BFKL and High-Energy Behaviour of HERA data

Motivation:

Confinement forces emerge from complicated gluon-gluon interaction,

experimental studies of confinement are difficult because everything happens very quickly, 10^{-23} sec

However, at very high energies time dilatation slows down all physics processes. This allows to describe complicated gluon structures from first principles

- BFKL resummation of all Feynman diagrams for gluon-gluon interaction
- Phenomenological investigation of HE-Behaviour of $\sigma^{\gamma p} \propto F_2$

Henri Kowalski

x-Fitter Meeting, Minsk, 19th of March 2019

the talk is based on:

Decoupling of the leading contribution in the discrete BFKL analysis of high-precision HERA data

Eur. Phys. J. C (2017) 77:777

H. Kowalski, L. N. Lipatov, D. A. Ross, O. Schulz

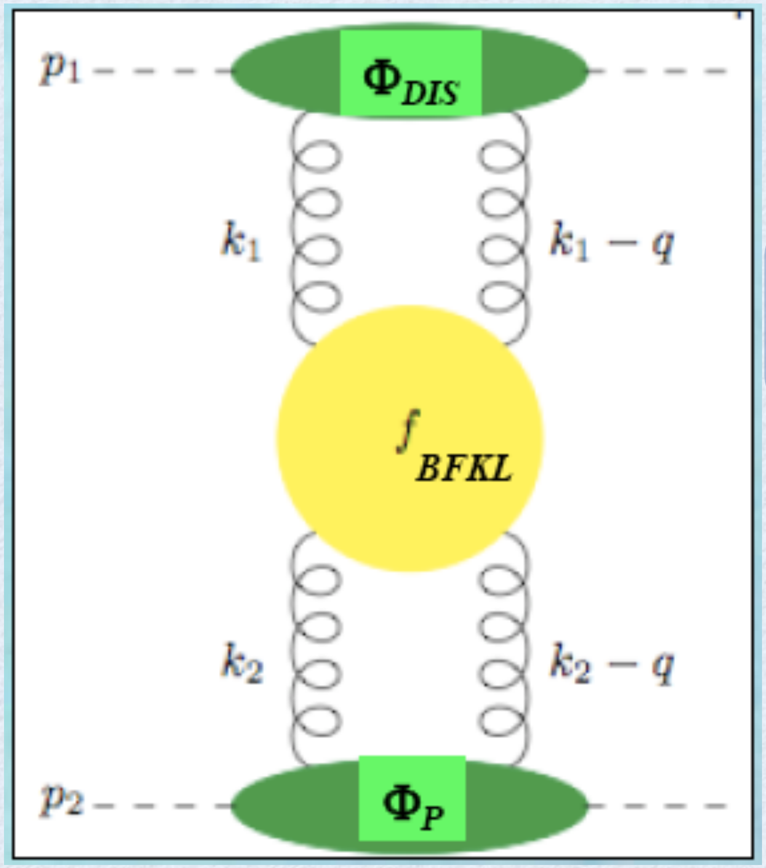
- ✦ Discrete spectrum of BFKL eigenstates
- ✦ Analysis of HERA-F_2 ☛ BFKL-Ground State has to be saturated
- ✦ Dipole Picture as a natural framework for analysis of saturated
- ✦ Ground State

Investigation of High Energy Behaviour of HERA data using λ-Fits

A. Luszczak, H. Kowalski

The dynamics of Gluon Density at low x is determined by the amplitude for the scattering of a gluon on a gluon, described by the BFKL equation

$$\frac{\partial}{\partial \ln s}\mathcal{A}(s,\mathbf{k},\mathbf{k}') \;=\; \delta(k^2-k'^2)+\int dq^2\mathcal{K}(\mathbf{k},\mathbf{q})\mathcal{A}(s,\mathbf{q},\mathbf{k}')$$



solved by the Green function method, in terms of the eigenfunctions of the kernel

$$\int dk'^2\mathcal{K}(\mathbf{k},\mathbf{k}')f_\omega(\mathbf{k}') \;=\; \omega f_\omega(\mathbf{k})$$

in LO, with fixed α_s

$$f_\omega(\mathrm{k}) = \exp(i\nu\ln k^2)/k$$
$$\omega \;=\; \alpha_s\chi_0(\nu)$$

Green f. method - preserves the scaling (conformal) invariance of BFKL

⇒ most consistent solution of BFKL

a possible bridge to Pomeron-Graviton?

In LO with fixed α_s

BFKL (saddle point aprox.)
predicts that F_2 is dominated by
the leading trajectory $F_2 \sim (1/x)^{\omega}$

$$\omega = 4\,\alpha_s \ln 2 \approx 0.5$$

independent of Q^2

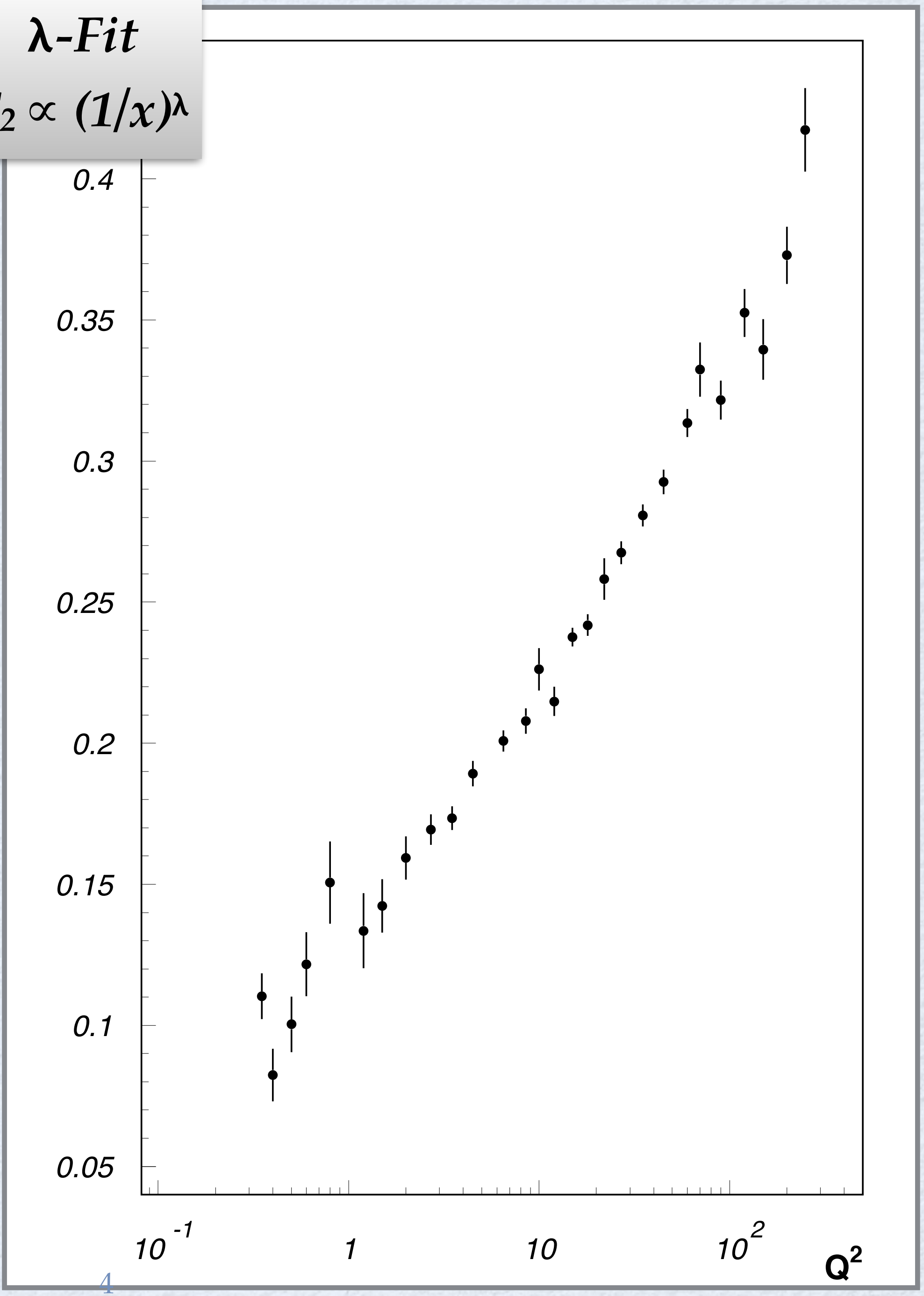
with NLO with resummation
$\omega \approx 0.3$
also independent of Q^2
$\Rightarrow$

clear contradiction to HERA data

but (Lipatov 1986)
BFKL solutions with running α_s
and infrared cutoff
are given by discrete
eigenfunction

λ-Fit

$F_2 \propto (1/x)^{\lambda}$



KLRO ⇒

the eigenfunctions of the BFKL Kernel are given by

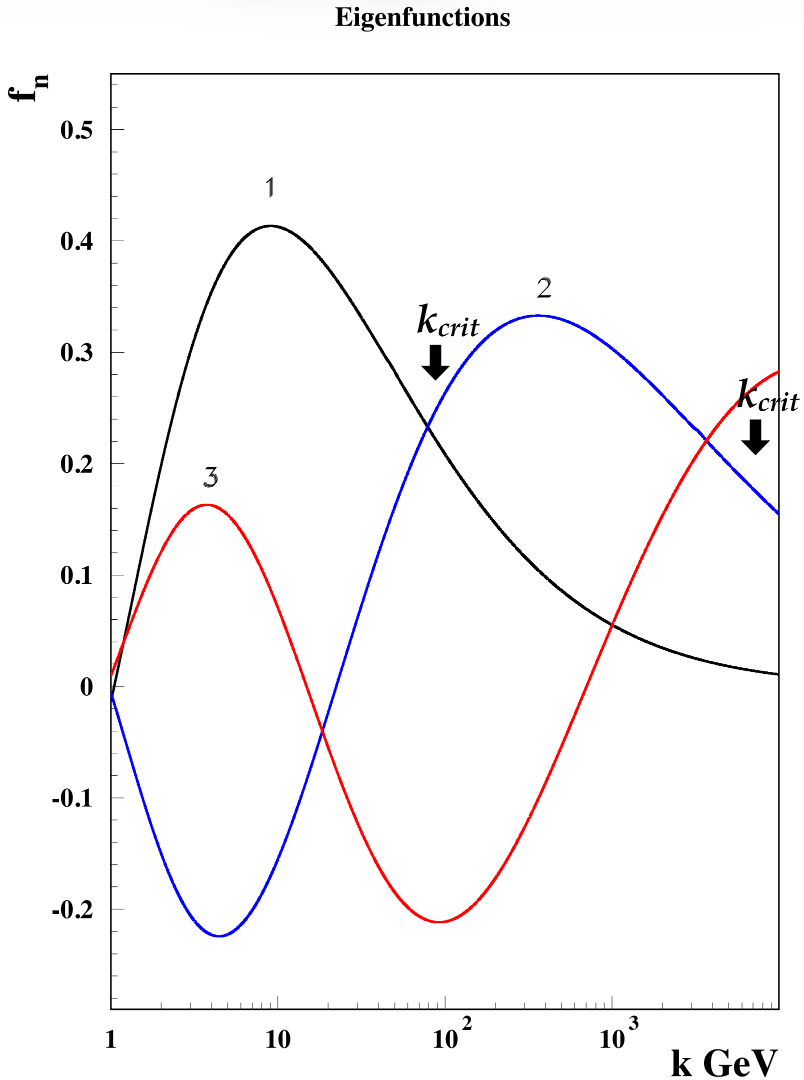
$$f_{\omega_n}(t) \;=\; \sqrt{\frac{\pi}{\phi'(\omega_n)}}N_{\omega_n}(t)Ai(z(t)).$$

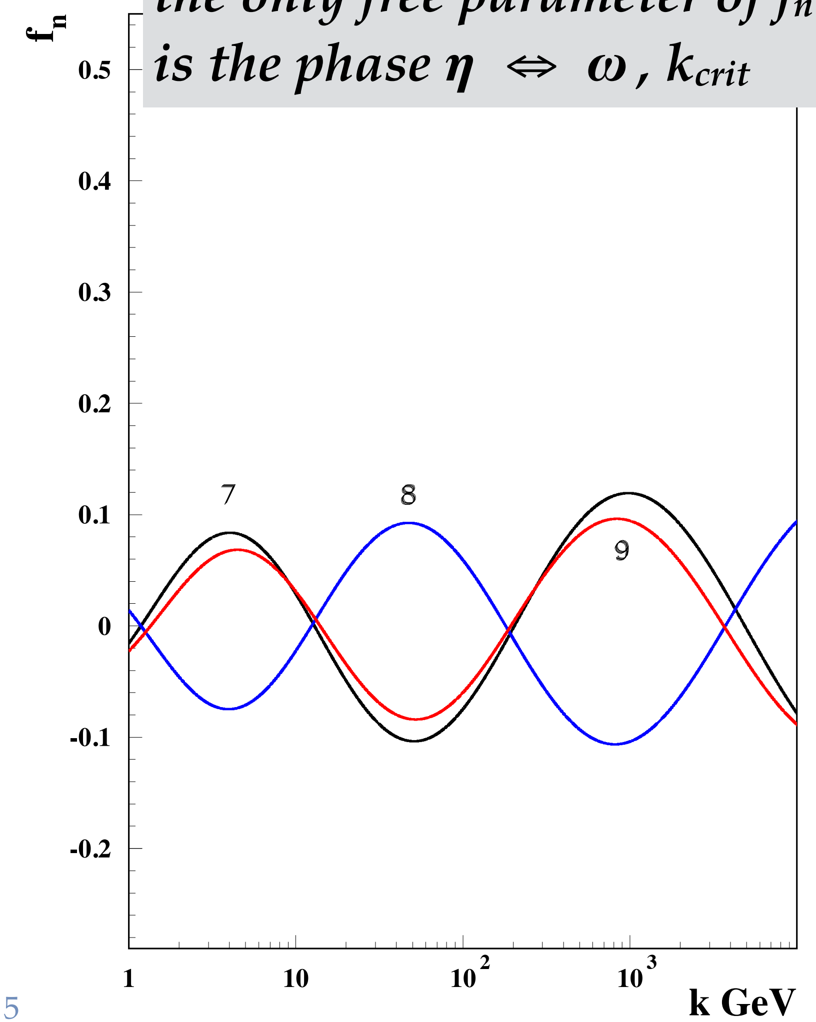
N_ω, z, ϕ are known

functions of $t = ln(k^2/\Lambda)$

the only free parameter of $f_n(t)$

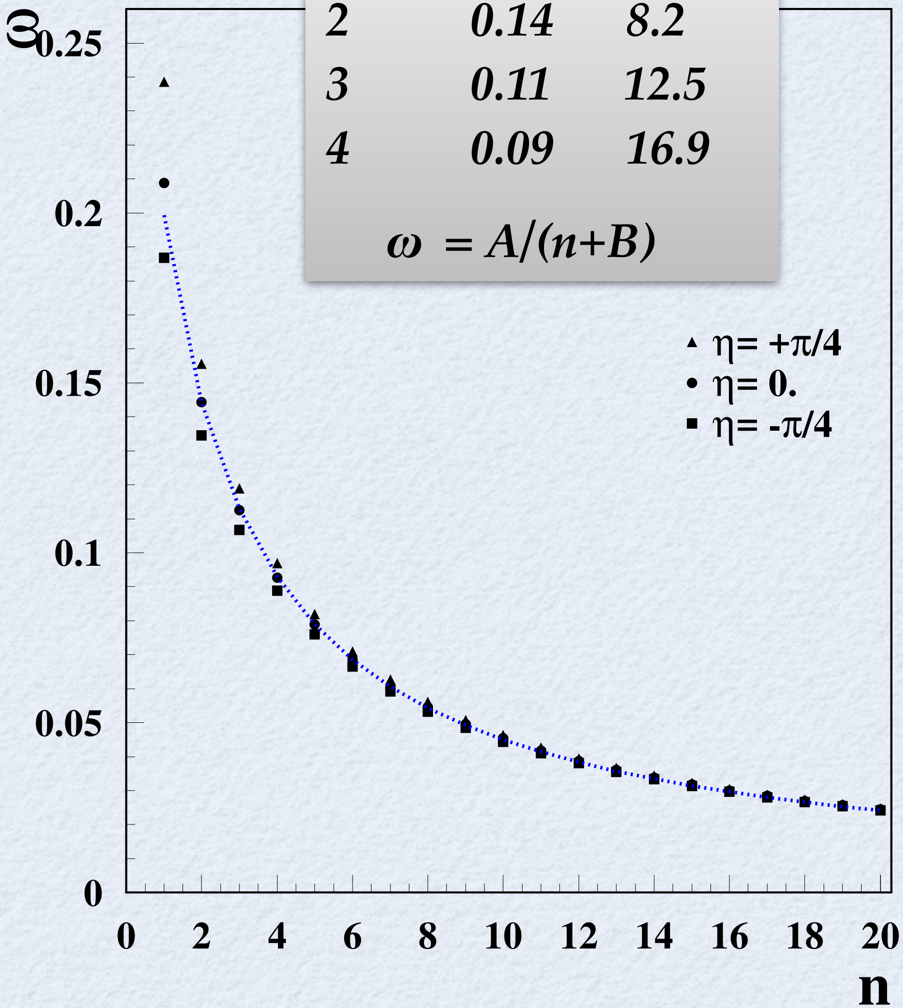
is the phase η ⇔ ω, k_{crit}



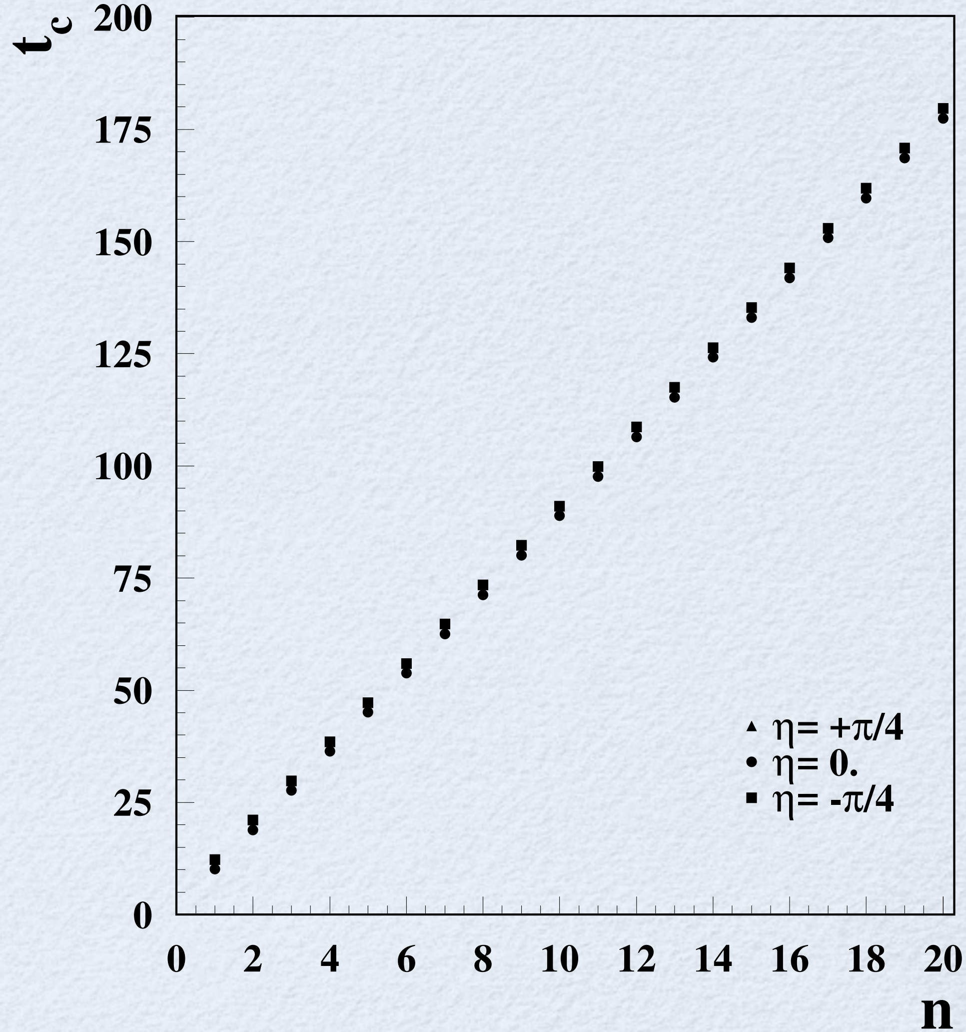


n	ω	$ln(k_{crit})$
1	0.2	3.8
2	0.14	8.2
3	0.11	12.5
4	0.09	16.9

$$\omega = A/(n+B)$$



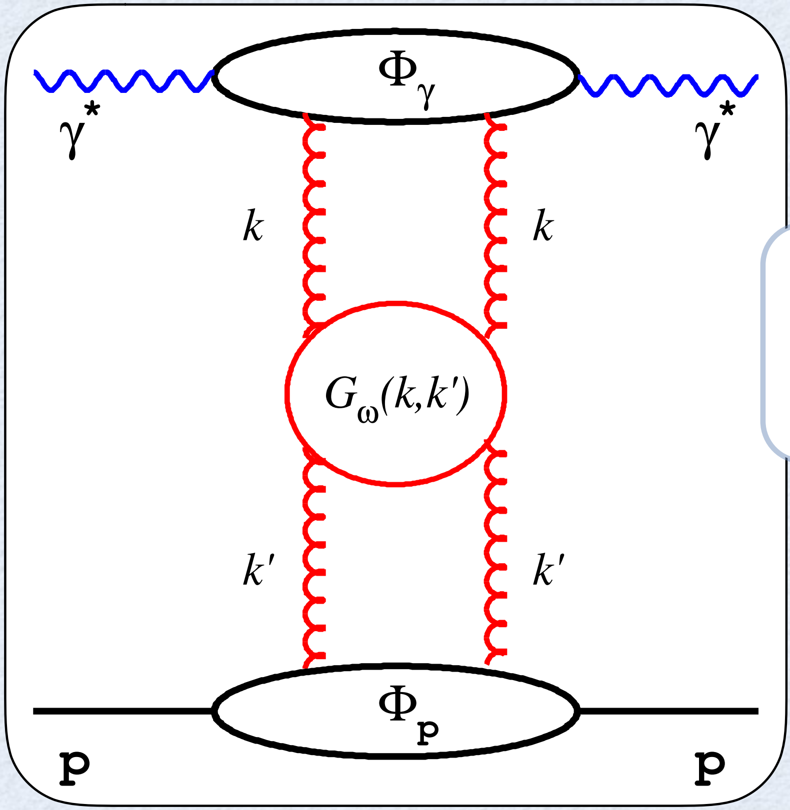
$$t_c = ln(k_{crit}^2 / \Lambda_{QCD})$$



Comparison with HERA data

Construct the Green Function and integrate it with the proton impact factor

$$xg(x,k) = \int \frac{dk'}{k'}\Phi_p(k')\left(\frac{k'\,x}{k}\right)^{-\omega_n} k^2\left(\sum_n f^*_{\omega_n}(k')f_{\omega_n}(k) + \int_{-\infty}^{0} d\omega x^{-\omega} f_{-|\omega|}(t)f_{-|\omega|}(t')\right)$$

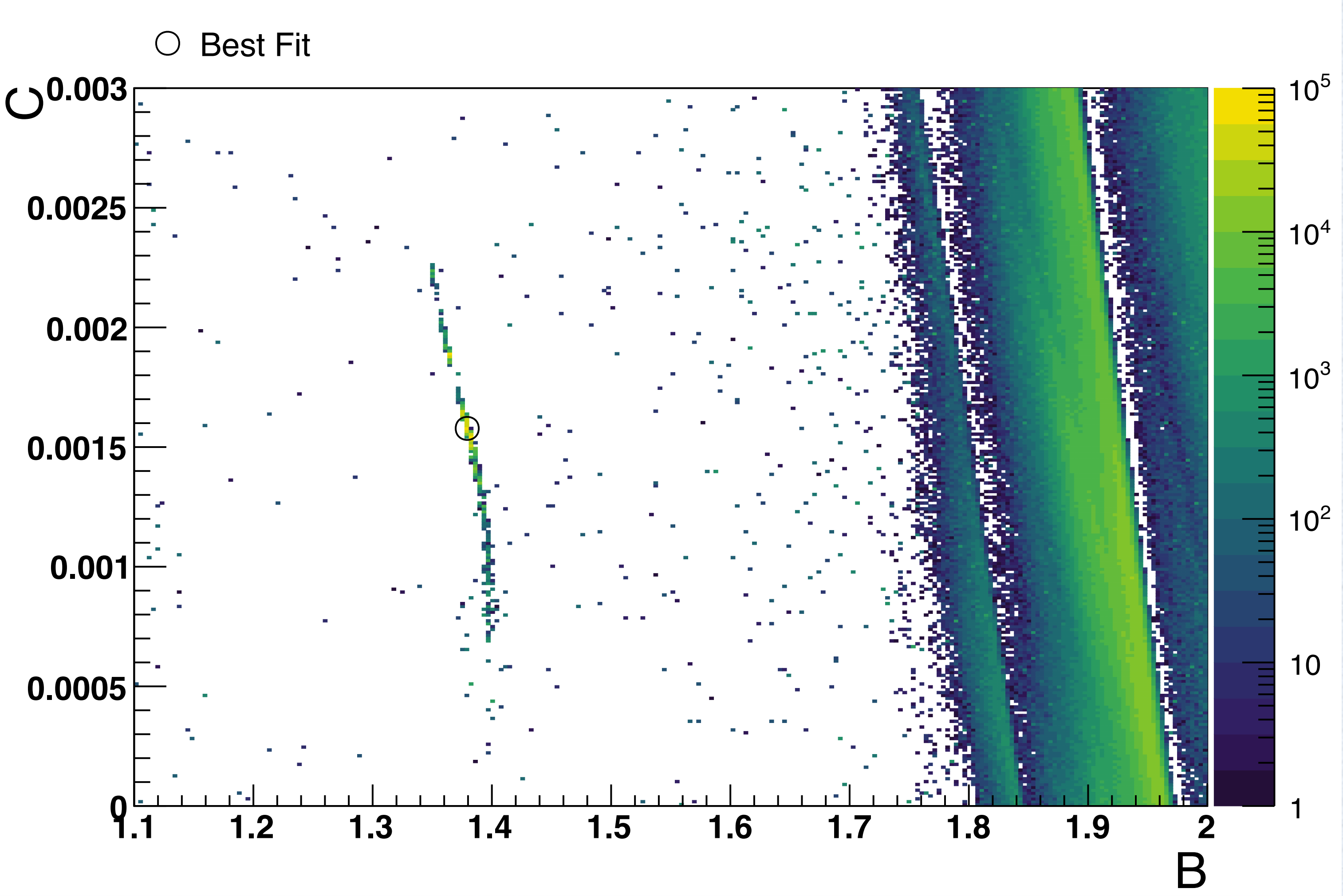


Integrate the gluon density with the photon impact factor

$$F_2(x,Q^2) = \int_x^1 d\zeta \int \frac{dk}{k}\Phi_\gamma(\zeta,Q,k)xg\left(\frac{x}{\zeta},k\right)$$

Markov-Chain MC
Bayesian Analysis Tool
probability density of the fit

$$\omega = A/(n+B) + C$$



$$\Phi_p(\mathbf{k}) = A k^2 e^{-bk^2},$$

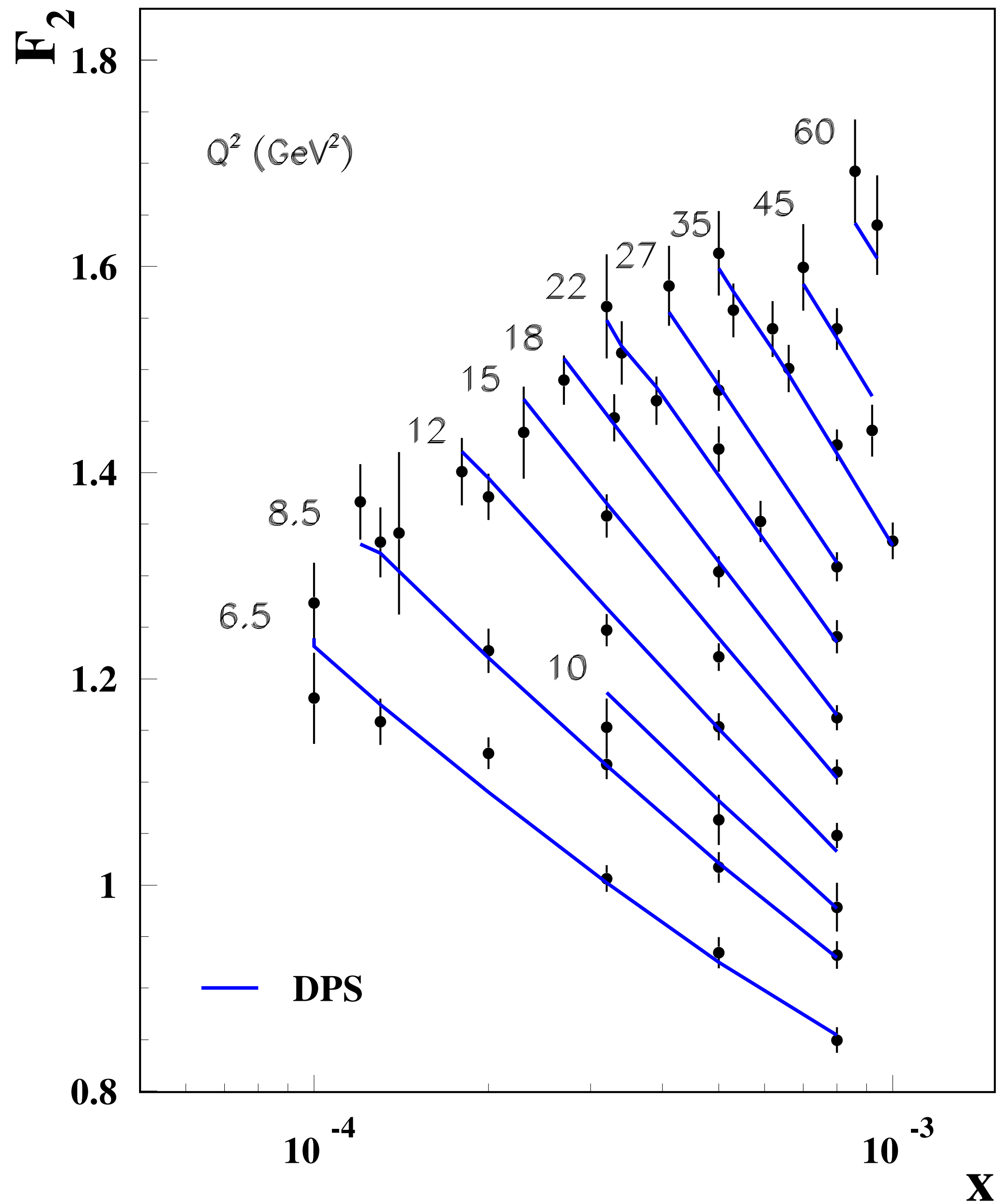
$Q^2 > 6$ GeV^2
$x < 0.001$

$N_p = 51$

b (GeV^{-2})	10	20
A	0.48771	0.47905
B	1.37933	1.34020
C	0.001578	0.002424
η_{neg}	-0.0754	-0.0518
χ^2	32.9	33.1

b (GeV^{-2})	10	20
A	0.51844	0.51913
B	1.58697	1.58657
η_{neg}	-0.0911	-0.0550
χ^2	33.9	33.3

$\eta_1 = 0.0707$ $\eta_1 = 0.0503$



very low $\chi^2/N_{df} = 0{,}72$
due to uncorrelated errors

Q^2 dependence

KMS photon impact factor, computed beyond LO

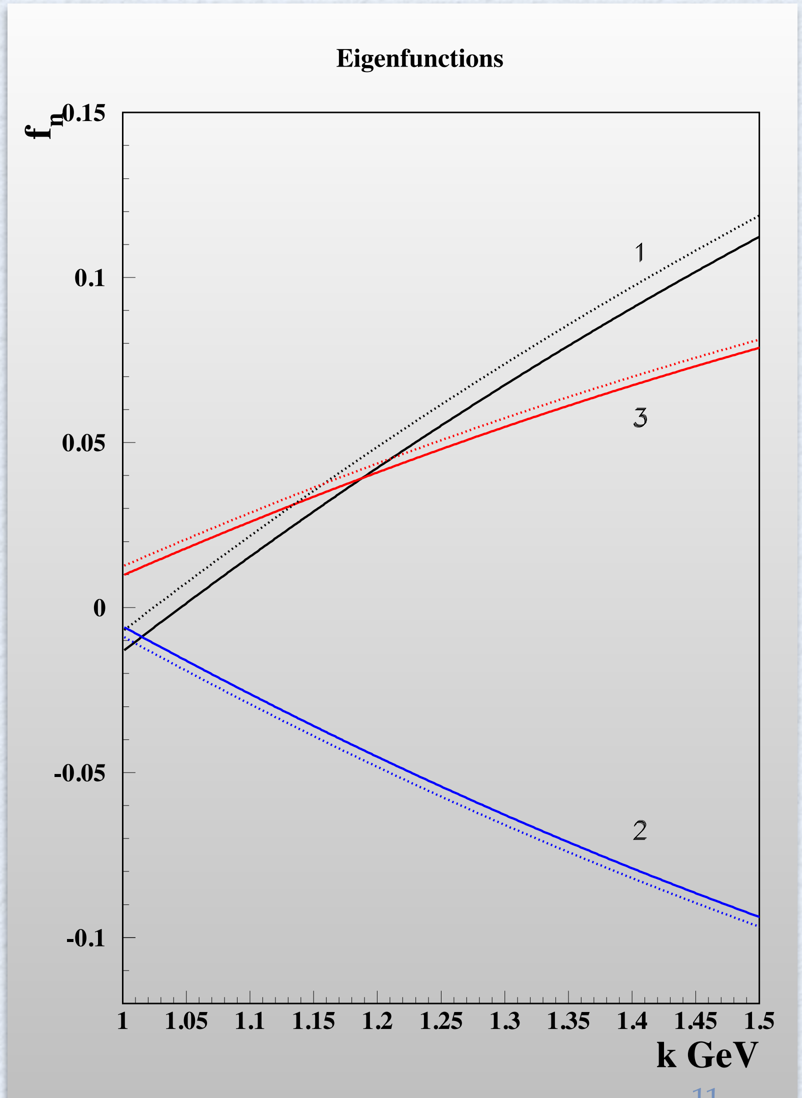
Q^2 cut (GeV^2)	4	6	9
A	0.51852	0.51844	0.51818
B	1.58847	1.58697	1.58356
η_{neg}	-0.0911	-0.0911	-0.0911
N_p	59	51	37
χ^2	68.5	33.9	17.4
χ^2/N_{df}	1.25	0.72	0,52

Dipole photon impact factor, ABC fit, with JULIA

χ^2 /N	**0.76**	**0.61**	**0.52**

Decoupling of the leading eigenfunction

the fit selected always a phase for which $\int \Phi_p(k)\, f_1(k)\, d(\ln k) \approx 0$



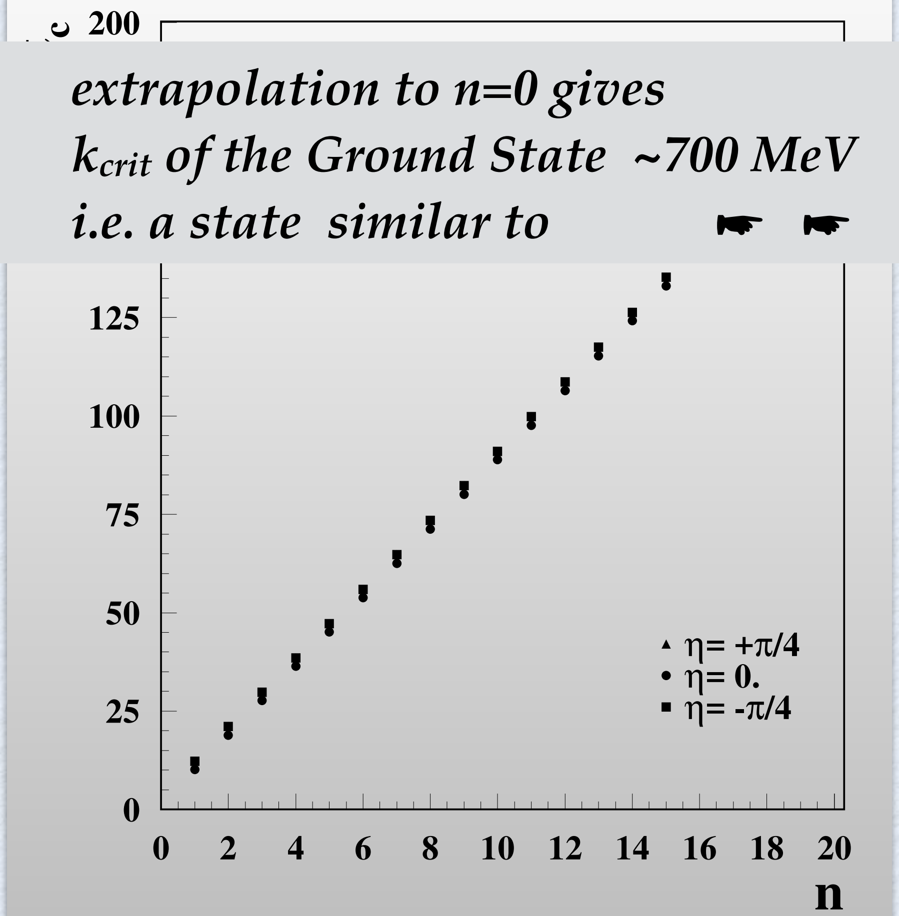
$$\Phi_p(k) = Ak^2 \exp(-bk^2)$$

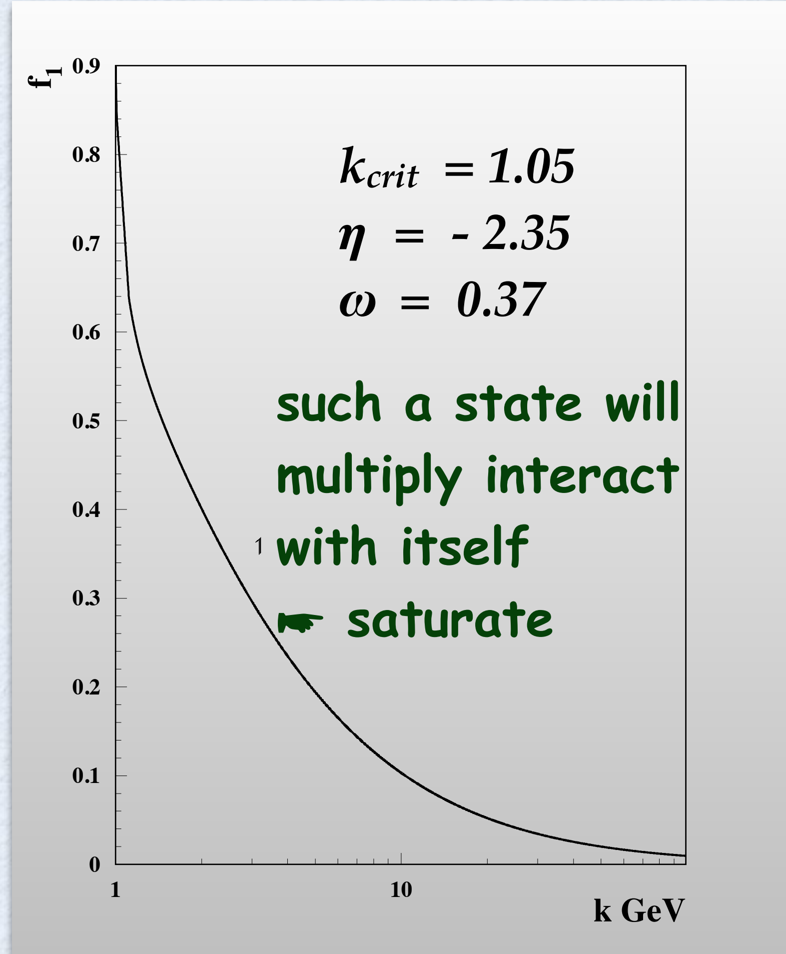
——— $b = 10\ GeV^{-2}$

.... $b = 20\ GeV^{-2}$

$\Rightarrow f_1(t)$ **cannot be the Ground State of BFKL**

extrapolation to n=0 gives
k_{crit} of the Ground State ~700 MeV
i.e. a state similar to ☛ ☛





$k_{crit} = 1.05$
$\eta = -2.35$
$\omega = 0.37$

such a state will multiply interact with itself
☛ saturate

→ Evaluate using Dipole Model

Dipole Model evaluation

$$\sigma_{T,L}^{\gamma^* p}(x,Q) = \sum_f \int d^2\vec{r} \int_0^1 \frac{dz}{4\pi} (\Psi^*\Psi)_{T,L}^f \, \sigma_{q\bar{q}}(x,r)$$

$$\frac{d\sigma_{q\bar{q}}}{d^2\vec{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_S(\mu^2) x g(x,\mu^2) T(b)\right)\right].$$

use BFKL Gluon Density with

$$\mu^2 = k^2 = C/r^2 + \mu_0^2$$

Ground State has some similarity to GBW Ansatz

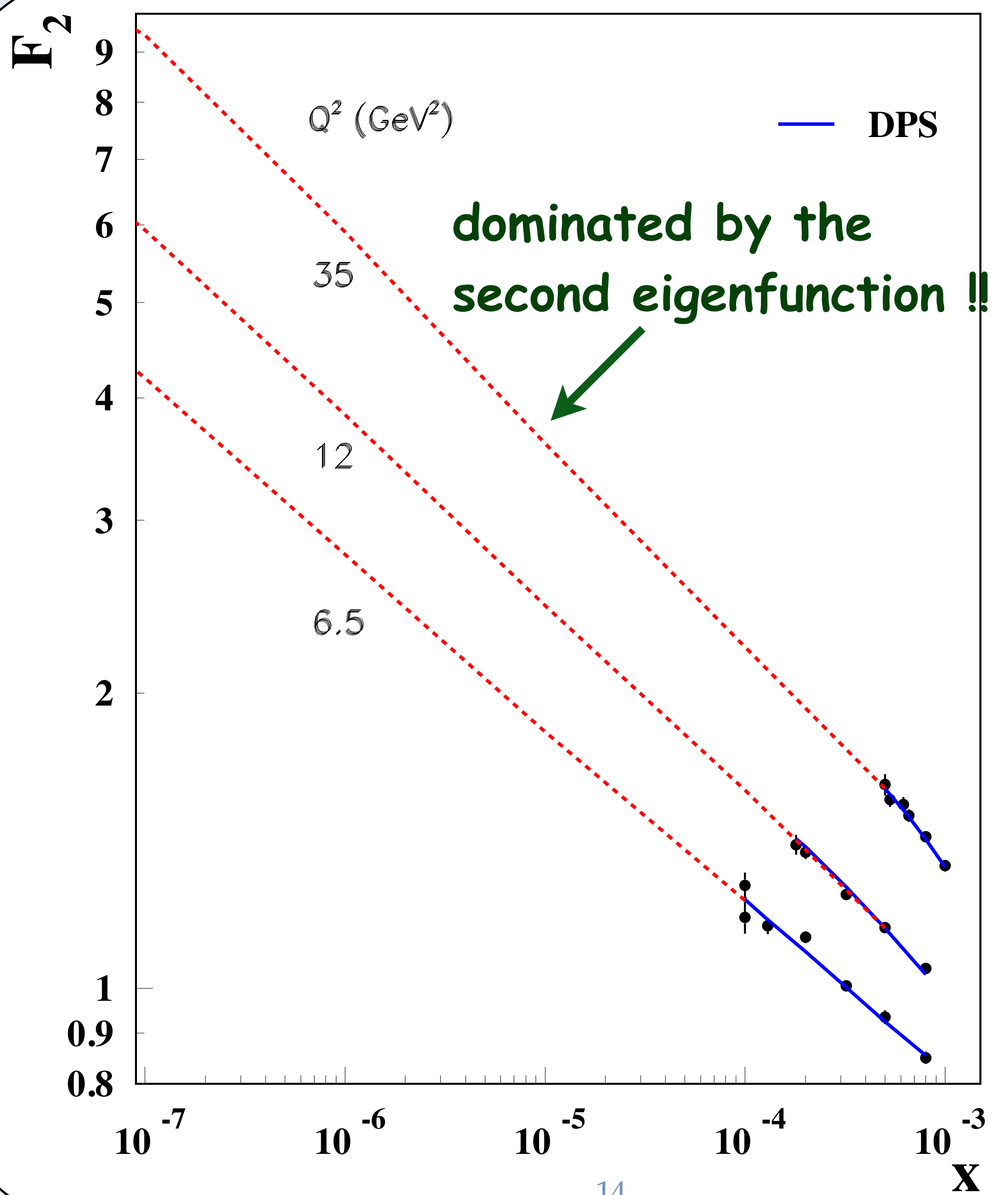
$$\sigma_{q\bar{q}}^{\mathrm{GBW}}(x,r) = \sigma_0 \left(1 - \mathrm{e}^{-r^2 Q_s^2(x)/4}\right)$$

$$Q_s^2(x) = (x_0/x)^{\lambda_{\mathrm{GBW}}}$$

~ exp(-1/r^2)

but exponentially suppressed at small r^2

Extrapolation of the BFKL fit to very low x



VHEep or LHeC range

Investigation of the High Energy Behaviour of F_2 using the highest precision HERA data

λ-Fit $F_2 \propto (1/x)^{\lambda}$

data are given as reduced cross sections

$$\sigma_{red} \approx F_2 - \frac{y^2}{Y_+} F_L,$$

Following the H1 and ZEUS F_L measurements we assumed: $F_L \propto F_2$

$$F_2 = \frac{\sigma_{red}}{1 - \frac{y^2 R}{Y_+(1+R)}},$$

$$Y_+ = 1 + (1-y)^2$$

$R = 0.23 \pm 0.04$ independent on x and Q^2; desy13-211, H1

$R \approx 0.20$ ZEUS result

we assumed R = 0.25 and as check used also R=0.2 and 0.3

$R \approx 0.25$ is in agreement with DGLAP NLO fits and Dipole Models

Note:

the agreement between DGLAP and Dipole Models is non-trivial because F_L and F_T are not determined by the (dynamical) properties of the gluon density, they are determined by the photon impact factor, which are different in DGLAP and DM (collinear vs transverse factorisation)

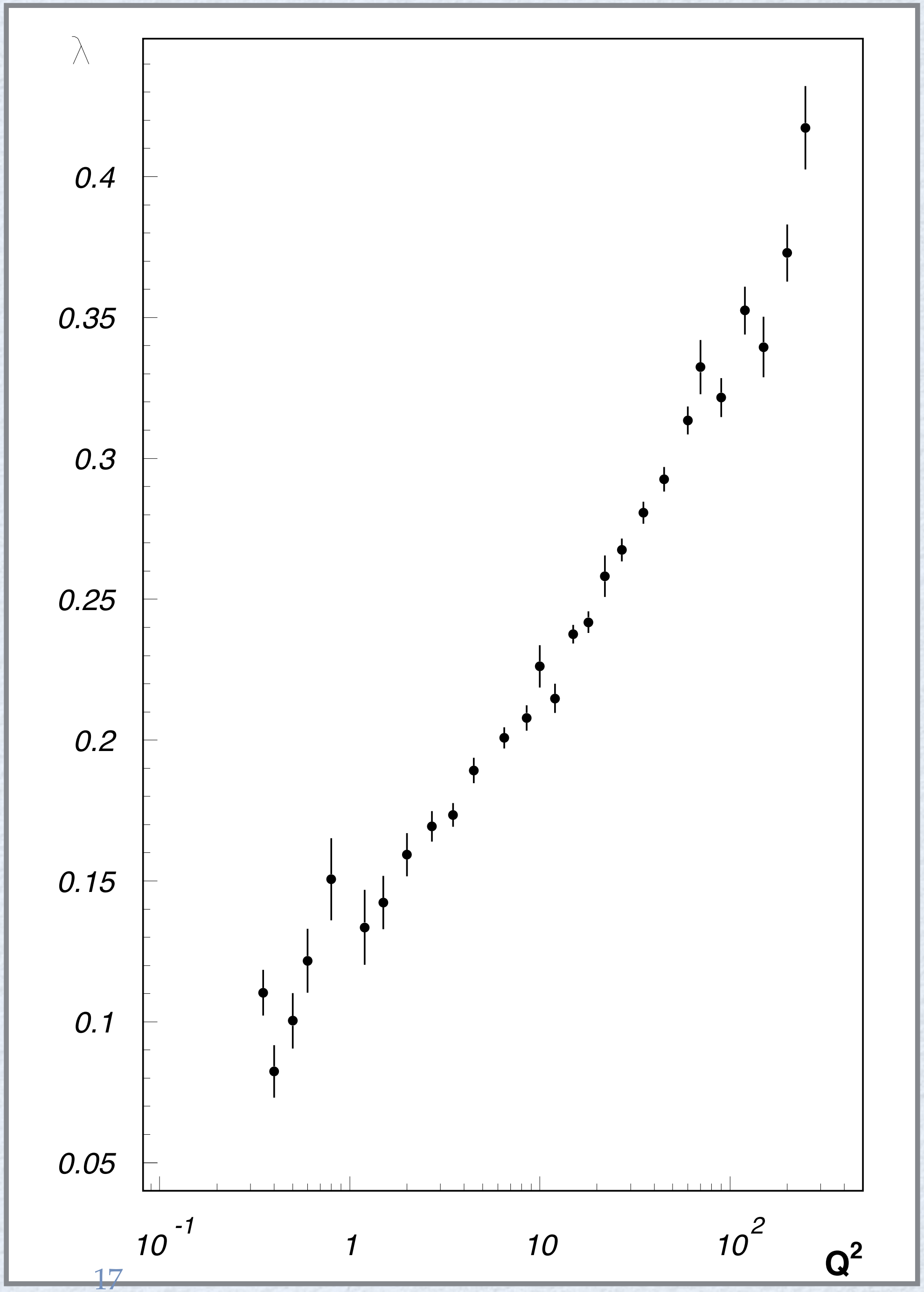
Results of the

$$\lambda\text{-}Fit: \quad F_2 \propto (1/x)^{\lambda}$$

good quality of fits
on average:

$$\chi^2/N_{df} \;=\; 1.05$$

can we see some signs of dominance of the second eigenfunction at HERA?

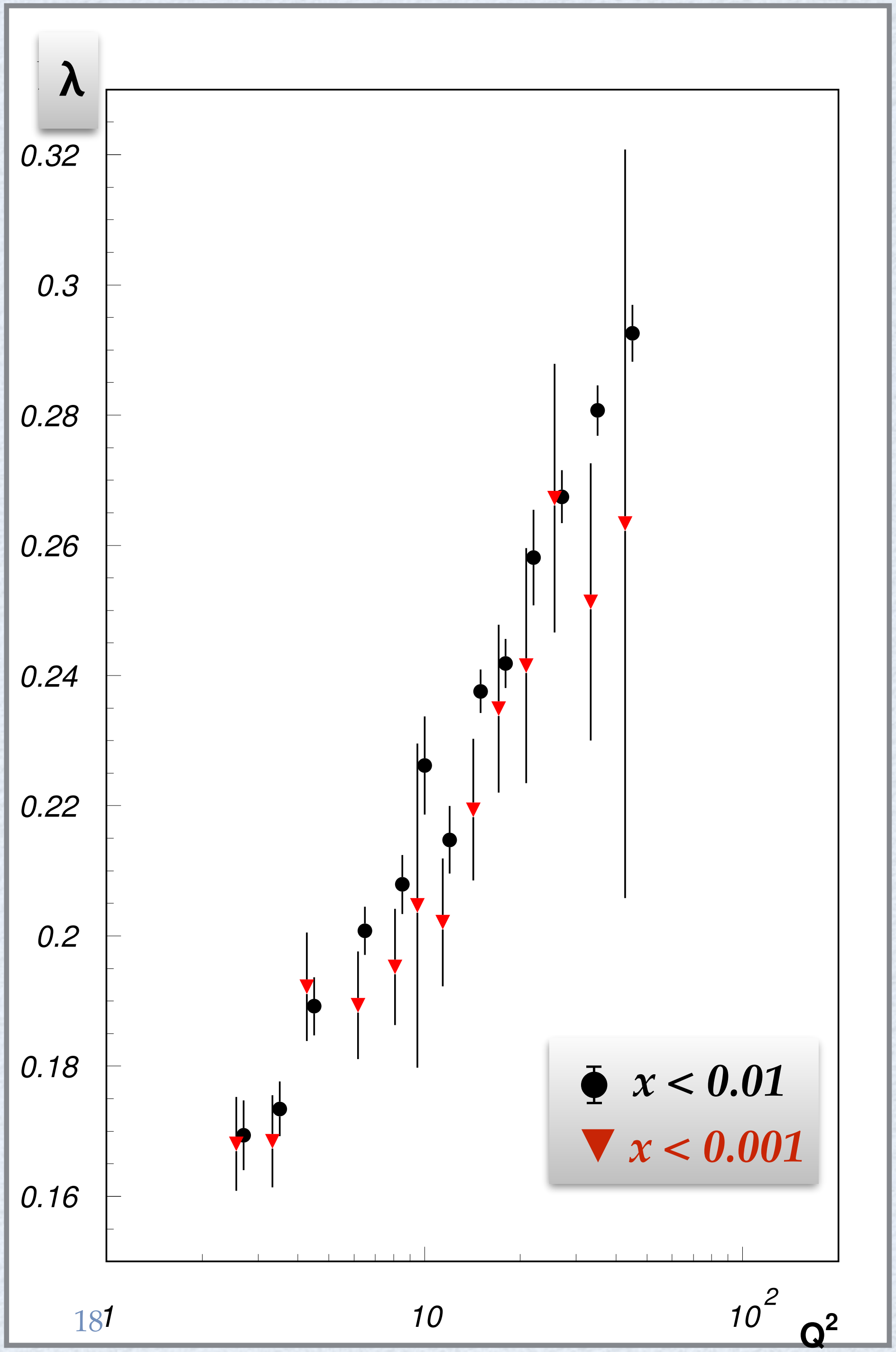


comparison of the rate of rise λ

in the low-x, $x < 0.01$

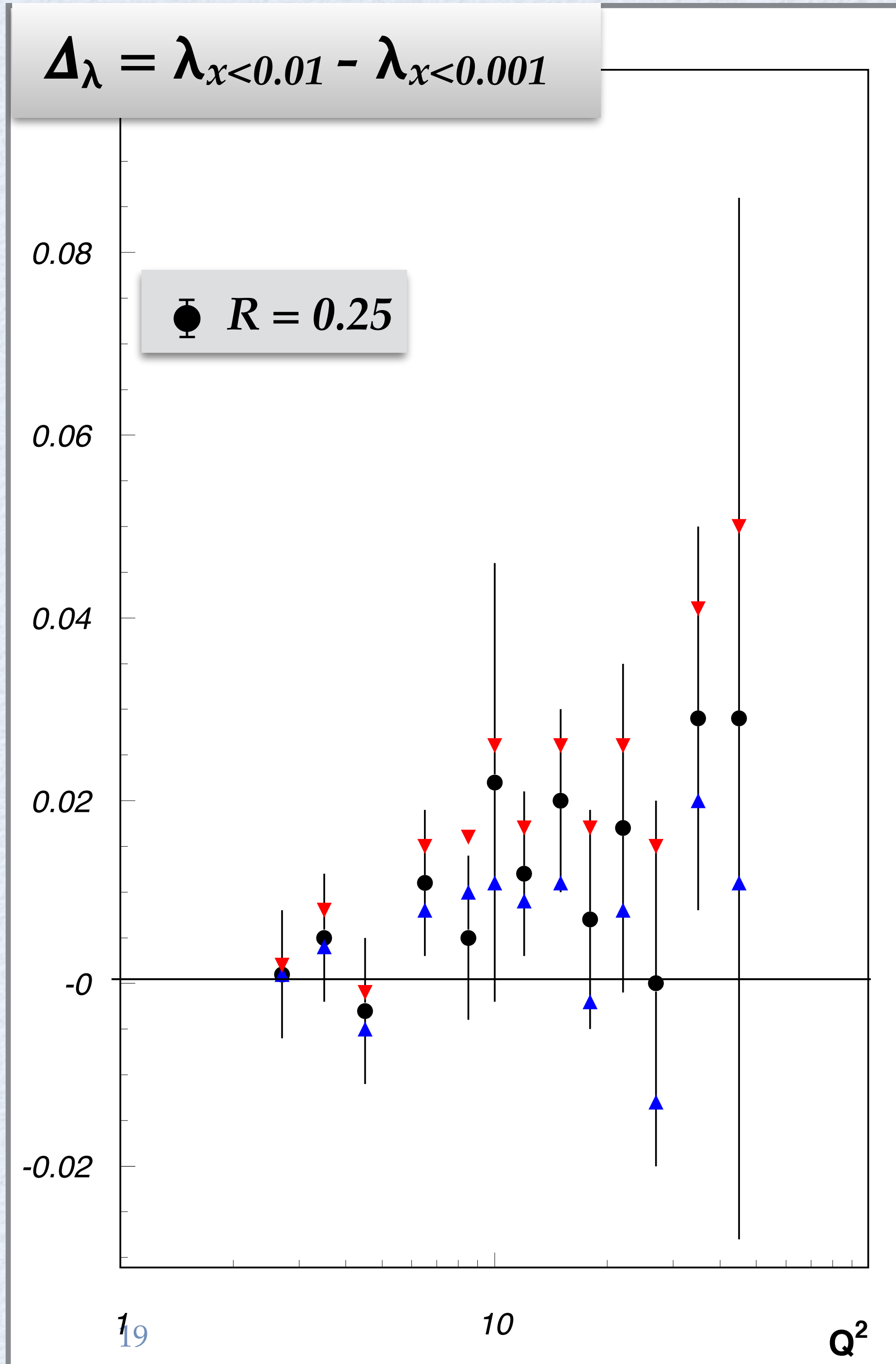
and very-low-x, $x< 0.001$

regions



there is a clear tendency to positive Δ_λ i.e. λ becomes smaller with decreasing x

some sensitivity to the assumed value of R is seen



● $R = 0.25$
▼ $R = 0.2$
▲ $R = 0.3$

Conclusions

λ Fits are a good phenomenological description of the rate of rise of $\sigma^{\gamma p}$ (or F_2)

HERA data indicate that the rate of rise λ is diminishing with decreasing x

This could indicate the onset of BFKL behaviour in the very-low-x region
or
the common BFKL and DGLAP asymptotic behaviour

$$xg(x, Q^2) = \exp \sqrt{C \ln \frac{\ln Q^2/\Lambda}{\ln Q_0^2/\Lambda} \; \ln 1/x}$$

Measurements in the very-very-low-x region will become possible at VHEeP or LHCeP (up to $x < 10^{-7}$)
Here, the confinement effects should become simpler (Feynman 1971)