A new simple PDF parametrization: improved description of the HERA data

M. Bonvini, F. Giuli

xFitter External Workshop, Minsk (Belarus)
18/03/2019

Università di Roma Tor Vergata

xFitter

Theoretical motivations

- Previous fit based on default HERAPDF parameterisation:

$$x f(x, \mu_0^2) = A\, x^B (1-x)^C \left[1 + Dx + Ex^2\right] - \underbrace{A'\, x^{B'} (1-x)^{C'}}_{\text{Negative term (only for gluon)}}$$

- Is this parameterisation flexible enough to describe small-x behaviour?
- Limited structure at small-x → shape strongly dominated by asymptotic behaviour of x^B
- **More flexibility in the small-x regime is needed**
- It is also very important in the light of future higher-energy colliders:
  - Large Hadron-electron Collider (LHeC)
  - Future Circular electron-hadron or hadron-hadron Colliders (FCC-eh and FCC-hh)

New proposed parametrization

- New general parametrization:
  - Multiplicative option:

$$xf(x,\mu_0^2) = A\,x^B(1-x)^C\left[1+Dx+Ex^2\right]\left[1+F\log x+G\log^2 x+H\log^3 x\right]$$

  - **Additive option:**

$$\boxed{xf(x,\mu_0^2) = A\,x^B(1-x)^C\left[1+Dx+Ex^2+F\log x+G\log^2 x+H\log^3 x\right]}$$

- Difference between the two options in the medium-x region e.g. **x ~ 0.1**

$$xg(x,\mu_0^2) = A_g\,x^{B_g}(1-x)^{C_g}\left[1+F_g\log x+G_g\log^2 x\right]$$
$$xu_v(x,\mu_0^2) = A_{u_v}\,x^{B_{u_v}}(1-x)^{C_{u_v}}\left[1+E_{u_v}x^2+F_{u_v}\log x+G_{u_v}\log^2 x\right]$$
$$xd_v(x,\mu_0^2) = A_{d_v}\,x^{B_{d_v}}(1-x)^{C_{d_v}}$$
$$x\bar{u}(x,\mu_0^2) = A_{\bar{u}}\,x^{B_{\bar{u}}}(1-x)^{C_{\bar{u}}}\left[1+D_{\bar{u}}x+F_{\bar{u}}\log x\right]$$
$$x\bar{d}(x,\mu_0^2) = A_{\bar{d}}\,x^{B_{\bar{d}}}(1-x)^{C_{\bar{d}}}\left[1+D_{\bar{d}}x+F_{\bar{d}}\log x\right],$$

Minimal parametrization

$$xs(x,\mu_0^2) = x\bar{s}(x,\mu_0^2) = r_s\,x\bar{d}(x,\mu_0^2) \qquad r_s = \frac{f_s}{1-f_s} \quad \text{with } \underline{f_s = 0.4 \text{ fixed}}$$

New proposed parametrization

- Our new PDF parametrization:
  - Depends on 18 free parameters that must be fitted
  - This is to be compared with HERAPDF2.0 (14 free parameters)
  - Two extra parameters for u_V
  - Two extra parameters for $\bar{u}$ and $\bar{d}$
  - Major improvement comes from the gluon PDF (same number of free parameters)

$$xg(x,\mu_0^2) = A_g\, x^{B_g}(1-x)^{C_g}\left[1+F_g\log x+G_g\log^2 x\right]$$

$$xu_v(x,\mu_0^2) = A_{u_v}\, x^{B_{u_v}}(1-x)^{C_{u_v}}\left[1+E_{u_v}x^2+F_{u_v}\log x+G_{u_v}\log^2 x\right]$$

$$xd_v(x,\mu_0^2) = A_{d_v}\, x^{B_{d_v}}(1-x)^{C_{d_v}}$$

$$x\bar{u}(x,\mu_0^2) = A_{\bar{u}}\, x^{B_{\bar{u}}}(1-x)^{C_{\bar{u}}}\left[1+D_{\bar{u}}x+F_{\bar{u}}\log x\right]$$

$$x\bar{d}(x,\mu_0^2) = A_{\bar{d}}\, x^{B_{\bar{d}}}(1-x)^{C_{\bar{d}}}\left[1+D_{\bar{d}}x+F_{\bar{d}}\log x\right],$$

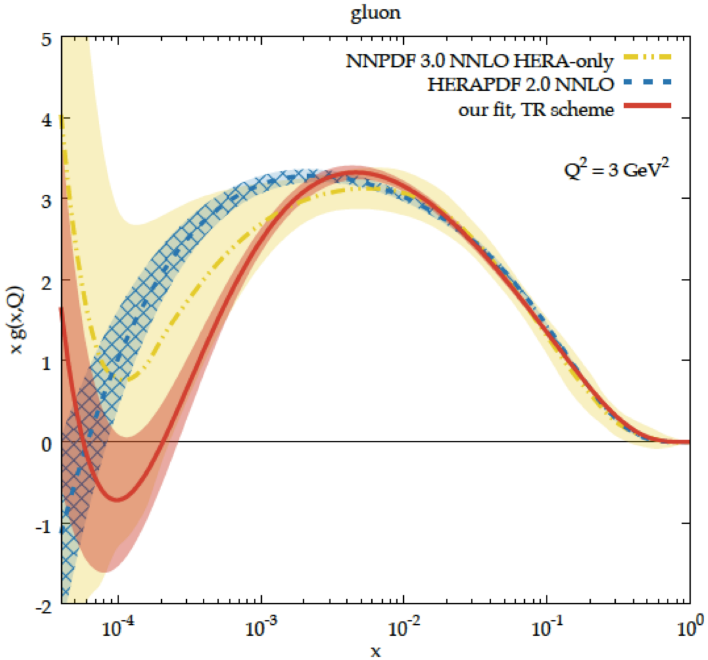
Minimal parametrization

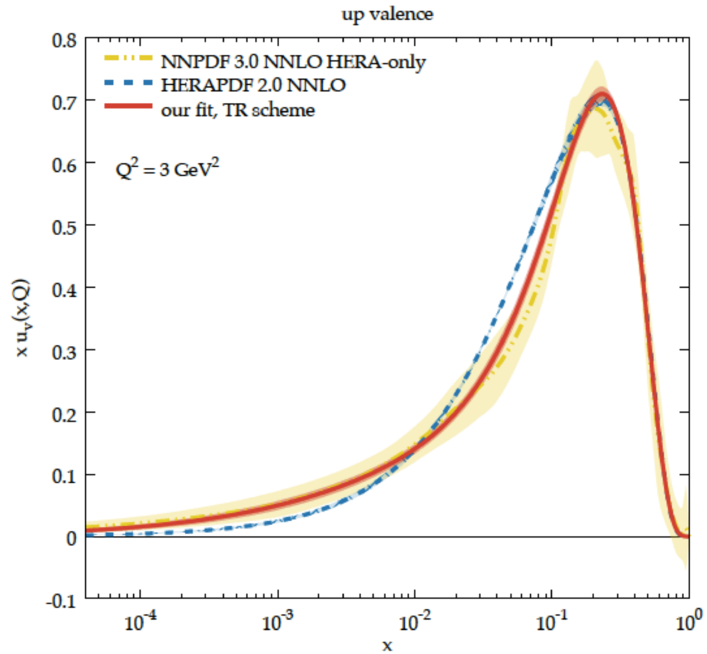
$$xs(x,\mu_0^2) = x\bar{s}(x,\mu_0^2) = r_s\, x\bar{d}(x,\mu_0^2) \qquad r_s = \frac{f_s}{1-f_s} \quad \text{with } \underline{f_s = 0.4 \text{ fixed}}$$

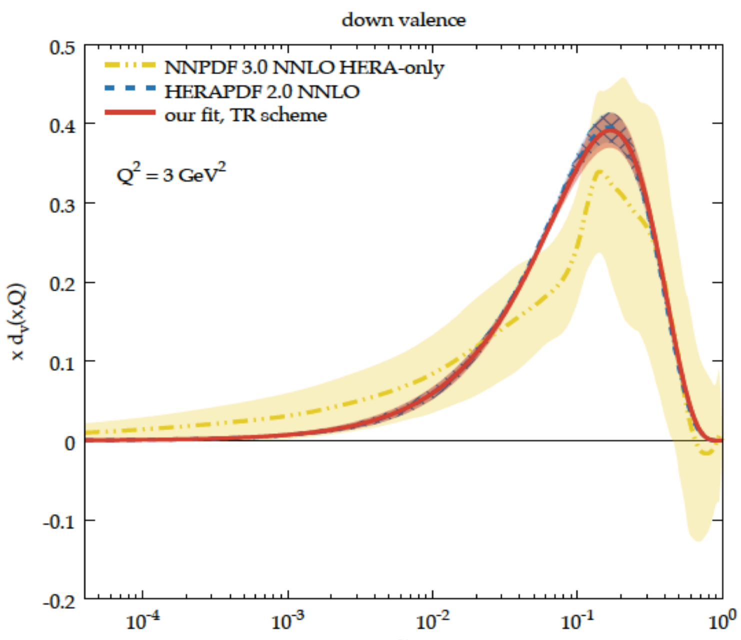
Comparison to HERAPDF2.0

Contribution to χ^2	HERAPDF2.0	Our fit (new parametrization)
subset NC e^+ 920 χ^2/n.d.p.	444/377	403/377
subset NC e^+ 820 χ^2/n.d.p.	66/70	74/70
subset NC e^+ 575 χ^2/n.d.p.	219/254	221/254
subset NC e^+ 460 χ^2/n.d.p.	217/204	222/204
subset NC e^- χ^2/n.d.p.	219/159	220/159
subset CC e^+ χ^2/n.d.p.	45/39	38/39
subset CC e^- χ^2/n.d.p.	56/42	50/42
correlation term + log term	91 + 5	75 – 3
Total χ^2/d.o.f.	**1363/1131**	**1301/1127**

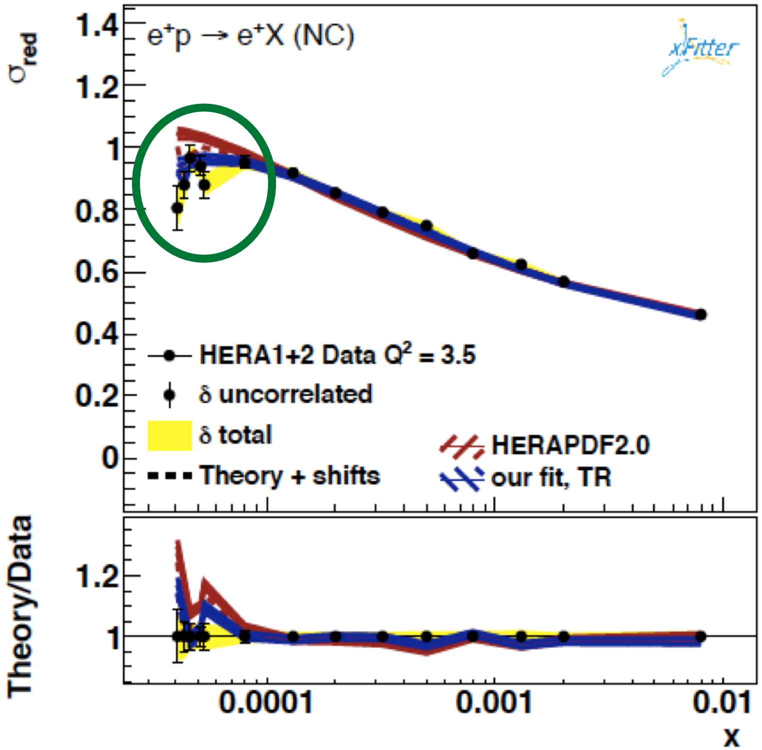
$$\chi^2 = \underbrace{\sum_i \frac{\left[D_i - T_i\left(1 - \sum_j \gamma_{ij} b_j\right)\right]^2}{\delta^2_{i,\mathrm{uncor}} T_i^2 + \delta^2_{i,\mathrm{stat}} D_i T_i}}_{\text{Exp. term}} + \underbrace{\sum_j b_j^2}_{\text{Corr. term}} + \underbrace{\sum_i \log \frac{\delta^2_{i,\mathrm{uncor}} T_i^2 + \delta^2_{i,\mathrm{stat}} D_i T_i}{\delta^2_{i,\mathrm{uncor}} D_i^2 + \delta^2_{i,\mathrm{stat}} D_i^2}}_{\text{Log term}}$$

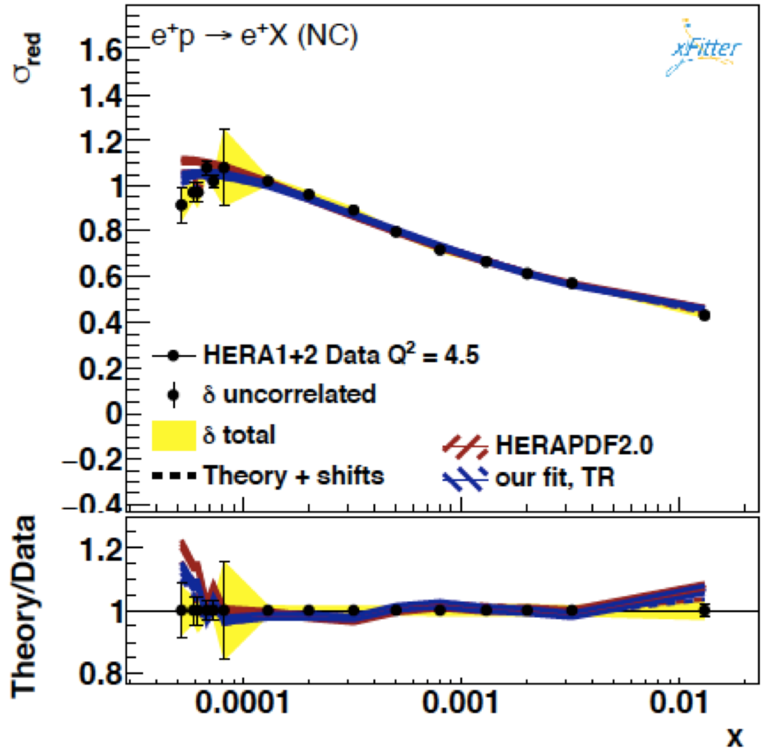






- Richer structure at medium-/high-x than HERAPDF2.0
- Gluon decreases more rapidly for x ~ 10^{-2} and starts rising again for x < 10^{-4}
- Up-valence rather different
- Down-valence is identical (same parametrization as in HERAPDF2.0)
- If compared to NNPDF3.0 (HERA data only), qualitatively same behavior





- **How could the fit quality improve so much?**
- In most of the cases the agreement is at the same level
- Exception for low-Q^2 and low-x data, where a **clear improvement of the theoretical description** is manifest
- This region is where the impact of log(1/x) terms is expected to be largest
- χ^2 improvement of the same size as the one found in our small-x resummation paper Eur. Phys. J. C78 (2018) 621 – **is resummation really needed?**

From TR to FONLL

- Various variations studied

- First of all, migration from TR scheme to FONLL (to include small-x resummation in a later stage) – as done in Eur. Phys. J. C78 (2018) 621

Differences in the fit setup		
heavy flavour scheme	TR	FONLL
initial scale μ_0	1.38 GeV	1.6 GeV
charm matching scale μ_c	m_c	$1.12 m_c$
charm mass m_c	1.43 GeV	1.46 GeV

- Raising the initial scale from the HERAPDF2.0 value (Q_0^2 = 1.9 GeV^2) to Q_0^2 = 2.56 GeV^2

- FONLL scheme prefers m_c = 1.46 GeV (while m_c^{HERA} = 1.43 $\pm$ 0.06 GeV)

- The charm PDF must be generated perturbatively at a matching scale $\mu_c > \mu_0 > m_c$ which needs to be larger than the default value $\mu_c = m_c$

- So μ_c = 1.12 m_c (adopted also in Eur. Phys. J. C78 (2018) 621)

From TR to FONLL

Contribution to χ^2	Old parametrization	New parametrization
subset NC e^+ 920 $\tilde{\chi}^2$/n.d.p.	451/377	406/377
subset NC e^+ 820 $\tilde{\chi}^2$/n.d.p.	68/70	74/70
subset NC e^+ 575 $\tilde{\chi}^2$/n.d.p.	220/254	222/254
subset NC e^+ 460 $\tilde{\chi}^2$/n.d.p.	218/204	225/204
subset NC e^- $\tilde{\chi}^2$/n.d.p.	215/159	217/159
subset CC e^+ $\tilde{\chi}^2$/n.d.p.	44/39	37/39
subset CC e^- $\tilde{\chi}^2$/n.d.p.	57/42	50/42
correlation term + log term	100 + 15	79 + 2
Total χ^2/d.o.f.	**1388/1131**	**1312/1127**

- Worse than the χ^2 presented in Slide 4 but:
  - Old parameterisation → deterioration of **25 units** wrt TR
  - New parameterisation → deterioration of **just 11 units** wrt TR
- **PDFs largely compatible** (backup)

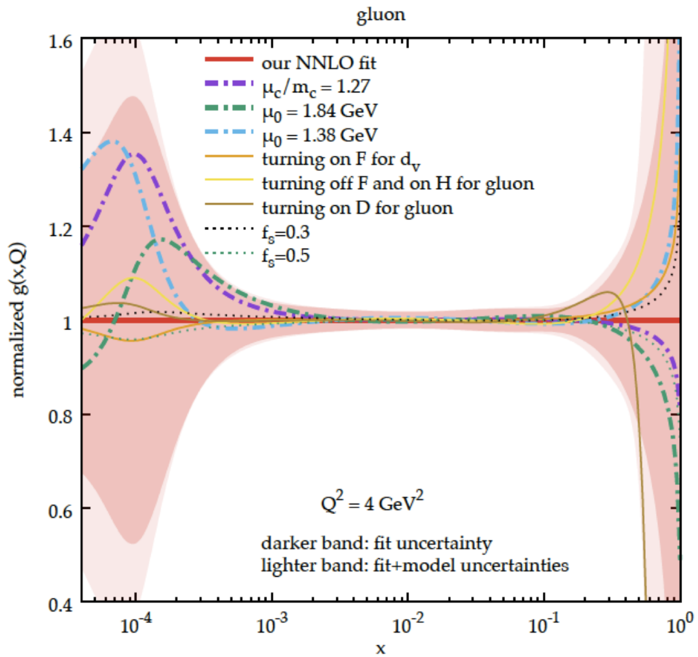
Stability of our fit

- First, we consider variations of the fit scale:
  - μ_0 = 1.38 GeV and μ_c/m_c = 1.12 (μ_c = 1.46 GeV) – **Down variation**
  - μ_0 = 1.60 GeV and μ_c/m_c = 1.27 (μ_c = 1.85 GeV) – Intermediate step
  - μ_0 = 1.84 GeV and μ_c/m_c = 1.27 (μ_c = 1.85 GeV) – **Up variation**

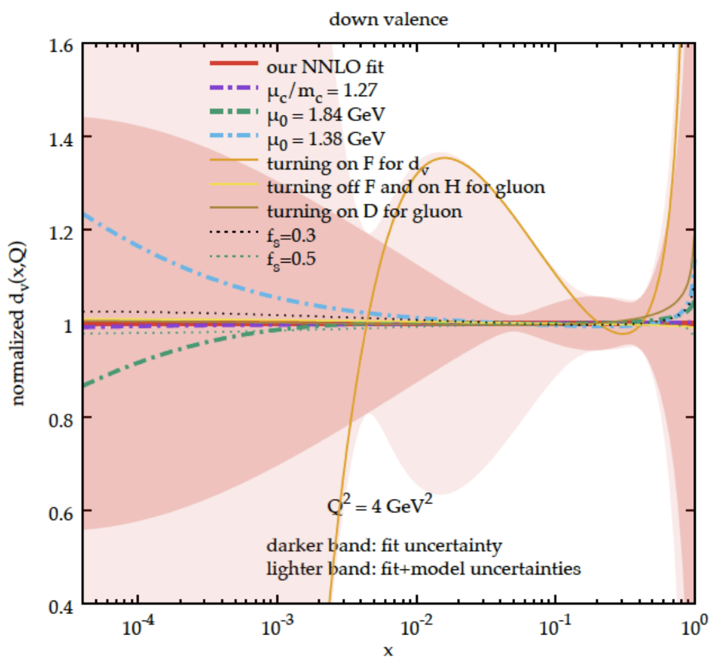
- Strange fraction variations:
  - $f_s = 0.5$ (up variation) and $f_s = 0.3$ (down variation) – same as HERAPDF2.0

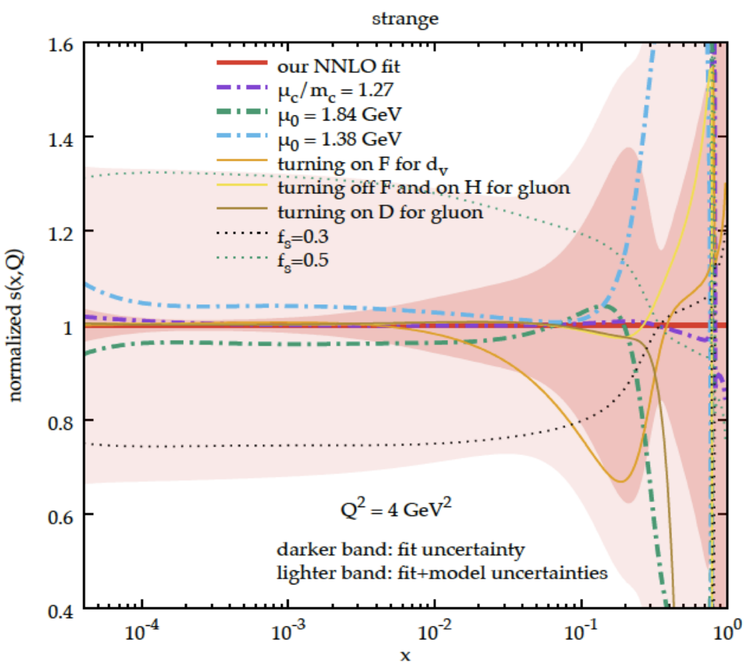
- Parametrization uncertainties addressed adding or removing parameters that do not change the fit quality. The ones giving the largest effect are:
  - Adding F_{d_v}
  - Adding D_g (more flexibility at large-x)
  - Adding H_g and removing F_g (possible effect at small-x)

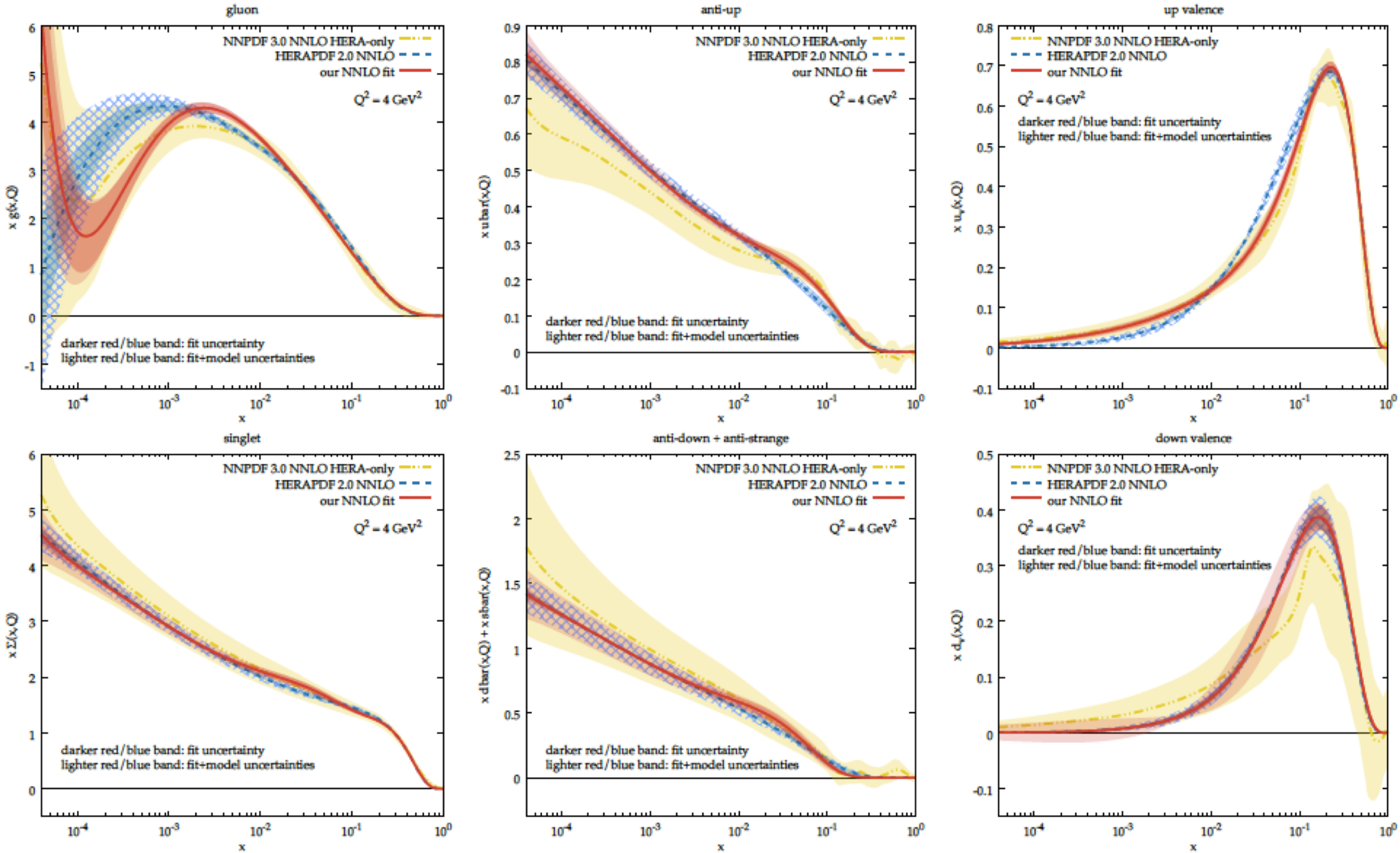
- The effect of all these variations is summarised in the next slide



- The addition of the log term to d_v has the largest effect (negative for $x \lesssim 10^{-3}$)
- When D_g is activated, large-x shape changes substantially, but in a region where the gluon PDF is very small and largely unconstrained
- Effect of H_g (without F_g) very mild
- Up/down variations of f_s have a larger effect on the strange PDF (as expected)
- μ_0 variations have small effects







- We combined all the uncertainties (**exp+th+model**) into **a (symmetric) uncertainty band**
- Our final PDF set including the full uncertainty is largely compatible with NNPDF30

Including small-x resummation

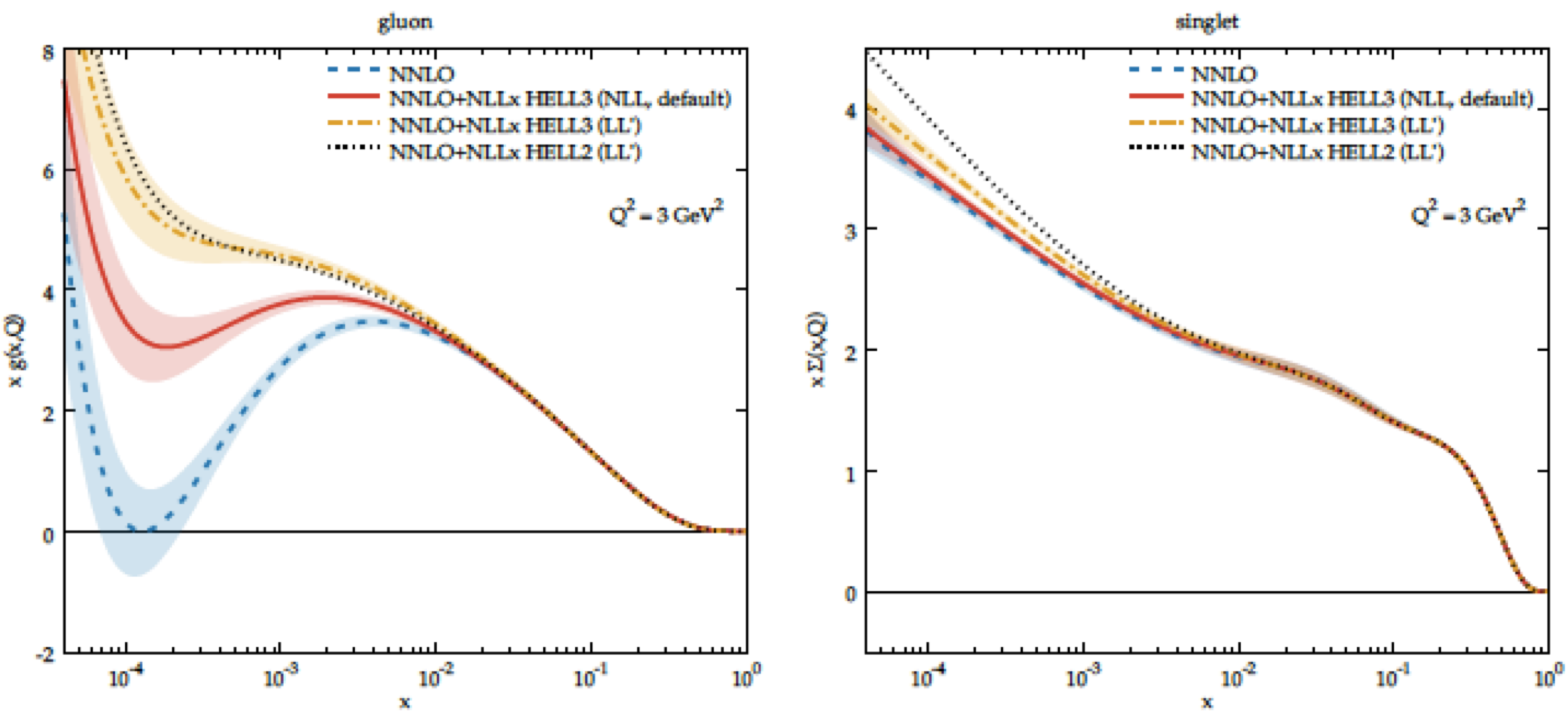
- Done using HELL (v3.0 for **the first time**) – 1805.08785, 1805.06460, 1708.07510
- It provides resummed contributions to the DGLAP splitting functions, the heavy quark matching conditions and the DIS coefficient functions at NLLx

Contribution to χ^2	HELL3.0 (NLL)	HELL3.0 (LL′)	HELL2.0 (LL′)
subset NC e^+ 920 $\tilde{\chi}^2$/n.d.p.	402/377	403/377	403/377
subset NC e^+ 820 $\tilde{\chi}^2$/n.d.p.	70/70	69/70	69/70
subset NC e^+ 575 $\tilde{\chi}^2$/n.d.p.	219/254	219/254	218/254
subset NC e^+ 460 $\tilde{\chi}^2$/n.d.p.	223/204	224/204	224/204
subset NC e^- $\tilde{\chi}^2$/n.d.p.	219/159	220/159	220/159
subset CC e^+ $\tilde{\chi}^2$/n.d.p.	38/39	38/39	38/39
subset CC e^- $\tilde{\chi}^2$/n.d.p.	49/42	49/42	49/42
correlation term + log term	73 – 7	72 – 11	72 – 10
Total χ^2/d.o.f.	**1284/1127**	**1283/1127**	**1283/1127**

- If compared to NNLO fit, further reduction of ~30 units in χ^2

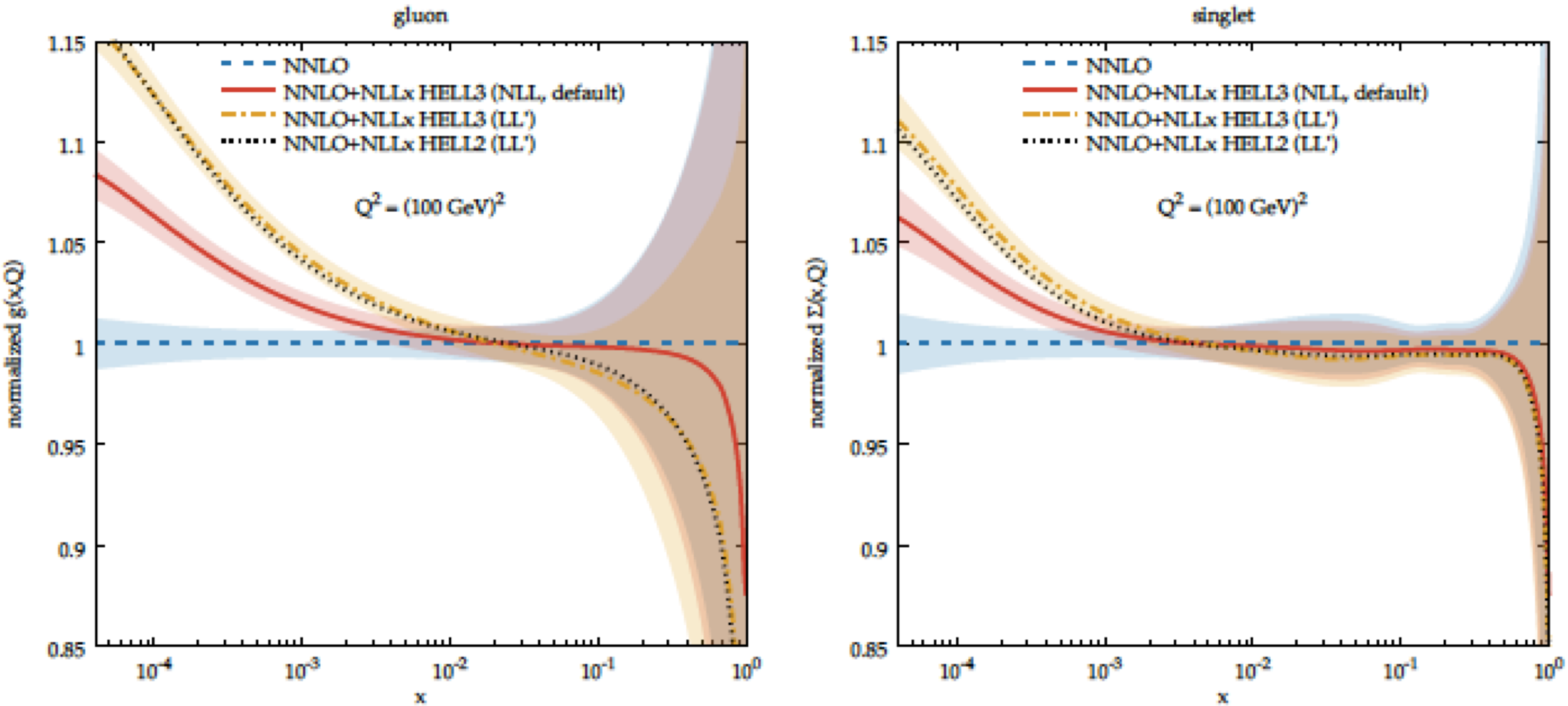
Including small-x resummation

- The difference between two versions of HELL v3.0 is the introduction of a new default treatment of subleading logarithmic contributions
- HELL v2.0 is the previous version of the HELL code



Including small-x resummation

- The difference between two versions of HELL v3.0 is the introduction of a new default treatment of subleading logarithmic contributions
- HELL v2.0 is the previous version of the HELL code

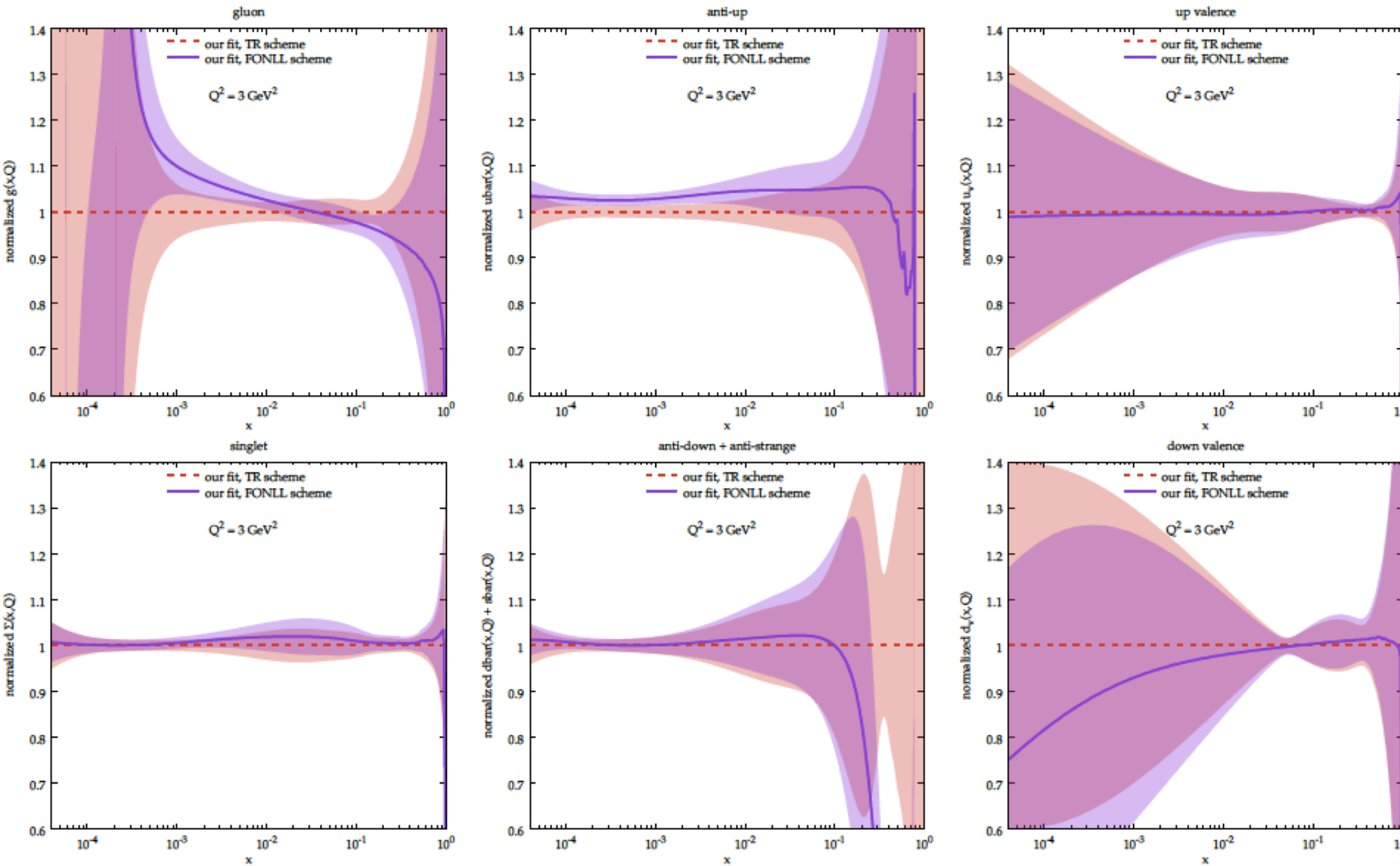


Conclusion and outlook

- Paper announced on arXiv and submitted to EPJC - 1902.11125
- We proposed a **new simple parametrization** for the PDFs at the initial scale that includes a low degree polynominal in log(x) – more flexibility at low-x
- Improvement of the fit quality (62 units reduction in χ^2 wrt HERAPDF2.0)
- Accomplished using **18 parameters** (only 4 more than HERAPDF2.0)
- Stability of our fit tested upon several model and parametrization variations → **results very robust**
- Flexibility of our parametrization allows for a more reliable determination of the uncertainties
- The impact of supplementing the theoretical predictions with the resummation of small-x logarithms investigated → gain of ~30 units in χ^2
- **ADD THIS NEW PARAMETRIZATION IN THE NEW XFITTER RELEASE**

LHAPDF grids available upon request

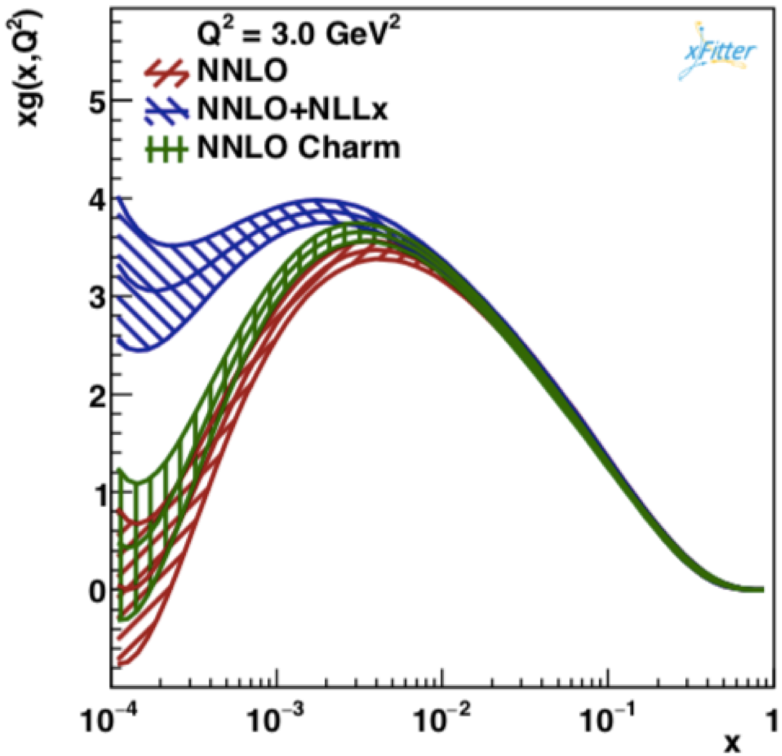
Backup Slides

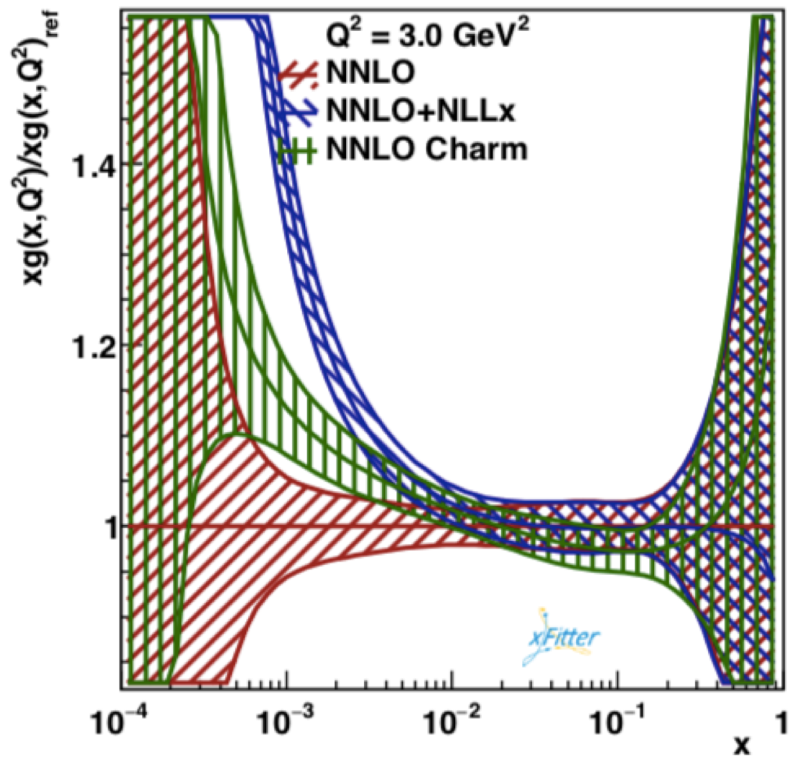


- Some differences are manifest (gluon/sea quarks)
- 1σ bands overlap or are very close to each other (apart from $\bar{d} + \bar{s}$)

More sensitivity to the gluon PDF

- We also studied the inclusion of HERA Charm combined data (Eur.Phys.J. C78 (2018) no.6, 473)
- These data are directly sensitive to $xg(x, Q^2)$

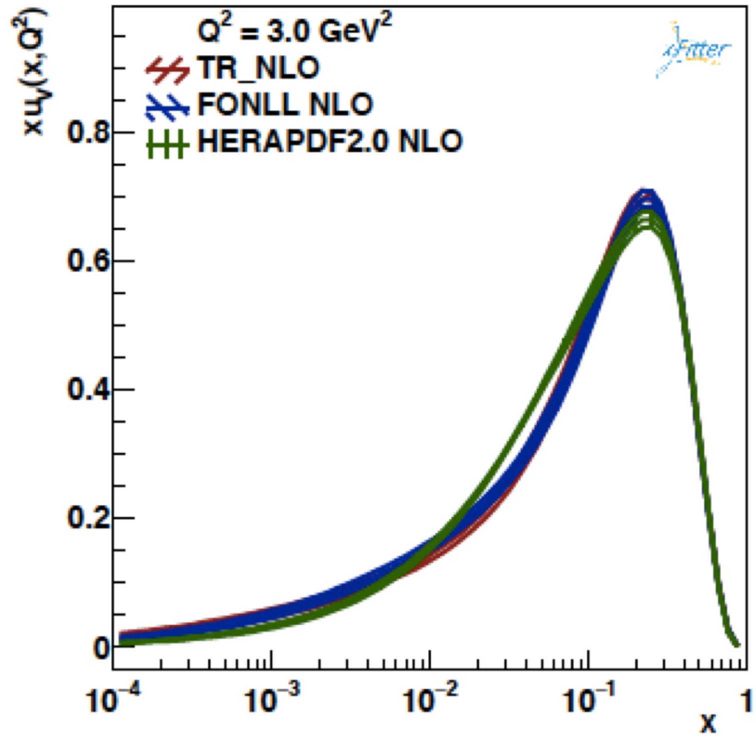


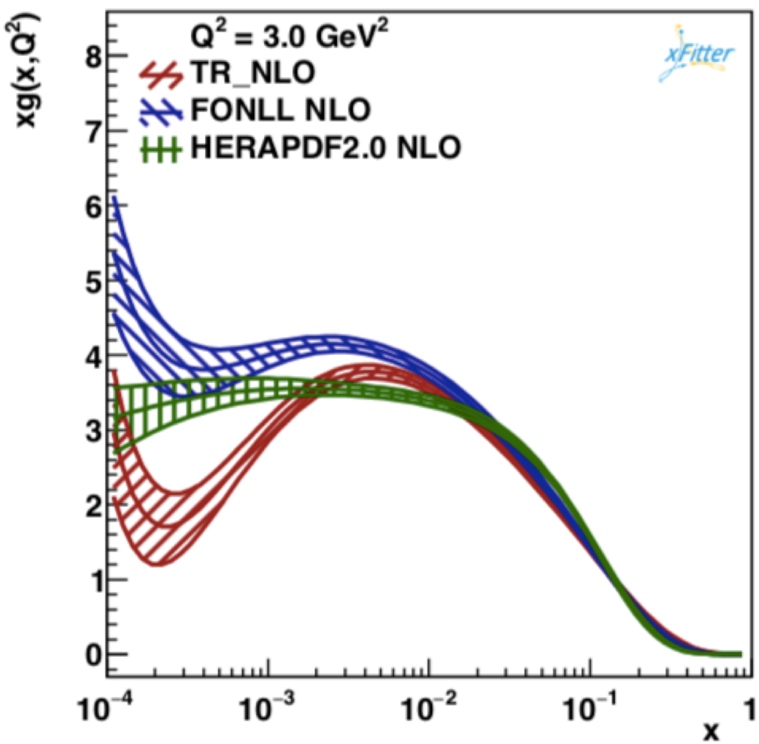


- It is remarkable that the two FO fits are in agreement within uncertainties

A NLO fit

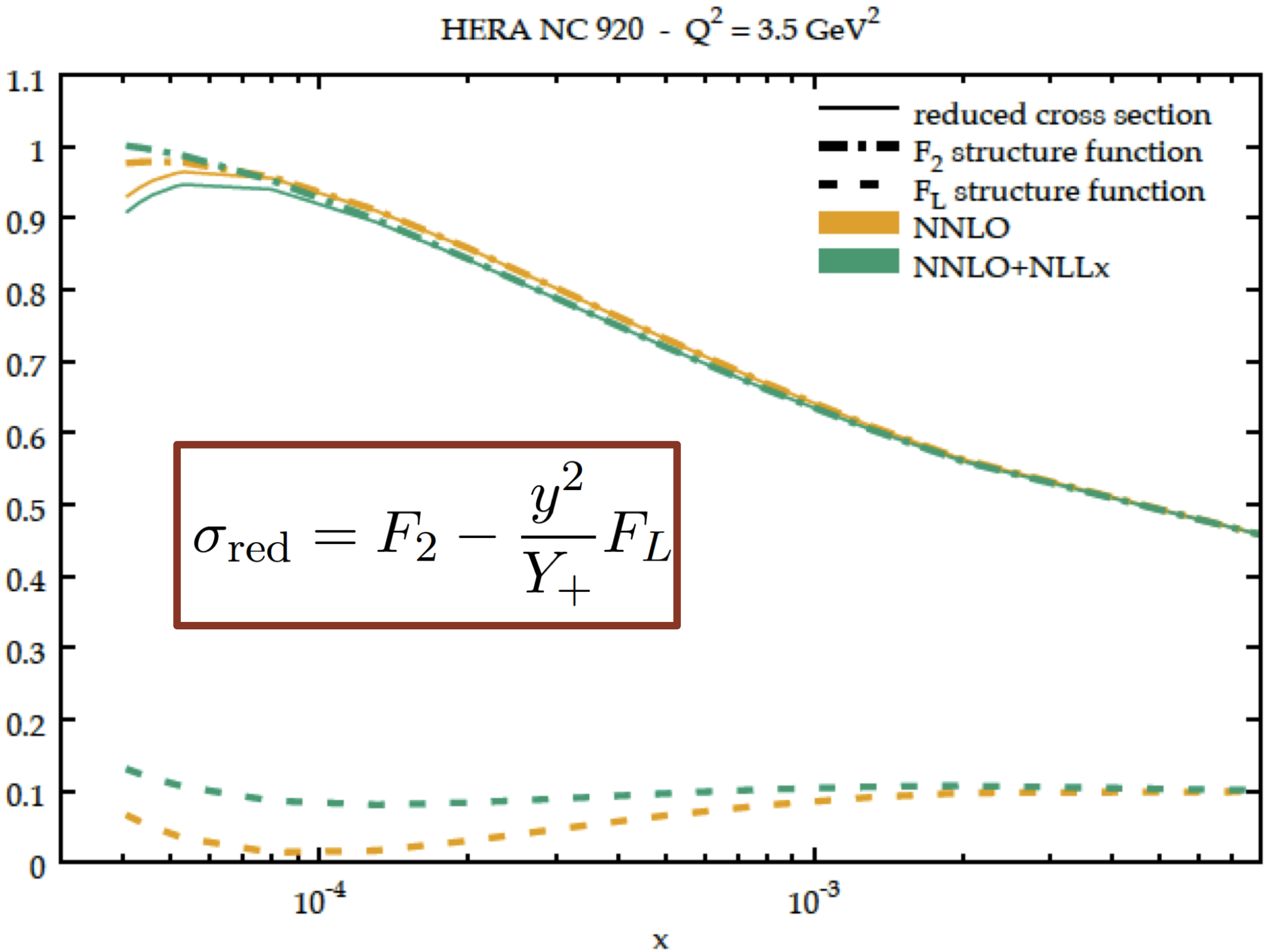
- We also tried a NLO fit (using both TR and FONLL-B) – **preliminary**
- FONLL-B provides a better description than TR
- At low-x, same structure in the gluon PDF





Dataset	TR NLO	FONLL NLO	HERAPDF2.0 NLO
HERA1+2 CCep	38 / 39	39 / 39	43 / 39
HERA1+2 CCem	49 / 42	48 / 42	54 / 42
HERA1+2 NCem	220 / 159	215 / 159	222 / 159
HERA1+2 NCep 820	72 / 70	68 / 70	68 / 70
HERA1+2 NCep 920	407 / 377	400 / 377	440 / 377
HERA1+2 NCep 460	223 / 204	225 / 204	217 / 204
HERA1+2 NCep 575	222 / 254	219 / 254	219 / 254
Correlated χ^2	76	67	86
Log penalty χ^2	-1.39	-6.17	+8.9
Total χ^2 / dof	1305 / 1127	1276 / 1127	1357 / 1145

Reduced cross section, F_2 and F_L



Where $Y_+ = (1 + (1-y)^2)$ and $y = Q^2/(sx)$

- FO and resumed calculations very similar for the reduced xsec
- NNLO prediction for F_2 decreases at small x (softer gluon and quark singlet), while it rises steadily at resumed level (larger singlet)
- Resummed F_L is quite flat in x and much larger than the NNLO one ($x \lesssim 10^{-3}$)
- Rise of F_L due to the gluon PDF shape (rising for $x \sim 10^{-4}$)

	B_g	C_g	F_g	G_g	B_{u_v}	C_{u_v}	E_{u_v}	F_{u_v}	G_{u_v}	B_{d_v}	C_{d_v}	$C_{\bar{U}}$	$D_{\bar{U}}$	$A_{\bar{d}}$	$B_{\bar{d}}$	$C_{\bar{d}}$	$D_{\bar{d}}$	$F_{\bar{d}}$
B_g	1.000	0.783	-0.508	-0.465	-0.055	0.055	0.074	-0.094	-0.098	0.000	-0.058	-0.176	0.043	-0.457	-0.525	-0.285		
C_g	0.783	1.000	-0.093	-0.070	-0.014	0.038	0.046	-0.100	-0.091	0.061	-0.061	-0.163	0.036	-0.352	-0.383	-0.345		
F_g	-0.508	-0.093	1.000	0.989	-0.072	0.117	0.142	-0.150	-0.119	0.051	0.063	0.011	-0.183	0.422	0.494	0.125		
G_g	-0.465	-0.070	0.989	1.000	-0.075	0.124	0.149	-0.157	-0.125	0.051	0.061	0.016	-0.218	0.488	0.558	0.110		
B_{u_v}	-0.055	-0.014	-0.072	-0.075	1.000	-0.202	-0.598	0.485	0.897	-0.226	-0.197	-0.127	-0.244	0.126	0.050	-0.634		
C_{u_v}	0.055	0.038	0.117	0.124	-0.202	1.000	0.846	-0.616	-0.381	-0.042	0.030	-0.535	-0.521	0.211	0.178	0.315		
E_{u_v}	0.074	0.046	0.142	0.149	-0.598	0.846	1.000	-0.871	-0.777	0.184	0.248	-0.462	-0.443	0.164	0.157	0.646		
F_{u_v}	-0.094	-0.100	-0.150	-0.157	0.485	-0.616	-0.871	1.000	0.806	-0.409	-0.445	0.356	0.523	-0.240	-0.206	-0.673		
G_{u_v}	-0.098	-0.091	-0.119	-0.125	0.897	-0.381	-0.777	0.806	1.000	-0.402	-0.384	0.002	0.048	-0.031	-0.064	-0.730		
B_{d_v}	0.000	0.061	0.051	0.051	-0.226	-0.042	0.184	-0.409	-0.402	1.000	0.940	0.390	0.133	0.075	0.069	0.383		
C_{d_v}	-0.058	-0.061	0.063	0.061	-0.197	0.030	0.248	-0.445	-0.384	0.940	1.000	0.262	0.013	0.123	0.112	0.437		
$C_{\bar{U}}$	-0.176	-0.163	0.011	0.016	-0.127	-0.535	-0.462	0.356	0.002	0.390	0.262	1.000	0.721	0.005	0.056	-0.126		
$D_{\bar{U}}$	0.043	0.036	-0.183	-0.218	-0.244	-0.521	-0.443	0.523	0.048	0.133	0.013	0.721	1.000	-0.595	-0.517	-0.083		
$A_{\bar{d}}$	-0.457	-0.352	0.422	0.488	0.126	0.211	0.164	-0.240	-0.031	0.075	0.123	0.005	-0.595	1.000	0.986	0.078		
$B_{\bar{d}}$	-0.525	-0.383	0.494	0.558	0.050	0.178	0.157	-0.206	-0.064	0.069	0.112	0.056	-0.517	0.986	1.000	0.122		
$C_{\bar{d}}$	-0.285	-0.345	0.125	0.110	-0.634	0.315	0.646	-0.673	-0.730	0.383	0.437	-0.126	-0.083	0.078	0.122	1.000		
$D_{\bar{d}}$	-0.042	0.022	0.029	-0.010	-0.665	0.241	0.571	-0.575	-0.706	0.188	0.215	-0.317	-0.048	-0.304	-0.252	0.752	1.000	
$F_{\bar{d}}$	0.562	0.363	-0.590	-0.652	0.013	-0.142	-0.142	0.166	0.086	-0.061	-0.095	-0.094	0.429	-0.941	-0.983	-0.147	0.200	1.000

- Correlation matrix between fit parameters
- Most of them are poorly correlated
- When present, F and G parameters strongly correlated (they probe the same x regime)
- They are also correlated to B parameters (same reason as above)
- Down-valence parameters highly correlated (same as for HERAPDF2.0)

Local minima

- While fitting data with fixed-order theory, we found a local minimum pretty far away from the global minimum presented in the paper
- Main difference in the gluon PDF: global minimum with $B_g < 0$, while local minimum with $B_g > 0$
- The fit converged in the local minimum has an extra parameter in it: cubic logarithmic term in the gluon PDF (H_g)
- Even though very significant differences in some parameters, χ^2 really similar
- When transitioning from one minimum to the other in the parameter space, the χ^2 becomes much larger → with a standard minimization routine it is highly unlikely that once the local minimum is found, it could converge to the global minimum
- The physical expectation $B_g < 0$ was crucial to guide us

Fitted parameter	NNLO (FONLL) local minimum	NNLO (FONLL) global minimum	NNLO+NLLx HELL 3.0 (NLL)
B_g	$\boxed{0.34 \pm 0.07}$	-0.55 ± 0.03	-0.52 ± 0.04
C_g	8.8 ± 1.0	4.5 ± 0.5	4.5 ± 0.5
F_g	0.76 ± 0.04	0.230 ± 0.003	0.217 ± 0.005
G_g	0.22 ± 0.02	0.0131 ± 0.0004	0.0112 ± 0.0005
H_g	0.017 ± 0.002		
B_{u_v}	0.85 ± 0.06	0.83 ± 0.06	0.76 ± 0.06
C_{u_v}	4.5 ± 0.1	4.6 ± 0.2	4.6 ± 0.1
E_{u_v}	1.7 ± 0.8	1.9 ± 1.0	2.6 ± 1.1
F_{u_v}	0.38 ± 0.04	0.37 ± 0.05	0.35 ± 0.04
G_{u_v}	0.062 ± 0.011	0.058 ± 0.012	0.049 ± 0.010
B_{d_v}	1.01 ± 0.09	0.98 ± 0.10	0.99 ± 0.09
C_{d_v}	4.7 ± 0.4	4.7 ± 0.5	4.7 ± 0.5
$A_{\bar{d}}$	0.070 ± 0.008	0.13 ± 0.02	0.14 ± 0.02
$B_{\bar{d}}$	-0.45 ± 0.02	-0.34 ± 0.02	-0.33 ± 0.02
$C_{\bar{d}}$	28 ± 3	24 ± 2	24 ± 3
$D_{\bar{d}}$	76 ± 17	40 ± 12	38 ± 10
$F_{\bar{d}}$	0.084 ± 0.001	0.072 ± 0.004	0.071 ± 0.004
$C_{\bar{u}}$	11 ± 1	11 ± 1	11 ± 1
$D_{\bar{u}}$	33 ± 6	20 ± 4	18 ± 4
χ^2/d.o.f.	1314/1126	1312/1127	1284/1127