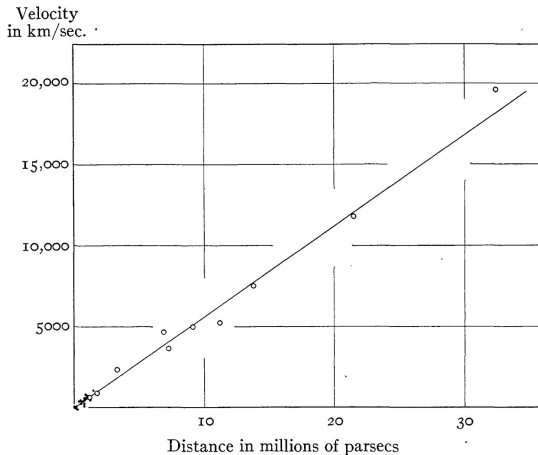




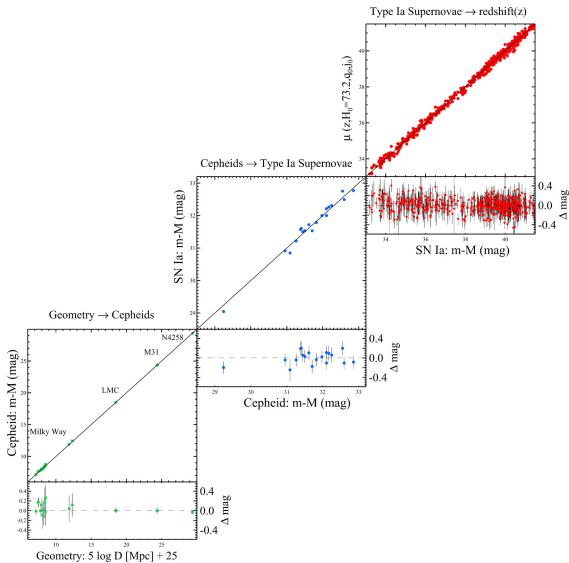
# Expansion Law



$$v = \frac{d}{1790} = 558 \text{ km/sec. per million parsecs.}$$

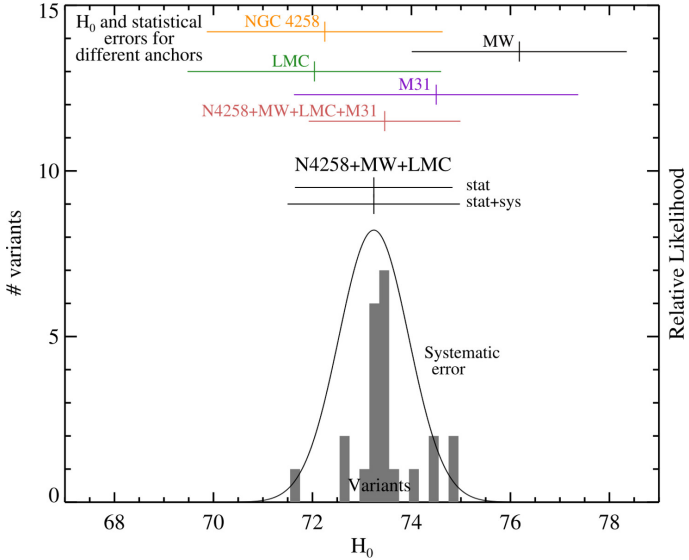
Hubble & Humason 1931

# Expansion Law



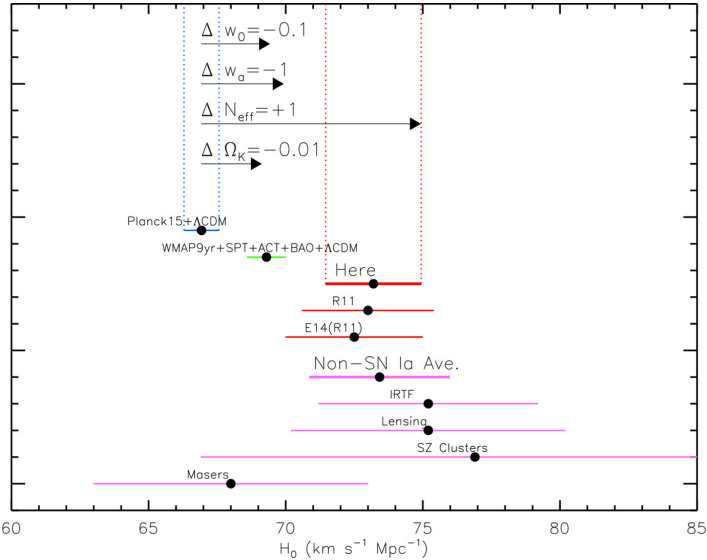
Riess et al. 2016

# Expansion Law



Riess et al. 2016

# Expansion Law



Riess et al. 2016

Metric (spatially homogeneous and isotropic)

$$ds^2 = -c^2 dt^2 + a^2(t) \left[ dw^2 + f_K^2(w) d\Omega^2 \right]$$

Scale factor  $a(t)$ ,  $f_K(w) = w$  for  $K = 0$

Hubble constant  $H_0$ , Hubble function  $H(a)$

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(a) = H_0^2 E^2(a)$$

Expansion function  $E(a)$

Expansion function  $E(a)$

$$E(a) = \left( \Omega_{r0} a^{-4} + \Omega_{m0} a^{-3} + \Omega_{K0} a^{-2} + \Omega_{\Lambda 0} \right)^{1/2}$$



Expansion function  $E(a)$

$$E(a) = \left( \Omega_{r0} a^{-4} + \Omega_{m0} a^{-3} + \Omega_{K0} a^{-2} + \Omega_{Q0} a^{-3(1+w)} \right)^{1/2}$$

Equation-of-state parameter  $w$

Expansion function  $E(a)$

$$E(a) = \left( \Omega_{r0} a^{-4} + \Omega_{m0} a^{-3} + \Omega_{K0} a^{-2} + \Omega_{Q0} e^{-3 \int_a^1 (1+w) d \ln a'} \right)^{1/2}$$

Equation-of-state parameter  $w$

Expansion function  $E(a)$

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Equation-of-state parameter  $w$

Chevallier-Polarski-Linder (CPL) parameterisation:

$$w(a) = w_0 + (1 - a)w_a$$

Light propagation,  $ds = 0$ , from the metric:

$$c|dt| = a(t)dw$$

Light propagation,  $ds = 0$ , from the metric:

$$w(a) = \frac{c}{H_0} \int_a^1 \frac{dx}{x^2 E(x)}$$

Light propagation,  $ds = 0$ , from the metric:

$$w(a) = \frac{c}{H_0} \int_a^1 \frac{dx}{x^2 E(x)}$$

Angular-diameter distance ( $K = 0$ )

$$D_{\text{ang}}(a) = \frac{\text{length scale}}{\text{angle spanned}} = a w(a)$$

Luminosity distance

$$D_{\text{lum}}(a) = \frac{\text{luminosity}}{\text{flux}} = \frac{w(a)}{a}$$

Etherington relation, independent of cosmological model

Euler-Poisson system, linearised, Fourier modes  $\delta_k$  of density contrast, sound speed  $c_s$

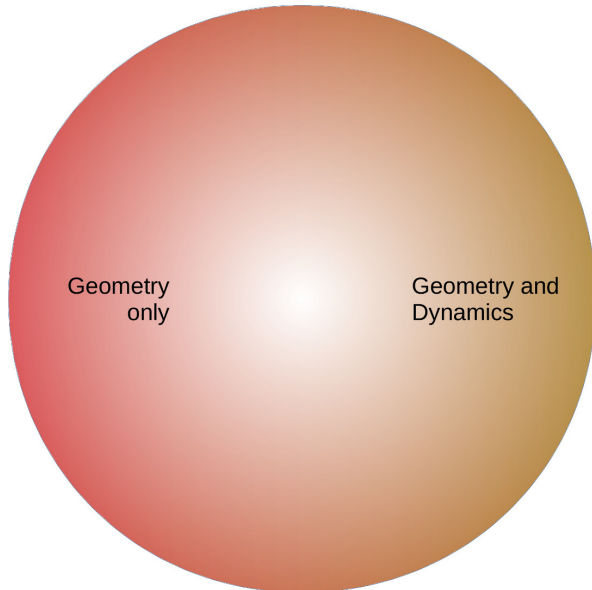
$$\ddot{\delta}_k + 2H\dot{\delta}_k = \left(4\pi G\bar{\rho} - \frac{c_s^2 k^2}{a^2}\right) \delta_k$$

dark matter:  $c_s = 0$

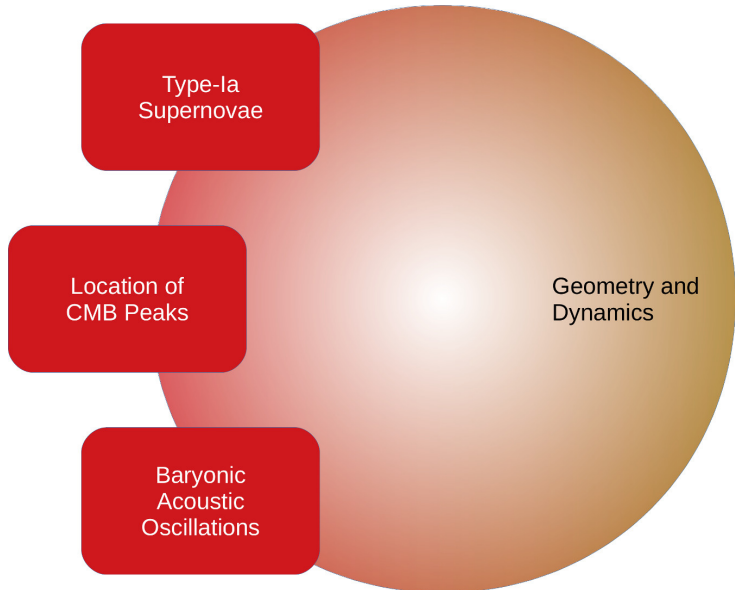
$$\ddot{\delta}_k + 2H\dot{\delta}_k = 4\pi G\bar{\rho} \delta_k$$

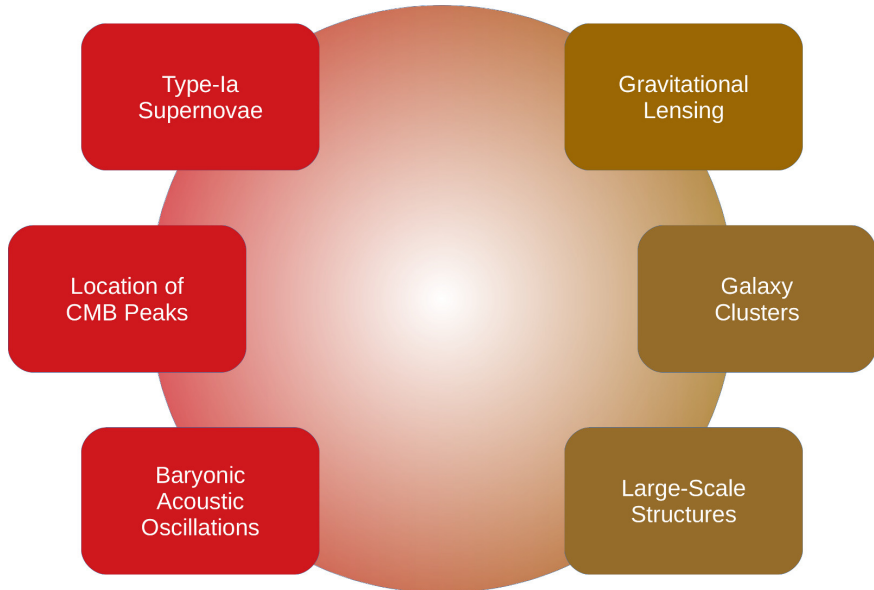
(growing) solution: linear growth factor  $D_+(a)$

baryonic matter:  $c_s \approx c / \sqrt{3} > 0$ , oscillations on small scales

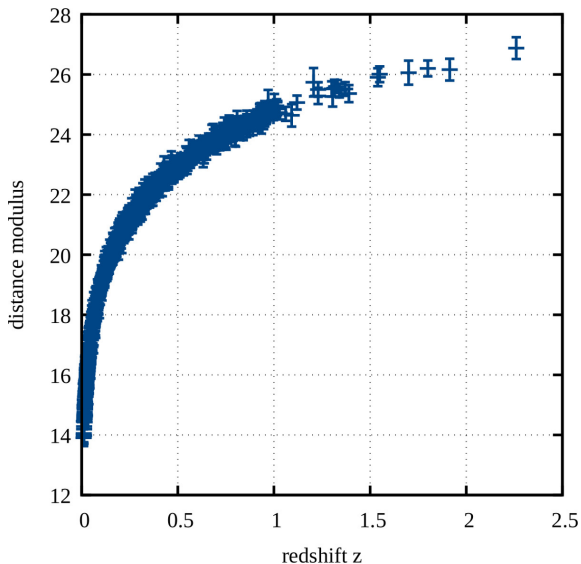






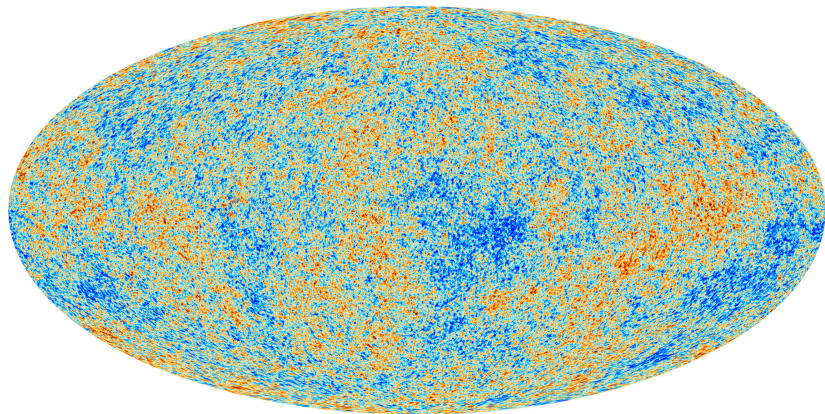


# Type-Ia Supernovae



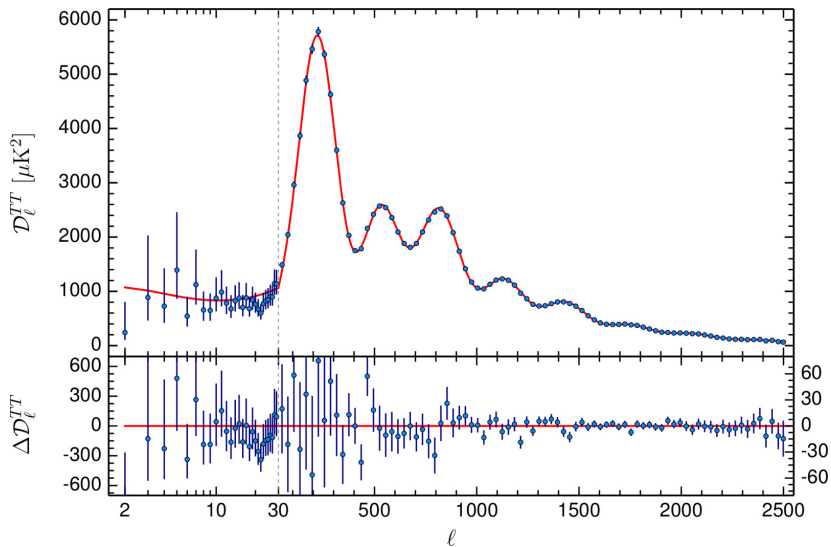
Pantheon sample, Scolnic et al. 2018

# Location of CMB Peaks



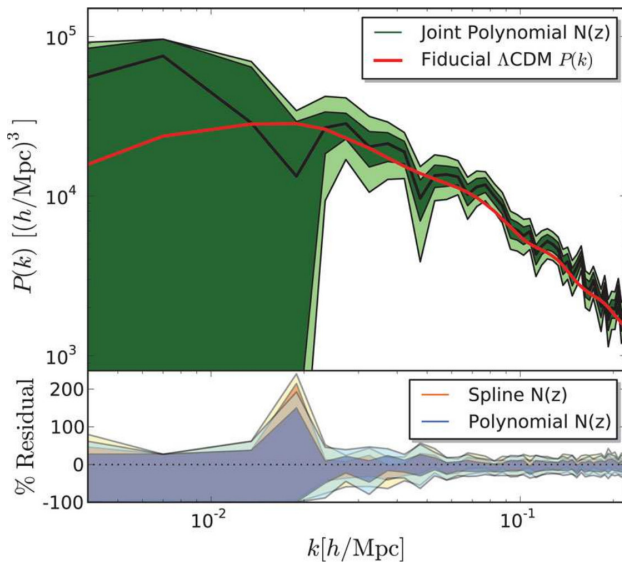
Planck 2015

# Location of CMB Peaks

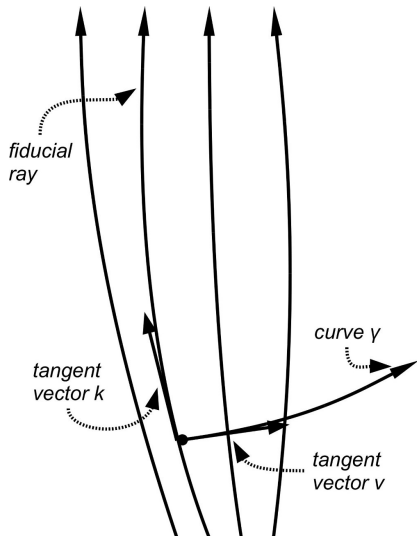


Planck 2015

# Baryonic Acoustic Oscillations



WiggleZ, Poole et al.



From Jacobi equation:

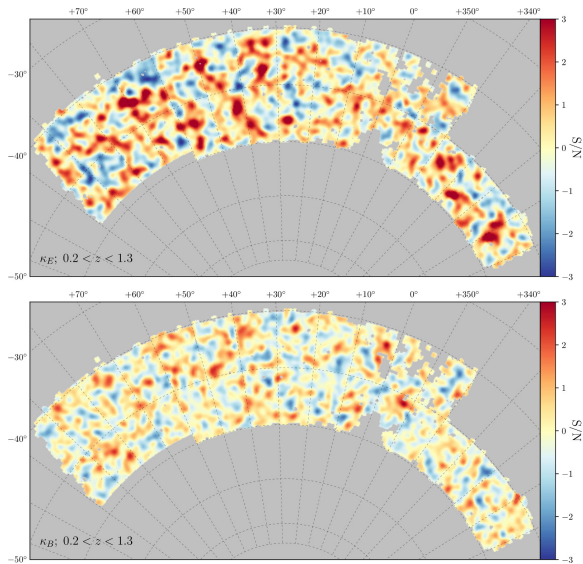
$$(d_w^2 + K)x^i = -2\partial^i\Phi$$

Astigmatism, image distortions  $\gamma$ , correlation function:

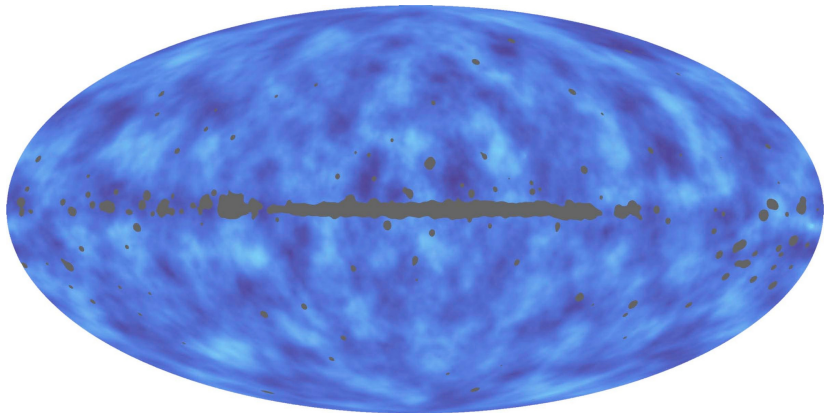
$$C_{\ell}^{\gamma} = \frac{9}{4} \left( \frac{H_0}{c} \right)^4 \Omega_{\text{m}0}^2 \int_0^{w_s} dw \underbrace{\left( \frac{w_s - w}{aw_s} \right)^2}_{E(a)} \underbrace{P_{\delta} \left( \frac{\ell}{w} \right)}_{D_+(a)}$$



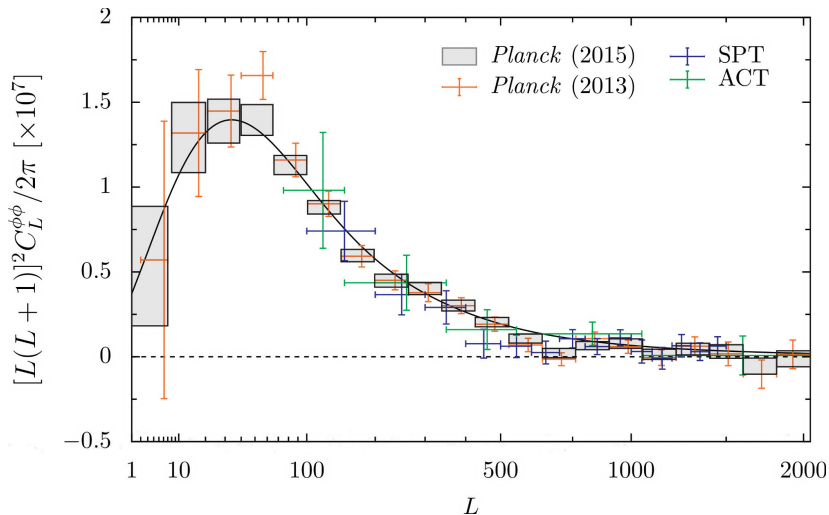
# Gravitational Lensing



DES, Chang et al. 2017

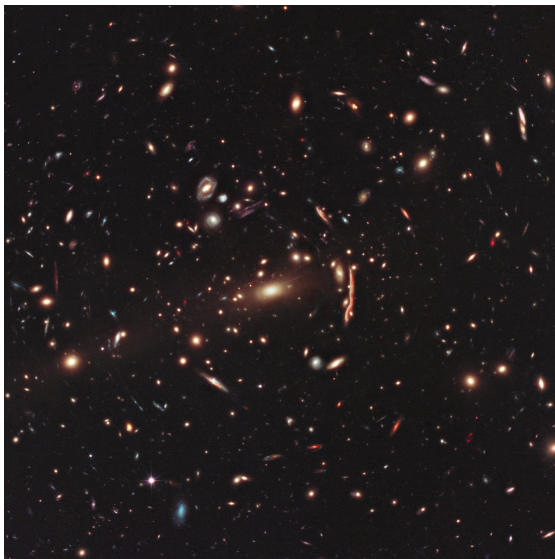


Planck 2015

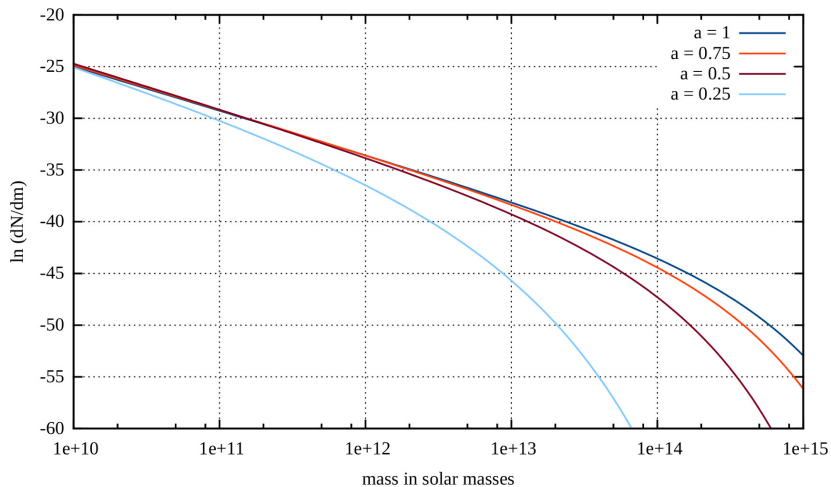


Planck 2015

# Galaxy Clusters

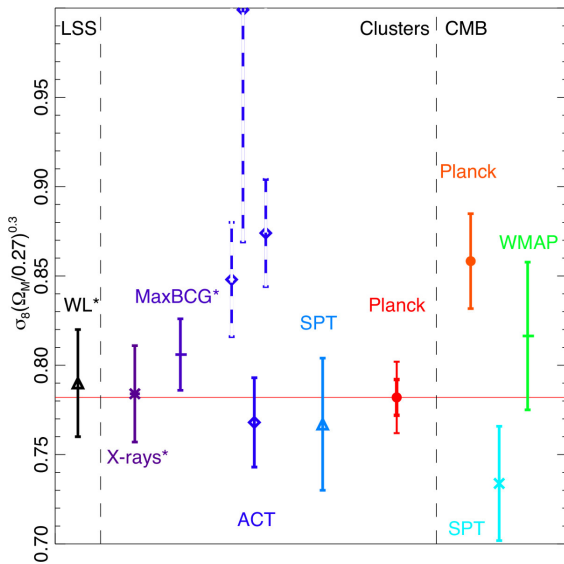


MACS 1206.2-0847

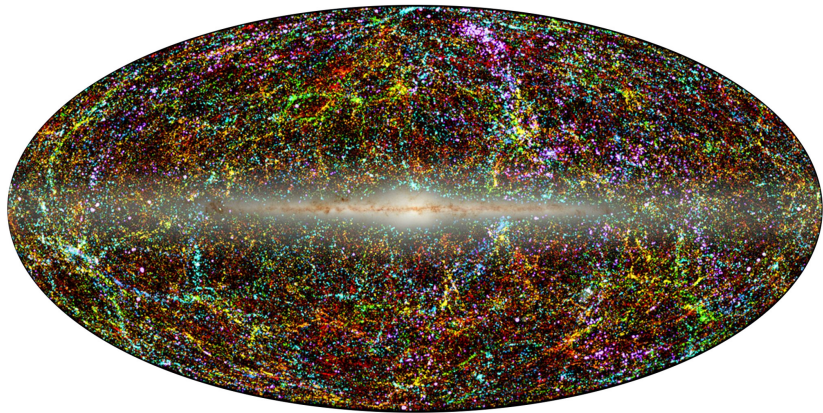


volume:  $E(a)$ , structure growth:  $D_+(a)$

# Galaxy Clusters

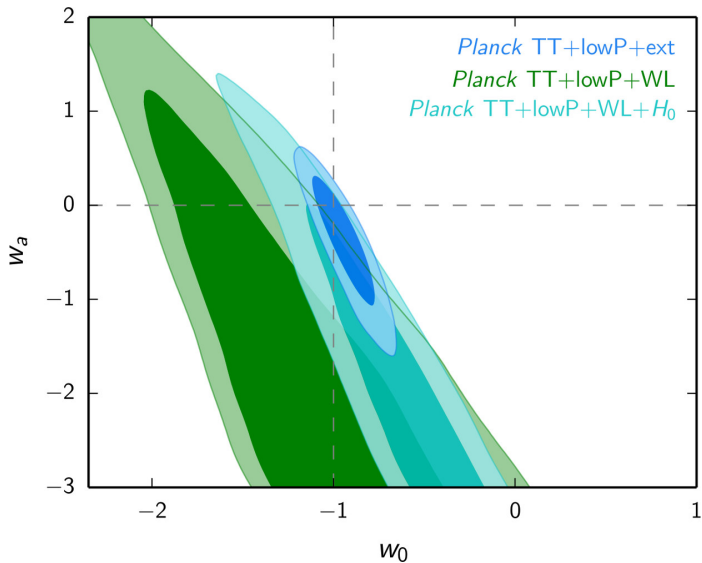


Planck 2015



2-Micron All-Sky Survey (2mass)

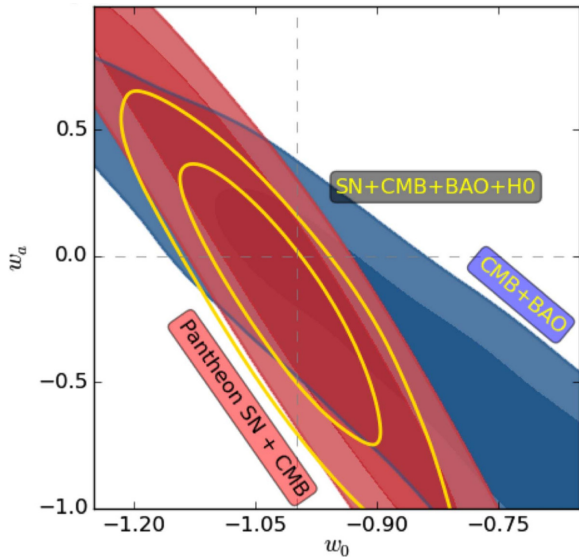
# Current Constraints



Planck 2015



# Current Constraints



Scolnic et al. 2018

Measurement: luminosity distance

$$D_{\text{lum}}(a) = \frac{1}{a} \int_a^1 \frac{dx}{x^2 E(x)}$$

Representation of  $E(a)$ :

$$e(a) := [aE(a)]^{-1}, \quad e(a) = -a^2 D'_{\text{lum}}(a) + \lambda \int_1^a \frac{dx}{x} e(x)$$

Measurement: luminosity distance

$$D_{\text{lum}}(a) = \frac{1}{a} \int_a^1 \frac{dx}{x^2 E(x)}$$

Representation of  $E(a)$ :

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Representation of  $D_{\text{lum}}(a)$ :

$$D_{\text{lum}}(a) = \sum_{j=1}^M c_j p_j(a)$$

with (orthonormal) basis functions  $p_j(a)$

Mignone & MB, Mignone & Maturi, Benítez Herrera et al., ...

Measurement: luminosity distance

$$D_{\text{lum}}(a) = \frac{1}{a} \int_a^1 \frac{dx}{x^2 E(x)}$$

Representation of  $E(a)$ :

$$e(a) := [aE(a)]^{-1}, \quad e(a) = -a^2 D'_{\text{lum}}(a) + \lambda \int_1^a \frac{dx}{x} e(x)$$

Reconstruction of  $D_+(a)$ :

$$D_+'' + \left( \frac{3}{a} + \frac{E'(a)}{E(a)} \right) D_+' = \frac{3}{2} \frac{\Omega_{\text{m}0}}{a^5} D_+$$

Initial conditions:

$$D_+(a_{\text{min}}) = 1, \quad D_+'(a_{\text{min}}) = \frac{D_+(a)}{a} f(a)$$

Measurement: luminosity distance

$$D_{\text{lum}}(a) = \frac{1}{a} \int_a^1 \frac{dx}{x^2 E(x)}$$

Representation of  $E(a)$ :

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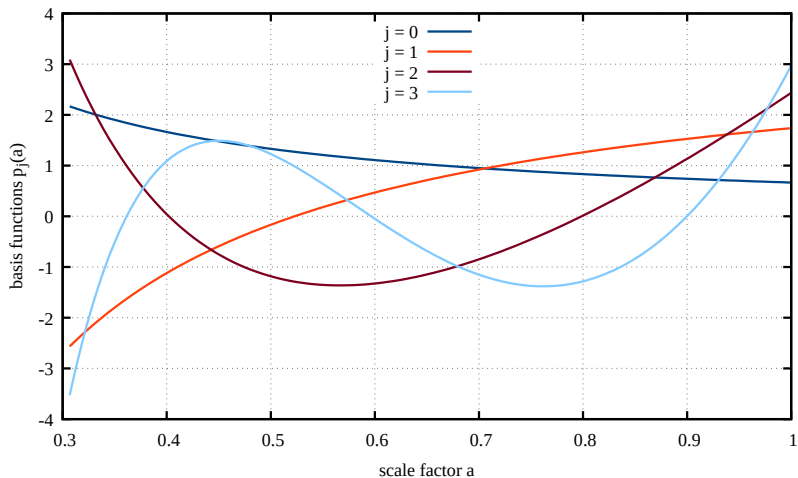
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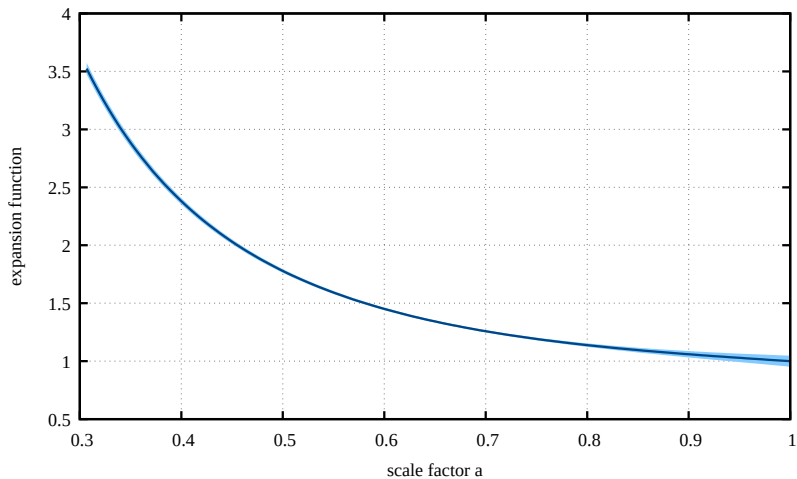
Initial conditions:

$$D_+(a_{\text{min}}) = 1, \quad D_+'(a_{\text{min}}) = \frac{D_+(a)}{a} \Omega_{\text{m}}^\gamma$$

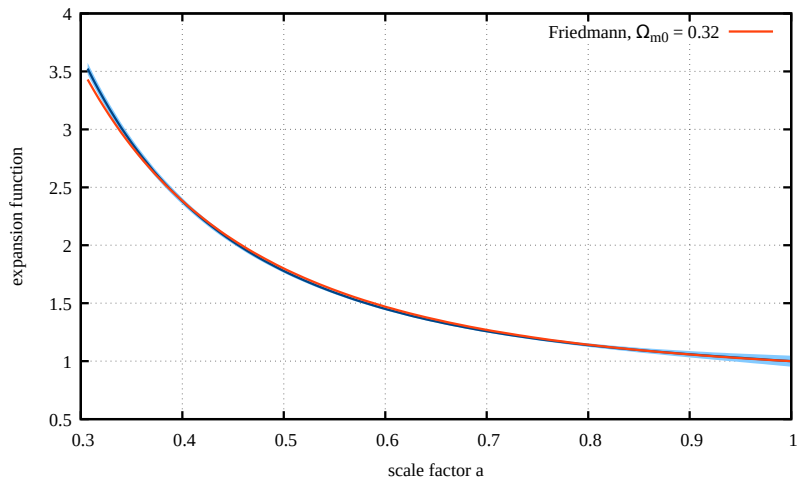
# Model-Independent Reconstruction of $E(a)$



# Model-Independent Reconstruction of $E(a)$

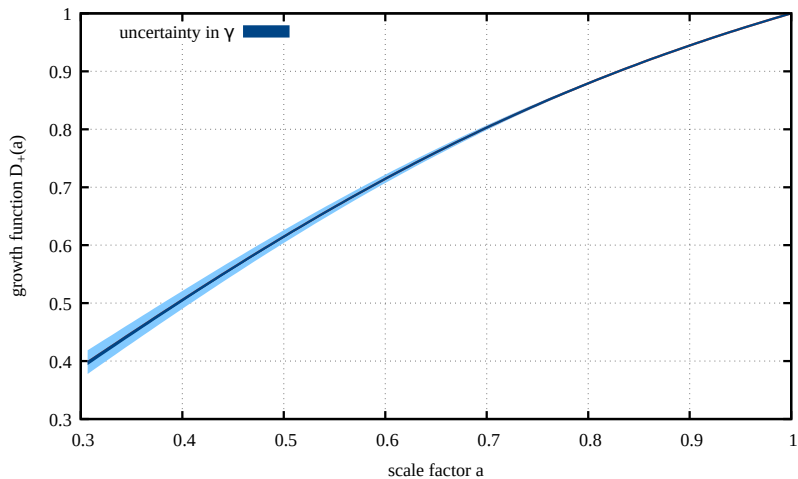


# Model-Independent Reconstruction of $E(a)$





# Model-Independent Reconstruction of $D_+(a)$



Haude, Maturi, MB 2019

- Multitude of cosmological observables is determined by expansion function  $E(a)$  and growth function  $D_+(a)$
- Constraints within framework of Friedmann models favour cosmological constant
- Model-independent reconstruction of  $E(a)$  is possible, result is tightly constrained
- (Almost) model-independent reconstruction of  $D_+(a)$  follows
- $E(a)$  and  $D_+(a)$  are empirically known within tight limits, based exclusively on metric theory of gravity and symmetry assumptions
- Generalisations of GR need to agree with both Heisenberg & MB, in preparation