Double-Real Radiation for Top Quark Pair Production at NNLO

or

A New Subtraction Scheme for NNLO

M. Czakon

RWTH Aachen

Loops & Legs 2010, April 28th 2010

Status of Tops at NNLO

No double-real radiation

Subtraction Schemes

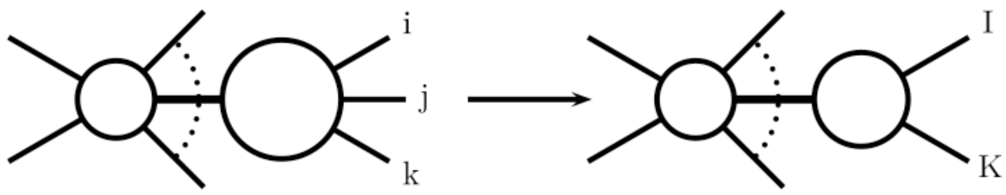
- NLO
  1) Catani-Seymour (smooth interpolation between limits, remapping of phase space allows for arbitrary phase space generators)
  2) FKS (Frixione-Kunszt-Signer) (decomposition of phase space according to collinear singularities, energy-angle parameterization, residue subtraction)

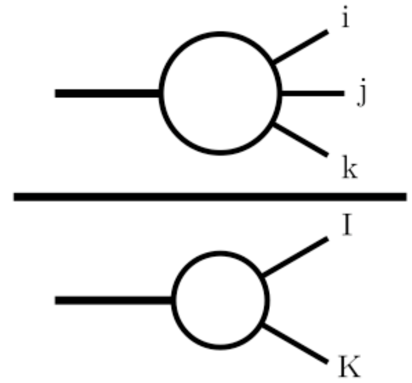
- NNLO general and successful
  1) Sector Decomposition (Binoth,Heinrich ’04, Anastasiou,Melnikov,Petriello ‘05)
  2) Antenna Subtraction (Gehrmann De-Ridder, Gehrmann, Glover ’05)

- NNLO special for colourless particles – Catani, Grazzini ’07

- NNLO in the making (main problem - integration of subtraction terms)
  1) “generalized” Catani-Seymour – Weinzierl ’03
  2) “generalized” FKS (?) – Somogyi, Trocsanyi, Del Duca ‘06

Antenna Subtraction





- use matrix elements of complete processes

- use remapping to drop number of particles

- obtain integrated subtraction terms with multi-loop methods (analytic results)

- drawbacks
  1) huge amount of analytic calculations
  2) reduced efficiency since no azimuthal correlations
     (hybrid method using lower cutoff)

Sector Decomposition

- process independent method working with abstract matrix element
- details tailored to the problem
- parameterize phase space completely depending on the singularity of given diagram and map to unit hypercube
- factorize divergences as in example

$$I = \int_0^1 \mathrm{d}x\mathrm{d}y \frac{x^\epsilon y^\epsilon}{(x+y)^2}$$

split into *y > x* and x > y

$$I_1 = \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \frac{x^\epsilon y^\epsilon}{(x+y)^2} F_J[s_{ab}(x,y)]$$

$$I_1 = \int_0^1 \mathrm{d}x\mathrm{d}y \frac{x^{-1+2\epsilon} y^\epsilon}{(1+y)^2} F_J[s_{ab}(x,xy)]$$

subtract at *x=0*
and expand

Sector Decomposition

- typical problems to solve is getting rid of

1) line singularities (occur inside phase space and not at boundaries)
2) quadratic singularities (would need higher orders of expansion in *x*)
3) complicated phase space parameterizations

$$\mathrm{d}\Pi_{\mathrm{R}} = N \int_0^1 \mathrm{d}\lambda_1 \mathrm{d}\lambda_2 \mathrm{d}\lambda_3 \mathrm{d}\lambda_4 \left[(1-\lambda_1)(1-\lambda_1 K_m/K_p)\right]^{-\epsilon} \left[\lambda_1\lambda_2(1-\lambda_2)\right]^{-\epsilon}$$

$$\times \left[\lambda_3(1-\lambda_3)\right]^{1-2\epsilon} \left[\lambda_4(1-\lambda_4)\right]^{-\epsilon-1/2} \left[K_p r/(1+u)^2\right]^{-1+\epsilon} \left[1 - \frac{\lambda_1 K_m}{r(1+z)}\right]$$

$$\begin{aligned}
s_{1h} &= -\lambda_3(1-z)\left[1-\lambda_1 r(1-rt)/(r+t)\right], \\
s_{2h} &= -(1-\lambda_3)(1-z)\left[1-\lambda_1(r-t)/r/(1+rt)\right], \\
s_{23} &= -\lambda_2\lambda_3(1-z)\left[1+\lambda_1(1-rt)/r/(r+t)\right], \\
s_{24} &= -(1-\lambda_2)\lambda_3(1-z)\left[1+\lambda_1(1-rt)/r/(r+t)\right], \\
s_{34} &= \lambda_1\lambda_3(1-\lambda_3)(1-z)^2(1+u)^2/K_p/r, \\
s_{13} &= -\frac{(1-\lambda_3)(1-z)}{K_p r\left[1+\lambda_1(1-rt)/r/(r+t)\right]}\left[A_1 + A_2 + 2(2\lambda_4-1)\sqrt{A_1 A_2}\right]
\end{aligned}$$

example from
Anastasiou, Melnikov, Petriello '05

Main Ideas

- at first, restrict problem to production of only massive states at LO, which for NNLO is equivalent to

$$p_1 + p_2 \to k_1 + k_2 + q_1 + \cdots + q_n$$

$$p_1^2 = p_2^2 = k_1^2 = k_2^2 = 0\,, \quad q_i^2 = m_i^2 \neq 0\,, \quad i = 1, \ldots, n\,, \quad n \geq 2$$

- work in the partonic COM system – cross section is no distribution
- decompose phase space into sectors with simplest singularities
- avoid remapping
- use specific parameterization only for relevant kinematics
- sector decompose according to physical singularities
- do not aim for analytic integration

Phase Space

- central role played by the two massless states, since they generate most of the singular configurations

- factorize phase space accordingly into three particle phase space and phase of the massive products

invariant mass of composite

$$\int d\Phi_{n+2} = \int \frac{dQ^2}{2\pi}$$
$$\times \int \frac{d^{d-1}k_1}{(2\pi)^{d-1}2k_1^0} \frac{d^{d-1}k_2}{(2\pi)^{d-1}2k_2^0} \frac{d^{d-1}Q}{(2\pi)^{d-1}2Q^0} (2\pi)^d \delta^{(d)}(k_1 + k_2 + Q - p_1 - p_2)$$
$$\times \int \prod_{i=1}^{n} \frac{d^{d-1}q_i}{(2\pi)^{d-1}2q_i^0} (2\pi)^d \delta^{(d)}(q_1 + \cdots + q_n - Q) \, .$$

irrelevant for now

$$\int d\Phi_{n+2} = \int \frac{d^{d-1}k_1}{(2\pi)^{d-1}2k_1^0} \frac{d^{d-1}k_2}{(2\pi)^{d-1}2k_2^0}$$
$$\times \int \prod_{i=1}^{n} \frac{d^{d-1}q_i}{(2\pi)^{d-1}2q_i^0} (2\pi)^d \delta^{(d)}(q_1 + \cdots + q_n - Q)$$

First Decomposition

- due to soft singularities involving the massive states, phase space split only useful if parameterization includes energies of massless states

- remaining parameters are best chosen to be relative angles

construction inspired by *idée fixe* of FKS

- simplify treatment by decomposing according to collinear singularities

- introduce functions that restrict divergences, e.g. $f_1(k)$ allows only divergences, when k parallel to p_1, but not p_2

- introduce function that screens divergence when k_1 parallel to k_2

First Decomposition

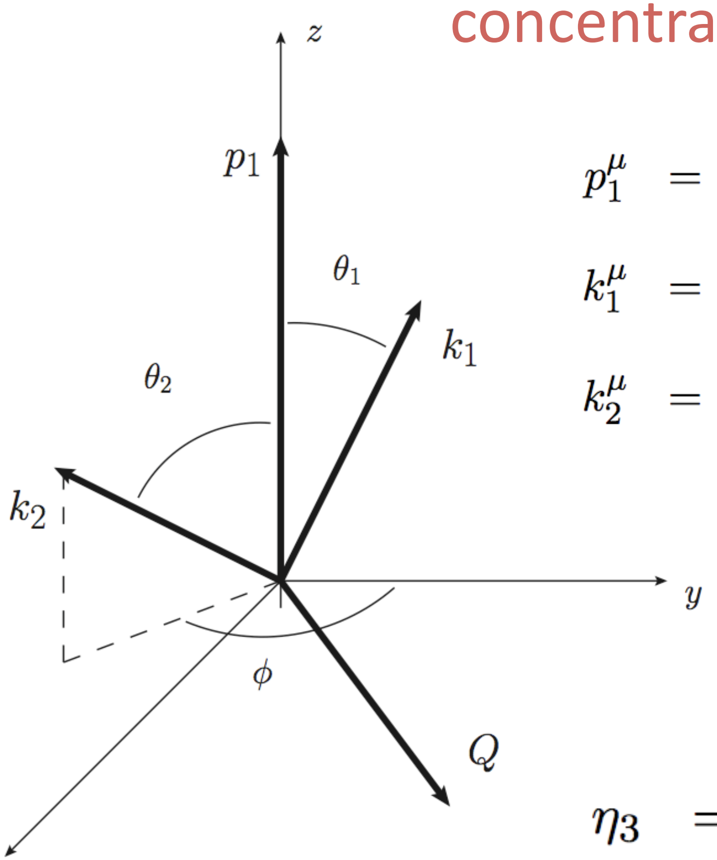
most difficult

non-trivial only because
of soft-collinear divergences

$$
\begin{aligned}
1 \quad = \quad & \\
& \left.\begin{array}{l} +f_1(k_1)f_1(k_2) \\ +f_2(k_1)f_2(k_2) \end{array}\right\} \text{non-factorizable double-collinear limits} \\
& \left.\begin{array}{l} +f_1(k_1)f_2(k_2)f_3(k_1,k_2) \\ +f_2(k_1)f_1(k_2)f_3(k_1,k_2) \end{array}\right\} \text{factorizable double-collinear limits} \\
& +(f_1(k_1)f_2(k_2)+f_2(k_1)f_1(k_2))(1-f_3(k_1,k_2)) \ \} \text{single-collinear limit}
\end{aligned}
$$

trivial, because NLO
attach to first sector (contains same divergences)

Simplest Parameterization Possible



concentrate on double-collinear limits involving p_1

$$\beta \equiv \sqrt{1 - \frac{(\sum_{i=1}^{n} m_i)^2}{s}}$$

$$p_1^\mu = \frac{\sqrt{s}}{2}(1, 0^{(d-2)}, 1) ,$$
$$k_1^\mu = \frac{\sqrt{s}}{2}\beta^2\xi_1(1, 0^{(d-3)}, \sin\theta_1, \cos\theta_1) ,$$
$$k_2^\mu = \frac{\sqrt{s}}{2}\beta^2\xi_2(1, 0^{(d-4)}, \sin\phi\sin\theta_2, \cos\phi\sin\theta_2, \cos\theta_2)$$

introduce

$$\eta_{1,2} = \frac{1}{2}(1 - \cos\theta_{1,2})$$

$$\eta_3 = \frac{1}{2}(1 - \cos\phi\sin\theta_1\sin\theta_2 - \cos\theta_1\cos\theta_2)$$
$$= \frac{1}{2}(1 - \cos(\theta_1 - \theta_2) + (1 - \cos\phi)\sin\theta_1\sin\theta_2)$$

source of problems, since singularities only when
$\Theta_1 = \Theta_2$ and at the same time $\varphi = 0$ (classic case of line singularity)

Three Particle Phase Space

$$\begin{aligned}\int d\Phi_3 &= \int \frac{d^{d-1}k_1}{(2\pi)^{d-1}2k_1^0}\frac{d^{d-1}k_2}{(2\pi)^{d-1}2k_2^0}\\ &= \frac{1}{8(2\pi)^{5-2\epsilon}\Gamma(1-2\epsilon)}s^{2-2\epsilon}\beta^{8-8\epsilon}\\ &\times \int_0^1 d\eta_1\,(\eta_1(1-\eta_1))^{-\epsilon}\int_0^1 d\eta_2\,(\eta_2(1-\eta_2))^{-\epsilon}\int_{-1}^1 d\cos\phi\,(1-\cos^2\phi)^{-\frac{1}{2}-\epsilon}\\ &\iint d\xi_1 d\xi_2\,\xi_1^{1-2\epsilon}\xi_2^{1-2\epsilon}\,,\end{aligned}$$

...and the offending propagators

$$\begin{aligned}-(p_1-k_1)^2 &= s\beta^2\xi_1\eta_1\,,\\ -(p_1-k_2)^2 &= s\beta^2\xi_2\eta_2\,,\\ (k_1+k_2)^2 &= s\beta^4\xi_1\xi_2\eta_3\,,\\ -(p_1-k_1-k_2)^2 &= s\beta^2(\xi_1\eta_1+\xi_2\eta_2-\beta^2\xi_1\xi_2\eta_3)\end{aligned}$$

ignored soft singular propagators of the massive states (treated by the same procedure)

How to Avoid Line Singularity

- known problem of sector decomposition

- illusory complication, since need only introduce a parametrization such that all phase space covered, but when $\Theta_1 \to \Theta_2$ then at the same time $\varphi \to 0$

- use variable inspired by considerations from
  Anastasiou, Melnikov, Petriello ’05

$$\zeta \equiv \frac{1}{2}\frac{(1-\cos(\theta_1-\theta_2))(1+\cos\phi)}{1-\cos(\theta_1-\theta_2)+(1-\cos\phi)\sin\theta_1\sin\theta_2} \in [0,1]$$

$$\eta_3 = \frac{1}{2}\frac{(\cos\theta_1-\cos\theta_2)^2}{1-\cos(\theta_1-\theta_2)+2\zeta\sin\theta_1\sin\theta_2}$$

$$= \frac{(\eta_1-\eta_2)^2}{\eta_1+\eta_2-2\eta_1\eta_2-2(1-2\zeta)\sqrt{\eta_1(1-\eta_1)\eta_2(1-\eta_2)}}$$

Final Form of Phase Space

$$\int d\Phi_3 = \frac{\pi^{2\epsilon}}{8(2\pi)^5\Gamma(1-2\epsilon)} s^{2-2\epsilon}\beta^{8-8\epsilon}\int_0^1 d\zeta\,(\zeta(1-\zeta))^{-\frac{1}{2}-\epsilon}$$

$$\times \iint_0^1 d\eta_1 d\eta_2\,(\eta_1(1-\eta_1))^{-\epsilon}(\eta_2(1-\eta_2))^{-\epsilon}\frac{\eta_3^{1-2\epsilon}}{|\eta_1-\eta_2|^{1-2\epsilon}}$$

$$\iint d\xi_1 d\xi_2\,\xi_1^{1-2\epsilon}\xi_2^{1-2\epsilon}\,.$$

replaced by

cancels collinear singularity
as in propagator

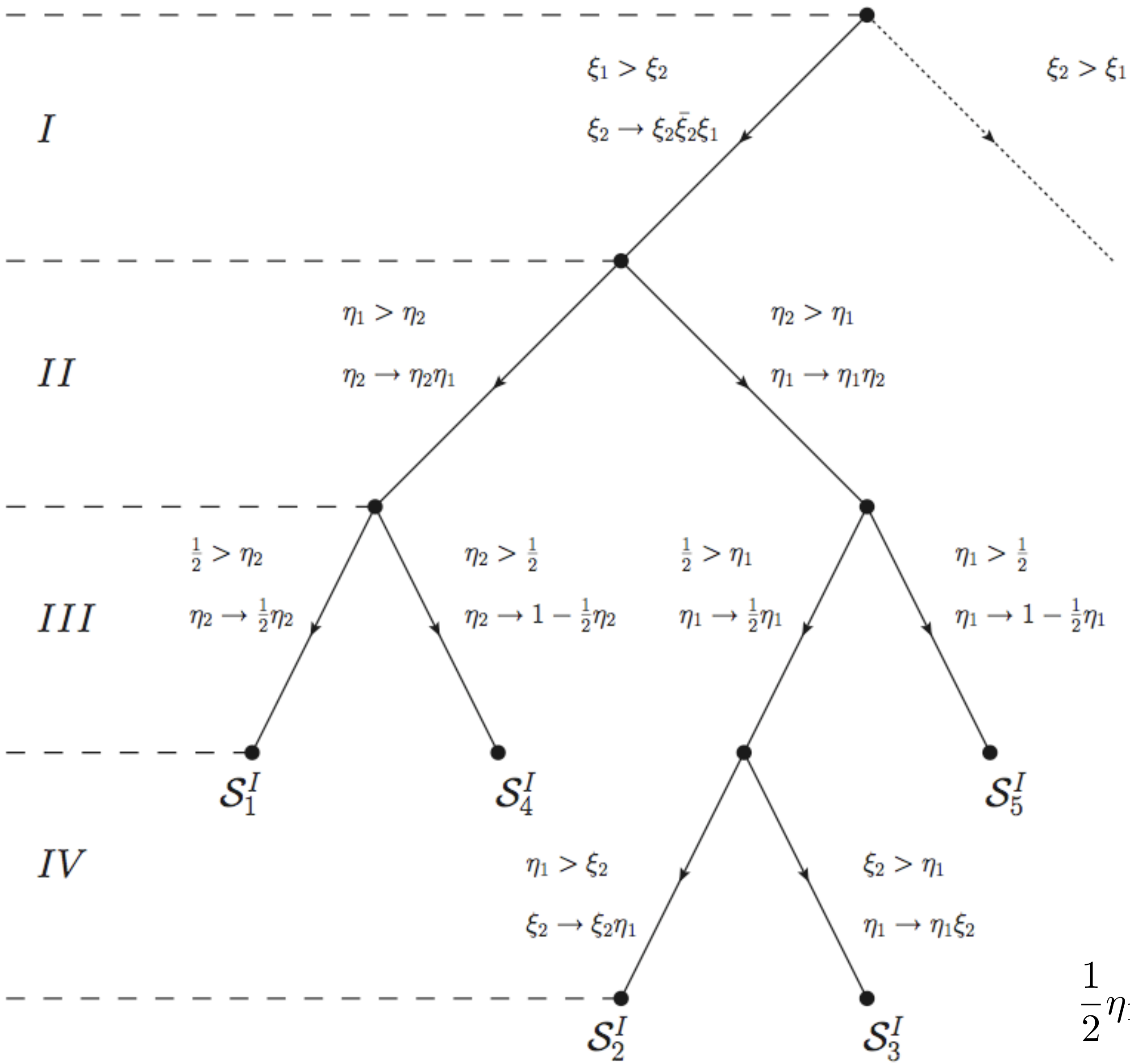
Energy integration range restricted as

$$\xi_1 > \xi_2 \;:\quad \xi_1 \in [0,1]\,,\quad \xi_2 \in \left[0, \min\left(\xi_1, \frac{1-\xi_1}{1-\beta^2\eta_3\xi_1}\right)\right]$$

$$\xi_2 > \xi_1 \;:\quad \xi_2 \in [0,1]\,,\quad \xi_1 \in \left[0, \min\left(\xi_2, \frac{1-\xi_2}{1-\beta^2\eta_3\xi_2}\right)\right]$$

natural symmetric decomposition

Sector Decomposition



Example:

$$\xi_1\eta_1 + \xi_2\eta_2 - \beta^2\xi_1\xi_2\eta_2$$

$$\frac{1}{2}\eta_1\xi_1\left(2 + \eta_2\xi_2\bar{\xi}_2 - 2\beta^2\xi_1\xi_2\eta_{31}\bar{\xi}_2\right)$$

Subtraction Terms

- subtraction terms generated iteratively with

$$\int_0^1 dx \frac{1}{x^{1-\alpha\epsilon}} f(x,\epsilon) = \left[\frac{1}{x^{1-\alpha\epsilon}} \left(f(x,\epsilon) - f(0,\epsilon) \right) \right]_{\epsilon=0} + \int_0^1 dx \frac{1}{x^{1-\alpha\epsilon}} f(0,\epsilon)$$

$$= \left[\frac{1}{x} \left(f(x,0) - f(0,0) \right) \right] + \frac{1}{\alpha\epsilon} f(0,\epsilon)$$

- each subtraction term corresponds to a physical limit with known asymptotic behaviour

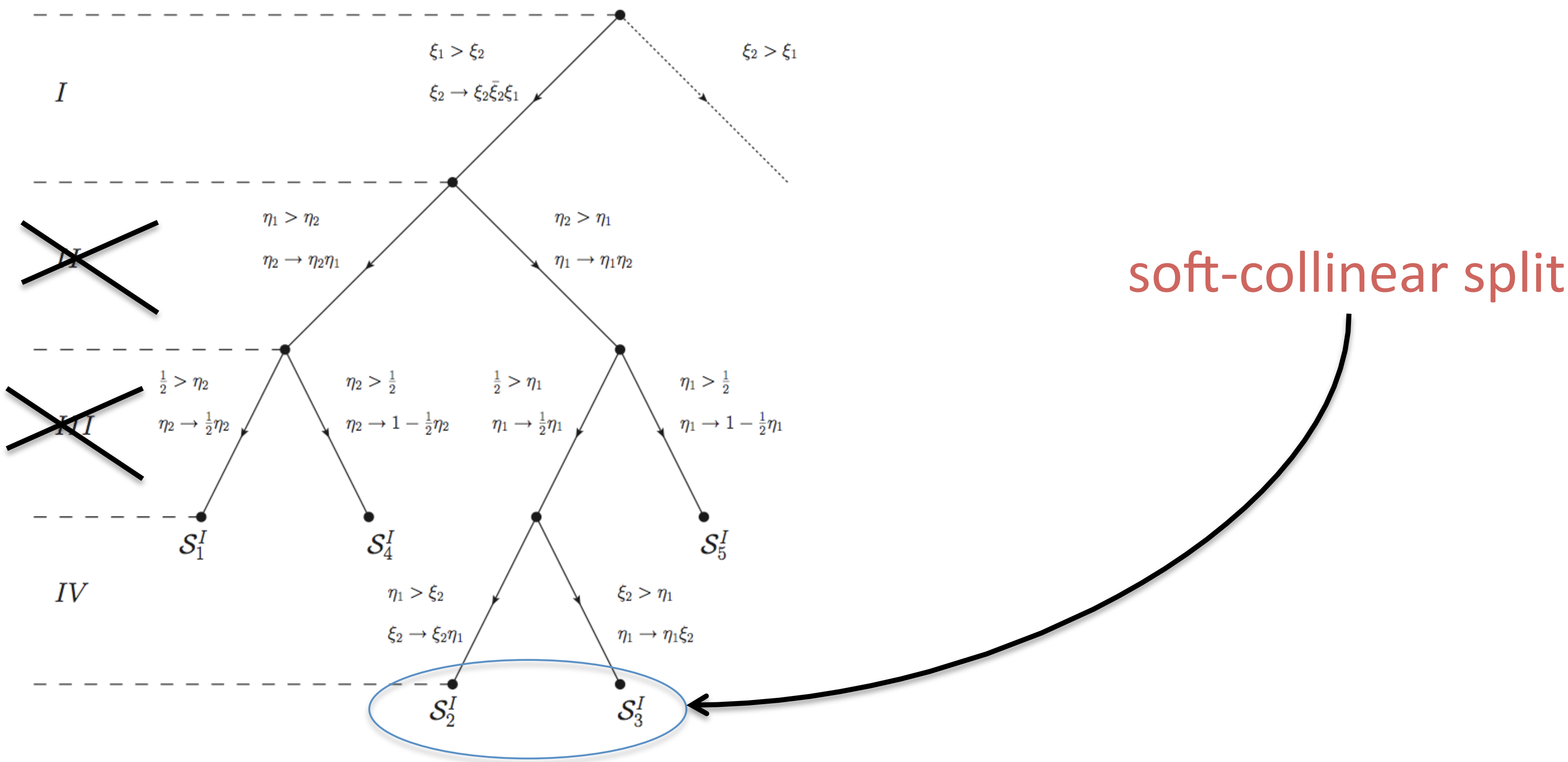
  Behrends, Giele '89 (double-soft)
  Campbell, Glover '98 (double-collinear without azimuthal correlations)
  Catani, Grazzini '99 (double-collinear with azimuthal correlations)

- no need to decompose the amplitude, just the splitting functions and eikonal terms !!!

- <u>guaranteed locality of subtraction terms and process independence</u>

Factorizable Double-Collinear Limits

- parametrization obtainable by substitution $\eta_{1,2}$ -> $1-\eta_{1,2}$
- no need for variable change for η_3, since no divergence
- only two sectors per branch in decomposition tree



Conclusions

- Presented general subtraction scheme for massive particle production

- Can be applied to get the last missing difficult contribution to top quark pair production

- Can be generalized to massless final states