Gluon scattering at NNLO

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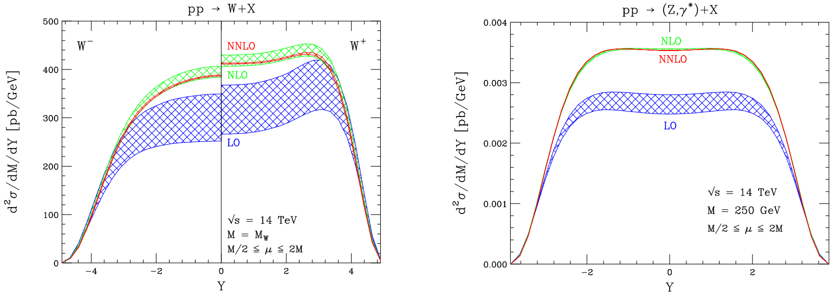
Based on:
Nigel Glover, J.P. [arXiv:1003.2824]

Talk structure

- Motivation - why go beyond NLO?
- Antenna subtraction at NNLO
- Antennae numerical implementation - unresolved emission from:
  - final-final emitters
  - initial-final emitters
  - initial-initial emitters
- Double real radiation counterterm
- Results
- Conclusions

Precise predictions at NNLO



Gauge boson production at the LHC
[C.Anastasiou, L.Dixon, K.Melnikov, F.Petriello '03]

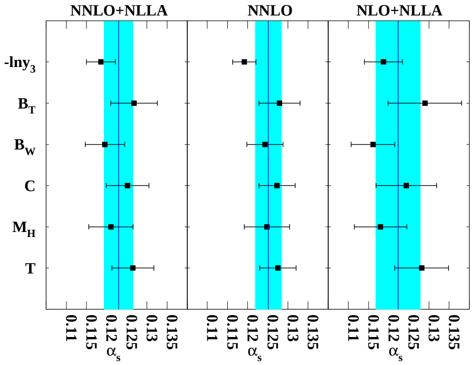
- complete stability against scale variations at NNLO
- convergence of the perturbative expansion

$e^+e^- \to 3$ jets and event shapes

Application of NNLO antenna subtraction

- parton-level event generator: EERAD3
  [A.Gehrmann-De Ridder, T.Gehrmann, E.W.N. Glover, G.Heinrich '07]
- computes jet cross sections and event shapes through to α_s^3
- independent implementation of the method by [S. Weinzierl '08]
- fixed order NNLO calculation for event shapes matched to NLLA
  [T.Gehrmann, G.Luisioni, H.Stenzel '08]
- new extractions of α_s based on NNLO or NNLO+NLLA
  [G.Dissertori, A.Gehrmann-De Ridder, T.Gehrmann, E.W.N. Glover, G.Heinrich, G.Luisioni, H.Stenzel '09]

$e^+e^- \to 3$ jets and event shapes



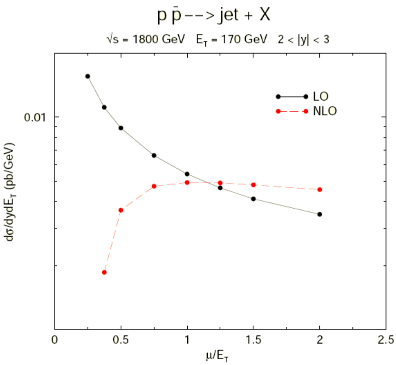
α_s measurments and total uncertainty

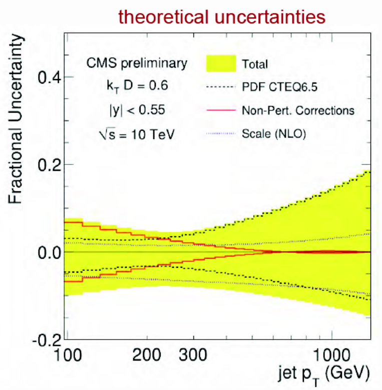
[G.Dissertori, A.Gehrmann-De Ridder, T.Gehrmann, E.W.N. Glover, G.Heinrich, G.Luisioni, H.Stenzel '08]

$$\alpha_s(M_Z) = 0.1224 \pm 0.0009(\text{stat}) \pm 0.0009(\text{exp}) \pm 0.0012(\text{stat}) \pm 0.0035(\text{theo})$$

One important observable - high-E_T jets

- high-E_T jet data helps to constrain α_s and the gluon pdf at large values of x





- each figure shows the theoretical uncertainties related to jet production [D. Stump et al. '03],[CMS Physics Analysis Summary '09]

One important observable - high-E_T jets

- if the cross section depends on the choice of scale, then as the scale is varied the pdf will have to change in order to be able to fit data
- doing a fit with a larger renormalization scale causes the high-x gluon to be larger since the high-E_T partonic cross section has decreased
- the scale dependence results in a shift of the pdf and, hence, makes a contribution to the pdf uncertainty
- better determination of the gluon pdf improves the theoretical predictions of any hadronic scattering process

$$\sigma = \sum_{ij} \int_0^1 dx_1 dx_2 f_i^{(h1)}(x_1, \mu) f_j^{(h2)}(x_2, \mu) \sigma_{ij \to pq}(x_1, x_2, \mu)$$

- dominant hard scattering process at LHC
- rich in potential signals of new physics

$pp \to j + X$ at NNLO

NNLO calculation for $pp \to j + X$ reaction contains:

$$
\begin{aligned}
\hat{\sigma}_{NNLO} \sim \quad & \int \left[\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle \right]_{n+2} \mathrm{d}\Phi_{n+2} \\
+ \quad & \int \left[\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle + \langle \mathcal{M}^{(1)} | \mathcal{M}^{(0)} \rangle \right]_{n+1} \mathrm{d}\Phi_{n+1} \\
+ \quad & \int \left[\langle \mathcal{M}^{(1)} | \mathcal{M}^{(1)} \rangle + \langle \mathcal{M}^{(0)} | \mathcal{M}^{(2)} \rangle + \langle \mathcal{M}^{(2)} | \mathcal{M}^{(0)} \rangle \right]_{n} \mathrm{d}\Phi_{n}
\end{aligned}
$$

- tree level $2 \to 4$ matrix elements [F.A. Berends, W.T. Giele '87], [M.Mangano, S.J.Parke, Z.Xu '87]
- 1-loop $2 \to 3$ matrix elements [Z.Bern, L.Dixon, D.A. Kosower '93]
- 2-loop $2 \to 2$ matrix elements [C. Anastasiou, E.W.N. Glover, C.Oleari, M.E. Tejeda-Yeomans '01], [Z.Bern, A.De Freitas, L.Dixon '02]

$pp \to j + X$ at NNLO

- three-loop splitting functions required for the evolution of parton distribution functions at NNLO

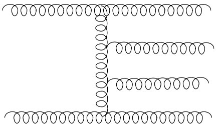
$$\frac{d}{dln\mu^2} f_i(x,\mu^2) = \sum_k \left[P_{ik}(\alpha_S(\mu^2)) \otimes f_k(\mu^2) \right](x)$$

$$P = \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \dots$$

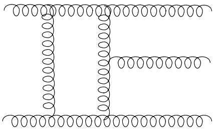
- DGLAP splitting kernels have been calculated to $\mathcal{O}(\alpha_s)^3$ and are needed for a consistent phenomenological treatment
  [S.Moch, J.A.M.Vermaseren, A.Vogt '04]

- NNLO parton distribution functions e.g.
  [A.D.Martin, R.Roberts, W.J.Stirling, R.S.Thorne, G.Watt]
  [S.Alekhin, J.Blümlein, S.Klein, S.Moch]

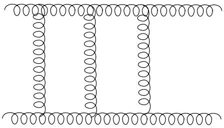
gluon-gluon channel



$A_6^{(0)}(gg \to gggg)$



$A_5^{(1)}(gg \to ggg)$



$A_4^{(2)}(gg \to gg)$

- explicit infrared poles from loop integrations
  - pole structure agrees with prediction of [S. Catani '98]
- implicit poles in phase space regions for single and double unresolved gluon emission
- procedure to extract the infrared singularities and assemble all the parts
  - sector decomposition [C.Anastasiou, K.Melnikov, F.Petriello '03],[T. Binoth, G.Heinrich '02]
  - NNLO subtraction [V. Del Duca, G.Somogyi, Z.Trocsanyi] $\to$ see Somogyi's talk, [S.Catani, M.Grazzini '07]
  - NNLO antenna subtraction [A.Gehrmann-De Ridder, T.Gehrmann, E.W.N. Glover, G. Heinrich '05]

NNLO subtraction

Structure of NNLO antenna subtraction [A.Gehrmann-De Ridder, T.Gehrmann, E.W.N. Glover, G. Heinrich '05]:

$$
\begin{aligned}
d\sigma_{NNLO} &= \int_{d\Phi_{m+2}} \left(d\sigma^{R}_{NNLO} - d\sigma^{S}_{NNLO} \right) \\
&+ \int_{d\Phi_{m+1}} \left(d\sigma^{V,1}_{NNLO} - d\sigma^{VS,1}_{NNLO} \right) \\
&+ \int_{d\Phi_{m}} d\sigma^{V,2}_{NNLO} + \int_{d\Phi_{m+2}} d\sigma^{S}_{NNLO} + \int_{d\Phi_{m+1}} d\sigma^{VS,1}_{NNLO}
\end{aligned}
$$

- $d\sigma^{S}_{NNLO}$: real radiation subtraction term for $d\sigma^{R}_{NNLO}$
- $d\sigma^{VS,1}_{NNLO}$: one-loop virtual subtraction term for $d\sigma^{V,1}_{NNLO}$
- $d\sigma^{V,2}_{NNLO}$: two-loop virtual corrections
- subtraction terms constructed using the antenna subtraction method at NNLO
- each line above is finite numerically and free of infrared ϵ-poles

Antenna functions and types

- colour-ordered pair of hard partons (radiators) with radiation in between
  - hard quark-antiquark pair
  - hard quark-gluon pair
  - hard gluon-gluon pair

- three-parton antenna $\rightarrow$ one unresolved parton

- four-parton antenna $\rightarrow$ two unresolved partons

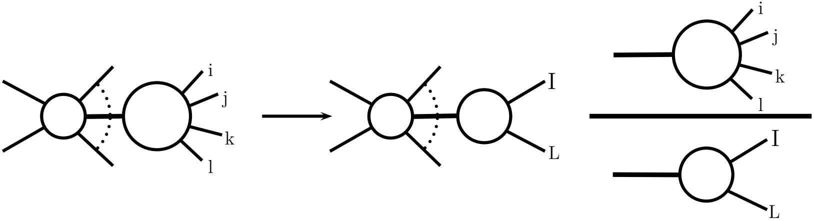
- can be at tree level or at one loop

- all have three antenna types
  - final-final antenna
  - initial-final antenna
  - initial-initial antenna

- all three-parton and four-parton antenna functions can be derived fom physical matrix elements, normalised to two-parton matrix elements

NNLO final-final antennae

- factorisation of both the squared matrix elements and the $(m+2)$- particle phase space → colour connected unresolved particles



- momentum mapping: [D.A. Kosower '02]

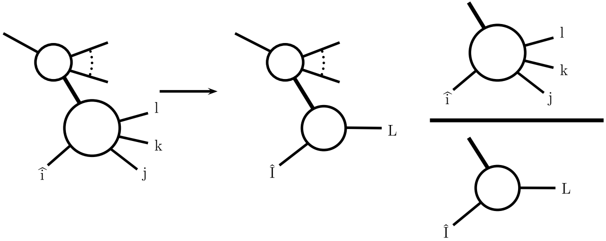
$$
\begin{aligned}
p_I^\mu &= x p_i^\mu + r_1 p_j^\mu + r_2 p_k^\mu + z p_l^\mu \\
p_L^\mu &= (1-x) p_i^\mu + (1-r_1) p_j^\mu + (1-r_2) p_k^\mu + (1-z) p_l^\mu
\end{aligned}
$$

- phase-space factorisation:

$$
\begin{aligned}
\mathrm{d}\Phi_{m+2}(p_a,\ldots,p_i,p_j,p_k,p_l,\ldots,p_{m+2}) &= \mathrm{d}\Phi_m(p_a,\ldots,p_I,p_L,\ldots,p_{m+2}) \\
&\quad \mathrm{d}\Phi_{X_{ijkl}}(p_i,p_j,p_k,p_l)
\end{aligned}
$$

NNLO initial-final antennae

- antenna factorisation for the initial-final situation → colour connected unresolved particles



- momentum-mapping: [A. Daleo, T. Gehrmann, D. Maître '06]

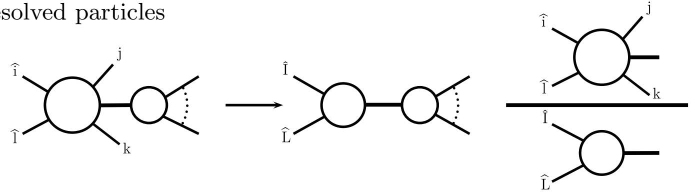
$$
\begin{aligned}
p^\mu_{\hat{I}} &= x\, p^\mu_i, \\
p^\mu_L &= p^\mu_j + p^\mu_k + p^\mu_l - (1-x)\, p^\mu_i
\end{aligned}
$$

- phase-space factorisation:

$$
\begin{aligned}
d\Phi_{m+2}(p_a,\ldots,p_j,p_k,p_l,\ldots,p_{m+2};p_i,r) &= d\Phi_m(p_a,\ldots,p_L,\ldots,p_{m+2};p_I,r) \\
&\quad \frac{Q^2}{2\pi} d\Phi_3(p_j,p_k,p_l;p_i,q)\,\frac{dx}{x}
\end{aligned}
$$

NNLO initial-initial antennae

- antenna factorisation for the initial-initial situation → colour connected unresolved particles



- momentum-mapping: [A. Daleo, T. Gehrmann, D. Maître '06]

$$
\begin{aligned}
p^{\mu}_{\hat{I}} &= \hat{x}_i \, p^{\mu}_i & \hat{x}_i &= \left(\frac{s_{il}+s_{jl}+s_{kl}}{s_{il}+s_{ij}+s_{ik}} \frac{s_{ij}+s_{ik}+s_{il}+s_{jk}+s_{jl}+s_{kl}}{s_{il}} \right)^{1/2} \\
p^{\mu}_{\hat{L}} &= \hat{x}_l \, p^{\mu}_l & \hat{x}_l &= \left(\frac{s_{il}+s_{ij}+s_{ik}}{s_{il}+s_{jl}+s_{kl}} \frac{s_{ij}+s_{ik}+s_{il}+s_{jk}+s_{jl}+s_{kl}}{s_{il}} \right)^{1/2}
\end{aligned}
$$

- phase-space factorisation:

$$
\begin{aligned}
\mathrm{d}\Phi_{m+2}(p_a,\ldots,p_j,p_k,\ldots,p_{m+2}) \quad = \quad & \mathrm{d}\Phi_m(\tilde{p_a},\ldots,\tilde{p}_{m+2};x_i p_i, x_l p_l) \\
& \delta(x_i-\hat{x_i})\,\delta(x_l-\hat{x_l})\,[\mathrm{d}k_j]\,[\mathrm{d}k_k]\,\mathrm{d}x_i\,\mathrm{d}x_l
\end{aligned}
$$

NNLO double real correction

NNLO real radiation contribution:

- all six parton tree level processes contributing to two jet final states
- gluon scattering contribution at leading coulour:

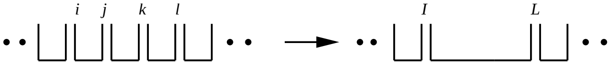
$$
\begin{aligned}
d\sigma^{R}_{NNLO} \quad = \quad & N^2\, N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 d\Phi_4(p_3,\ldots,p_6;p_1,p_2)\Bigg(\\
& \frac{2}{4!} \sum_{P(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,\hat{2}_g,i_g,j_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l) \\
& + \frac{2}{4!} \sum_{P(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,i_g,\hat{2}_g,j_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l) \\
& + \frac{2}{4!} \sum_{P_C(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,i_g,j_g,\hat{2}_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l)\Bigg)
\end{aligned}
$$

- three topologies according to position of the initial state gluons

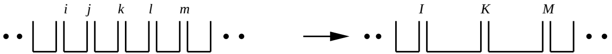
Counterterm

$$\mathrm{d}\sigma^{S}_{NNLO} = \mathrm{d}\sigma^{S,a}_{NNLO} + \mathrm{d}\sigma^{S,b}_{NNLO} + \mathrm{d}\sigma^{S,c}_{NNLO} + \mathrm{d}\sigma^{S,d}_{NNLO} + \mathrm{d}\sigma^{A}_{NNLO}$$

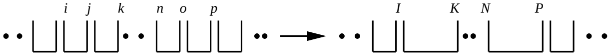
- (a) one unresolved parton → three parton antenna function X^0_{ijk}
- (b) two colour-connected unresolved partons → four parton antenna function X^0_{ijkl}



- (c) two almost colour-unconnected unresolved partons → strongly ordered product of non-independent three parton antenna functions



- (d) two colour-unconnected unresolved partons → product of independent three parton antenna functions



- (A) subtracts large angle soft radiation

Counterterm - IIFFFF topology

$$d\sigma^{R}_{NNLO} = N^2\ N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 d\Phi_4(p_3,\ldots,p_6;p_1,p_2)\ \frac{2}{4!} \sum_{P(3,4,5,6)} A_6^0(\hat{1}_g,\hat{2}_g,i_g,j_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l)$$

$$d\sigma^{S,b}_{NNLO} = N^2\ N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 d\Phi_4(p_3,\ldots,p_6;p_1,p_2)\ \frac{2}{4!} \Bigg($$

$$\Big(F_4^0(\hat{2}_g,i_g,j_g,k_g) - f_3^0(\hat{2}_g,i_g,j_g) F_3^0(\hat{\hat{2}}_g,\widetilde{(ij)}_g,k_g) - f_3^0(i_g,j_g,k_g) F_3^0(\hat{2}_g,\widetilde{(ij)}_g,\widetilde{(jk)}_g)$$

$$-f_3^0(j_g,k_g,\hat{2}_g) F_3^0(\hat{\hat{2}}_g,i_g,\widetilde{(kj)}_g)\Big) A_4^0(\hat{1}_g,\hat{\hat{2}}_g,\widetilde{(ijk)}_g,l_g) J_2^{(2)}(p_{\widetilde{ijk}},p_l))$$

$$+\Big(F_{4,a}^0(i_g,j_g,k_g,l_g) - f_3^0(i_g,j_g,k_g) f_3^0(\widetilde{(ij)}_g,\widetilde{(jk)}_g,l_g) - f_3^0(j_g,k_g,l_g) f_3^0(i_g,\widetilde{(jk)}_g,\widetilde{(kl)}_g)\Big)$$

$$A_4^0(\hat{1}_g,\hat{2}_g,\widetilde{(ijk)}_g,\widetilde{(lkj)}_g) J_2^{(2)}(p_{\widetilde{ijk}},p_{\widetilde{lkj}})$$

$$+\Big(F_{4,b}^0(i_g,j_g,k_g,l_g) - f_3^0(i_g,j_g,k_g) f_3^0(\widetilde{(ij)}_g,l,\widetilde{(jk)}_g)\Big) A_4^0(\hat{1}_g,\hat{2}_g,\widetilde{(ijl)}_g,\widetilde{(klj)}_g) J_2^{(2)}(p_{\widetilde{ijl}},p_{\widetilde{klj}})$$

$$+\Big(F_4^0(\hat{1}_g,l_g,k_g,j_g) - f_3^0(\hat{1}_g,l_g,k_g) F_3^0(\hat{\hat{1}}_g,\widetilde{(lk)}_g,j_g) - f_3^0(l_g,k_g,j_g) F_3^0(\hat{1}_g,\widetilde{(lk)}_g,\widetilde{(kj)}_g)$$

$$-f_3^0(k_g,j_g,\hat{1}_g) F_3^0(\hat{\hat{1}}_g,l_g,\widetilde{(jk)}_g)\Big) A_4^0(\hat{\hat{1}}_g,\hat{2}_g,i_g,\widetilde{(lkj)}_g) J_2^{(2)}(p_i,p_{\widetilde{lkj}}) + \text{cyclic} + \text{l.reversal} + \ldots\Bigg)$$

Counterterm - IFIFFF topology

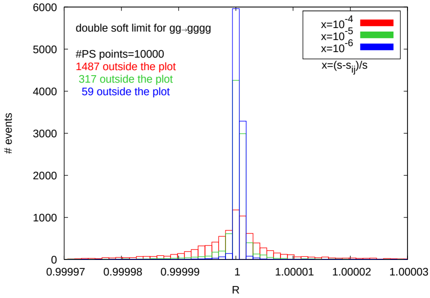
$$\mathrm{d}\sigma^{R}_{NNLO} = N^2\ N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 \mathrm{d}\Phi_4(p_3,\ldots,p_6;p_1,p_2)\,\frac{2}{4!}\sum_{P(3,4,5,6)} A_6^0(\hat{1}_g,i_g,\hat{2}_g,j_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l)$$

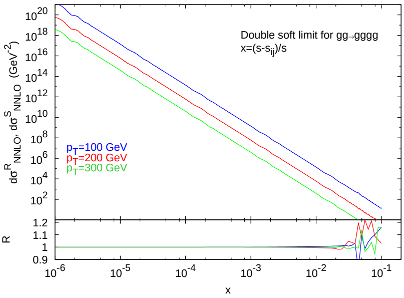
$$\begin{aligned}
\mathrm{d}\sigma^{S,b}_{NNLO} = N^2\ N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 \mathrm{d}\Phi_4(p_3,\ldots,p_6;p_1,p_2)\,\frac{2}{4!}\Bigg(\\
\Big(F_4^0(\hat{2}_g,j_g,k_g,l_g) - f_3^0(\hat{2}_g,j_g,k_g)\,F_3^0(\hat{\hat{2}}_g,\widetilde{(jk)}_g,l_g) - f_3^0(j_g,k_g,l_g)\,F_3^0(\hat{2}_g,\widetilde{(jk)}_g,\widetilde{(kl)}_g)\\
-f_3^0(k_g,l_g,\hat{2}_g)\,F_3^0(\hat{\hat{2}}_g,j_g,\widetilde{(kl)}_g)\Big) A_4^0(\hat{1}_g,i_g,\hat{\hat{2}}_g,\widetilde{(jkl)}_g)\,J_2^{(2)}(p_i,\widetilde{p_{jkl}})\\
+\Big(F_4^0(\hat{1}_g,l_g,k_g,j_g) - f_3^0(\hat{1}_g,l_g,k_g)\,F_3^0(\hat{\hat{1}}_g,\widetilde{(kl)}_g,j_g) - f_3^0(l_g,k_g,j_g)\,F_3^0(\hat{1}_g,\widetilde{(lk)}_g,\widetilde{(kj)}_g)\\
-f_3^0(k_g,j_g,\hat{1}_g)\,F_3^0(\hat{\hat{1}}_g,l_g,\widetilde{(jk)}_g)\Big) A_4^0(\hat{\hat{1}}_g,i_g,\hat{2}_g,\widetilde{(lkj)}_g)\,J_2^{(2)}(p_i,\widetilde{p_{lkj}})\\
+\Big(F_4^0(\hat{1}_g,i_g,\hat{2}_g,j_g) - F_3^0(\hat{1}_g,3_g,\hat{2}_g)\,F_3^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{j}_g)\\
-F_3^0(\hat{2}_g,j_g,\hat{1}_g)\,F_3^0(\hat{\hat{1}}_g,\tilde{i}_g,\hat{\hat{2}}_g)\Big) A_4^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{k}_g,\tilde{l}_g)\,J_2^{(2)}(\widetilde{p_k},\widetilde{p_l})\\
+\Big(F_4^0(\hat{1}_g,i_g,\hat{2}_g,l_g) - F_3^0(\hat{1}_g,3_g,\hat{2}_g)\,F_3^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{l}_g)\\
-F_3^0(\hat{2}_g,l_g,\hat{1}_g)\,F_3^0(\hat{\hat{1}}_g,\tilde{i}_g,\hat{\hat{2}}_g)\Big) A_4^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{k}_g,\tilde{j}_g)\,J_2^{(2)}(\widetilde{p_k},\widetilde{p_j}) + \ldots\Bigg)
\end{aligned}$$

Counterterm - IFFIFF topology

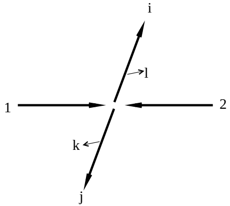
$$d\sigma^{R}_{NNLO} = N^2 \; N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 d\Phi_4(p_3,\ldots,p_6;p_1,p_2)\,\frac{2}{4!} \sum_{P(3,4,5,6)} A_6^0(\hat{1}_g,i_g,j_g,\hat{2}_g,k_g,l_g) J_2^{(4)}(p_i,\ldots,p_l)$$

$$\begin{aligned}
d\sigma^{S,b}_{NNLO} = N^2 \; N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 d\Phi_4(p_3,\ldots,p_6;p_1,p_2)\,\frac{2}{4!} \Bigg(\\
\Big(F_4^0(\hat{1}_g,i_g,j_g,\hat{2}_g) - f_3^0(\hat{1}_g,i_g,j_g)\, F_3^0(\hat{\hat{1}}_g,\widetilde{(ij)}_g,\hat{2}_g) - f_3^0(i_g,j_g,\hat{2}_g)\, F_3^0(\hat{1}_g,\widetilde{(ji)}_g,\hat{\hat{2}}_g)\Big) \\
A_4^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{k}_g,\tilde{l}_g)\, J_2^{(2)}(\widetilde{p_k},\widetilde{p_l}) \\
+\Big(F_4^0(\hat{2}_g,k_g,l_g,\hat{1}_g) - f_3^0(\hat{2}_g,k_g,l_g)\, F_3^0(\hat{\hat{2}}_g,\widetilde{(kl)}_g,\hat{1}_g) - f_3^0(k_g,l_g,\hat{2}_g)\, F_3^0(\hat{1}_g,\widetilde{(lk)}_g,\hat{\hat{2}}_g)\Big) \\
A_4^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{j}_g,\tilde{i}_g)\, J_2^{(2)}(\widetilde{p_j},\widetilde{p_i}) \\
+\Big(F_4^0(\hat{1}_g,j_g,\hat{2}_g,k_g) - F_3^0(\hat{1}_g,j_g,\hat{2}_g)\, F_3^0(\hat{\hat{1}}_g,\hat{\hat{2}}_g,\tilde{k}_g) - F_3^0(\hat{2}_g,k_g,\hat{1}_g)\, F_3^0(\hat{\hat{1}}_g,\tilde{j}_g,\hat{\hat{2}}_g)\Big) \\
A_4^0(\hat{\hat{1}}_g,\tilde{i}_g,\hat{\hat{2}}_g,\tilde{l}_g)\, J_2^{(2)}(\widetilde{p_i},\widetilde{p_l}) \\
+\ldots \Bigg)
\end{aligned}$$

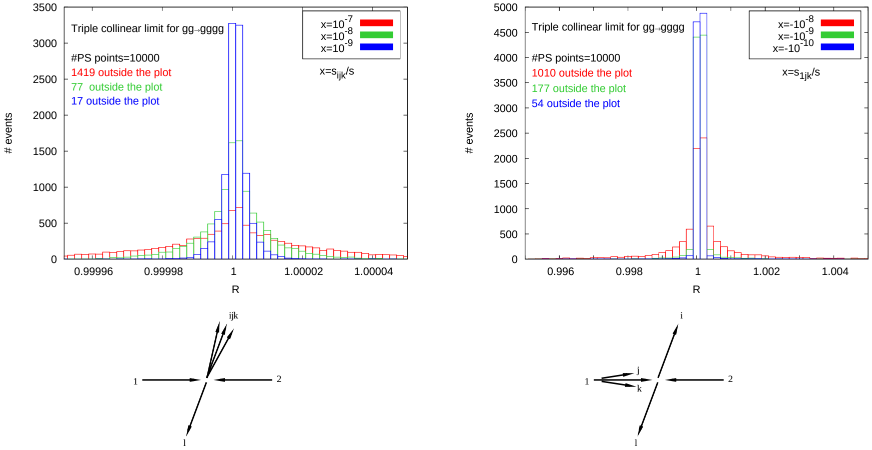




- double k, l soft limit when $s_{ij} \approx s$
- infrared behaviour of subtraction term coincides with the matrix element
- $R = \frac{\mathrm{d}\sigma^R_{NNLO}}{\mathrm{d}\sigma^S_{NNLO}} \xrightarrow{l_g, k_g \to 0} 1$

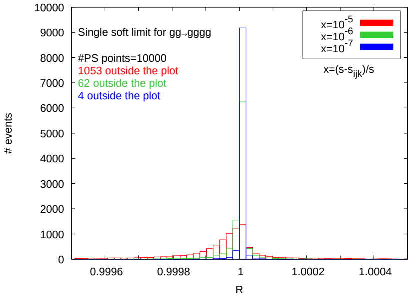


Triple collinear limit



- generate phase space points with small triple invariant s_{ijk} or s_{1jk} mass
- soft and collinear limit ✓
- double collinear limit ✓

Singly unresolved limits

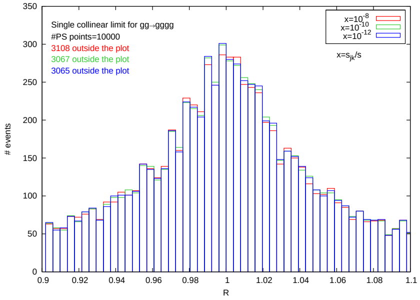


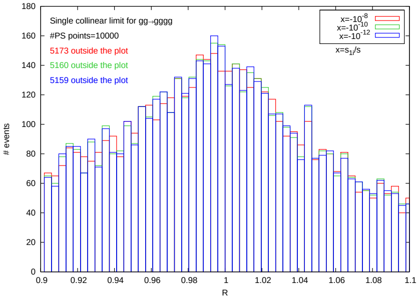
Single soft limit:

- generate phase space points with $s_{ijk} \approx s$
- iterative structure of $d\sigma^{b,c,d}_{NNLO}$ introduces remnant soft gluon behaviour associated with the phase space mappings
- $d\sigma^{A}_{NNLO}$ piece subtracts large angle soft gluon radiation

- $d\sigma^{A}_{NNLO}$ needed to cancel explicit poles in five-parton configuration [S. Weinzierl]

Singly unresolved limits





Single collinear limits:

- generate phase space points with s_{jk} or s_{1i} small

Distributions with a broader shape due to:

- angular correlations in matrix elements and antenna functions when an initial/final state gluon splits into two gluons not accounted for by the subtraction term → non-locality of subtraction term

Angular terms subtraction - Solution I

- reconstruct the angular structure of F_4^0 by contracting tensorial splitting function and tensorial three parton antenna function:

$$
\begin{aligned}
F_4^0(1_g, i_g, j, 2_g) \quad &\xrightarrow{i_g \| j_g} \quad \frac{1}{s_{ij}} P^{\mu\nu}_{gg\to G}(z) (F_3^0)_{\mu\nu}(1_g, (ij)_g, 2_g) \\
&= \quad \frac{1}{s_{ij}} P_{gg\to G}(z) F_3^0(1_g, (ij)_g, 2_g) + \text{ang.}
\end{aligned}
$$

- subtract the following local counterterm:

$$
\begin{aligned}
\Theta_{F_3^0}(i, j, z, k_\perp) \quad &= \quad \left[\frac{1}{s_{ij}} P^{\mu\nu}_{ij\to(ij)}(z, k_\perp)(F_3^0)_{\mu\nu} - \frac{1}{s_{ij}} P_{ij\to(ij)}(z) F_3^0(1_g, (ij)_g, 2_g) \right] \\
&= \quad \frac{4}{s_{ij}^2 s_{1p2}^2} \left(\frac{s_{12}^2 s_{1p2}^2 + s_{1p}^2 s_{p2}^2}{s_{12}^2 s_{1p}^2 s_{p2}^2} \right) \Big[s_{12} s_{1p} s_{p2} \; k_\perp \cdot k_\perp \\
&\quad -4 p_1 \cdot k_\perp p_2 \cdot k_\perp s_{1p} s_{p2} + 2(p_1 \cdot k_\perp)^2 s_{p2}^2 + 2(p_2 \cdot k_\perp)^2 s_{1p}^2 \Big]
\end{aligned}
$$

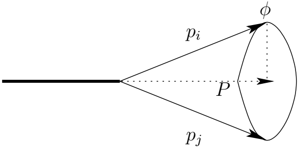
- integrates to zero over the unresolved phase space ✓
- local subtraction of angular terms ✓
- new singularities in PS regions where $s_{1p}, s_{2p}, s_{12} \to 0$ ✗

Angular terms subtraction - Solution II

- angular terms vanish after averaging over the azimuthal angle

$$\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\,(p_l\cdot k_\perp) = 0\,, \qquad \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\,(p_l\cdot k_\perp)^2 = -k_\perp^2\,\frac{p\cdot p_l\; n\cdot p_l}{p\cdot n}$$

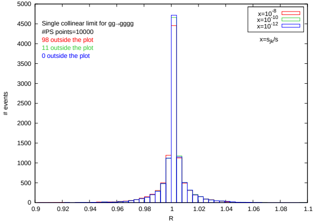
- combine phase space points related to each other by a rotation of the system of unresolved partons $\{p_i, p_j\} \to \{p_i', p_j'\}$



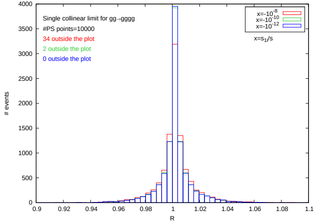
$$p_i^\mu = z p^\mu + k_\perp^\mu - \frac{k_\perp^2}{z}\frac{n^\mu}{2p\cdot n}\,, \qquad p_j^\mu = (1-z)p^\mu - k_\perp^\mu - \frac{k_\perp^2}{1-z}\frac{n^\mu}{2p\cdot n}\,,$$

$$\text{with } 2p_i\cdot p_j = -\frac{k_\perp^2}{z(1-z)}\,, \qquad p^2 = n^2 = k_\perp.p = k_\perp.n = 0$$

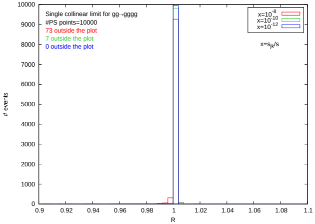
Angular terms subtraction



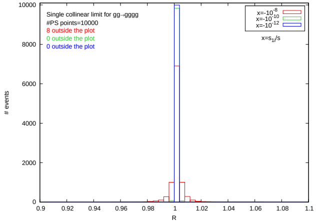
FF solution I



IF solution I

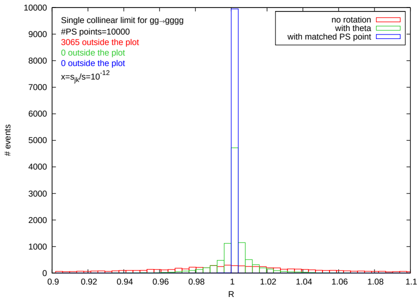


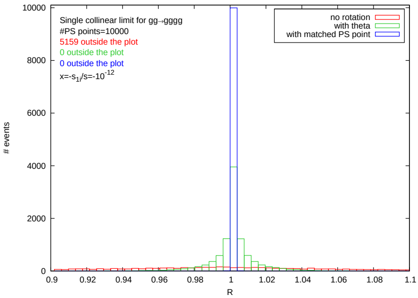
FF solution II

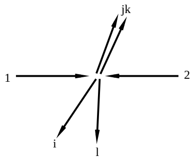


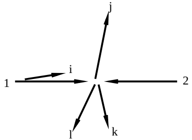
IF solution II

Angular terms subtraction









- in both collinear limits combining phase space points largely cancels angular dependent terms

Conclusions

$$
\begin{aligned}
d\sigma_{NNLO} &= \int_{d\Phi_{m+2}} \left(d\sigma^{R}_{NNLO} - d\sigma^{S}_{NNLO}\right) + \int_{d\Phi_{m+2}} d\sigma^{S}_{NNLO} \\
&+ \int_{d\Phi_{m+1}} \left(d\sigma^{V,1}_{NNLO} - d\sigma^{VS,1}_{NNLO}\right) + \int_{d\Phi_{m+1}} d\sigma^{VS,1}_{NNLO} \\
&+ \int_{d\Phi_{m}} d\sigma^{V,2}_{NNLO}
\end{aligned}
$$

- $d\sigma^{S}_{NNLO}$ for gluon channel [this talk]
- infrared structure of the double real (NNLO) emission written in terms antenna functions
- $\int_{d\Phi_{m+2}} \left(d\sigma^{R}_{NNLO} - d\sigma^{S}_{NNLO}\right)$ finite and integrable in four dimensions

Conclusions

- inclusive integrals needed for $\int_{d\Phi_{m+2}} d\sigma^S_{NNLO}$

	final-final	initial-final	initial-initial
F_3^0	✓[1]	✓[2]	✓[2]
F_4^0	✓[1]	✓[3]	(in progress)[4]
$F_3^0 \otimes F_3^0$	✓[1]	(in progress)	(in progress)

[1] A. Gehrmann-De Ridder, T. Gehrmann and E. W. N. Glover, *JHEP* **09** (2005) 056 [hep-ph/0505111];
[2] A. Daleo, T. Gehrmann and D. Maître, *JHEP* **04** (2007) 016 [hep-ph/0612257];
[3] A. Daleo, A. Gehrmann-De Ridder, T. Gehrmann and G. Luisoni, *JHEP* **01** (2010) 118 [0912.0374];
[4] R. Boughezal, A. Gehrmann-De Ridder and M. Ritzmann, 1001.2396;

Outlook

Future work:

- derive mixed real-virtual counterterm
- go beyond leading colour approximation
- include remaining channels
  - 4g2q processes
  - 2g4q processes
  - 6q processes
- construction of a numerical program to compute NNLO QCD estimates of di-jet production in hadron-hadron collisions