

# Monte Carlo modelling of NLO DGLAP QCD Evolution in the fully unintegrated form

Reinventing the Parton Shower Monte Carlo

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More on <http://jadach.web.cern.ch/>



# DGLAP Collinear QCD ISR Evolution in the Monte Carlo

1970

1980

1990

2000

2010

Moments OPE

(74) QCD: Georgi+Politzer

Diagramatic

(72) QED: Gribov+Lipatov

(77) Altarelli+Parisi

Monte Carlo

10 years

(85) Sjostrand

(88) Marchesini, Webber

LO

Moments OPE

(78) Floratos+Ross+Sachrajda

WE ARE HERE!!!

Diagramatic

(81) Curci+Furmanski+Petronzio

Monte Carlo

27 years later

(08) Jadach Skrzypek

NLO

Moments

(03) Moch+Verm.+Vogt

Diagramatic

(03) Moch+Verm.+Vogt

Monte Carlo

(15) ???

NNLO

# Why 20 years time lag?

## Many reasons:

- 1 Lack of motivation – poor exp. data from hadron colliders
- 2 QCD Parton shower Monte Carlo (PSMC) main objective was (is?) hadronization  
– until recently not involved in pQCD @ hard process
- 3 **Conceptual barriers: Factorisation theorems (EGMPR, CFP, CSS, Bodwin,...) not suited for MC beyond LO:**
  - non-conservation of 4-momenta
  - over-subtractions → huge cancellations
  - non-positive distributions
  - real emissions irreversibly integrated over
- 4 CPU time: in 1985 No. of MC events  $< 50k$ ,  $\sim 3\%$  precision; now 2010,  $2 \cdot 10^{10}$  events and  $\sim 0.01\%$  errors are routine.

**Prediction:** NLO PS era in QCD MC is coming soon!

After completing NLO and NNLO calculations for hard processes  
NLO PSMC may/will become main front of activity!

## Synonyms:

Fully Unintegrated  $\equiv$  Exclusive  $\equiv$  MonteCarlo Event Generation

# Possible profits/gains from NLO PSMC

- Complete set of “unintegrated soft counterterms” for combining hard process ME at NNLO with NLO PS MC
- Natural extensions towards BFKL/CCFM at low  $x$
- Better modelling of low scale phenomena,  $Q < 10\text{GeV}$ , quark thresholds, primordial  $k^T$ , underlying event, etc.
- Porting information on parton distributions from DIS (HERA) to W/Z/DY (LHC) in the MC itself, instead in form of collinear PDFs (universality must be preserved)
- and more...

MC modelling of NLO DGLAP is not the aim in itself – it will be a starting platform for many developments in many directions.



# The aim of the present exercise (KRKMC project)

## Constructing NLO Parton Shower Monte Carlo for QCD Initial State Radiation for one initial parton:

- based on the collinear factorisation (EGMPR, CFP, CSS, Bodwin...) as rigorously as we can,
- CFP=Curci-Furmanski-Petronzio scheme as a main reference/guide (axial gauge,  $\overline{MS}$  dim. regulariz.),
- implementing *exactly* NLO DGLAP evolution,
- and fully unintegrated exclusive PDFs (ePDFs),
- with NLO evolution done by the MC itself, using new Exclusive NLO kernels

We are going to show that it is feasible

– the proof of the concept for non-singlet NLO DGLAP.



# More details on the project are available here:

- Epiphany 2009 Proceedings,  
article <http://arxiv.org/abs/0905.1399>  
slides <http://home.cern.ch/jadach/public/epip09.pdf>  
More on 1st version of methodology of re-inserting NLO corrections into LO ladder in the unintegrated form ( $\sim C_F^2$ ), 2-nd improved version presented here.
- RADCOR 2009 Proceedings,  
article <http://arxiv.org/abs/1002.0010>  
slides <http://home.cern.ch/jadach/public/RADCOR09.pdf>  
More on the “factorization scheme” used in the NLO PSMC, on the differences with the orthodox CFP and how they are corrected.

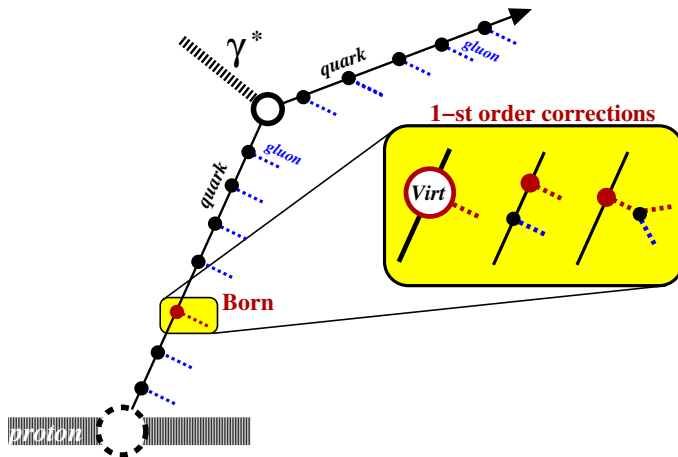
**To cover at the above additional material I would need at least another 30min.**



# Leading Order (LO) ladder vertex is our “Born”

Emission of gluons out of quark

The aim is to implement in the Monte Carlo complete NLO DGLAP in the initial state ladder, using unintegrated Feynman diagrams of Curci-Furmanski-Petronzio scheme (axial gauge).

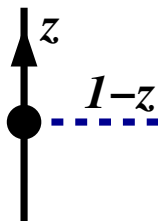




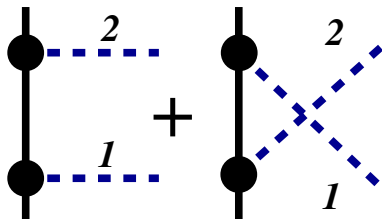
# 1-st order virtual and real correction diagrams

Virtual :

$$\left(1 + \Delta_{ISR}^{(1)}(z)\right)$$



Real :



# NOTATION: squared MEs = cut-diagrams, $C_F^2$ only

$$\left| \begin{array}{c} \bullet \text{---} 2 \text{---} \bullet \\ | \quad | \\ \bullet \text{---} 1 \text{---} \bullet \end{array} + \begin{array}{c} \bullet \text{---} 2 \text{---} \bullet \\ | \quad | \\ \bullet \text{---} 1 \text{---} \bullet \end{array} \right|^2 = \begin{array}{c} \bullet \text{---} 2 \text{---} \bullet \\ | \quad | \\ \bullet \text{---} 1 \text{---} \bullet \end{array} + \begin{array}{c} \bullet \text{---} 1 \text{---} \bullet \\ | \quad | \\ \bullet \text{---} 2 \text{---} \bullet \end{array} + 2 \begin{array}{c} \bullet \text{---} 2 \text{---} \bullet \\ | \quad | \\ \bullet \text{---} 1 \text{---} \bullet \end{array}$$

The diagram shows the squared magnitude of the sum of two diagrams. The first diagram has two vertices on the left and two on the right, with a top line labeled '2' and a bottom line labeled '1'. The second diagram is similar but with the lines crossed. The result is the sum of three diagrams: two with a vertical dashed red line between the vertices, and one with two vertical dashed red lines, each with a coefficient of 2.

$$\left| \begin{array}{c} \uparrow z \\ \bullet \text{---} 1-z \text{---} \bullet \\ | \quad | \\ \bullet \end{array} \right|^2 = \begin{array}{c} \uparrow \\ \bullet \text{---} \bullet \\ | \quad | \\ \bullet \end{array}$$

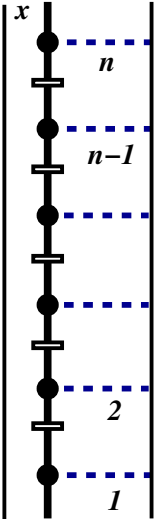
The diagram shows the squared magnitude of a diagram with a vertical line on the left, a vertex with an upward arrow labeled 'z', and a horizontal line labeled '1-z' connecting to a vertex on the right. The result is a diagram with a vertical line on the left, a horizontal line connecting to a vertex on the right, and a vertical line on the right with a downward arrow.

$$\left| \begin{array}{c} \bullet \text{---} 2 \text{---} \bullet \\ \text{---} \text{---} \text{---} \\ \bullet \text{---} 1 \text{---} \bullet \end{array} \right|^2 = \begin{array}{c} \bullet \text{---} 2 \text{---} \bullet \\ \text{---} \text{---} \text{---} \\ \bullet \text{---} 1 \text{---} \bullet \end{array} \mathbf{P}$$

The diagram shows the squared magnitude of a diagram with two vertices on the left and two on the right, with a top line labeled '2' and a bottom line labeled '1'. A horizontal bar is drawn between the two vertices on the left. The result is the same diagram with a larger horizontal bar between the two vertices on the left, labeled with a bold 'P'.



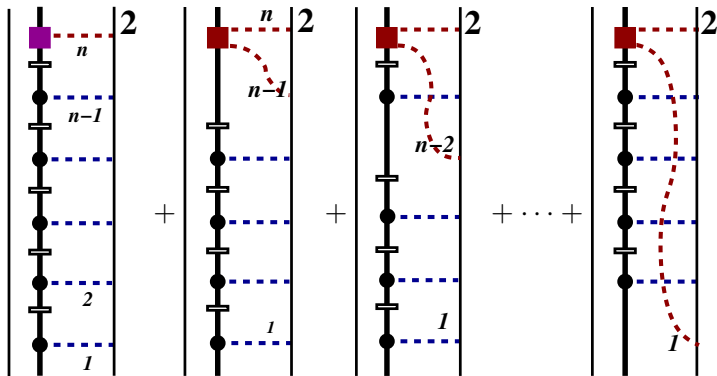
# LO ladder = parton shower MC



$$\sum_{n=0}^{\infty} = e^{-S_{ISR}} \sum_{n=0}^{\infty} \prod_{i=1}^n \frac{d^3 k_i}{k_i^0} \theta_{Q > a_i > a_{i-1}} \rho_{1B}^{(0)}(k_i) \delta_{x=\prod z_i}$$

$$a_i = \frac{k_i^T}{\alpha_i}, \quad \alpha_i = \frac{k_i^+}{2E_h}, \quad \rho_{1B}^{(0)}(k_i) = \frac{2C_F^2 \alpha_s}{\pi} \frac{1}{k_i^{T2}} \frac{1+z^2}{2}$$

# LO with NLO-corrected kernel at the end of the ladder



Virt. multiplicative

Undoing LO simplificat.

Sum over trailing LO spectators, essential (BE, YFS61)

$$\left| \begin{array}{c} \uparrow \\ \text{---} \\ \square \\ \text{---} \\ \uparrow \end{array} \right|^2 = (1 + 2\Re(\Delta_{ISR}^{(1)})) \left| \begin{array}{c} \uparrow \\ \text{---} \\ \bullet \\ \text{---} \\ \uparrow \end{array} \right|^2,$$

$$\left| \begin{array}{c} \uparrow \\ \text{---} \\ \square \\ \text{---} \\ \uparrow \end{array} \right|^2 = \left| \begin{array}{c} \bullet \\ \text{---} \\ \bullet \\ \text{---} \\ \uparrow \end{array} \right|^2 + \left| \begin{array}{c} \bullet \\ \text{---} \\ \bullet \\ \text{---} \\ \uparrow \end{array} \right|^2 - \left| \begin{array}{c} \bullet \\ \text{---} \\ \square \\ \text{---} \\ \bullet \\ \text{---} \\ \uparrow \end{array} \right|^2$$



# LO with NLO-corrected end-ladder kernel, $\sim C_F^2$

With more details:

$$\bar{D}_B^{[1]}(x, Q) = e^{-S_{ISR}} \left[ \text{Diagram 1} + e^{-S_{ISR}} \left[ \text{Diagram 2} + e^{-S_{ISR}} \sum_{j=1}^{n-1} \text{Diagram 3} \right] \right] = e^{-S_{ISR}} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left( \prod_{i=1}^n \int_{Q > a_i > a_{i-1}} d^3 \eta_i \rho_{1B}^{(1)}(k_i) \right) \left[ \beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^n x_j} \right\},$$

where  $d\eta_i = \frac{d^3 k_i}{k_i^0}$ ,  $\beta_0^{(1)} = \frac{\text{Diagram 4}}{\text{Diagram 5}}$ ,  $W(k_2, k_1) = \frac{\text{Diagram 6}}{\text{Diagram 7}} = \frac{\text{Diagram 8} + \text{Diagram 9}}{\text{Diagram 7}} - 1$ .

Mapping  $k_i \rightarrow \tilde{k}_i$  instrumental.  $S_{ISR}$  = double-log Sudakov,  $W$  is non-singular!

# Algebraic crosscheck

Analytical integration of NLO part  $\sum_j W(\tilde{k}_n, \tilde{k}_j)$  can be done leading to:

$$\sum_{n=1}^{\infty} \int du \int_{Q > a_n > a_{n-1}} \frac{da_n}{a_n} \mathcal{P}_{qq}^{(1)}(u) \left( \prod_{i=1}^{n-1} \int_{a_{i+1} > a_i > a_{i-1}} \frac{da_i}{a_i} \mathcal{P}_{qq}^{(0)}(z_i) \right) \delta_{x=u \prod_{j=1}^{n-1} z_j}$$

where we recover precisely NLO part (including virtuals) of standard DGLAP kernel  $\mathcal{P}_{qq}^{(1)}(u)$  defined according to:

$$\mathcal{P}_{qq}^{(1)}(u) \ln \frac{Q}{q_0} = \int_{Q > a_n > a_0} d^3 \eta_n \rho_{1B}^{(1)}(k_n) \beta_0^{(1)}(z_n) \delta_{u=z_n} + \int_{Q > a_n > a_0} d^3 \eta_n \int_{a_n > a_{n'} > 0} d^3 \eta_{n'} \beta_1^{(1)}(\tilde{k}_n, \tilde{k}_{n'}) \delta_{u=z_n z_{n'}}$$

One NLO standard inclusive kernel of DGLAP truly reproduced.



# NLO-corrected middle-of-the-ladder kernel, $\sim C_F^2$

Position of the NLO correction/insertion  $p$  can be anywhere in the ladder and we sum up over  $p$ :

$$\bar{D}_B^{[1]}(x, Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \left\{ \begin{array}{l} \text{Diagram 1: Ladder with } n \text{ rungs, levels } 1, 2, \dots, n-1, n, \text{ and } x. \\ \text{Diagram 2: Ladder with } n \text{ rungs, level } p \text{ highlighted in purple.} \\ \text{Diagram 3: Ladder with } n \text{ rungs, level } p \text{ highlighted in red, with a dashed red loop between levels } p \text{ and } j. \end{array} \right\} = e^{-S_{ISR}} \left\{ \delta_{x=1} + \right. \\ \left. + \sum_{n=1}^{\infty} \left( \prod_{i=1}^n \int_{Q > a_i > a_{i-1}} d^3 \eta_i \rho_{1B}^{(1)}(k_i) \right) \left[ \sum_{p=1}^n \beta_0^{(1)}(z_p) + \sum_{p=1}^n \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^n x_j} \right\},$$

Next step is to add more “NLO insertions”,  
 for instance 2 at positions  $p_1$  and  $p_2$  and sum up over them...  
 then 3 insertions at  $p_1, p_2, p_3$  and so on  
 – LO+NLO kernels built up all over along the ladder!



# NLO-corrected kernels all over the ladder, $\sim C_F^2$

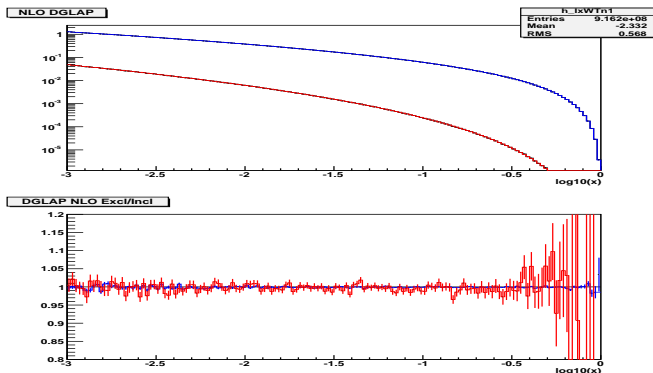
$$\begin{aligned}
 \bar{D}_B^{[1]}(x, Q) &= e^{-S_{ISR}} \sum_{n=0}^{\infty} \left\{ \begin{array}{l} \text{Diagram 1: Ladder with } n \text{ rungs, rungs } p, 2, \dots, 1 \\ \text{Diagram 2: Ladder with } n \text{ rungs, rungs } p_1, j_1, \dots, 1 \\ \text{Diagram 3: Ladder with } n \text{ rungs, rungs } p_1, p_2, j_1, j_2, \dots, 1 \end{array} \right\} \\
 &= e^{-S_{ISR}} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left( \prod_{i=1}^n \int_{Q > a_i > a_{i-1}} d^3 \eta_i \rho_{1B}^{(1)}(k_i) \beta_0^{(1)}(z_p) \right) \left[ 1 + \sum_{p=1}^n \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) + \right. \right. \\
 &\quad \left. \left. + \sum_{p_1=1}^n \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1 \\ j_1 \neq p_2}}^{p_1-1} \sum_{\substack{j_2=1 \\ j_2 \neq p_1, j_2}}^{p_2-1} W(\tilde{k}_{p_1}, \tilde{k}_{j_1}) W(\tilde{k}_{p_2}, \tilde{k}_{j_2}) + \dots \right] \delta_{x=\prod_{j=1}^n x_j} \right\},
 \end{aligned}$$

The above has been tested with 3-digit precision in the MC prototype, see next slide.





# Numerical test of ISR pure $C_F^2$ NLO MC



Numerical results for  $D(x, Q)$  from inclusive and exclusive **two** Monte Carlos. **Blue curve** is single NLO insertion, **red curve** is double insertion component. LO+NLO is off scale. Evolution  $10\text{GeV} \rightarrow 1\text{TeV}$  starting from  $\delta(1-x)$ . The ratio demonstrates 3-digit agreement, in units of LO.



# THE PROBLEM WITH GLUON PAIR COMPONENT OF the NLO KERNEL, $\sim C_F C_A$ (FSR)

Straightforward inclusion of gluon pair diagram in the previous method would ruin Monte Carlo weight due to presence of Sudakov double logarithmic  $+S_{FSR}$  in 2-real correction:

$$\left| \text{diagram with red square} \right|^2 = \left| \text{diagram 1} + \text{diagram 2} + \text{diagram 3} \right|^2 - \left| \text{diagram with white box} \right|^2$$

and  $-S_{FSR}$  in the virtual correction:

$$\left| \text{diagram with purple square} \right|^2 = (1 + 2\Re(\Delta_{ISR} + V_{FSR})) \left| \text{diagram with black dot} \right|^2.$$

SOLUTION: Resummation/exponentiation of FSR, see next slides for details of the scheme and numerical test of the prototype MC.



# NLO FSR corr. at the end of the ladder, $\sim C_F C_A$

Additional NLO FSR corr. at the end of the ladder:

$$e^{-S_{ISR} - S_{FSR}} \sum_{n,m=0}^{\infty} \sum_{r=1}^m \left| \begin{array}{c} \text{Diagram with } n-2, n-1, 1, 2, r, m \text{ labels} \end{array} \right|^2$$

where Sudakov  $S_{FSR}$  is subtracted in the virtual part:

$$\left| \begin{array}{c} \text{Diagram with } z \text{ label} \end{array} \right|^2 = (1 + 2\Re(\Delta_{ISR} + V_{FSR} - S_{FSR})) \left| \begin{array}{c} \text{Diagram with } 1-z \text{ label} \end{array} \right|^2$$

and FSR counterterm is subtracted in the 2-real-gluon part:

$$\left| \begin{array}{c} \text{Diagram with } 2 \text{ label} \end{array} \right|^2 = \left| \begin{array}{c} \text{Diagram 1} \end{array} \right|^2 + \left| \begin{array}{c} \text{Diagram 2} \end{array} \right|^2 + \left| \begin{array}{c} \text{Diagram 3} \end{array} \right|^2 - \left| \begin{array}{c} \text{Diagram 4} \end{array} \right|^2 - \left| \begin{array}{c} \text{Diagram 5} \end{array} \right|^2$$

The miracle: both are free of any collinear or soft divergency!!!



# ISR+FSR NLO scheme, NLO corr. at end of the ladder

$$\bar{D}_{NS}^{[1]}(x, Q) =$$

$$e^{-S} \sum_{n,m=0}^{\infty} \left\{ \left| \begin{array}{c} \text{Diagram 1: Ladder with } n \text{ rungs, } m \text{ external lines. Top rung is purple square.} \\ \text{Diagram 2: Ladder with } n \text{ rungs, } m \text{ external lines. Top rung is red square.} \\ \text{Diagram 3: Ladder with } n \text{ rungs, } m \text{ external lines. Top rung is red square, with } r \text{ rungs in red.} \end{array} \right. \right\}$$

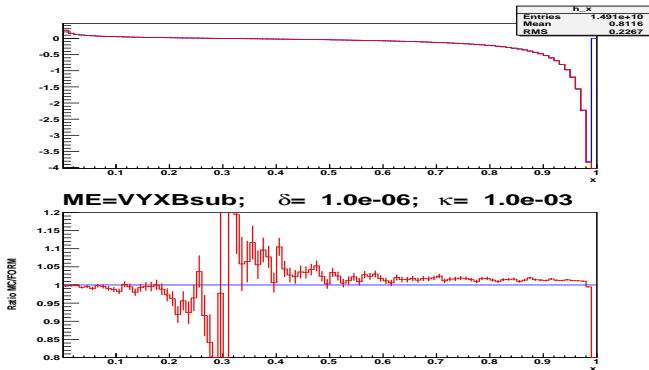
$$= e^{-S_{ISR}} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left( \prod_{i=1}^n \int_{Q > a_i > a_{i-1}} d^3 \eta_i \rho_{1B}^{(1)}(k_i) \right) e^{-S_{FSR}} \sum_{m=0}^{\infty} \left( \prod_{j=1}^m \int_{Q > a_{nj} > a_{n(l-1)}} d^3 \eta'_j \rho_{1V}^{(1)}(k'_j) \right) \right. \\ \left. \times \left[ \beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) + \sum_{r=1}^m W(\tilde{k}_n, \tilde{k}'_r) \right] \delta_{x=\prod_{j=1}^n x_j} \right\}$$

$$\beta_0^{(1)} \equiv \frac{\left| \begin{array}{c} \text{Diagram: Ladder with } n \text{ rungs, } m \text{ external lines. Top rung is purple square.} \end{array} \right|^2}{\left| \begin{array}{c} \text{Diagram: Ladder with } n \text{ rungs, } m \text{ external lines.} \end{array} \right|^2}, \quad W(k_2, k_1) \equiv \frac{\left| \begin{array}{c} \text{Diagram: Ladder with } n \text{ rungs, } m \text{ external lines. Top rung is red square.} \end{array} \right|^2}{\left| \begin{array}{c} \text{Diagram: Ladder with } n \text{ rungs, } m \text{ external lines.} \end{array} \right|^2} - 1.$$



# 3-digit precision numerical test of FSR methodology

Numerical test done for single NLO ISR+FSR insertion  
for  $n = 1, 2$  ISR gluons and infinite no. of FSR gluons:



because in this case analytical integration is feasible.  
MC agrees precisely with the analytical result.



# Summary and Prospects

- First serious **feasibility study** of the true NLO exclusive MC parton shower is almost complete for non-singlet NLO DGLAP. It works!!!
- Short range aim: Complete non-singlet.
- Middle range aim: Complete singlet (Q-G transitions).
- Optimise MC weight evaluation (CPU time).
- Adding NLO hard process into the game (similar to MC@NLO but different).
- Complete NLO MC for DIS@HERA and W/Z prod. @LHC.
- Extensions towards CCFM/BFKL, quark masses, fitting PDFs with Monte Carlo.

