Monte Carlo modelling of NLO DGLAP QCD Evolution in the fully unintegrated form

Reinventing the Parton Shower Monte Carlo

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Partly supported by EU grant MRTN-CT-2006-035505, and Polish Government grant N N202 128937.

To be presented at Loops&Legs, Woerlitz DE, April 26-30, 2010

More on http://jadach.web.cern.ch/

DGLAP Collinear QCD ISR Evolution in the Monte Carlo

1970 1980 1990 2000 2010

LO

Moments OPE: (74) QCD: Georgi+Politzer

Diagramatic: (72) QED: Gribov+Lipatov; (77) Altarelli+Parisi

Monte Carlo: 10 years → (85) Sjostrand; (88) Marchesini,Webber

NLO

WE ARE HERE!!!

Moments OPE: (78) Floratos+Ross+Sachrajda

Diagramatic: (81) Curci+Furmanski+Petronzio

Monte Carlo: 27 years later → (08) Jadach Skrzypek

NNLO

Moments: (03) Moch+Verm.+Vogt

Diagramatic: (03) Moch+Verm.+Vogt

Monte Carlo: (15) ???

Why 20 years time lag?

Many reasons:

1. Lack of motivation – poor exp. data from hadron colliders
2. QCD Parton shower Monte Carlo (PSMC) main objective was (is?) hadronization
   – until recently not involved in pQCD @ hard process
3. Conceptual barriers: Factorisation theorems (EGMPR, CFP, CSS, Bodwin,...) not suited for MC beyond LO:
   - non-conservation of 4-momenta
   - over-subtractions $\rightarrow$ huge cancellations
   - non-positive distributions
   - real emissions irreversibly integrated over
4. CPU time: in 1985 No. of MC events <50k, $\sim$ 3% precision; now 2010, $2 \cdot 10^{10}$ events and $\sim$ 0.01% errors are routine.

Prediction: NLO PS era in QCD MC is coming soon!

After completing NLO and NNLO calculations for hard processes NLO PSMC may/will become main front of activity!

TERMINOLOGY

Synonyms:

Fully Unintegrated $\equiv$ Exclusive $\equiv$ MonteCarlo Event Generation

Possible profits/gains from NLO PSMC

- Complete set of "unitegrated soft counterterms" for combining hard process ME at NNLO with NLO PS MC
- Natural extensions towards BFKL/CCFM at low x
- Better modelling of low scale phenomena, $Q < 10 GeV$, quark thresholds, primordial k^T, underlying event, etc.
- Porting information on parton distributions from DIS (HERA) to W/Z/DY (LHC) in the MC itself, instead in form of collinear PDFs (universality must be preserved)
- and more...

MC modelling of NLO DGLAP is not the aim in itself – it will be a starting platform for many developments in many directions.

The aim of the present exercise (KRKMC project)

Constructing NLO Parton Shower Monte Carlo for QCD Initial State Radiation for one initial parton:

- based on the collinear factorisation (EGMPR, CFP, CSS, Bodwin...) as rigorously as we can,
- CFP=Curci-Furmanski-Petronzio scheme as a main reference/guide (axial gauge, $\overline{MS}$ dim. regulariz.),
- implementing *exactly* NLO DGLAP evolution,
- and fully unintegrated exclusive PDFs (ePDFs),
- with NLO evolution done by the MC itself, using new Exclusive NLO kernels

We are going to show that it is feasible
– the proof of the concept for non-singlet NLO DGLAP.

More details on the project are available here:

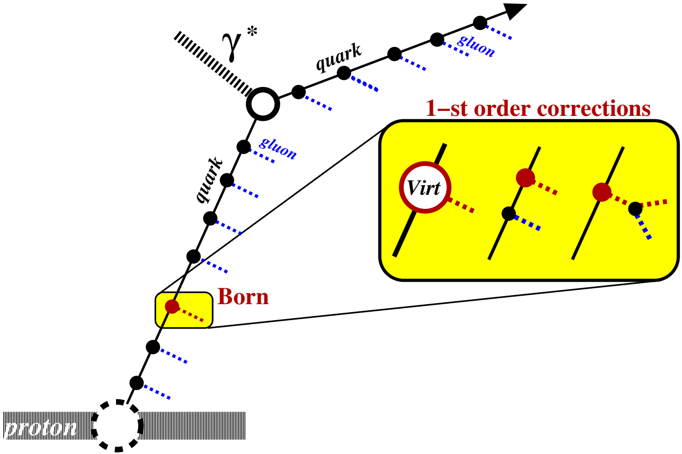
- Epiphany 2009 Proceedings,
  article http://arxiv.org/abs/0905.1399
  slides http://home.cern.ch/jadach/public/epip09.pdf
  More on 1st version of methodology of re-inserting NLO corrections into LO ladder in the unintegrated form ($\sim C_F^2$), 2-nd improved version presented here.
- RADCOR 2009 Proceedings,
  article http://arxiv.org/abs/1002.0010
  slides http://home.cern.ch/jadach/public/RADCOR09.pdf
  More on the "factorization scheme" used in the NLO PSMC, on the differences with the orthodox CFP and how they are corrected.

To cover at the above additional material I would need at least another 30min.

Leading Order (LO) ladder vertex is our “Born”

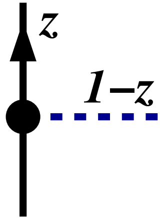
Emission of gluons out of quark

The aim is to implement in the Monte Carlo complete NLO DGLAP in the initial state ladder, using unintegrated Feynman diagrams of Curci-Furmanski-Petronzio scheme (axial gauge).

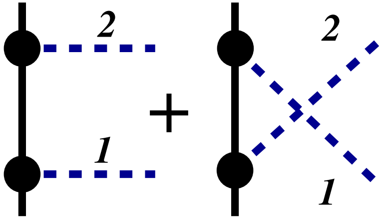


1-st order virtual and real correction diagrams

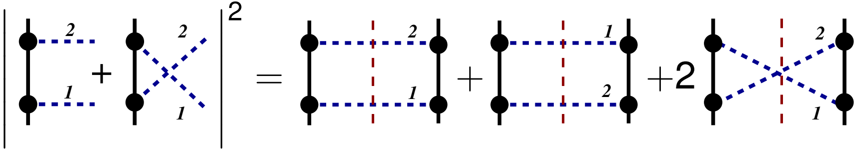
Virtual : $\left(1 + \Delta^{(1)}_{ISR}(z)\right)$

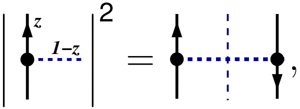


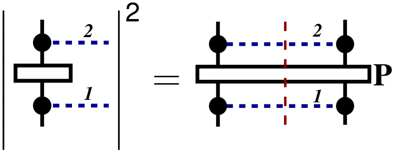
Real :



NOTATION: squared MEs = cut-diagrams, C_F^2 only

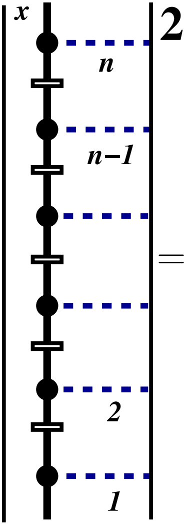






LO ladder = parton shower MC

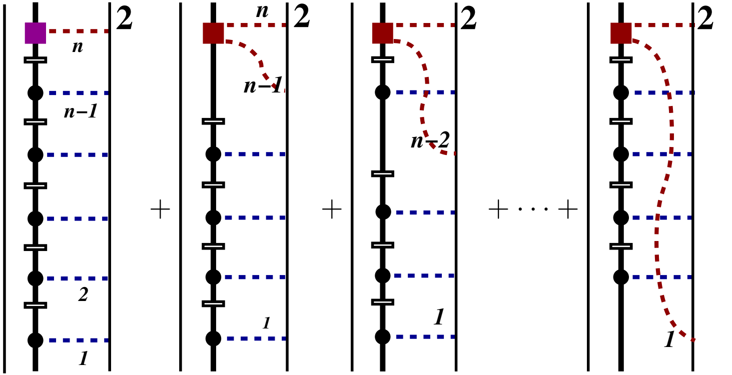
$$\sum_{n=0}^{\infty}$$



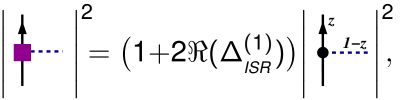
$$= e^{-S_{ISR}} \sum_{n=0}^{\infty} \prod_{i=1}^{n} \frac{d^3 k_i}{k_i^0}\, \theta_{Q>a_i>a_{i-1}}\, \rho^{(0)}_{1B}(k_i)\, \delta_{x=\prod z_i}$$

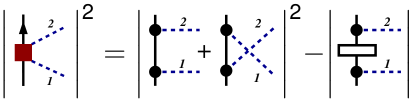
$$a_i = \frac{k_i^T}{\alpha_i}, \quad \alpha_i = \frac{k_i^+}{2E_h}, \quad \rho^{(0)}_{1B}(k_i) = \frac{2C_F^2 \alpha_s}{\pi} \frac{1}{k_i^{T2}} \frac{1+z^2}{2}.$$

LO with NLO-corrected kernel at the end of the ladder



Virt. multiplicative   Undoing LO simplificat.   Sum over trailing LO spectators, essential (BE, YFS61)

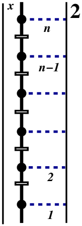




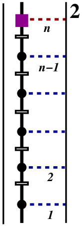
LO with NLO-corrected end-ladder kernel, $\sim C_F^2$

With more details:

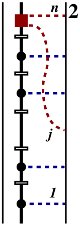
$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}}$



$+\, e^{-S_{ISR}}$



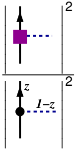
$+\, e^{-S_{ISR}} \sum_{j=1}^{n-1}$



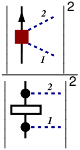
$= e^{-S_{ISR}} \Big\{ \delta_{x=1} +$

$$+\sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \left[\beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Big\},$$

where $d\eta_i = \frac{d^3 k_i}{k_i^0}$, $\beta_0^{(1)} =$



, $W(k_2, k_1) =$



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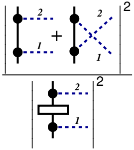


$-\,1.$

Mapping $k_i \to \tilde{k}_i$ instrumental. S_{ISR} = double-log Sudakov, W is non-singular!

Algebraic crosscheck

Analytical integration of NLO part $\sum_j W(\tilde{k}_n, \tilde{k}_j)$ can be done leading to:

$$\sum_{n=1}^{\infty} \int du \int\limits_{Q>a_n>a_{n-1}} \frac{da_n}{a_n} \mathcal{P}^{(1)}_{qq}(u) \left(\prod_{i=1}^{n-1} \int\limits_{a_{i+1}>a_i>a_{i-1}} \frac{da_i}{a_i} \mathcal{P}^{(0)}_{qq}(z_i) \right) \delta_{x=u\prod_{j=1}^{n-1} z_j}.$$

where we recover precisely NLO part (including virtuals) of standard DGLAP kernel $\mathcal{P}^{(1)}_{qq}(u)$ defined according to:

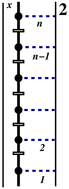
$$\mathcal{P}^{(1)}_{qq}(u) \ln \frac{Q}{q_0} = \int\limits_{Q>a_n>a_0} d^3\eta_n \; \rho^{(1)}_{1B}(k_n) \; \beta^{(1)}_0(z_n) \delta_{u=z_n} + \int\limits_{Q>a_n>a_0} d^3\eta_n \int\limits_{a_n>a_{n'}>0} d^3\eta_{n'} \; \beta^{(1)}_1(\tilde{k}_n, \tilde{k}_{n'}) \; \delta_{u=z_n z_{n'}}$$

One NLO standard inclusive kernel of DGLAP truly reproduced.

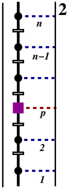
NLO-corrected middle-of-the-ladder kernel, $\sim C_F^2$

Position of the NLO correction/insertion p can be anywhere in the ladder and we sum up over p:

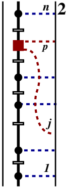
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$$



$$+ \sum_{p=1}^{n}$$



$$+ \sum_{p=1}^{n} \sum_{j=1}^{p-1}$$



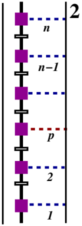
$$\Bigg\} = e^{-S_{ISR}} \Bigg\{ \delta_{x=1} +$$

$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \left[\sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

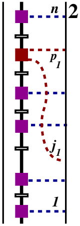
Next step is to add more "NLO insertions",
for instance 2 at positions p_1 and p_2 and sum up over them...
then 3 insertions at p_1, p_2, p_3 and so on
– LO+NLO kernels built up all over along the ladder!

NLO-corrected kernels all over the ladder, $\sim C_F^2$

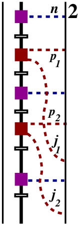
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$$



$$+ \sum_{p_1=1}^{n} \sum_{j_1=1}^{p_1-1}$$



$$+ \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1 \\ j_1\neq p_2}}^{p_1-1} \sum_{\substack{j_2=1 \\ j_2\neq p_1,j_2}}^{p_2-1}$$



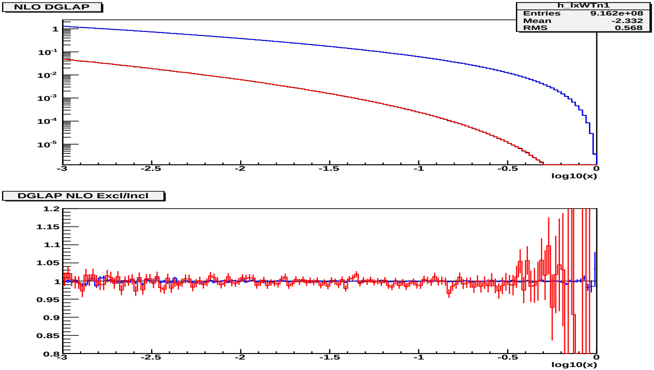
$$\Bigg\}$$

$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \Bigg(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \beta_0^{(1)}(z_p) \Bigg) \Bigg[1 + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p,\tilde{k}_j) +$$

$$+ \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1 \\ j_1\neq p_2}}^{p_1-1} \sum_{\substack{j_2=1 \\ j_2\neq p_1,j_2}}^{p_2-1} W(\tilde{k}_{p_1},\tilde{k}_{j_1}) W(\tilde{k}_{p_2},\tilde{k}_{j_2}) + \dots \Bigg] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

The above has been tested with 3-digit precision in the MC prototype, see next slide.

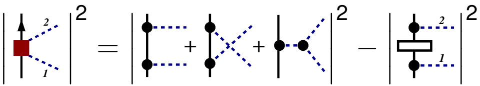
Numerical test of ISR pure C_F^2 NLO MC



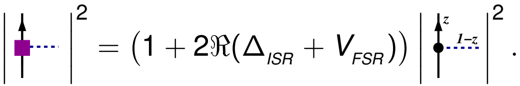
Numerical results for $D(x, Q)$ from inclusive and exclusive **two** Monte Carlos.
Blue curve is single NLO insertion, red curve is double insertion component.
LO+NLO is off scale. Evolution 10GeV→1TeV starting from $\delta(1-x)$.
The ratio demonstrates 3-digit agreement, in units of LO.

THE PROBLEM WITH GLUON PAIR COMPONENT OF the NLO KERNEL, $\sim C_F C_A$ (FSR)

Straightforward inclusion of gluon pair diagram in the previous method would ruin Monte Carlo weight due to presence of Sudakov double logarithmic $+S_{FSR}$ in 2-real correction:



and $-S_{FSR}$ in the virtual correction:

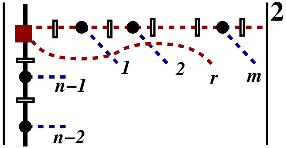


SOLUTION: Resummation/exponentiation of FSR, see next slides for details of the scheme and numerical test of the prototype MC.

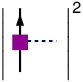
NLO FSR corr. at the end of the ladder, $\sim C_F C_A$

Additional NLO FSR corr. at the end of the ladder:

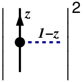
$$e^{-S_{ISR}-S_{FSR}} \sum_{n,m=0}^{\infty} \sum_{r=1}^{m}$$



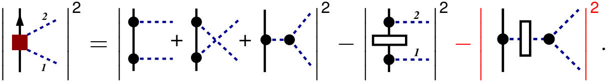
where Sudakov S_{FSR} is subtracted in the virtual part:



$$= \big(1 + 2\Re(\Delta_{ISR} + V_{FSR} - S_{FSR})\big)$$



and FSR counterterm is subtracted in the 2-real-gluon part:

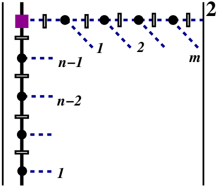


The miracle: both are free of any collinear or soft divergency!!!

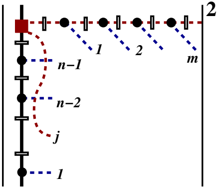
ISR+FSR NLO scheme, NLO corr. at end of the ladder

$\bar{D}^{[1]}_{NS}(x,Q) =$

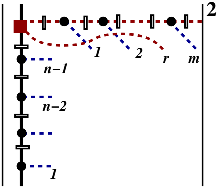
$e^{-S}\sum_{n,m=0}^{\infty}\Bigg\{$



$+\sum_{j=1}^{n-1}$



$+\sum_{r=1}^{m}$

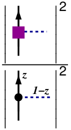


$\Bigg\}$

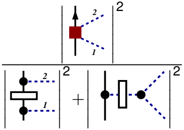
$$= e^{-S_{ISR}}\Bigg\{\delta_{x=1}+\sum_{n=1}^{\infty}\Bigg(\prod_{i=1}^{n}\int_{Q>a_i>a_{i-1}} d^3\eta_i\;\rho^{(1)}_{1B}(k_i)\Bigg)e^{-S_{FSR}}\sum_{m=0}^{\infty}\Bigg(\prod_{j=1}^{m}\int_{Q>a_{nj}>a_{n(l-1)}} d^3\eta'_j\;\rho^{(1)}_{1V}(k'_j)\Bigg)$$

$$\times\Bigg[\beta_0^{(1)}(z_n)+\sum_{j=1}^{n-1}W(\tilde{k}_n,\tilde{k}_j)+\sum_{r=1}^{m}W(\tilde{k}_n,\tilde{k}'_r)\Bigg]\delta_{x=\prod_{j=1}^{n}x_j}\Bigg\}$$

$\beta_0^{(1)} \equiv$



$,\quad W(k_2,k_1) \equiv$



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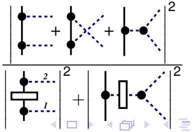

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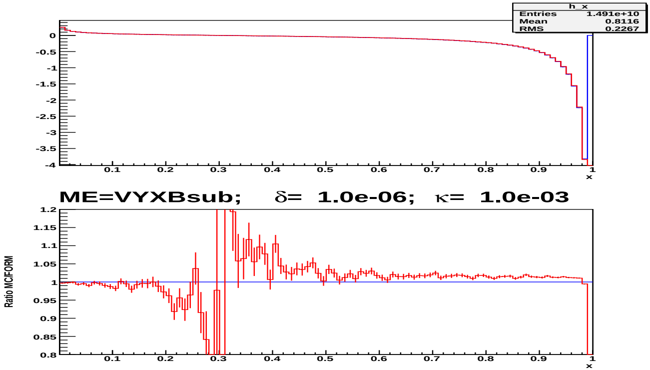
3-digit precision numerical test of FSR methodology

Numerical test done for single NLO ISR+FSR insertion
for $n = 1, 2$ ISR gluons and infinite no. of FSR gluons:



because in this case analytical integration is feasible.
MC agrees precisely with the analytical result.

Summary and Prospects

- First serious feasibility study of the true NLO exclusive MC parton shower is almost complete for non-singlet NLO DGLAP. It works!!!
- Short range aim: Complete non-singlet.
- Middle range aim: Complete singlet (Q-G transitions).
- Optimise MC weight evaluation (CPU time).
- Adding NLO hard process into the game (similar to MC@NLO but different).
- Complete NLO MC for DIS@HERA and W/Z prod. @LHC.
- Extensions towards CCFM/BFKL, quark masses, fitting PDFs with Monte Carlo.