Parton Branching TMD PDFs using xFitter

`xFitter` workshop

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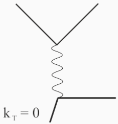
We want to develop an approach in which transverse momentum kinematics will be treated without any mismatch between matrix element (ME) and PS

Standard MC predictions

Eur. Phys. J. C19, 351 (2001)



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Mismatch between PDF used by $\hat{\sigma}$ and PS

No mismatch: $\hat{\sigma}$ and PS follow the same TMD

Goal: to construct TMDs in a wide range of x, k_T and μ^2

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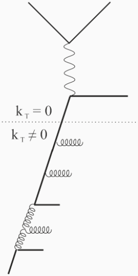

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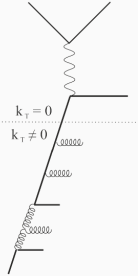

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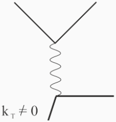

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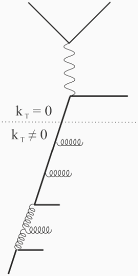

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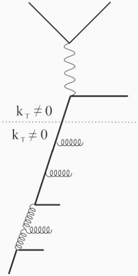

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Plan for today

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- brief reminder of the Parton Branching (PB) method
- TMDs from PB using `xFitter`
- comparison of PB with other existing approaches

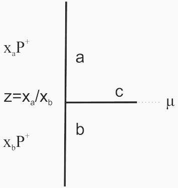
PB in a nutshell

PB as a method to solve DGLAP

DGLAP evolution equation

$$\frac{d\ \widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 dz\ P_{ab}(\mu^2,z)\ \widetilde{f}_b(x/z,\mu^2)$$

$xf(x,\mu^2) = \widetilde{f}(x,\mu^2)$



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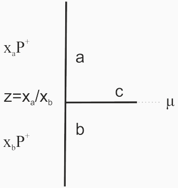

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We use:

- momentum sum rule to have only real splitting functions $P^R_{ab} = K_{ab}\frac{1}{(1-z)} + R_{ab}$
- $\int_0^1 \rightarrow \int_0^{z_M}$, $z_M \approx 1$: to define resolvable $z < z_M$ and non-resolvable $z > z_M$ branchings
- Sudakov form factor $\Delta_a(\mu^2) = \exp\left(-\int_{\mu_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2}\sum_b\int_0^{z_M} dz z P^R_{ba}\left(\mu'^2, z\right)\right)$ which is the probability of an evolution without any resolvable branching

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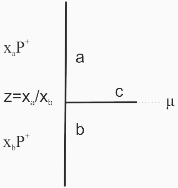

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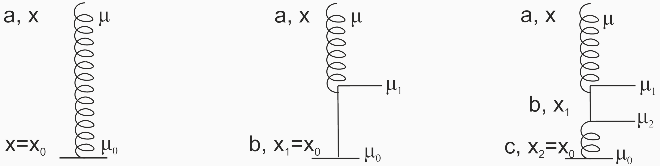
$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{\ln\mu_0^2}^{\ln\mu^2} d\ln\mu_1^2\,\frac{\Delta_a(\mu^2)}{\Delta_a(\mu_1^2)}\sum_b\int_x^{z_M} dz_1 P^R_{ab}\left(\mu_1^2,z_1\right)\widetilde{f}_b\left(\frac{x}{z_1},\mu_0^2\right)\Delta_b(\mu_1^2) + \ldots$$

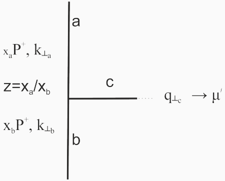


Transverse momentum in PB

How to connect branching scale μ'^2 and $q^2_{\perp,c}$?

resolvable & non-resolvable $\Rightarrow$ condition on min $q^2_{\perp,c}$ $\Rightarrow$ z_M

The argument of α_s should be $q^2_{\perp,c}$



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- PB method: effect of every individual part of the ordering definition can be studied separately
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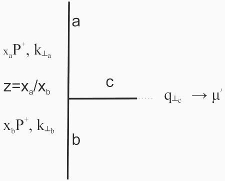

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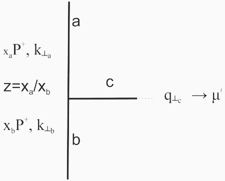

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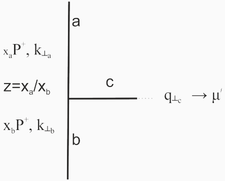

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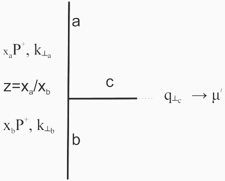

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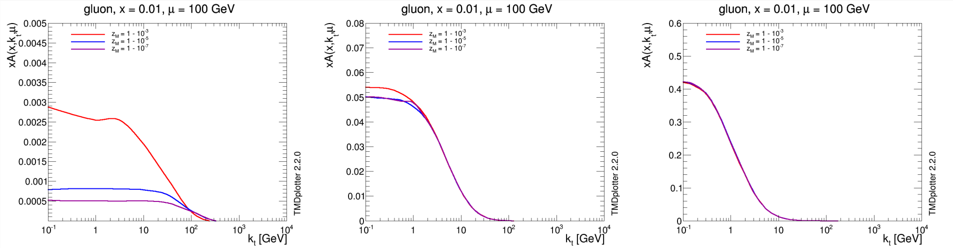
Highlights

Effect of ordering choice and z_M on TMDs



$p_\perp$- ordering	virtuality ordering	angular ordering
$q_\perp^2 = 1\mu'^2$	$q_\perp^2 = (1-z)\mu'^2$	$q_\perp^2 = (1-z)^2\mu'^2$
NOT stable TMDs		stable TMDs

Note1: Everywhere $\alpha_s\left(\mu'^2\right)$

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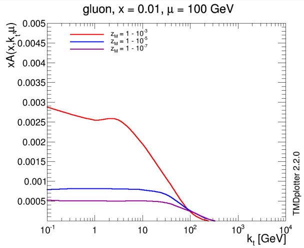

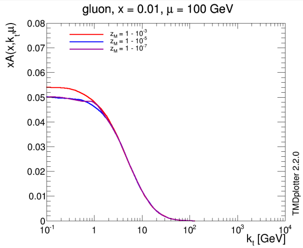

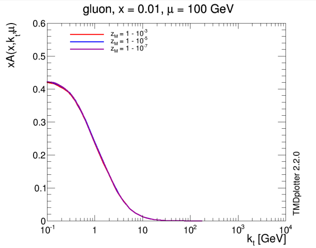

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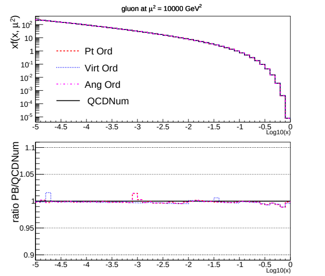
Note2: All these TMDs after integration over $k_\perp$ give the same collinear PDF

More details:
Phys.Lett. B772 (2017) 446-451, JHEP 1801 (2018) 070



Renormalization scale

angular ordering:

$\alpha_s\left((1-z)^2\mu^2\right) = \alpha_s(\mu^2) - \alpha_s^2(\mu^2)\beta_0 \ln\left((1-z)^2\right) + \ldots$

$P_{ab}(\mu^2, z) = \frac{\alpha_s(\mu^2)}{2\pi} P_{ab}^{LO}(\mu^2, z) - \frac{\alpha_s^2(\mu^2)}{4\pi^2}\beta_0 \ln((1-z)^2) P_{ab}^{LO}(\mu^2, z) + \frac{\alpha_s^2(\mu^2)}{4\pi^2} P_{ab}^{NLO}(\mu^2, z) + \ldots .$

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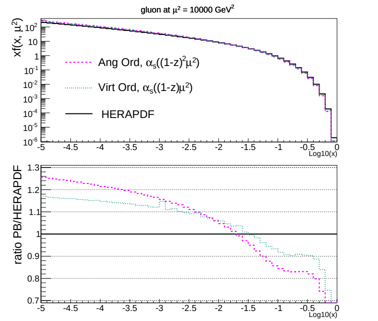
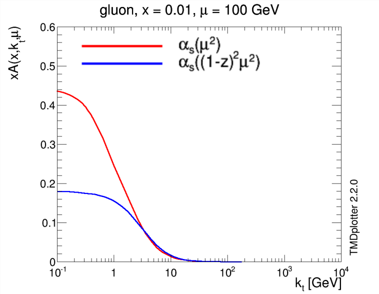


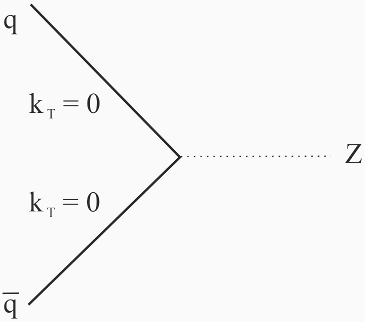


angular ordering, the same conclusions for virtuality ordering.

virtuality ordering, the same conclusions for angular ordering

Collinear and TMD PDFs affected significantly by the change of renormalization scale

Prediction for Z boson $p_\perp$ spectrum using TMDs



Procedure:

- DY collinear ME

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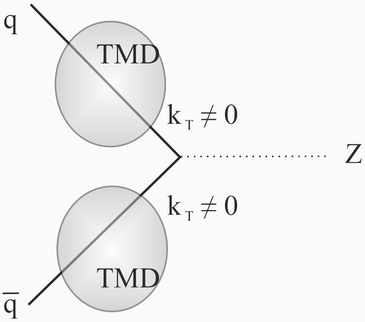
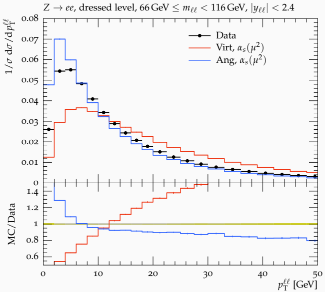


Procedure:

- DY collinear ME
- Generate $k_\perp$ of $q\overline{q}$ according to TMDs (m_{DY} fixed, x_1, x_2 change)
- compare with the 8 TeV ATLAS measurement

Prediction for Z boson $p_\perp$ spectrum using TMDs

here: DY LO matrix element from Pythia: $q\bar{q} \rightarrow Z$



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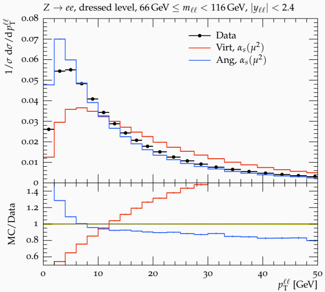

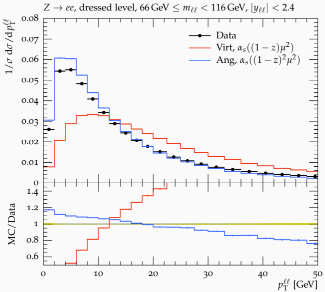

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- angular ordering: the shape of Z boson $p_\perp$ spectrum reproduced
- with $\alpha_s\left((1-z)^2\mu'^2\right)$ agreement within the data much better than for $\alpha_s(\mu'^2)$
- All the $p_\perp$ dependence directly from the PB method
- prediction for the whole spectrum from one method
- no tuning/adjustment of free parameters

Fit

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Based on the facts that PB with the angular ordering allows to:

- define stable (z_M independent) TMDs
- predict Z boson $p_\perp$ spectrum

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- Set1: $\alpha_s\left(\mu'^2\right)$, reproduces HERAPDF2.0 ✓
- Set2: $\alpha_s\left((1-z)^2\mu'^2\right)$, different HERAPDF2.0 ✓

details of the fit presented last year by Hannes Jung and given in arXiv:1804.11152 (to be published in Physical Review D soon)
https://indico.desy.de/indico/event/19213/session/12/contribution/27/material/slides/0.pdf

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- data: HERA H1 and ZEUS combined DIS measurement [Eur.Phys.J. C75 (2015) no.12, 580]
- range: $3.5 < Q^2 < 50000\ \mathrm{GeV}^2$, $4 \cdot 10^{-5} < x < 0.65$
- systematic uncertainty: in the χ^2 definition in `xFitter`
- experimental uncertainties: Hessian method in `xFitter`
- model uncertainties: variation of m_c, m_b, μ_0 (Set2: q_{cut} in α_s)
- initial parametrization in a form of HERAPDF2.0

Fit method

First iTMDs are fitted:

- kernel $K_{ba}(x'', \mu^2)$ obtained from PB for every initial parton species of flavour b [1] and final parton a. initial parametrization at μ_0^2: $x = 1 - 10^{-6}$
- convolution of the kernel with the starting distribution $f_{0,b}$

$$\begin{aligned}\widetilde{f}_a(x, \mu^2) &= x \int \mathrm{d}x' \int \mathrm{d}x'' f_{0,b}(x', \mu_0^2) K_{ba}(x'', \mu^2, \mu_0^2) \delta(x'x'' - x) \\ &= \int \mathrm{d}x' f_{0,b}(x', \mu_0^2) \frac{x}{x'} K_{ba}\left(\frac{x}{x'}, \mu^2, \mu_0^2\right)\end{aligned}$$

- $\widetilde{f}_a(x, \mu^2)$ convoluted with ME to obtain the structure function at NLO, which can be fitted to experimental data
- the procedure repeated with different values of the initial parameters until the minimal χ^2 is found.

[1] enough one light quark and gluon

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$$\begin{aligned} \widetilde{f}_a(x, \mu^2) &= x \int \mathrm{d}x' \int \mathrm{d}x'' f_{0,b}(x', \mu_0^2) K_{ba}(x'', \mu^2, \mu_0^2) \delta(x'x'' - x) \\ &= \int \mathrm{d}x' f_{0,b}(x', \mu_0^2) \frac{x}{x'} K_{ba}\left(\frac{x}{x'}, \mu^2, \mu_0^2\right) \end{aligned}$$

- $\widetilde{f}_a(x, \mu^2)$ convoluted with ME to obtain the structure function at NLO, which can be fitted to experimental data
- the procedure repeated with different values of the initial parameters until the minimal χ^2 is found.

To obtain TMDs:

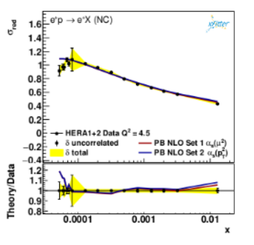
- A new kernel $K_a^b(x'', k_\perp, k_{\perp 0}^2, \mu^2, \mu_0^2)$ obtained from PB
- convoluted with the initial distribution from the fit of iTMDs

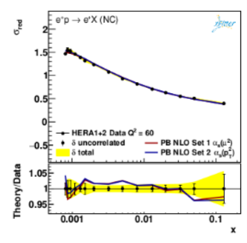
$$\begin{aligned} xA_a(x, k_\perp, \mu^2) &= x \int \mathrm{d}x' \int \mathrm{d}x'' A_{0,b}(x', k_{\perp 0}^2, \mu_0^2) K_{ba}(x'', k_\perp^2, k_{\perp 0}^2, \mu^2, \mu_0^2) \delta(x'x'' - x) \\ &= \int \mathrm{d}x' A_{0,b}(x', k_{\perp 0}^2, \mu_0^2) \frac{x}{x'} K_{ba}\left(\frac{x}{x'}, k_\perp^2, k_{\perp 0}^2, \mu^2, \mu_0^2\right) \end{aligned}$$

where $A_{0,b}(x', k_{\perp,0}^2, \mu_0^2) = f_{0,b}(x, \mu_0^2) \exp\left(-\frac{|k_{\perp,0}^2|}{\sigma^2}\right)$

[1] enough one light quark and gluon

Highlights from the fit (presented in detail last year)





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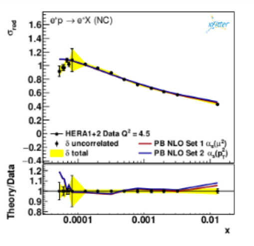
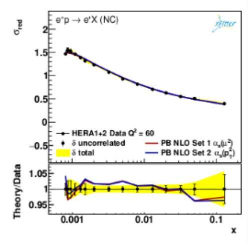
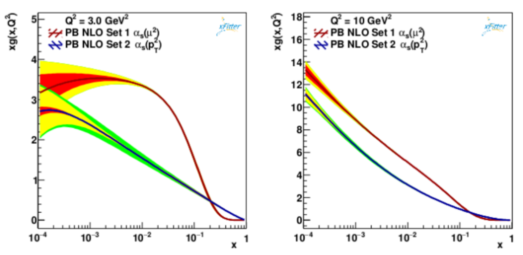




experimental, model, qcut for Set2



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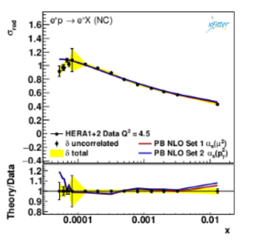

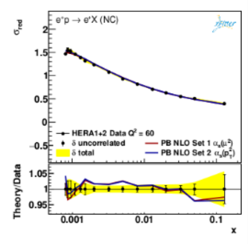

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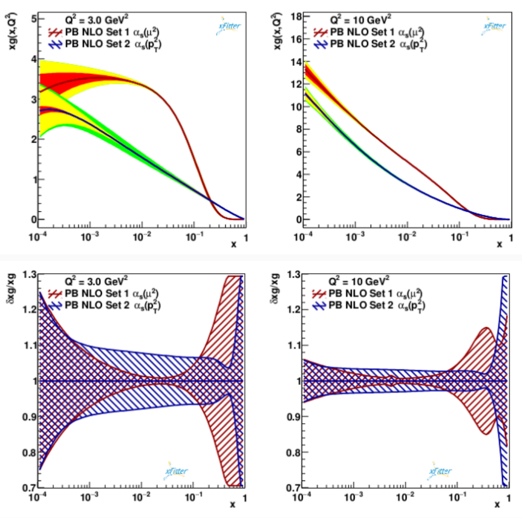
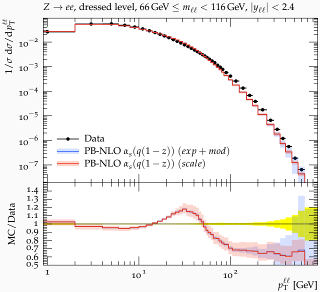
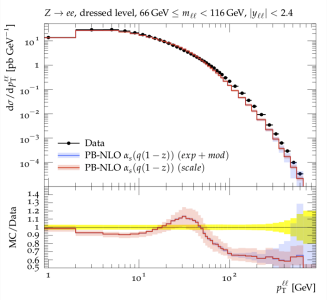


Application of TMDs to Z boson $p_\perp$

Application to the Z boson $p_\perp$ spectrum



Results after the fit. Experimental and model uncertainty here: PYTHIA LO ME



Results after the fit. Experimental and model uncertainty here: MCatNLO ME

PB successful

Possible improvements

To obtain TMDs:

$$
\begin{aligned}
xA_a(x, k_\perp, \mu^2) &= x \int \mathrm{d}x' \int \mathrm{d}x'' A_{0,b}(x', k_{\perp 0}^2, \mu_0^2) K_{ba}(x'', k_\perp^2, k_{\perp 0}^2, \mu^2, \mu_0^2) \delta(x'x'' - x) \\
&= \int \mathrm{d}x' A_{0,b}(x', k_{\perp 0}^2, \mu_0^2) \frac{x}{x'} K_{ba}\left(\frac{x}{x'}, k_\perp^2, k_{\perp 0}^2, \mu^2, \mu_0^2\right)
\end{aligned}
$$

where $A_{0,b}(x', k_{\perp,0}^2, \mu_0^2) = f_{0,b}(x, \mu_0^2) \exp\left(-\frac{|k_{\perp,0}^2|}{\sigma^2}\right)$

Intrinsic $k_{\perp,0}$ NOT fitted !

Come from Gauss distribution with $\sigma = 0.5\mathrm{GeV}$. The same for all flavours.

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To obtain TMDs:

$$\begin{aligned} xA_a(x, k_\perp, \mu^2) &= x \int \mathrm{d}x' \int \mathrm{d}x'' A_{0,b}(x', k^2_{\perp 0}, \mu_0^2) K_{ba}(x'', k_\perp^2, k^2_{\perp 0}, \mu^2, \mu_0^2) \delta(x'x'' - x) \\ &= \int \mathrm{d}x' A_{0,b}(x', k^2_{\perp 0}, \mu_0^2) \frac{x}{x'} K_{ba}\left(\frac{x}{x'}, k_\perp^2, k^2_{\perp 0}, \mu^2, \mu_0^2\right) \end{aligned}$$

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- to fit also intrinsic $k_\perp$ $\rightarrow$ use datasets sensitive to low $k_\perp$ (low mass DY)

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Intrinsic $k_{\perp,0}$ NOT fitted !

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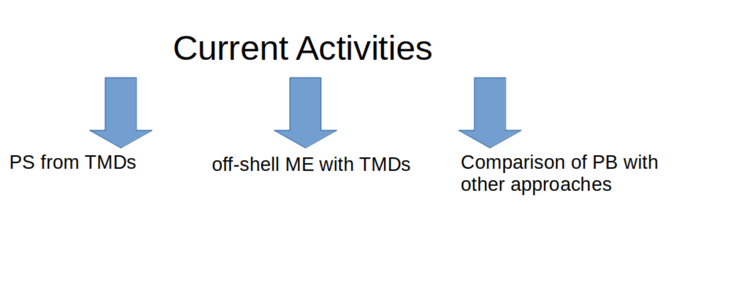
Possible improvements from `xFitter` side:

- to fit also intrinsic $k_\perp$ $\rightarrow$ use datasets sensitive to low $k_\perp$ (low mass DY)
- use also LHC and Tevatron data to perform global fit
  notice: fit only to HERA data, nevertheless we describe LHC measurement well!
- to use $k_\perp$-dependent ME and TMD to calculate the structure function (ongoing developments in off-shell ME calculations, e.g. KaTie)

PB and other approaches

Current Activities



→ I concentrate now on comparison of PB with other approaches

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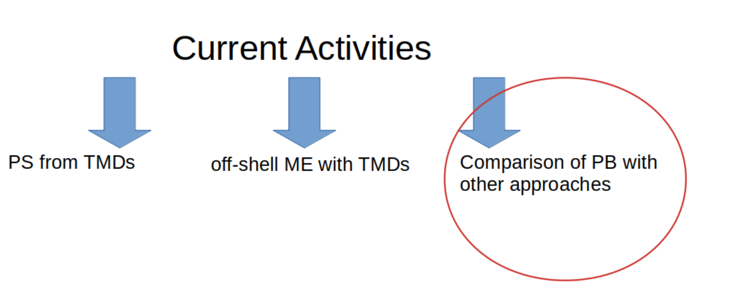

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PB and Marchesini, Webber

PB with angular ordering is very successful

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PB for angular ordering:

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \sum_b \int_x^{1-\frac{q_0}{\mu'}} dz P_{ab}^R\left(\alpha_s\left((1-z)^2\mu'^2\right), \mu'^2, z\right) \widetilde{f}_b\left(\frac{x}{z}, \mu'^2\right) \quad (1)$$

where

$$q_{\perp,i}^2 = (1-z_i)^2\mu'^2$$

Eq. (1) is identical to the Marchesini and Webber (MarWeb1988) prescription
Nuclear Physics B310 (1988) 461-526

PB and Kimber- Martin- Ryskin- Watt (KMRW)

PB for angular ordering written in terms of integral over $q_\perp$ (identical to MarWeb1988):

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{q_0^2}^{(1-x)^2\mu^2} \frac{dq_\perp^2}{q_\perp^2} \sum_b \int_x^{1-\frac{q_\perp}{\mu}} dz \Delta_a\left(\mu^2, \frac{q_\perp^2}{(1-z)^2}\right) P_{ab}^R\left(\alpha_s\left(q_\perp^2\right), \frac{q_\perp^2}{(1-z)^2}, z\right) \widetilde{f}_b\left(\frac{x}{z}, \frac{q_\perp^2}{(1-z)^2}\right)$$

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KMRW: TMDs (unintegrated PDFs) obtained from the integrated PDFs and the Sudakov form factors

Phys. Rev. D63 (2001) 114027

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2)$$
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at last step of the evolution the unintegrated distribution becomes dependent on two scales: $q_\perp$ and μ

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at last step of the evolution the unintegrated distribution becomes dependent on two scales: $q_\perp$ and μ

In KMRW:

- "Strong ordering": $q_M^2 = (1-x)^2\mu^2$ and $z_M = 1 - \frac{q_\perp}{\mu}$
- "Angular ordering" $q_M^2 = \left(\frac{1-x}{x}\right)^2 \mu^2$ and $z_M = 1 - \frac{\mu}{q_\perp + \mu}$

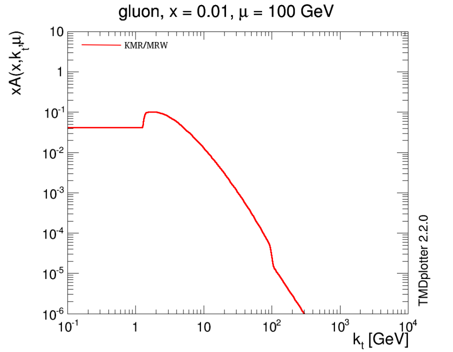
PB and KMRW: distributions

PB: intrinsic $k_\perp$ is a Gauss distribution with width=0.5 GeV

KMRW parametrization for $k_\perp < k_0 = 1\mathrm{GeV}$:

$$\frac{\widetilde{f}_a(x, k_\perp, \mu^2)}{k_\perp^2} = \frac{1}{\mu_0^2}\widetilde{f}_a(x, k_\perp, \mu_0^2)\Delta_a(\mu^2, \mu_0^2) = \mathrm{const}$$

TMD sets obtained according to KMRW formalism with angular ordering included in TMDlib in TMD set called MRW-ct10nlo Eur.Phys.J.C78(2018)no.2,137



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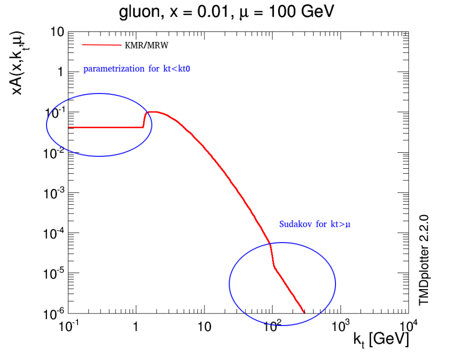

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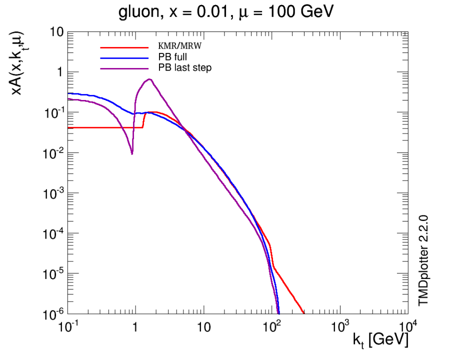


exercise:

PB last Step: try to obtain KMR from PB: take PB with angular ordering but take $k_\perp$ only from the last emission

do $\vec{k}_{\perp,a} = -\vec{q}_{\perp,c}$ instead $\vec{k}_{\perp,a} = \vec{k}_{\perp,b} - \vec{q}_{\perp,c}$ (PB full)

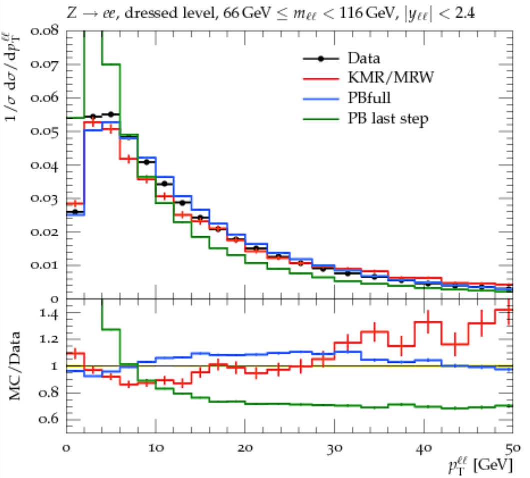
$k_t < 1\mathrm{GeV}$:

- KMRW: initial parametrization
- PB last Step: matching of intrinsic $k_\perp$ and evolution clearly visible
- PB full: matching of intrinsic $k_\perp$ and evolution smeared during evolution

For $k_t \in (\approx 10\mathrm{GeV}, \approx \mu)$:

PB full and KMRW very similar!

Z boson $p_\perp$ spectrum



- PB with angular ordering and full evolution works very well
- KMRW works well for small and middle-range $k_\perp$ but for higher $k_\perp$ it overestimates the data
- PB with last step evolution not sufficient

PB and Collins, Soper and Sterman

WORK IN PROGRESS

CSS: TMD factorization formula for the DY cross section:

Nuclear Physics B250 (1985) 199-224

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}y\mathrm{d}Q_T^2} \sim \frac{4\pi^2\alpha^2}{9Q^2 s}\frac{1}{(2\pi)^2}\int \mathrm{d}^2 b \exp(iQ_T\cdot b)\sum_j e_j^2 \cdot \sum_a \int_{x_A}^1 \frac{\mathrm{d}\xi_A}{\xi_A} f_{a/A}(\xi_A, 1/b)$$
$$\sum_b \int_{x_B}^1 \frac{\mathrm{d}\xi_B}{\xi_B} f_{b/B}(\xi_B, 1/b) \exp\left(-\int_{1/b^2}^{Q^2}\frac{\mathrm{d}\overline{\mu}^2}{\overline{\mu}^2}\left[\ln\left(\frac{Q^2}{\overline{\mu}^2}\right)A(g(\overline{\mu})) + B(g(\overline{\mu}))\right]\right) \tag{2}$$
$$\cdot C_{ja}\left(\frac{x_A}{\xi_A}, g(1/b)\right) C_{jb}\left(\frac{x_B}{\xi_B}, g(1/b)\right) + \frac{4\pi^2\alpha^2}{9Q^2 s} Y(Q_T, Q, x_A, x_B)$$

where $A = \sum_i \left(\frac{\alpha_s(\mu)}{\pi}\right)^i A^i$, the same for B and C.

PB and Collins, Soper and Sterman

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- one scale evolution up to a scale $1/b$
- in the last step of the evolution the dependence on the second scale enters

PB and CSS

WORK IN PROGRESS

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PB: Sudakov form factor with P^R_{ba} but possible also with P^V_a (momentum sum rule) .
For angular ordering:

$$\Delta_a(\mu^2) = \exp\left(-\int_{q_0^2}^{\mu^2} \frac{dq_\perp^2}{q_\perp^2}\left(\int_0^{1-\frac{q_\perp}{\mu}} dz \left(k_a \frac{1}{1-z}\right) - d\right)\right) .$$

notice: $2\int_0^{1-\frac{q_\perp}{\mu}} dz \left(\frac{1}{1-z}\right) = \ln\left(\frac{\mu}{q_\perp}\right)^2$

PB with angular ordering: in Sudakov the same coefficients as $\underbrace{\frac{1}{2}A^1}_{\text{LL}}, \underbrace{\frac{1}{2}A^2 \text{ and } \frac{1}{2}B^1}_{\text{NLL}}$ in CSS

NNLL: difference of CSS and PB B_2 comes from renormalization group
$2d_{2\mathrm{PB}} + B_{2\mathrm{CSS}} = 16\pi\beta_0\left(\frac{\pi^2}{6} - 1\right)$

Summary and Conclusions

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- different ordering definitions studied; TMDs with angular ordering stable
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- possible improvement: to fit also intrinsic $k_\perp$
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- studies on comparison with Marchesini and Webber, KMRW and CSS ongoing
- results in: Phys.Lett. B772 (2017) 446-451, JHEP 1801 (2018) 070, arXiv:1804.11152 (to be published in Physical Review D soon), new paper in preparation

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Outlook:

new level of precision in obtaining predictions for QCD observables (hard ME and PS follow the same TMD) for LHC and future colliders

Summary and Conclusions

- PB: collinear PDFs and TMDs obtained
- different ordering definitions studied; TMDs with angular ordering stable
- fit of integrated TMDs to HERA H1 and ZEUS combined F_2 data, two TMD sets obtained (with model and experimental uncertainties) with angular ordering for two different renormalization scales
- application of the TMDs to the Z boson $p_\perp$, a very good description of the 8 TeV data with angular ordering
- possible improvement: to fit also intrinsic $k_\perp$
- many different activities ongoing (PS from TMDs, off-shell ME and TMDs, etc....)
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Outlook:

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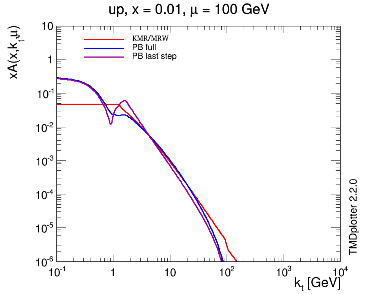
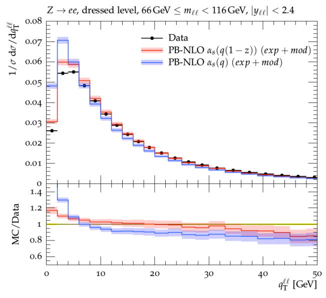
Thank you!

MWR-ct10nlo and PB for quarks





Results after the fit. Experimental and model uncertainty

Fit method

The fits to HERA measurements are performed using a χ^2 minimization

- an evolution kernel $K_a^b(x'', \mu^2)$ is obtained from the PB method for every initial parton (enough one light quark and gluon) of flavour b and final parton a. The initial parametrization at the scale μ_0^2 is given by $x = 1 - 10^{-6}$
- convolution of the kernel with the starting distribution $A_{0,b}$

$$\begin{aligned} \widetilde{f}_a(x, \mu^2) &= x \int \mathrm{d}x' \int \mathrm{d}x'' A_{0,b}(x') K_a^b(x'', \mu^2) \delta(x'x'' - x) \\ &= \int \mathrm{d}x' A_{0,b}(x') \frac{x}{x'} K_a^b \left(\frac{x}{x'}, \mu^2 \right) \end{aligned}$$

- The obtained distribution $\widetilde{f}_a(x, \mu^2)$ is convoluted with the matrix element to obtain the structure function at NLO, which can be fitted to experimental data
- the procedure is repeated with different values of the initial parameters until the minimal χ^2 is found.

To obtain TMDs:

- A new kernel $K_a^b(x'', k_\perp, \mu^2)$, depending now also on $k_\perp$, obtained from the PB method
- and convoluted with the initial distribution from the fit of iTMDs

$$\begin{aligned} xA_a(x, k_\perp, \mu^2) &= x \int \mathrm{d}x' \int \mathrm{d}x'' A_{0,b}(x') K_a^b(x'', k_\perp, \mu^2) \delta(x'x'' - x) \\ &= \int \mathrm{d}x' A_{0,b}(x') \frac{x}{x'} K_a^b \left(\frac{x}{x'}, k_\perp, \mu^2 \right) \end{aligned}$$

Parametrization and the parameters from the fit

The parametrization used:

$$
\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g} - A'_g x^{B'_g}(1-x)^{C'_g} \\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}(1+E_{u_v}x^2) \\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}} \ , \\
x\overline{U}(x) &= A_{\overline{U}} x^{B_{\overline{U}}}(1-x)^{C_{\overline{U}}}(1+D_{\overline{U}}x) \\
x\overline{D}(x) &= A_{\overline{D}} x^{B_{\overline{D}}}(1-x)^{C_{\overline{D}}}
\end{aligned}
$$

At μ_0^2 assume: $x\overline{U} = x\overline{u}$, $x\overline{D} = x\overline{d} + x\overline{s}$, strange quark at μ_0^2: $x\overline{s} = f_s x\overline{D}$ with $f_s = 0.4$, $B_{\overline{U}} = B_{\overline{D}}$, $A_{\overline{U}} = A_{\overline{D}}(1-f_s)$. The normalization parameters A_{u_v}, A_{d_v}, A_g and $A_{g'}$ are constrained by the quark number and momentum sum rules. $x\overline{U}$,, xU, $x\overline{D}$, xD - sums of parton distributions for up-type and down-type quarks and anti-quarks

	A	B	C	D	E	A'	B'	C'
Set 1								
xg	4.32	−0.015	9.15			1.040	−0.166	25
xu_v	4.07	0.714	4.84		13.5			
xd_v	3.15	0.806	4.07					
$x\bar{U}$	0.107	−0.173	8.05	11.8				
$x\bar{D}$	0.178	−0.173	4.89					
Set 2								
xg	0.42	−0.047	0.96			0.008	−0.58	25
xu_v	2.49	0.65	3.44		13.7			
xd_v	2.02	0.75	2.47					
$x\bar{U}$	0.14	−0.16	5.29	1.5				
$x\bar{D}$	0.24	−0.16	5.83					

Parameters of the initial distributions at NLO obtained from the fit. The parameter $C' = 25$ was fixed, as in HERAPDF2.0. The parameters correspond to a starting scale $\mu_0^2 = 1.9(1.4)\ \mathrm{GeV}^2$ for Set 1 (Set 2).

Experimental uncertainties- χ^2 definition

Def. of χ^2 includes treatment of correlated and uncorrelated systematic uncertainties.
In total 162 systematic uncertainties plus procedural uncertainties from the combination of H1 and ZEUS are treated as correlated uncertainties.
leading systematic uncertainties on the cross-section measurements from the uncertainties on the acceptance corrections and luminosity determinations
Procedural uncertainties: Multiplicative versus additive treatment of systematic uncertainties, Correlations between systematic uncertainties on different data sets

$$\chi^2_{\rm exp}(\mathbf{m},\mathbf{s}) = \sum_i \frac{\left(m^i - \sum_j \gamma^i_j m^i s_j - \mu^i\right)^2}{\delta^2_{i,stat}\mu^i m^i + \delta^2_{i,uncor}(m^i)^2} + \sum_j s_j^2 + \sum_i \ln \frac{\delta^2_{i,stat}\mu^i m^i + \left(\delta_{i,uncor} m^i\right)^2}{\delta^2_{i,stat} + \delta^2_{i,uncor}(\mu^i)^2}$$

s-systematic shifts
μ^i - value measured at point i
γ^i_j - relative correlated systematic uncertainties
$\delta_{i,stat}$- relative statistical uncertainties
$\delta_{i,uncor}$ - relative uncorrelated systematic uncertainties The method imposes that there is one and only one correct value for the cross section of each process at each point of the phase space. These values are obtained by optimising vector **m**

The experimental uncertainties of the resulting parton densities are determined by the Hessian method (as implemented in xFitter) with $\Delta\chi^2 = 1$

Hessian method in the Fit procedure

Hessian method - method to quantify the uncertainties of PDFs and their physical predictions
Quality of the fit between theory and experiment: $\chi^2_{\mathrm{global}} = \sum_n w_n \chi^2_n$
n - different data sets, w_n - weight for data set,
generic form of individual contribution: $\chi^2_n = \sum_l \left(\frac{D_{nl} - T_{nl}}{\sigma_{nl}}\right)^2$, l- data point, D- data value, T- theory value, σ - uncertainty of the data point. In practice: χ^2_n generalised (to include correlated errors, correlation matrix)
theory contains free parameters: $\{a_i\} = \{a_1, a_2, ..., a_d\}$
fit determines $\{a_i\}$
χ^2_{global} - depends on the PDF set S: how well data are fit by theory when PDF is defined by set of parameters $\{a_i(S)\}$
S_0 - best estimate Next step:
Variation of χ^2_{global} in the neighbourhood of the minimum : $\Delta\chi^2 = \chi^2 - \chi^2_0$
where $\chi^2 = \chi^2(S)$, $\chi^2_0 = \chi^2(S_0)$
Assumption:what is the allowed range of $\Delta\chi^2$? $\Delta\chi^2 \leq T^2$

Hessian method in the Fit procedure

$\Delta\chi^2 = \chi^2 - \chi_0^2 = \sum_{i=1}^{d}\sum_{j=1}^{d} H_{ij}(a_i - a_i^0)(a_j - a_j^0)$,
$\{a_j^0\} = \{a_j(S_0)\}$, $\{a_j\} = \{a_j(S)\}$, H_{ij}- Hessian matrix

Transformation between the original parameter space to the eigenvector basis:
$a_i - a_i^0 = \sum_{k=1}^{d} M_{ik} z_k$

Construct Eigenvector Basis sets $\{S_1^{\pm}, ..., S_d^{\pm}\}$:
displacement of a magnitude t up and down along each of the d eigenvector directions
$z_k(S_l^{\pm}) = \pm t\delta_{kl}$
The parameters that specify Eigenvector Basis sets: $a_i(S_l^{\pm}) - a_i^0 = \pm t M_{il}$

Uncertainty of any variable $X(S)$ (e.g. cross section):
best fit estimate: $X^0 = X(S^0)$
uncertainity: evaluate X for each of the $2d$ sets $\{S_l^{\pm}\}$
$\frac{\partial X}{\partial z_k} = \frac{X(S_k^+) - X(S_k^-)}{2t} = \frac{D_k(X)}{2t}$
$D(X) = \sum_k (D_k(X))^2$, $\hat{D}_k(X) = \frac{D_k(X)}{D(X)}$, $\Delta X = \sum_{k=1}^{d}(T\hat{D}_k \frac{\partial X}{\partial z_k})$
$\Delta X = \frac{T}{2t} D(X)$

Model uncertainties

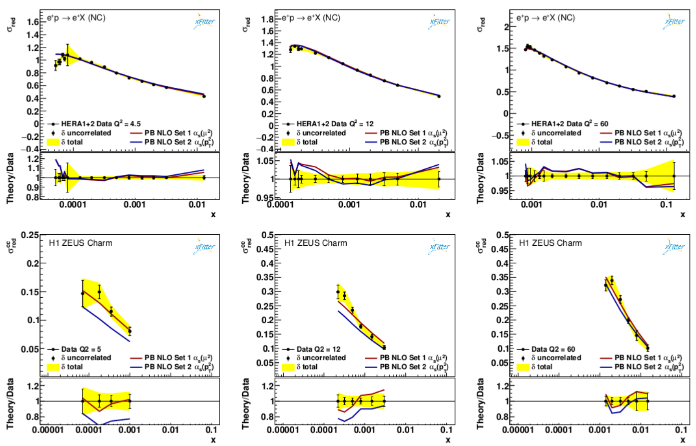
Model uncertainties:

- variation of m_c, m_b
- variation of μ_0
- Set2: variation of *qcut* in α_s (to protect situation when the scale in $\alpha_s\left((1-z)^2\mu^2\right)$ too small)

	Central value	Lower value	Upper value
Set 1 μ_0^2 (GeV^2)	1.9	1.6	2.2
Set 2 μ_0^2 (GeV^2)	1.4	1.1	1.7
Set 2 q_{cut} (GeV)	1.0	0.9	1.1
m_c (GeV)	1.47	1.41	1.53
m_b (GeV)	4.5	4.25	4.75

Fit to HERA F_2 data

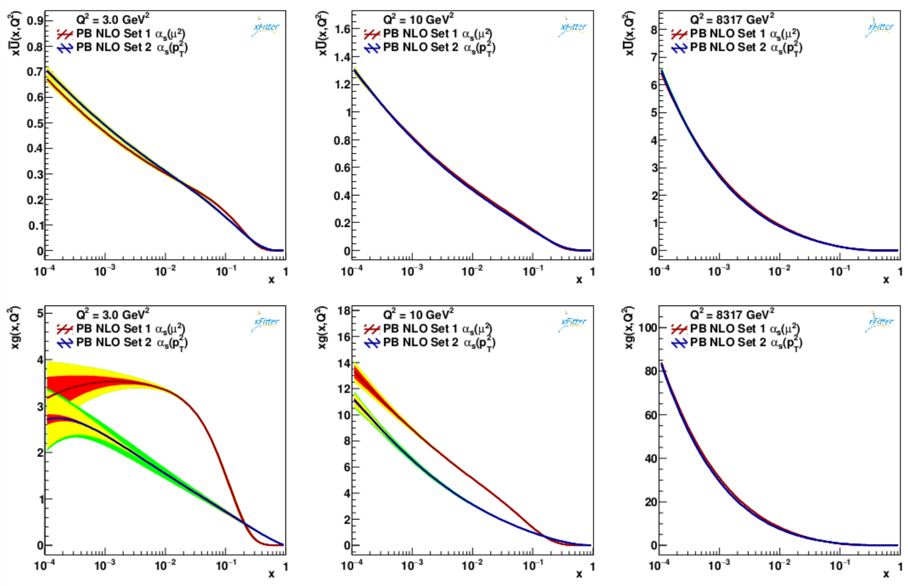
Measurement of the reduced cross section obtained at HERA compared to predictions using Set 1 and Set 2



predictions for the inclusive DIS cross section (top) and the inclusive charm cross section (bottom) obtained from the two different parton distributions compared to the measurements from HERA

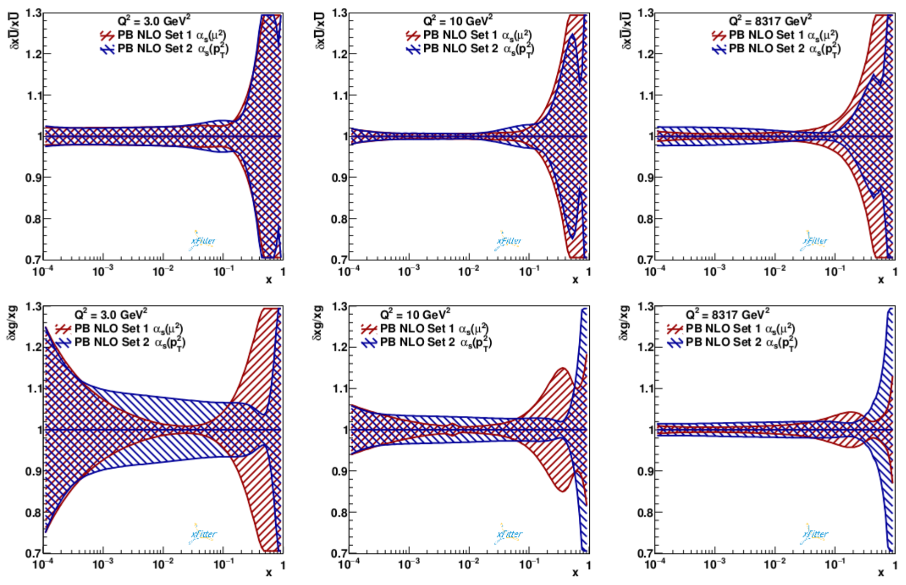
It has been checked explicitly that including the charm measurements in the fits does not significantly change the fit result (the charm data have too large uncertainty compared to the precise inclusive measurements)

Parton densities from the fit

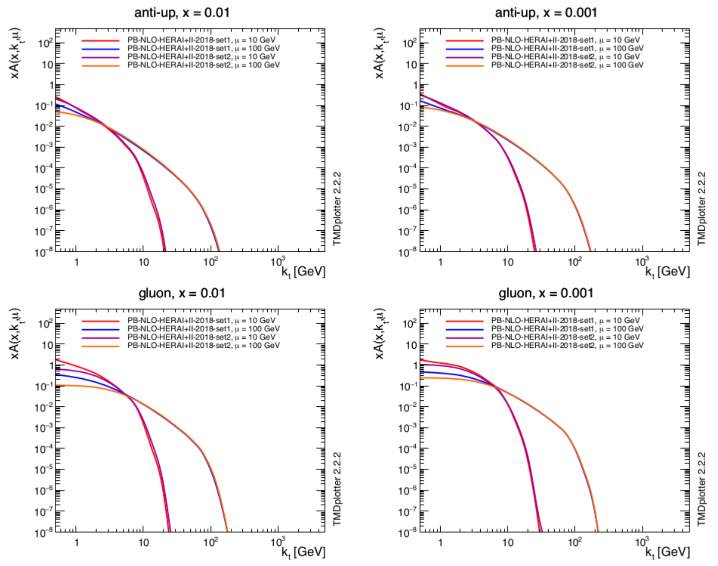


experimental, model, qcut for Set2

Total uncertainties

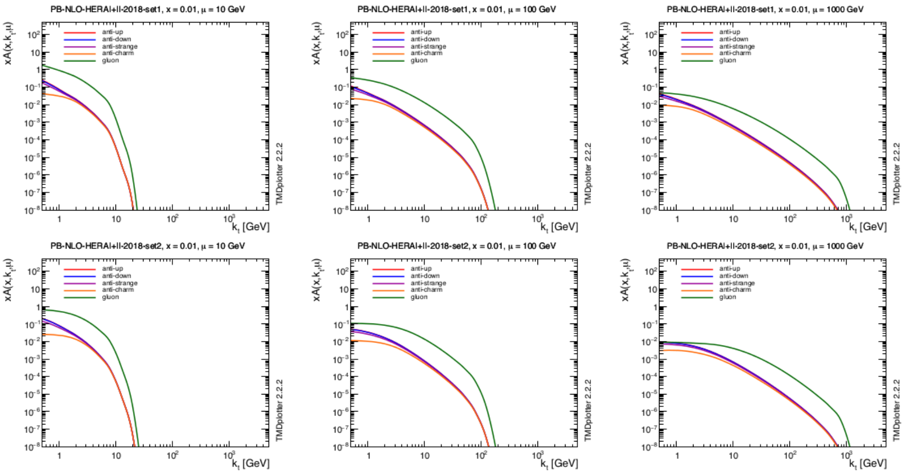


TMDs from the fit



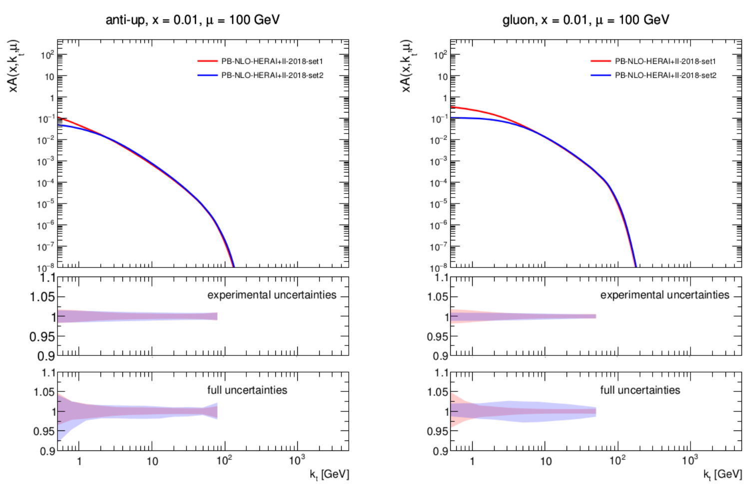
TMDs from the fit

TMDs from the fit

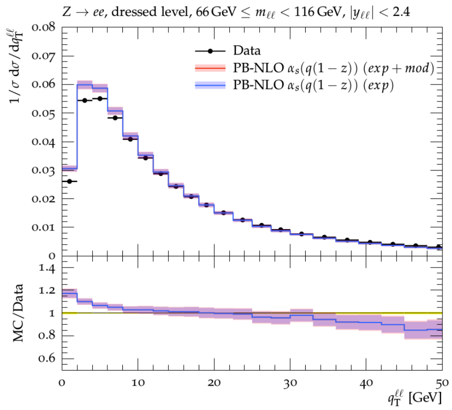


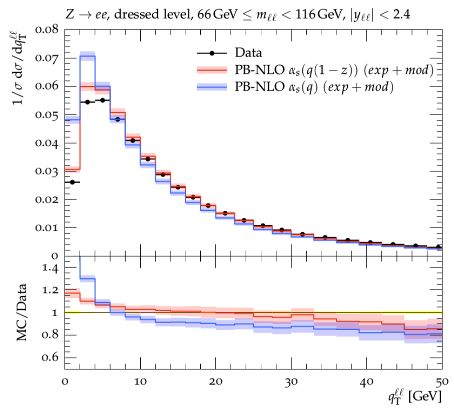
TMDs from the fit

TMD uncertainties



Only collinear splitting functions are used and the fit was obtained with collinear parton densities, but a $k_\perp$ dependence of the uncertainties is obtained, which comes from the different contributions to the spectrum. The experimental uncertainties are small over the whole range, while the model dependent uncertainties dominate.





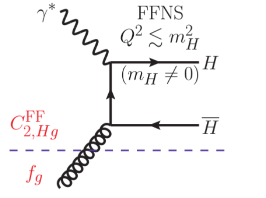
The difference between the full and experimental uncertainties from the fit is very small
no adjustment of any parameter is made, the TMDs are entirely constrained by the fits to inclusive DIS data

Heavy quark treatment

Correct treatment of heavy flavours in PDFs essential for precision measurements at hadron colliders

Two ideas: c, b - massive particles produced in the hard scattering or c, b-massless particles in the proton

- Fixed Flavour Number Scheme (FFNS)
  based on: $Q^2 \lesssim m_H^2 \; \rightarrow$ heavy quarks are final state particles, not partons inside a proton
  n_f- number of flavours in PDFs (different versions of FFNS: $n_f = 3,\ 4,\ 5$)
  $F_i\left(x, Q^2\right) = \sum_k C_{i,k}^{FF,n_f}\left(\frac{Q^2}{m_H^2}\right) \otimes f_k^{n_f}\left(Q^2\right)$
  Problem:
  1.) it does not sum $\alpha_s^m \ln^l(Q^2/m_H^2)$ $(l \leq m)$ in perturbative expansion $\rightarrow$ accuracy for $Q^2 > m_H^2$ uncertain
  2.) σ with mass dependence only for few processes at NLO
- Zero-Mass Variable Flavour Number Scheme (ZM-VFNS)
- General-Mass Variable Flavour Number Scheme (GM-VFNS)



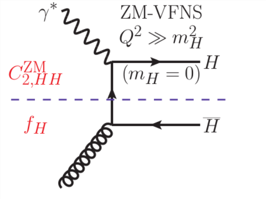
H - heavy quark

Heavy quark treatment

Correct treatment of heavy flavours in PDFs essential for precision measurements at hadron colliders

Two ideas: c, b - massive particles produced in the hard scattering or c, b-massless particles in the proton

- Fixed Flavour Number Scheme (FFNS)
- Zero-Mass Variable Flavour Number Scheme (ZM-VFNS)
  based on: $Q^2 \gg m_H^2$ heavy quarks behave like massless partons
  heavy quarks evolve according to splitting functions for massless quarks
  $n_f - 3$- number of active heavy flavours
  $F_i\left(x, Q^2\right) = \sum_j C_{i,j}^{ZM,n_f} \otimes f_j^{n_f}\left(Q^2\right)$
  mass dependence in the boundary conditions for evolution
  $f_j^{n+1}(Q^2) = \sum_k A_{jk}\left(Q^2/m_H^2\right) \otimes f_k^n(Q^2)$
  $A_{jk}\left(Q^2/m_H^2\right)$ perturbative matrix element containing $\ln\left(Q^2/m_H^2\right)$
  Problems:
  1.) it ignores $\mathcal{O}(m_H^2/Q^2)$ in C, innacurate for $Q^2 \gtrsim m_H^2$
- General-Mass Variable Flavour Number Scheme (GM-VFNS)



H - heavy quark

Heavy quark treatment

Correct treatment of heavy flavours in PDFs essential for precision measurements at hadron colliders

Two ideas: c, b - massive particles produced in the hard scattering or c, b-massless particles in the proton

- Fixed Flavour Number Scheme (FFNS)
- Zero-Mass Variable Flavour Number Scheme (ZM-VFNS)
- General-Mass Variable Flavour Number Scheme (GM-VFNS)
  smooth connection of in limits $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$
  equivalence of the descriptions:$n_f = n$ (FFNS) and $n_f = n + 1$ (GM-VFNS) above the transition point
  $F_i\left(x, Q^2\right) = \sum_k C_{i,k}^{FF,n}\left(\frac{Q^2}{m_H^2}\right) \otimes f_k^n\left(Q^2\right) = \sum_j C_{i,j}^{VF,n+1}\left(\frac{Q^2}{m_H^2}\right) \otimes f_j^{n+1}\left(Q^2\right)$
  Problems:
  1.) uniquely defined for $Q^2/m_H^2 \to \infty$
  for finite Q^2/m_H^2 one can swap terms $\mathcal{O}(Q^2/m_H^2)$ between different $C \to$ different versions of GM-VFNS: ACOT, TR (impose the correct kinematical requirement that (in neutral current DIS) one must have enough energy to create a pair of massive quarks in the final state by demanding continuity of $\frac{\mathrm{d}F_{2,H}}{\mathrm{d}\ln Q^2}$)

Measurement of heavy quark structure functions is a direct test of heavy flavour schemes