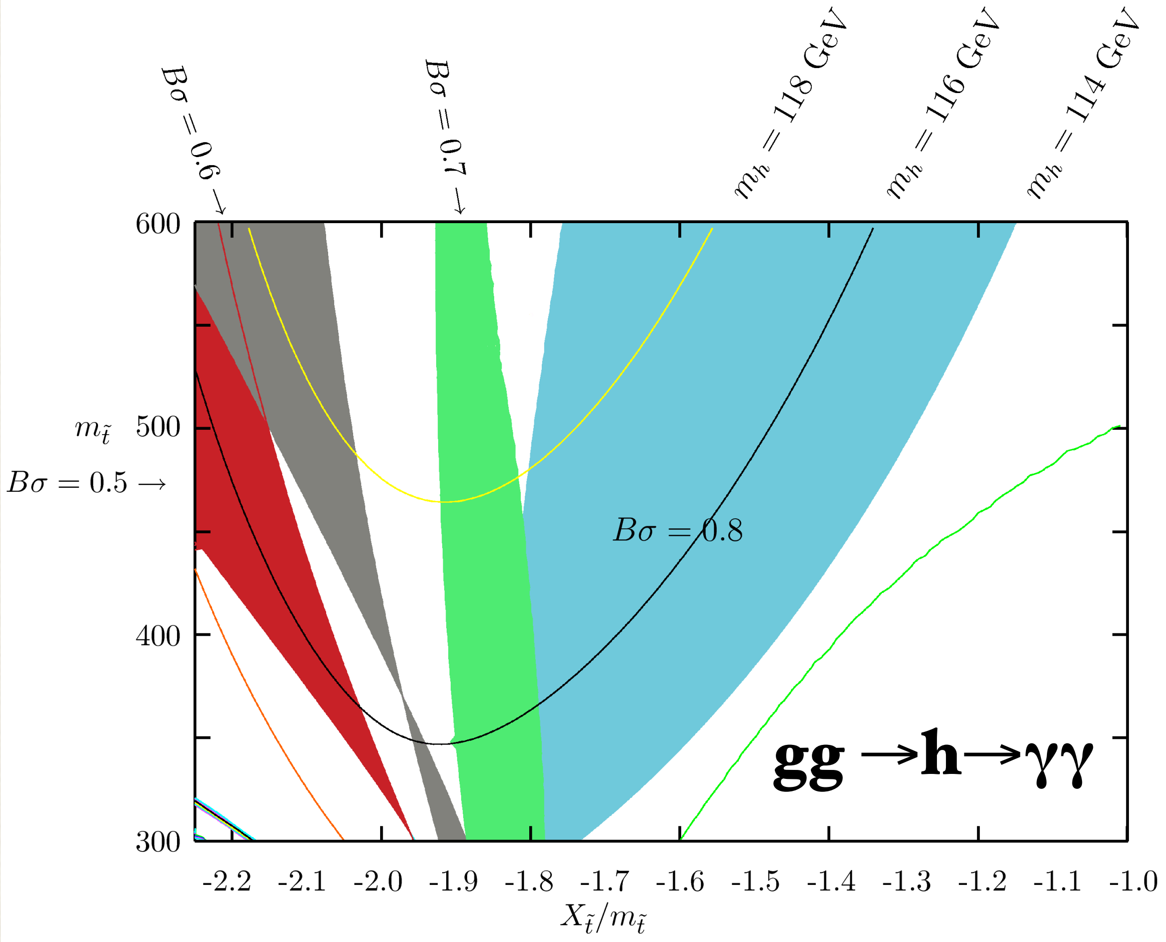


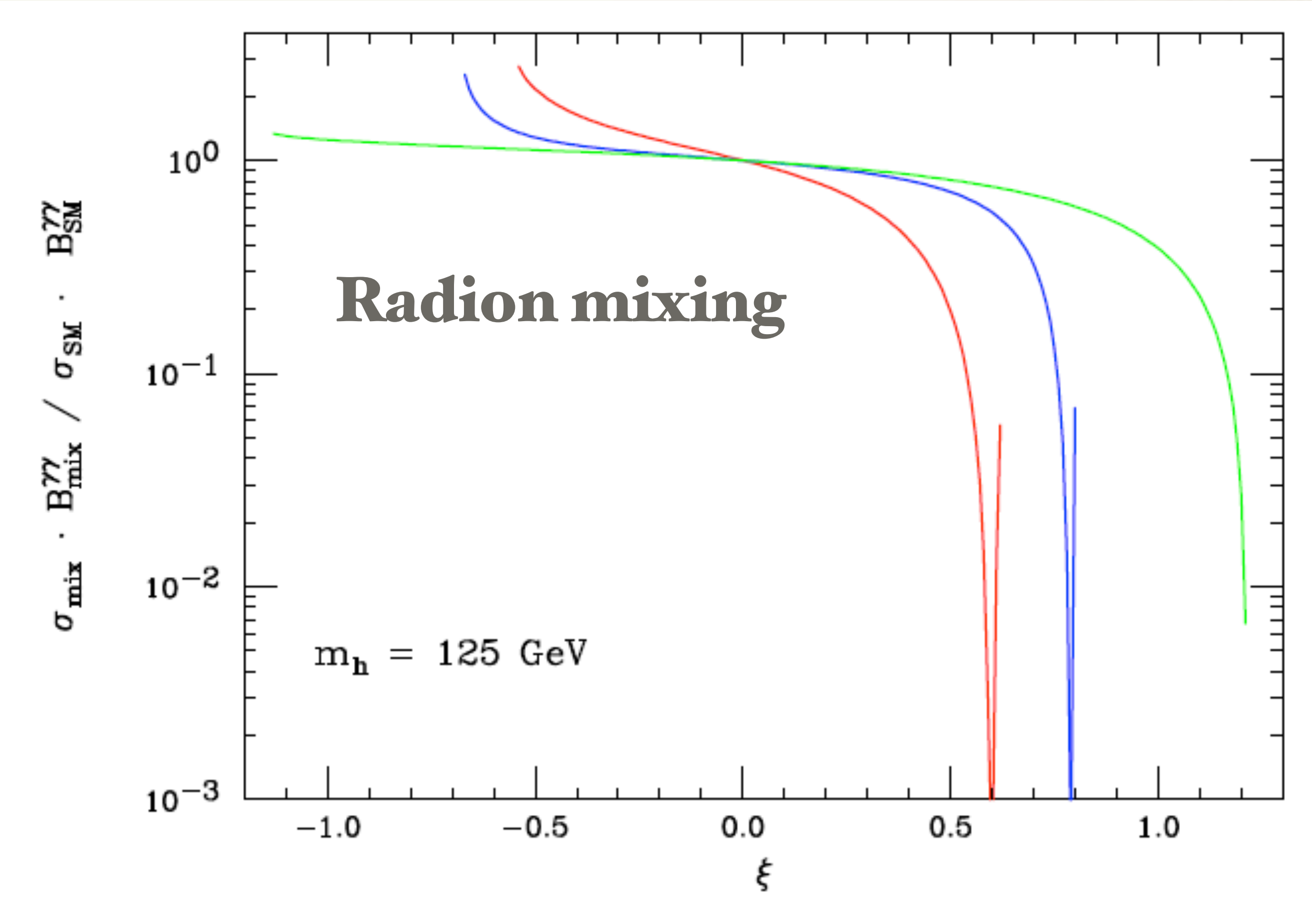
$$a_0^0 \rightarrow -\frac{s}{32\pi v^2}$$

Higgs in SM extensions

- The uncertainty in EWSB mechanism makes Higgs a portal into new physics at the TEV scale

MSSM





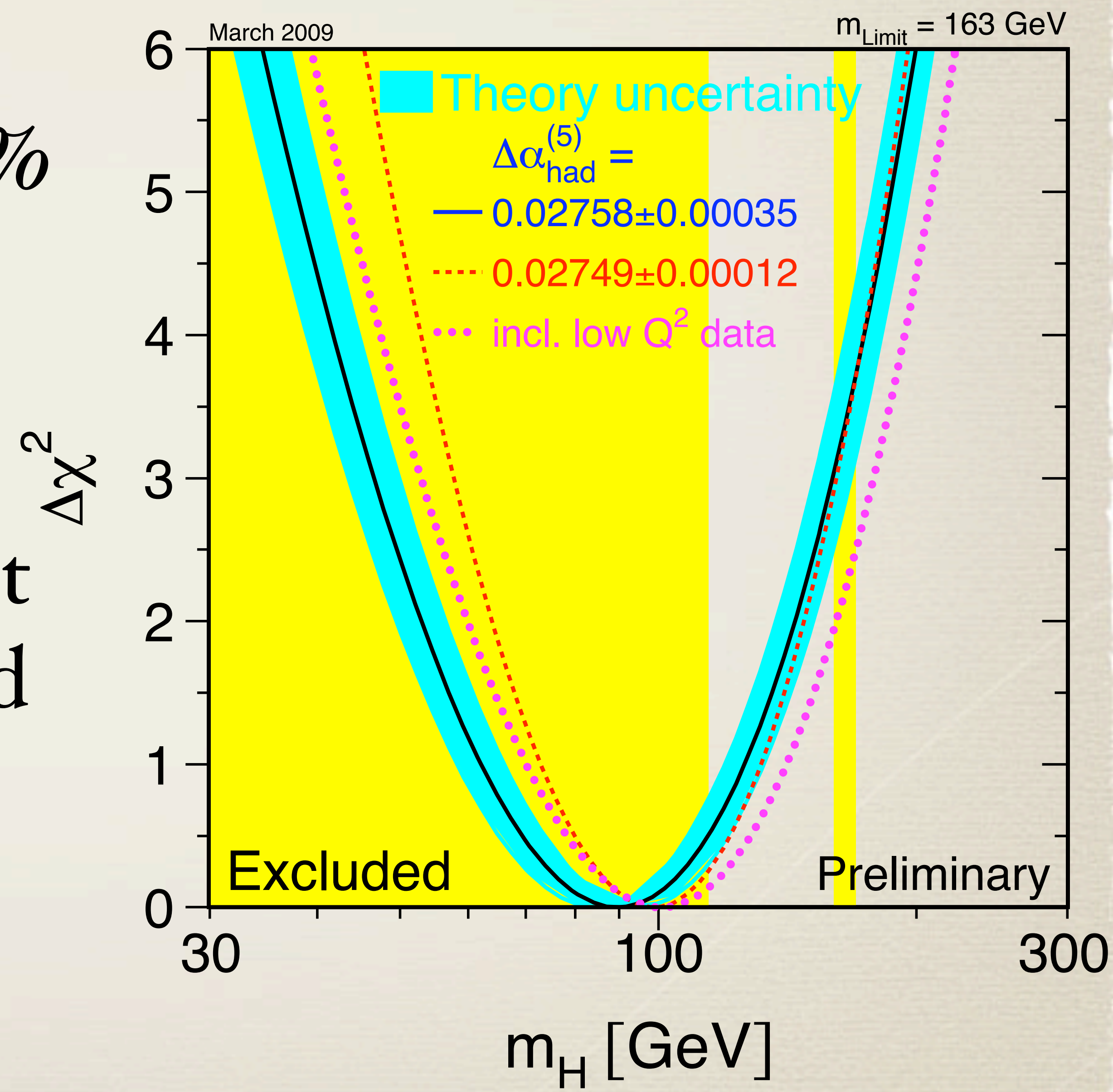
Low, Shalgar 2009

Hewett, Rizzo 2002

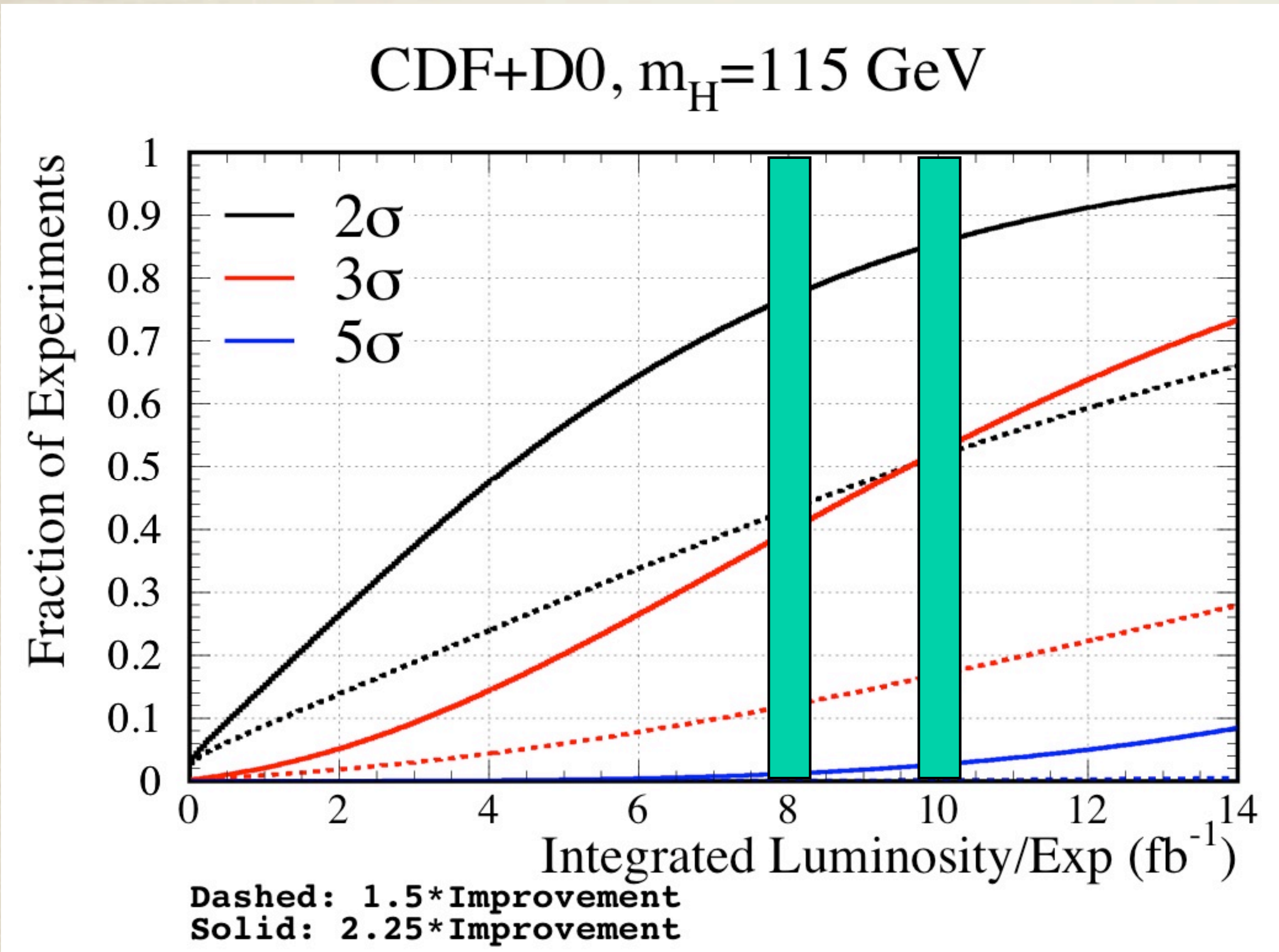
SM Higgs circa 2010

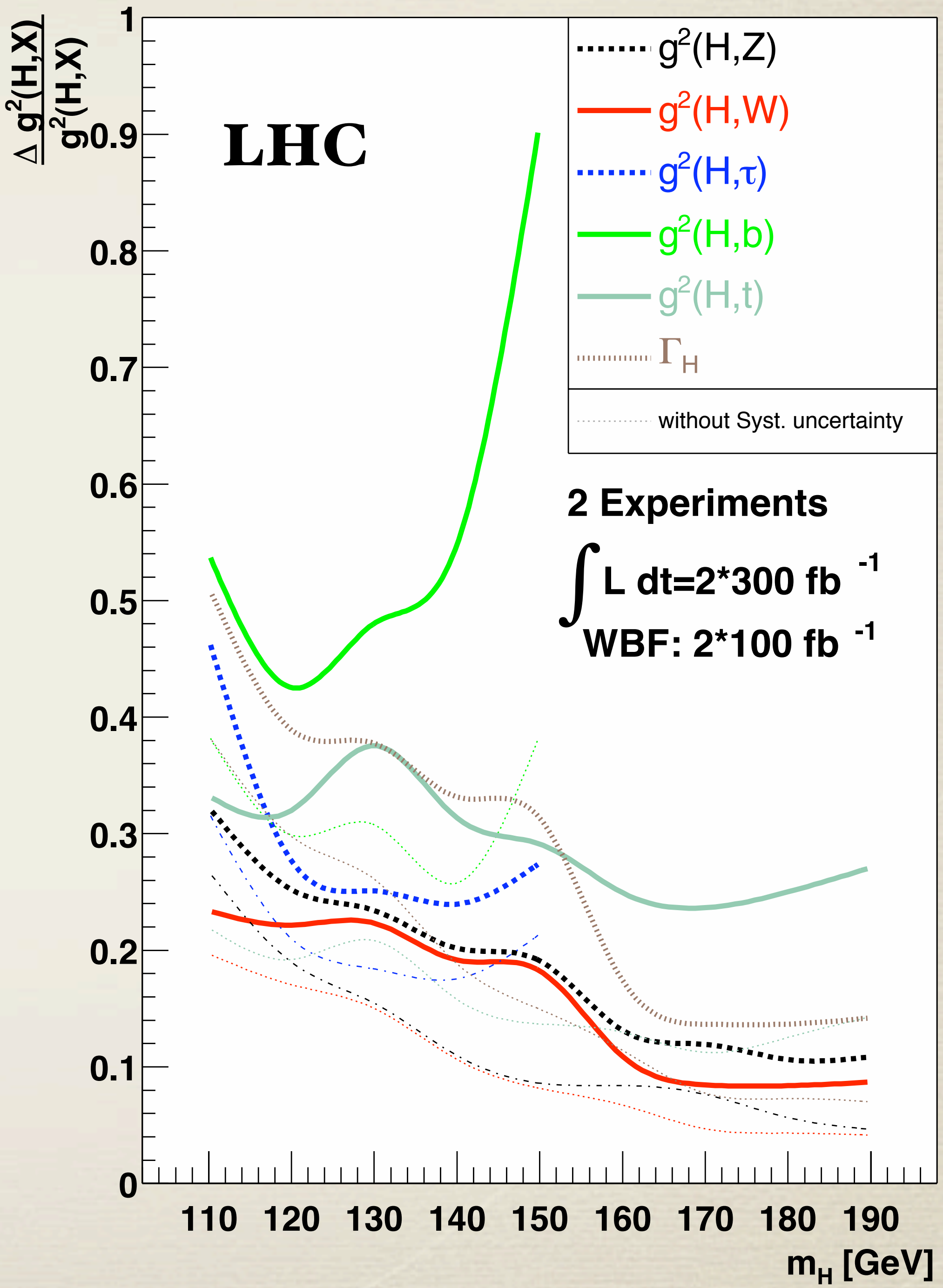
- Precision EW upper bound and direct search lower bound at 95% CL: 114 GeV ≤ M_H ≤ 190 GeV

- **News from the Tevatron**: First exclusion in 2008; new combined results exclude 162-166 GeV SM Higgs at 95% CL



Higgs in the future

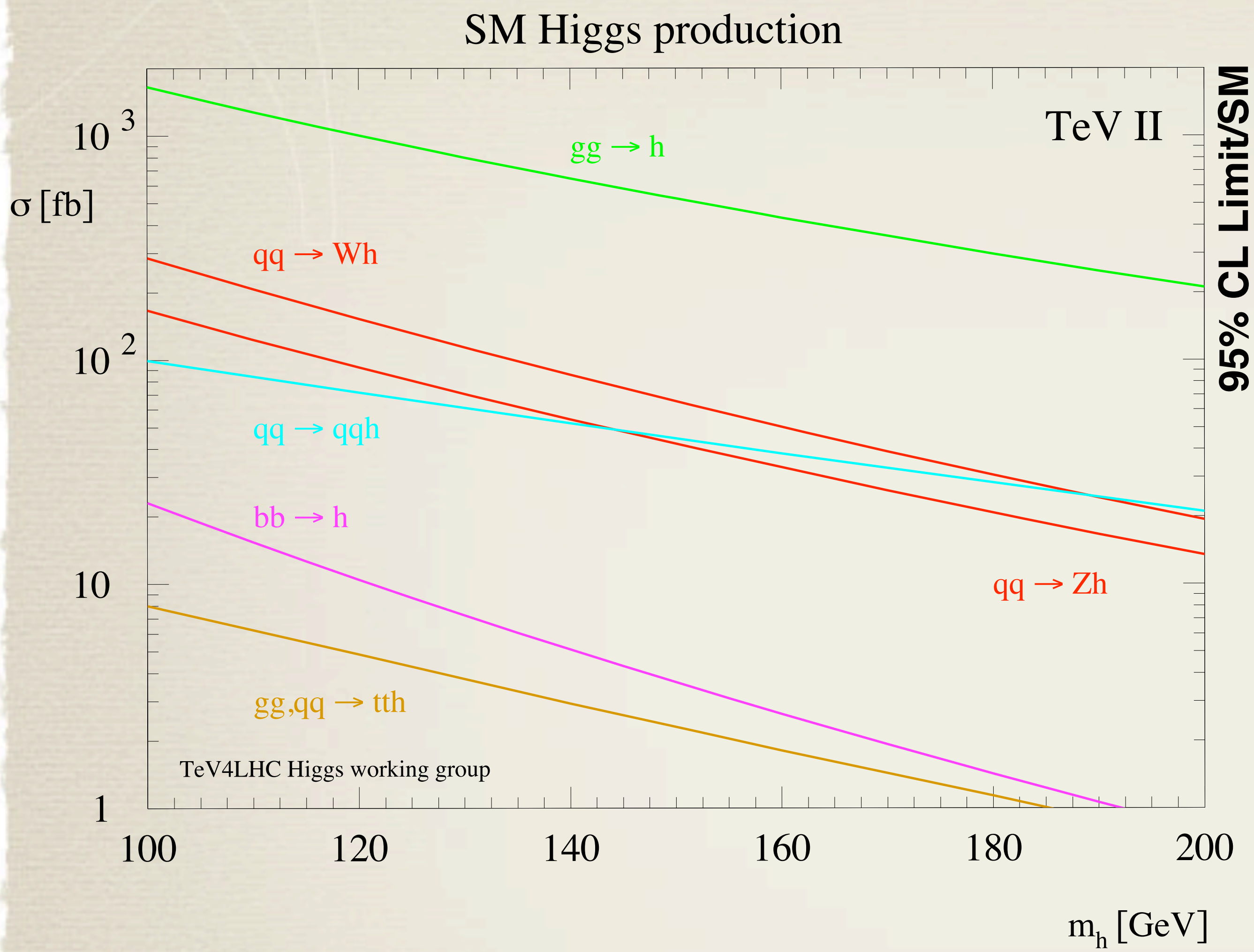


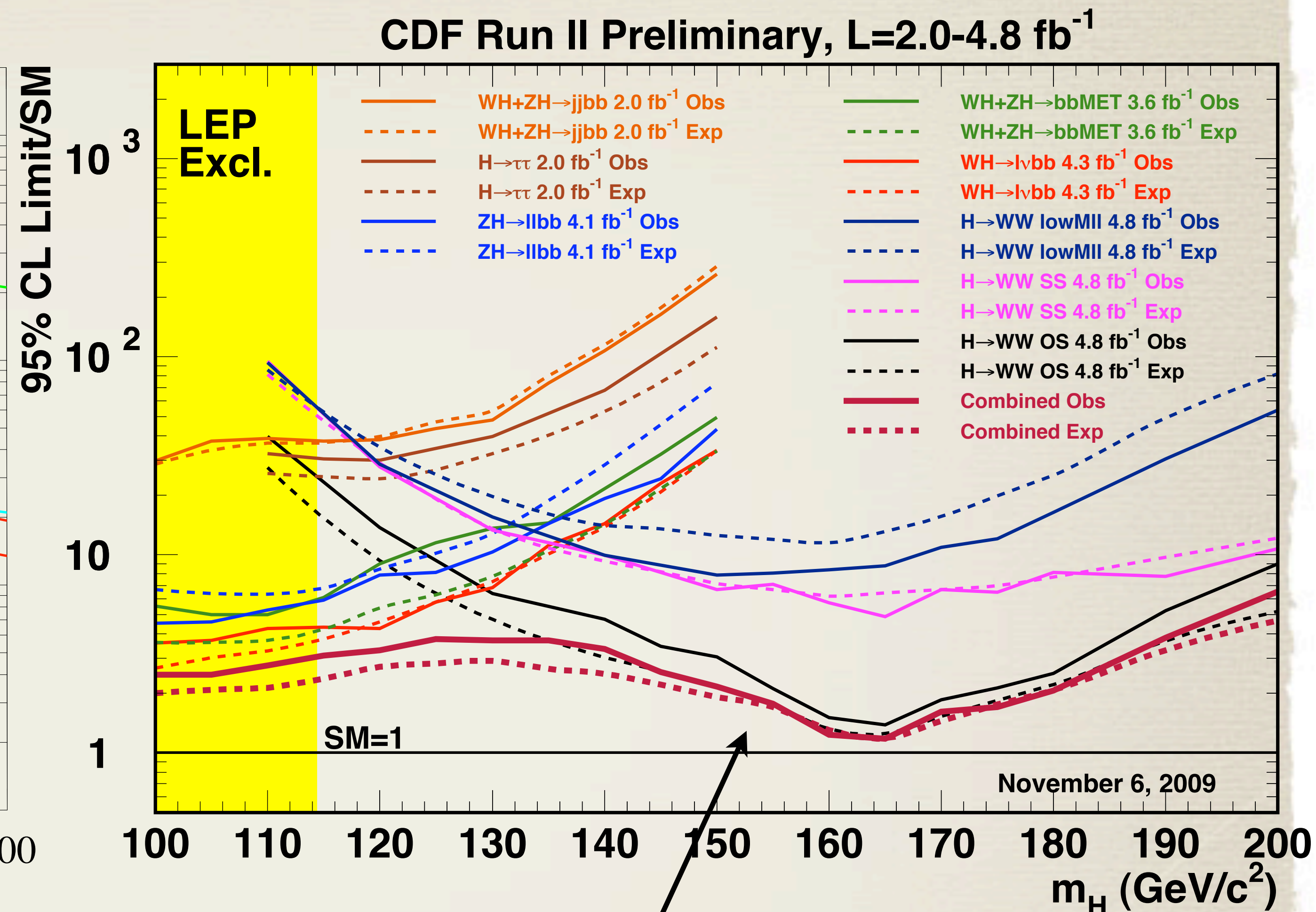


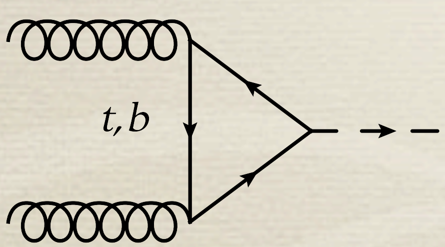
Tevatron can likely exclude entire range $M_H \leq 190$ GeV

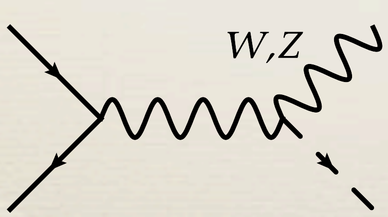
Duhrssen et al. 2004

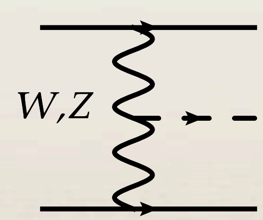
SM Higgs production











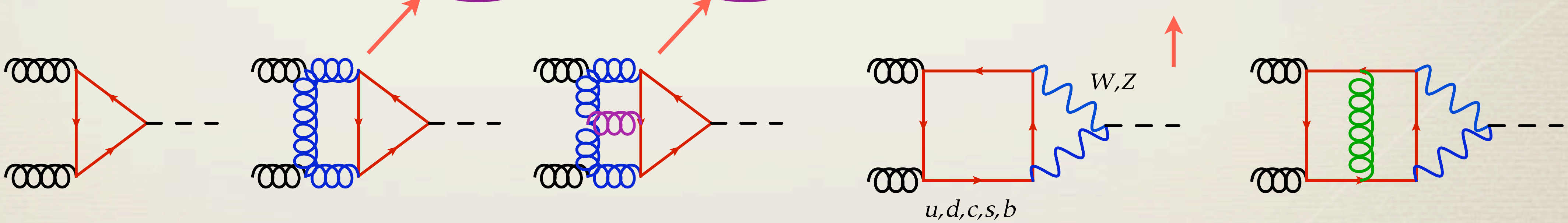
Tevatron exclusion limit entirely from gg→H→WW above 130 GeV

The Higgs Lagrangian

Summarized in an "effective Lagrangian" for Higgs-gluon interactions

$$\mathcal{L}_{eff} = \alpha_s \frac{C_1}{4v} H \, G^a_{\mu\nu} G_a^{\mu\nu}$$

$$C_1 = -\frac{1}{3\pi}\left\{1 + \alpha_s C_{1t} + \alpha_s^2 C_{2t} + \lambda_{EW}\left[1 + C_{1w}\right]\right\}$$



Inami, Kubota, Okada 1982

Chetyrkin, Kniehl, Steinhauser 1997

Anastasiou, Boughezal, FP 2009

$$C_{1q} = \frac{11}{4}, \quad C_{2q} = \frac{2777}{288} + \frac{19}{16}L_t + N_F\left(-\frac{67}{96} + \frac{1}{3}L_t\right)$$

Unreasonably effective EFT

NLO in the EFT:

analytic continuation to time-like form factor

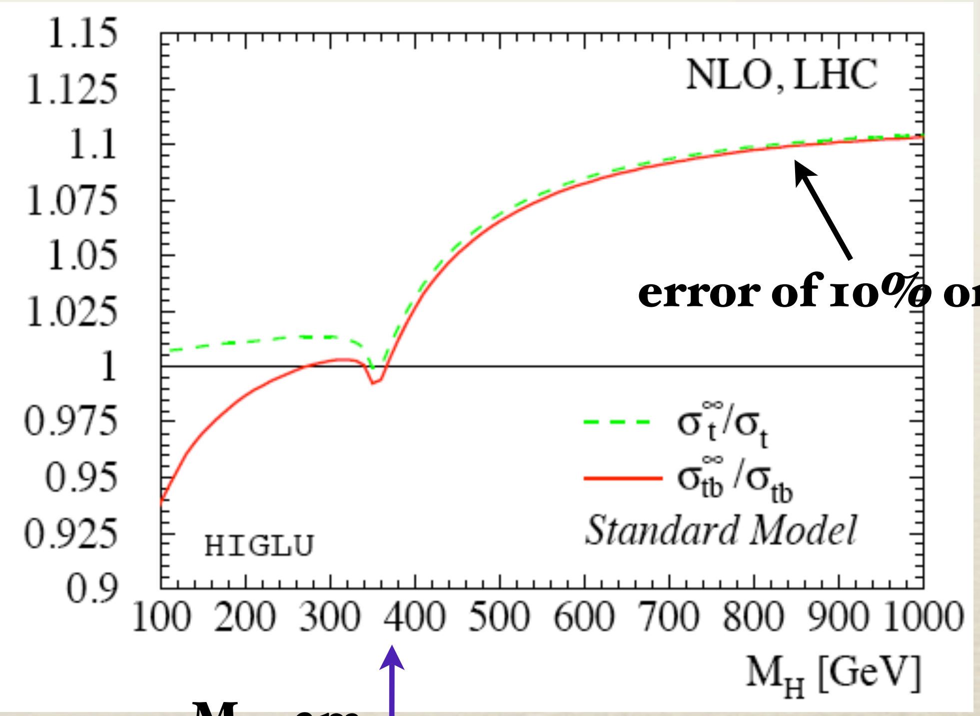
$z = M_H^2/(x_1 x_2 s)$

$$\Delta\sigma = \sigma_0 \frac{\alpha_s}{\pi}\left\{\left(\frac{11}{2}+\pi^2\right)\delta(1-z) + 12\left[\frac{\ln(1-z)}{1-z}\right]_+ - 12z(-z+z^2+2)\ln(1-z) - 6\frac{(z^2+1-z)^2}{1-z}\ln(z) - \frac{11}{2}(1-z)^3\right\}$$

eikonal emission of soft gluons

Identical factors in full theory with $\sigma_0 \rightarrow \sigma_{LO,\text{ full theory}}$

$$\sigma_{NLO}^{approx} = \left(\frac{\sigma_{NLO}^{EFT}}{\sigma_{LO}^{EFT}}\right)\sigma_{LO}^{QCD}$$



error of 10% on 100% correction

$M_H=2m_t$

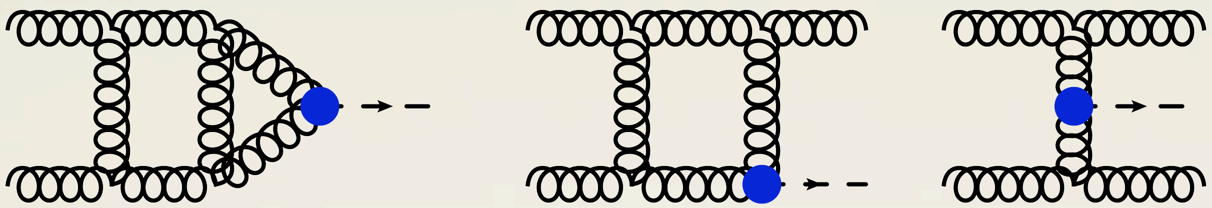
Harlander, 2009

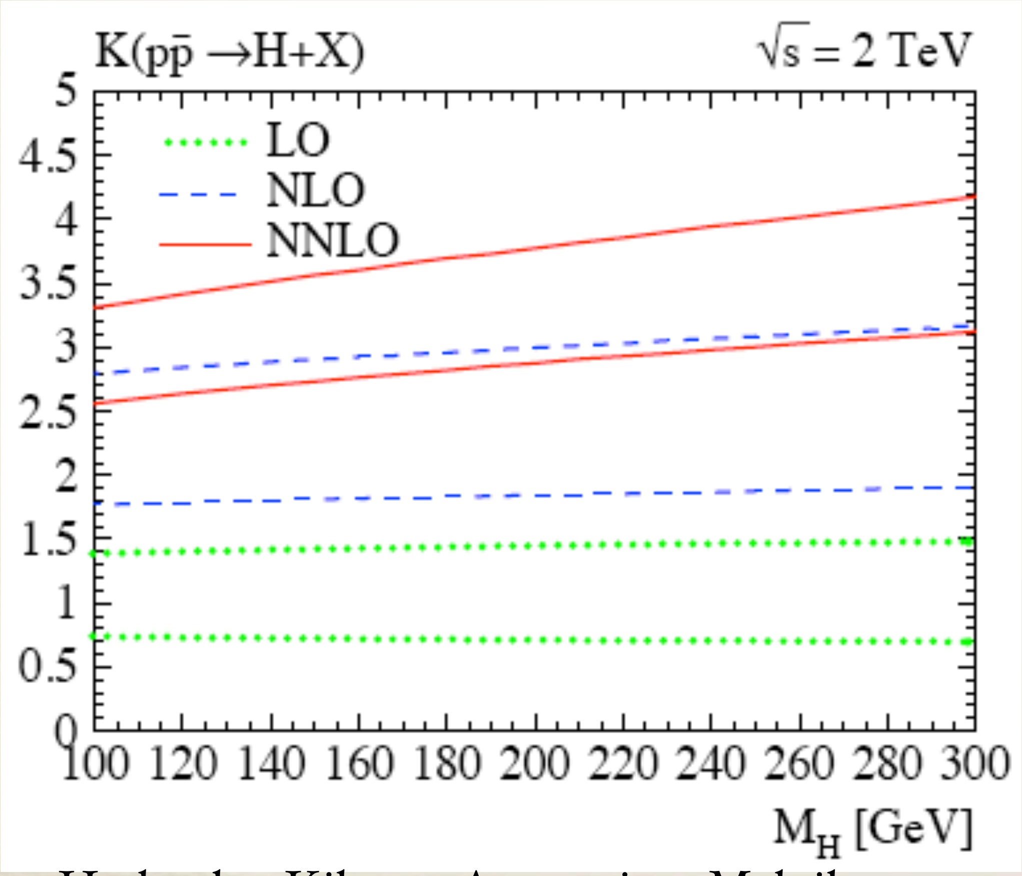
Initial NNLO study of $1/m_t$ suppressed operators indicates this persists

Harlander, Mantler, Marzani, Ozeren; Pak, Rogal, Steinhauser 2009

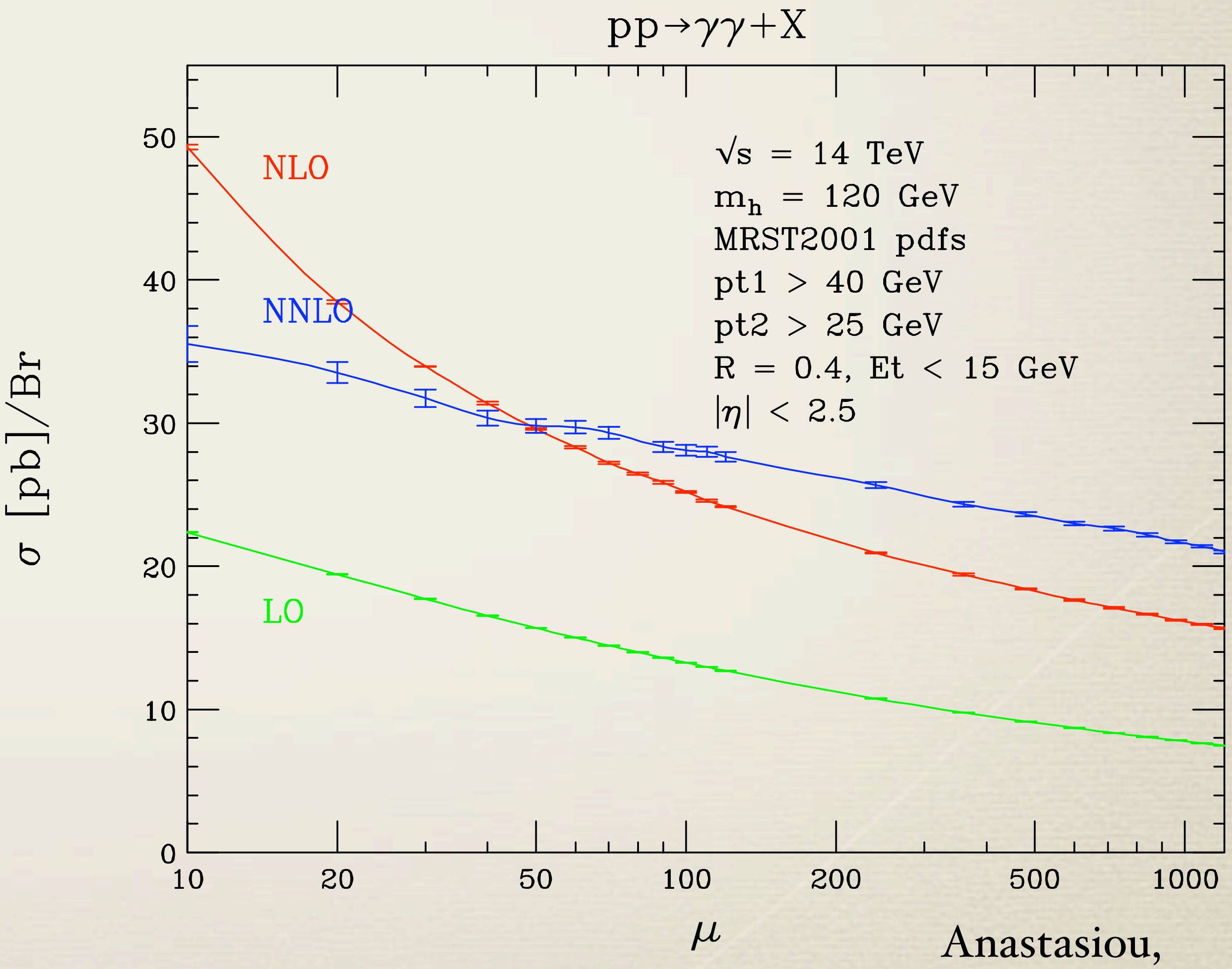
NNLO in the EFT

- Motivates calculation to NNLO in the EFT





Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven 2002-2003

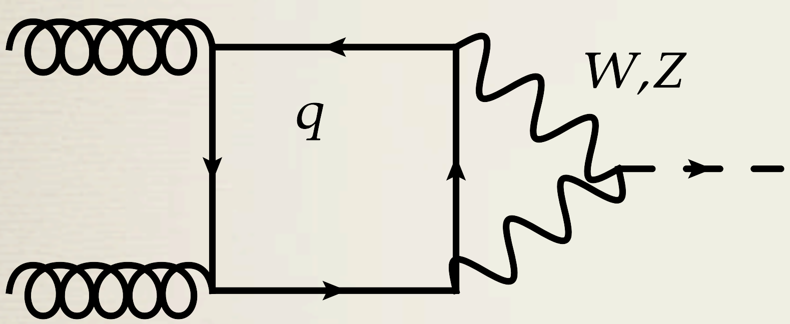


Anastasiou, Melnikov, FP 2005

K-factor: 2 at LHC, 3.5 at Tevatron

Electroweak corrections

- Residual QCD uncertainty ~10% ⇨ EW corrections potentially important to match QCD and experimental precision
- NF-enhanced sourced of 2-loop light-quark corrections



⇨ Up to ~6-7% at threshold relative to LO QCD

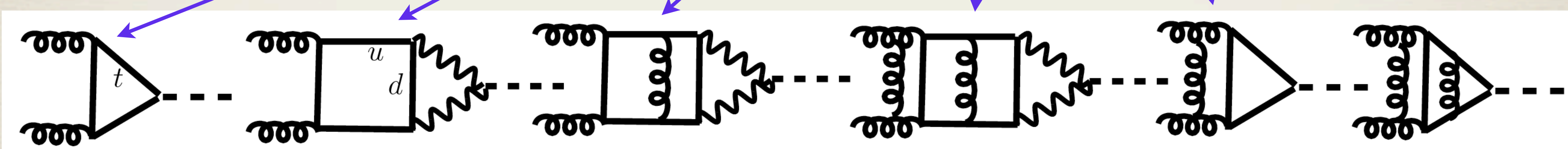
Aglietti, Bonciani, Degrassi, Vicini 2004
Actis, Passarino, Sturm, Uccirati 2008

- K-factor? Values between 1-3.5 assumed in literature; do these get same K-factor of top-quark piece? (Initial Tevatron exclusion limit assumed K=3.5)
- First goal: check with 3-loop calculation in EFT

EFT formulation

$$\mathcal{L} = -\alpha_s \frac{C_1}{4v} H G^a_{\mu\nu} G^{a\mu\nu}$$

$$C_1 = -\frac{1}{3\pi}\left\{1 + \lambda_{EW}\left[1 + a_s C_{1w} + a_s^2 C_{2w}\right] + a_s C_{1q} + a_s^2 C_{2q}\right\}$$



- Radius of convergence: $M_H \leq M_W$...
- *However,* dominant corrections from threshold logs and analytic continuation identical in full and EFT
- Calculate K-factor in EFT, normalize to exact 2-loop EW result

Factorization in the EFT

- If the K-factor for light-quark pieces is the same as the top quark, then the Wilson coefficient in the EFT "factorizes"

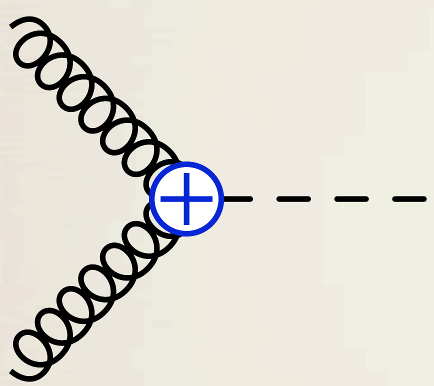
$$C_1 = -\frac{1}{3\pi}\left\{1 + \lambda_{EW}\left[1 + a_s C_{1w} + a_s^2 C_{2w}\right] + a_s C_{1q} + a_s^2 C_{2q}\right\}$$

$$C_1^{fac} = -\frac{1}{3\pi}\left(1 + \lambda_{EW}\right)\left\{1 + a_s C_{1q} + a_s^2 C_{2q}\right\}$$

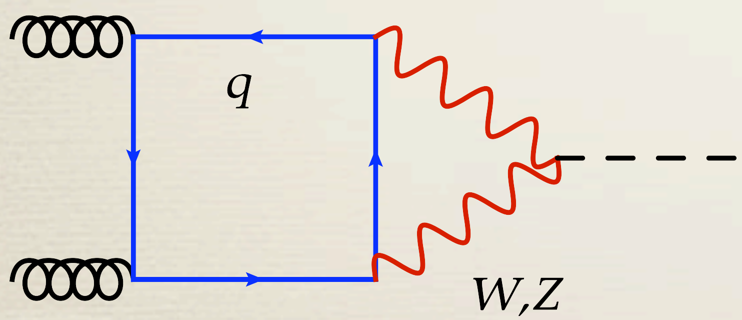
- Factorization holds if $C_{1w} = C_{1q}$; $C_{1q} = 11/4$
- Calculate C_{1w} from 3-loop diagrams, check deviation from C_{1q}, study numerical effect

Deriving the Higgs EFT

- Matching at $O(\alpha\alpha_S)$:



$$= \quad -\frac{1}{3\pi}\frac{\alpha_s}{v}\lambda_{EW}\mathcal{M}_0$$

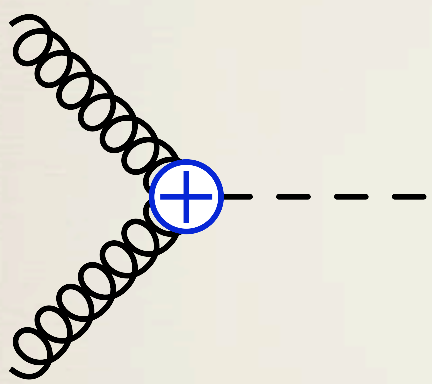


$$= \quad \mathcal{A}^{(2)}(M_H^2 = 0)\mathcal{M}_0 + \mathcal{O}\left(\frac{M_H^2}{M_W^2}\right)$$

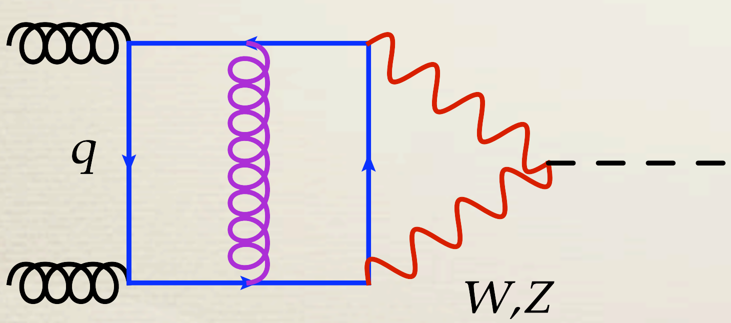
- Equate to get λ_{EW}

Deriving the Higgs EFT II

- Matching at O($\alpha\alpha_S^2$): (Other EFT graphs scaleless after expansion)



$$= \quad -\frac{1}{3\pi}\frac{\alpha_s}{v}(\lambda_{EW}C_{1w})\mathcal{M}_0$$

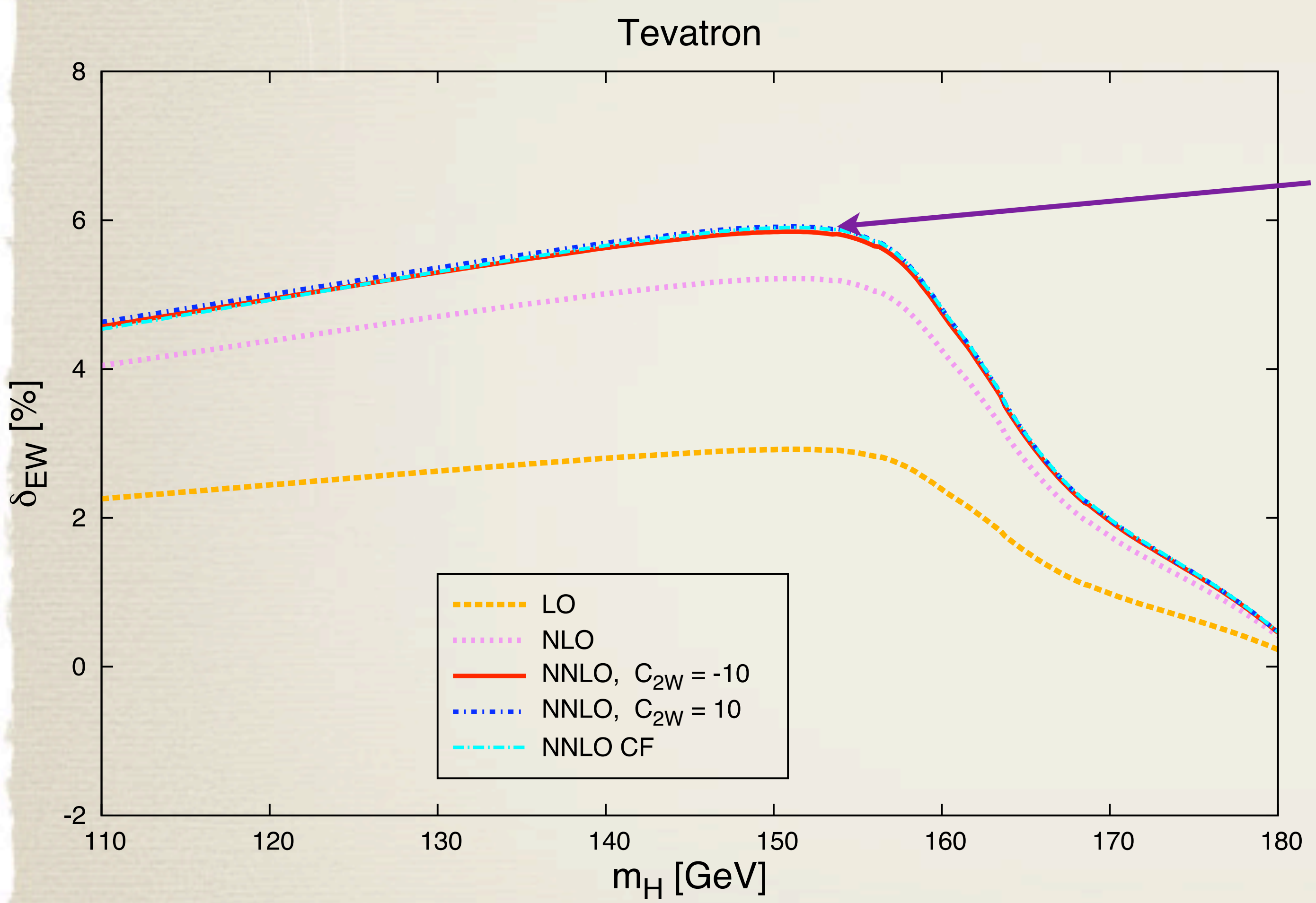


$$= \quad \mathcal{A}^{(3)}(M_H^2 = 0)\mathcal{M}_0 + \mathcal{O}\left(\frac{M_H^2}{M_W^2}\right)$$

☑ Equate to get C_{1W}

Factorization violation

- Analytical result: $\mathbf{C_{1w} = 7/6}$, compared to $\mathbf{C_{1q} = 11/4}$



Anastasiou, Boughezal, FP 2009

Difference between factorization hypothesis and actual result irrelevantly small (weak violation ⇒ same sign, order of magnitude)

$$\sigma_{3-loop} = \sigma_{2-loop}\left\{\frac{\alpha_s}{\pi}C_{1w} + G^{(1)}_{EFT}\right\}$$

- $\alpha_s(C_{1w}-C_{1q})/\pi \approx 5\%$
- $G^{(1)}_{EFT} \approx 100\%$; contains π^2, $\ln(1-z)/(1-z)$

- K-factor of 3.5 at Tevatron appropriate

Tevatron numerics summary

Anastasiou, Boughezal, FP 2009

m_H[GeV]	σ^{best}[pb]	m_H[GeV]	σ^{best}[pb]
110	1.417 (±7% pdf)	160	0.4344 (±9% pdf)
115	1.243 (±7% pdf)	165	0.3854 (±9% pdf)
120	1.094 (±7% pdf)	170	0.3444 (±10% pdf)
125	0.9669 (±7% pdf)	175	0.3097 (±10% pdf)
130	0.8570 (±8% pdf)	180	0.2788 (±10% pdf)
135	0.7620 (±8% pdf)	185	0.2510 (±10% pdf)
140	0.6794 (±8% pdf)	190	0.2266 (±11% pdf)
145	0.6073 (±8% pdf)	195	0.2057 (±11% pdf)
150	0.5439 (±9% pdf)	200	0.1874 (±11% pdf)
155	0.4876 (±9% pdf)	–	–

+8%, -11% scale uncertainty

- Correct K-factors for top, EW contributions (normalize EW to exact 2-loop of Actis et al.)
- Reduced K-factor for top-bottom interference
  K_{tb}~1.5, K_{tt}~3.5
- Update to MSTW 2008 PDFs
  significant changes in heavy-quark threshold treatment

Missing: New MSTW 2009 allows for α_S uncertainty study

±9% (pdf) ➟ ±13% (pdf+α_S)

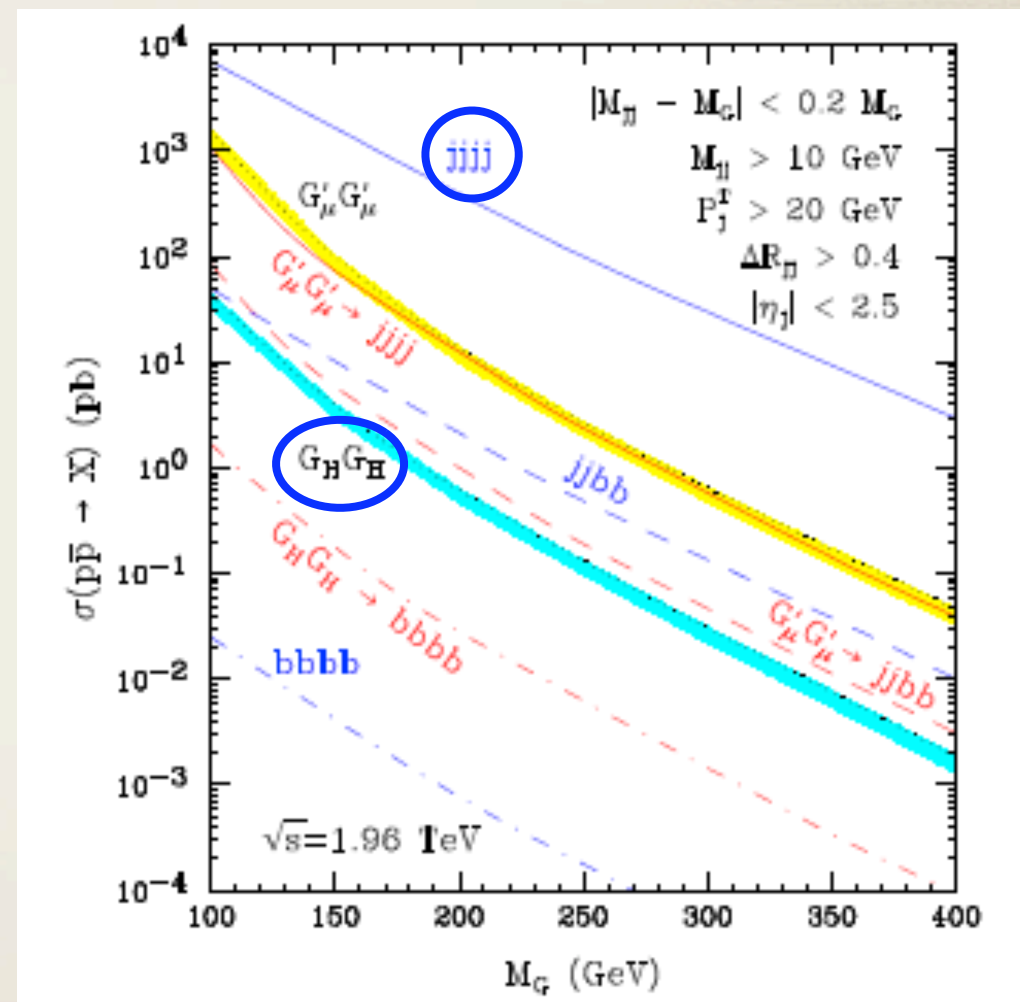
Should be updated in Tevatron analysis!

Color-octet scalars and gg→h

- Tevatron limit already imposing strong constraints on SM extensions
- Example: $(8,1)_0$ scalars

$$\mathcal{L}^{full} = \mathcal{L}_{SM} + \mathrm{Tr}\left[D_\mu S D^\mu S\right] - m_S'^2\,\mathrm{Tr}\left[S^2\right] - g_s^2\,G_{4S}\,\mathrm{Tr}\left[S^2\right]^2 - \lambda_1 H^\dagger H\,\mathrm{Tr}\left[S^2\right] - \lambda_h\left(H^\dagger H - \frac{v^2}{2}\right)^2.$$

  - Arise in technicolor, universal extra dimension theories
  - Pair-produced: difficult 4 b-jet final state
  - Estimated Tevatron direct search reach: $m_S > 280$ GeV



Dobrescu, Kong, Mahbubani 2007

Color-octet effective theory

- Can integrate out both top quark, color octet to derive effective ggh operator

$$\mathcal{L}^{full} = \mathcal{L}_{SM} + \mathrm{Tr}\left[D_\mu S D^\mu S\right] - m_S'^2 \,\mathrm{Tr}\left[S^2\right] - g_s^2 G_{4S} \,\mathrm{Tr}\left[S^2\right]^2 - \lambda_1 H^\dagger H \,\mathrm{Tr}\left[S^2\right] - \lambda_h \left(H^\dagger H - \frac{v^2}{2}\right)^2 .$$

$$\mathcal{L}^{eff} = \mathcal{L}_{QCD}^{n_l,eff} - C_1 \frac{H}{v} \mathcal{O}_1$$

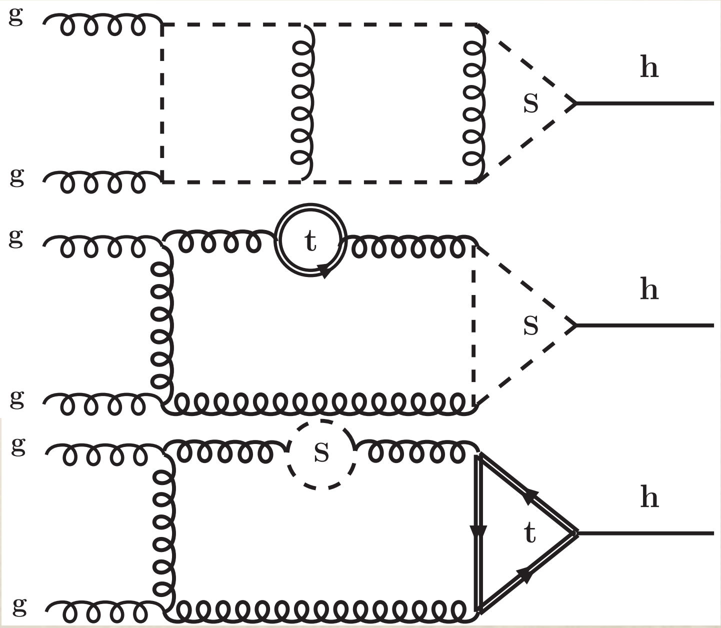
$$\mathcal{O}_1 = \frac{1}{4} G'^{a}_{\mu\nu} G'^{a\mu\nu}$$

Gluon decoupling constant

$$\frac{\zeta_3^0 C_1^0}{v} = \frac{\delta^{a_1 a_2}\left(g^{\mu_1\mu_2}(p_1 \cdot p_2) - p_1^{\mu_2} p_2^{\mu_1}\right)}{(N^2-1)(d-2)(p_1 \cdot p_2)^2} \mathcal{M}^{0,a_1a_2}_{\mu_1\mu_2}(p_1,p_2)\Big|_{p_1=p_2=0}$$

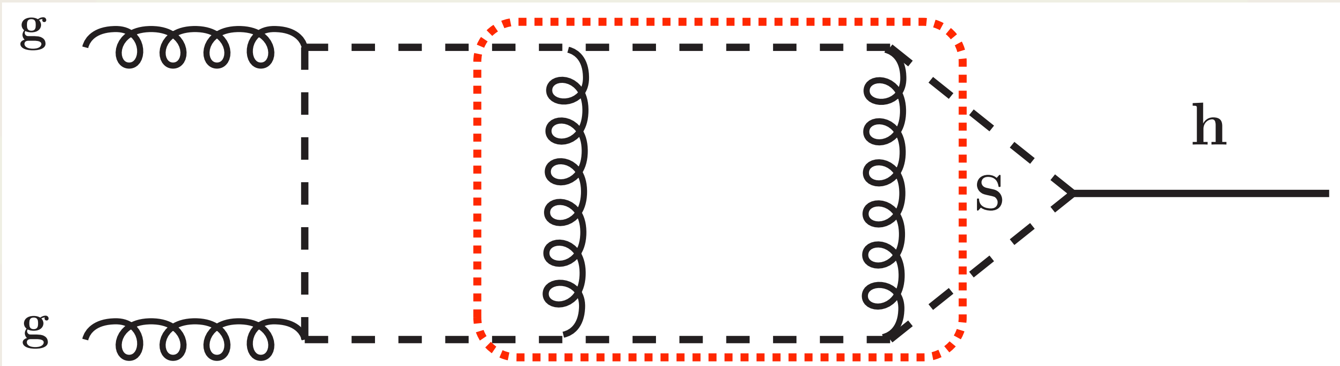
Bare Wilson coefficient

(Reviewed by Steinhauser, 2002)



Quartic-scalar potential at NNLO

- New feature at NNLO: must introduce quartic-scalar potential due to divergent subdiagrams

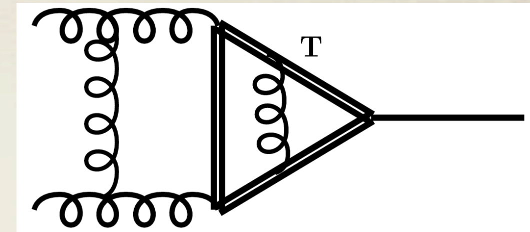


- Single extra Lagrangian term needed here; complication in models with more intricate potentials

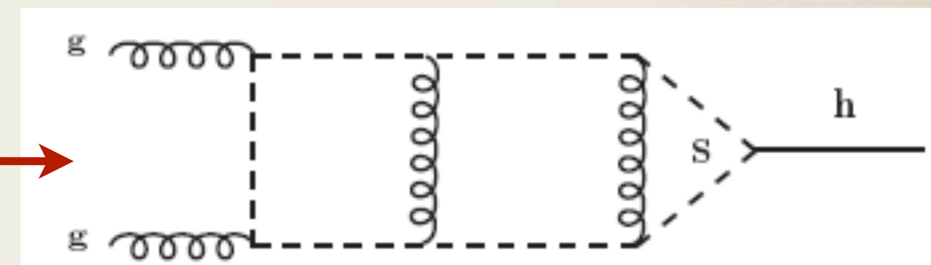
Analytic result

$$C_1 = C_{TTH} + C_{SSH} + C_{TS}$$

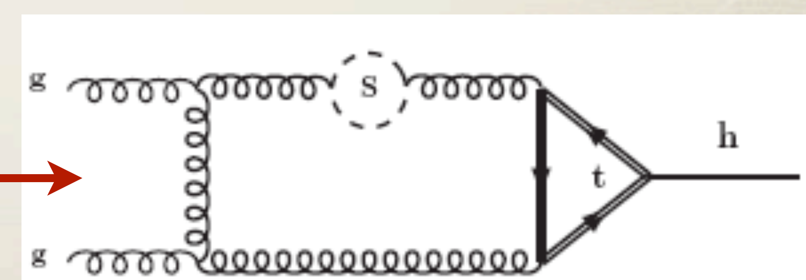
$$C_{TTH} = -\frac{a'}{3} - \frac{11\,a'^2}{12} + a'^3\left[\frac{1}{864}\,(-2777+684\,L_T) + \frac{1}{288}\,(67+64\,L_T)\,n_l\right]$$



$$\begin{aligned}
C_{SSH} = -\frac{\lambda_1 v^2}{2\,m_S^2}\Bigg\{ & \frac{a'}{4} + a'^2\left[\frac{33}{16} + \frac{5\,G_{4S}}{8}\right] + a'^3\Bigg[n_l\left(\frac{-101}{288} + \frac{7\,L_S}{24}\right) \qquad (3.22)\\
& + G_{4S}^2\left(\frac{-35}{16} + 5\,L_S\right) + \frac{9\,L_S\,(-43+8\,x^2)}{64} - \frac{3\,(76-3895\,x^2+257\,x^4)}{1024\,x^2}\\
& - G_{4S}\left(\frac{-705}{64} + \frac{575\,L_S}{96} + \frac{5\ln(x)}{24}\right) + \frac{3\,(76+37\,x^2+86\,x^4+225\,x^6)}{2048\,x^3}\times\\
& \left(\mathrm{Li}_3(x) - \mathrm{Li}_3(-x)\right)\\
& + \ln^2(x)\left\{-\frac{-228+41\,x^2-192\,x^4+675\,x^6}{2048\,(-1+x)\,x^2\,(1+x)} + \frac{3\,(76+37\,x^2+86\,x^4+225\,x^6)}{4096\,x^3}\times\right.\\
& \left.\left(\ln(1+x) - \ln(1-x)\right)\right\}\\
& + 3\ln(x)\left\{\frac{76-111\,x^2+159\,x^4}{1024\,x^2} - \frac{76+37\,x^2+86\,x^4+225\,x^6}{2048\,x^3}\left(\mathrm{Li}_2(x) - \mathrm{Li}_2(-x)\right)\right\}\Bigg]\Bigg\}.
\end{aligned}$$

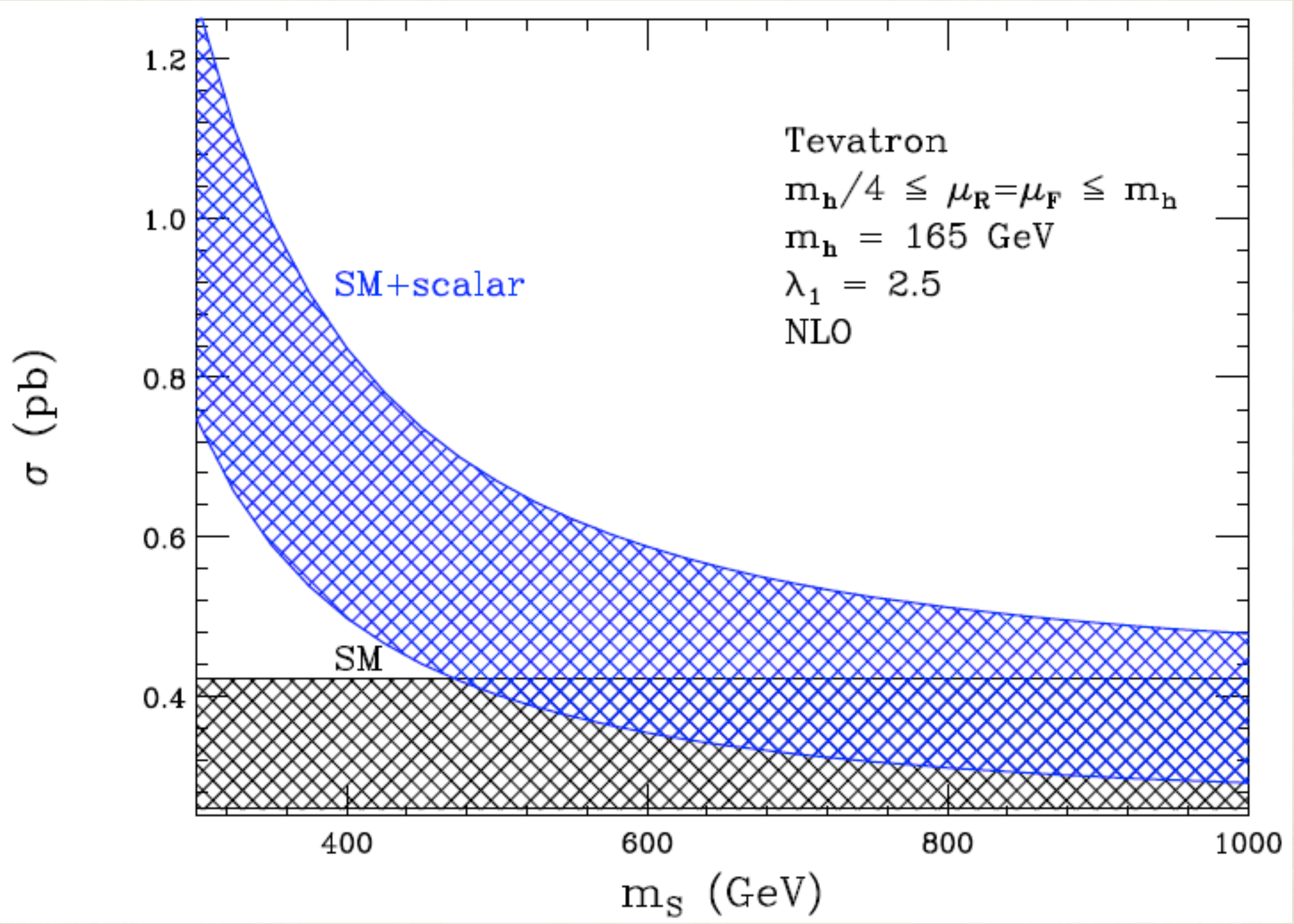


$$\begin{aligned}
C_{TS} = a'^3\Bigg[& \frac{9\,L_S\,x^2}{8} - \frac{2052+1075\,x^2+1755\,x^4}{9216\,x^2} \qquad (3.23)\\
& + \ln(x)\left\{\frac{684+409\,x^2+1431\,x^4}{3072\,x^2} - \frac{3\,(76+37\,x^2+86\,x^4+225\,x^6)}{2048\,x^3}\left(\mathrm{Li}_2(x) - \mathrm{Li}_2(-x)\right)\right\}\\
& + \ln^2(x)\left\{-\frac{-228+41\,x^2-192\,x^4+675\,x^6}{2048\,(-1+x)\,x^2\,(1+x)} + \frac{3\,(76+37\,x^2+86\,x^4+225\,x^6)}{4096\,x^3}\times\right.\\
& \left.\left(\ln(1+x) - \ln(1-x)\right)\right\} + \frac{3\,(76+37\,x^2+86\,x^4+225\,x^6)}{2048\,x^3}\left(\mathrm{Li}_3(x) - \mathrm{Li}_3(-x)\right)\Bigg]
\end{aligned}$$

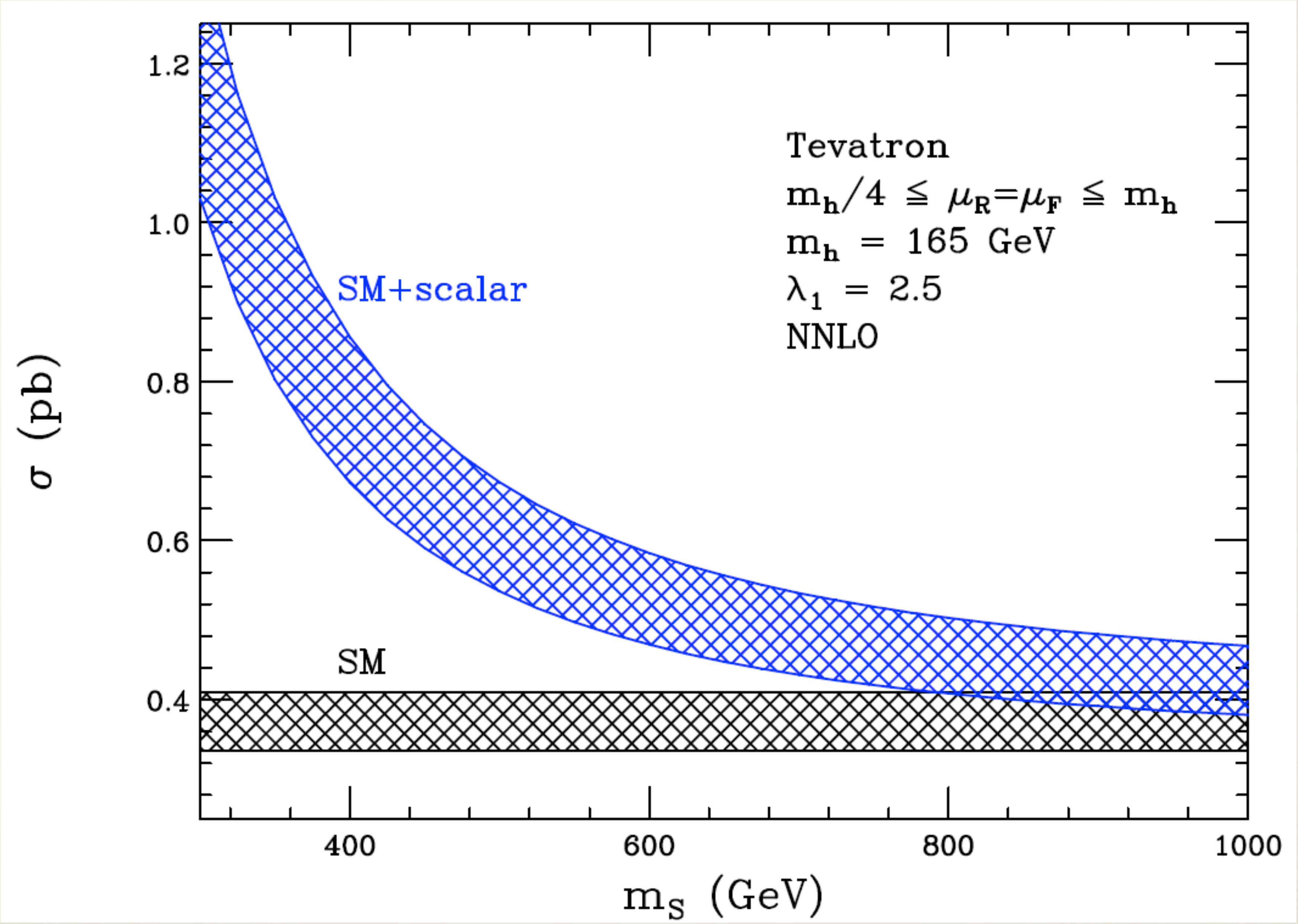


(use MIs from Bekavac et al. 2009)

Tevatron: NLO

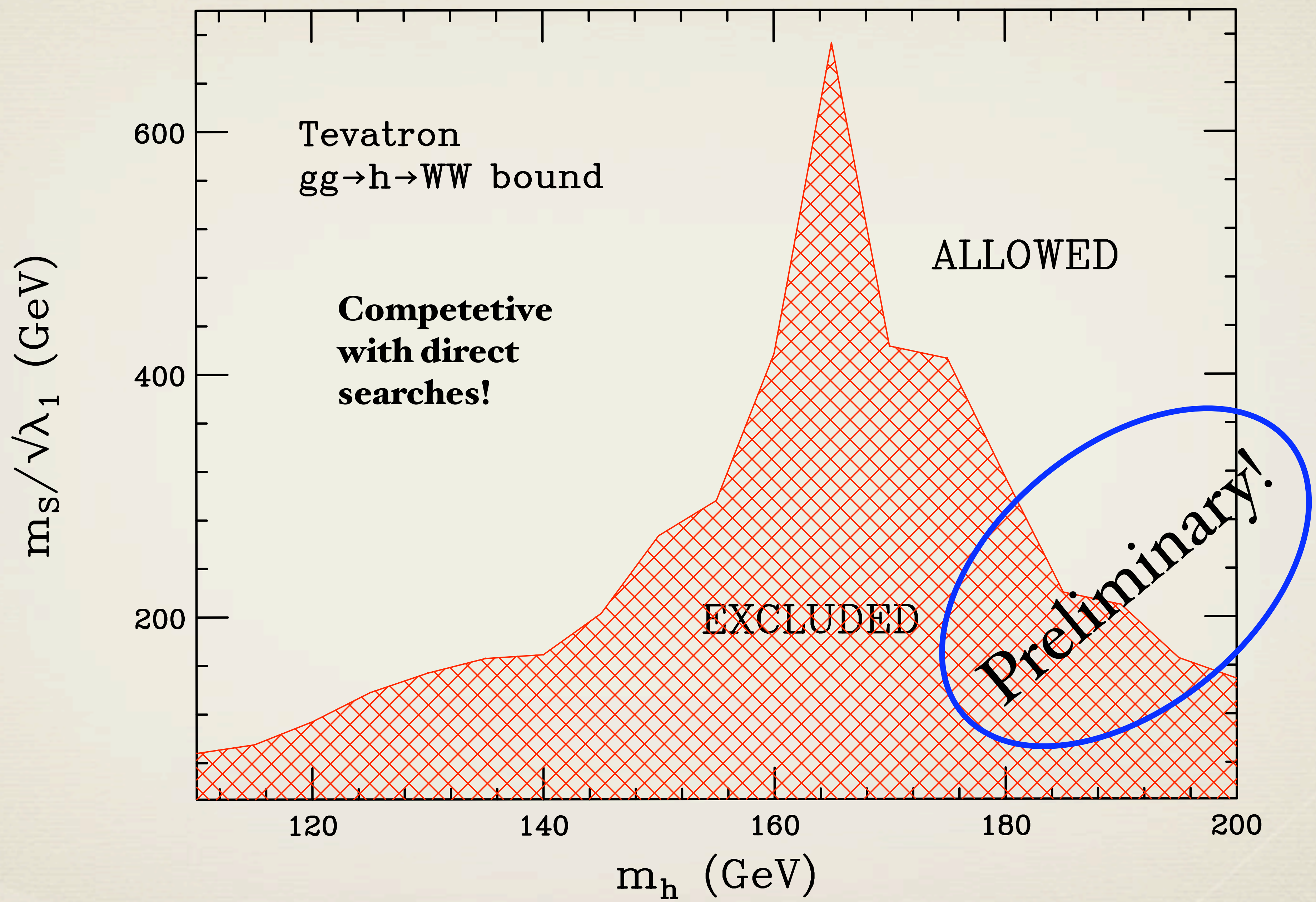


Tevatron: NNLO



- Only at NNLO is a precise prediction suitable for use in indirect searches obtained

Excluded region



Boughezal, FP 2010

Conclusions

- Mixed QCD-EW corrections justify complete factorization approach to inclusion of EW corrections
- Theory under control, with ±10% scale errors and PDF+α_S errors (±13% Tevatron, ±4% LHC)
- Limit already imposing strong constraints on SM extensions
- Example: color-octet scalar effects on $gg \rightarrow h$ through NNLO