

Transferring two massive mirrors into a single quantum object

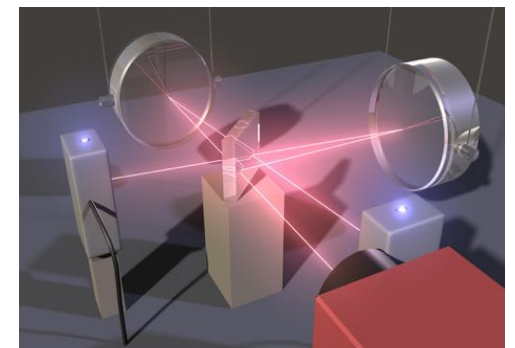
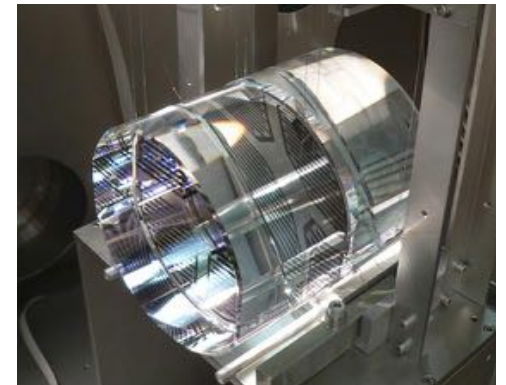
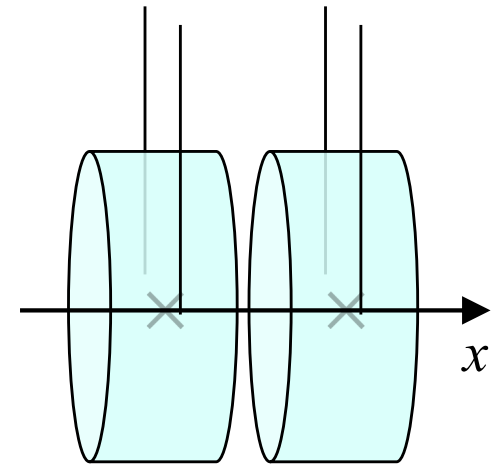
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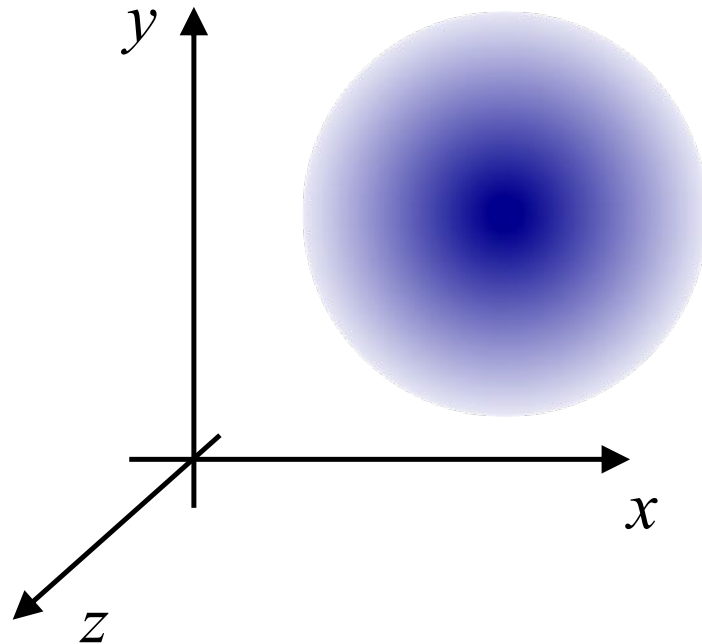


Outline

- A single quantum object and Einstein-Podolsky-Rosen (EPR) entanglement
- The aim: Two massive mirrors as a single quantum object of (1dim) centre of mass motion.
- Motivation for such an experiment
- The experimental concept
- Proposal for a table-top experiment with two pendulum-suspended 100g mirrors
- Reducing the coupling rate with the thermal bath



A single (isolated) quantum object



Position uncertainty
in three dimensions
of e.g.

- H-atom or
- H₂-molecule or
- U²³⁸-nucleus ...

Definition:

A single quantum object may potentially decay but it still has to be considered as a *single quantum object* if (after decay) it turns out that the interaction rate between the constituents (before decay) was much larger than the interaction rate of the constituents with the environment. → In this case measurements on the decay products will show *entanglement!*

→ Entanglement proves that the previous object was a *single quantum*.

Spontaneous decay of a photon

Typ II–spontaneous down-conversion in a nonlinear crystal probabilistically produces pairs of two photons, which show *entanglement* (with respect to several degrees of freedom, first of all existence altogether.)

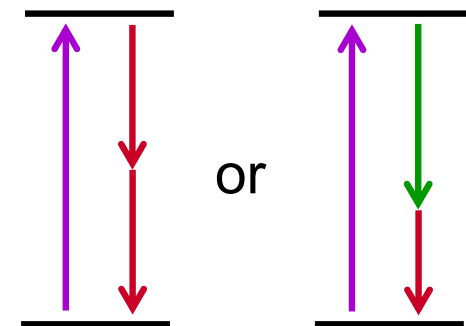
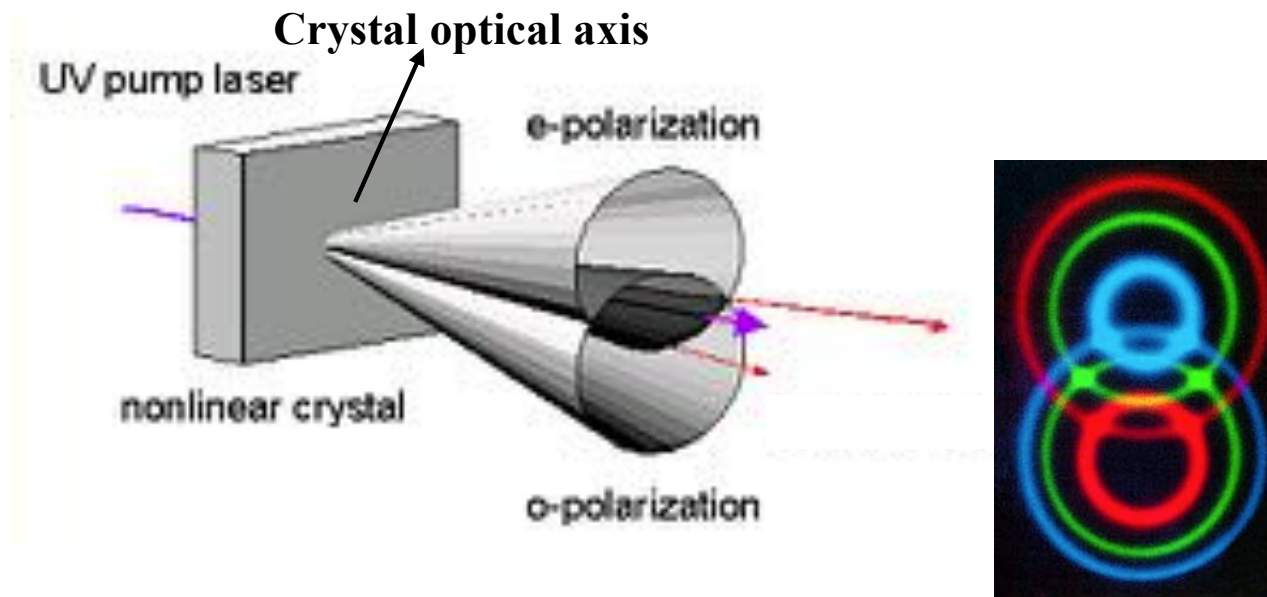
The event of a pair production is unpredictable but iff one photon is found in the upper emission cone there is a 2nd photon in the lower one.

Energy conservation

$$\hbar\omega_{pump} = \hbar\omega_o + \hbar\omega_e$$

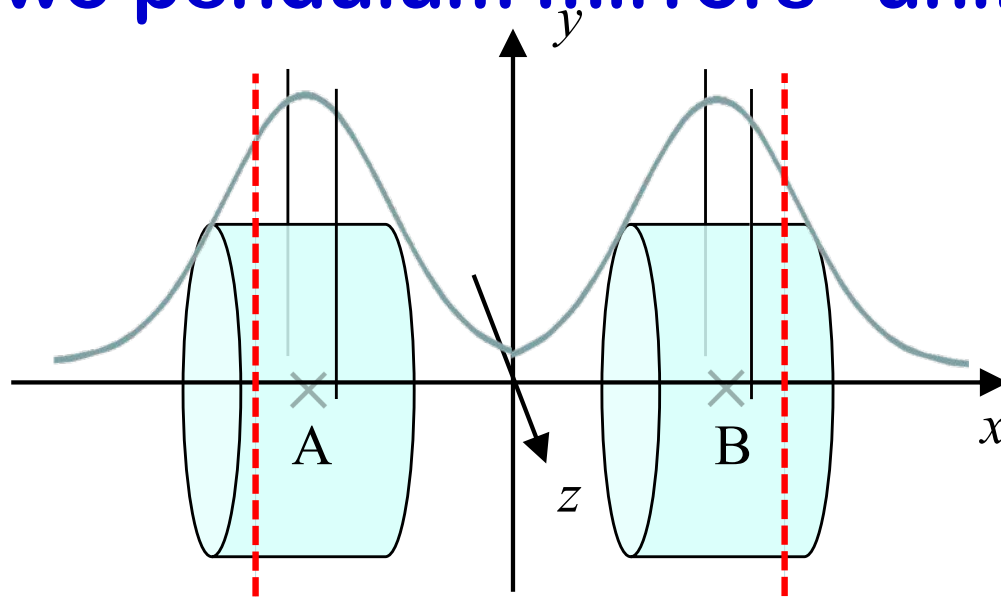
Momentum conservation

$$\hbar\vec{k}_{pump} = \hbar\vec{k}_o + \hbar\vec{k}_e$$



[P. G. Kwiat *et al.*, Phys. Rev. Lett. **75**, 4337 (1995)]

Two pendulum mirrors “unified” in one dimension



$$[(\hat{x}_A + \hat{x}_B), (\hat{p}_A - \hat{p}_B)] = 0$$

If their motion is that of a single quantum object, we will get the following *ensemble measurement statistics* :

- If mirror A is farther away from the origin, then also mirror B is (and vice versa)
- If mirror A moves faster to the left, then also B moves faster to the left (and vice versa).

→ Correlations and anti-correlations are tighter than the width of the individual mirrors' ground state uncertainties.



Quantum theory is complete

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in

quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Valid gedanken experiment by EPR:

There are systems ('single quantum objects') that can decay into components A and B such

$$\Delta^2(x_A + x_B) \wedge \Delta^2(p_A - p_B) \rightarrow 0$$

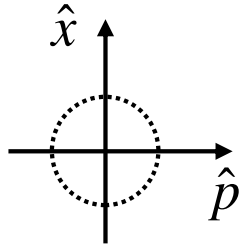
since $[x_A + x_B, p_A - p_B] = 0$

Incorrect assumption of EPR

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

Prior to decay, the information x_A , x_B , p_A , and p_B do not exist (in the universe)!

Einstein-Podolsky-Rosen-Entanglement



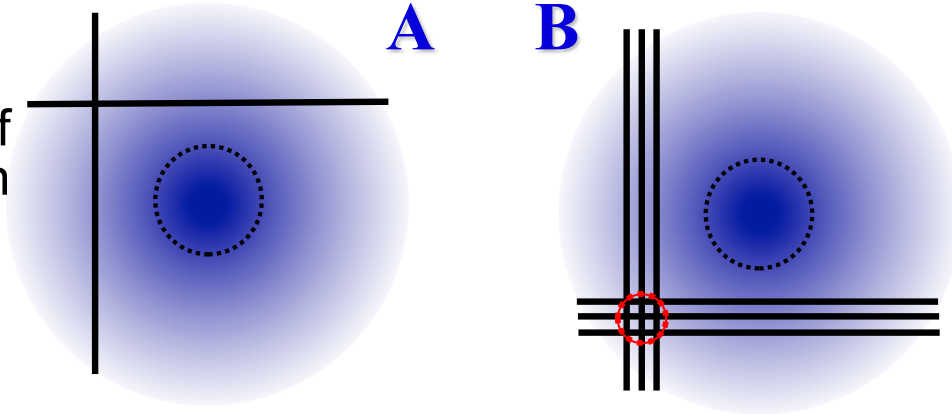
Ground state uncertainty

$$\Delta \hat{x} \cdot \Delta \hat{p} = \hbar/2$$

$$[\hat{x}_{A,B}, \hat{p}_{A,B}] = i\hbar$$

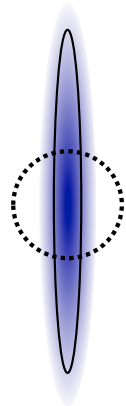
Example measurement of A's momentum

Example measurement of A's position

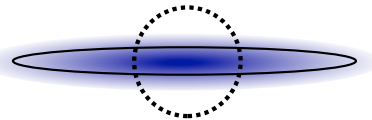


Precise inference at B is always possible through measurement at A

Differential momentum squeezed



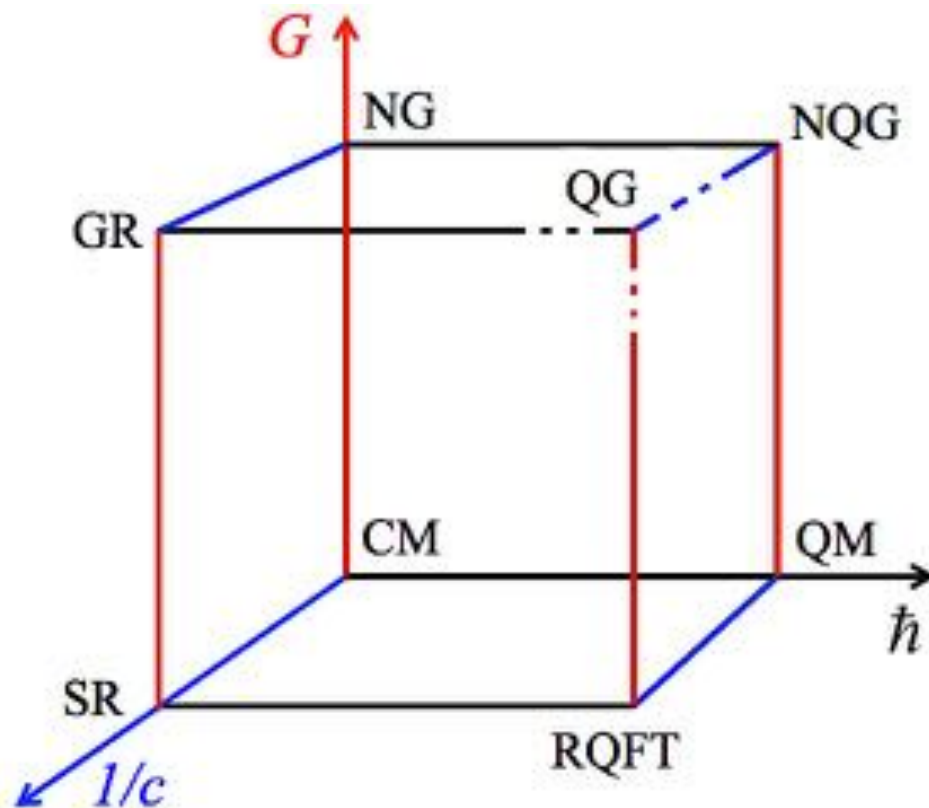
Sum of positions squeezed



$$\Delta(\hat{x}_A + \hat{x}_B) \cdot \Delta(\hat{p}_A - \hat{p}_B) \rightarrow 0$$

if correlations get stronger

Possible Routes to Quantum Gravity (QG)



Newtonian Quantum Gravity

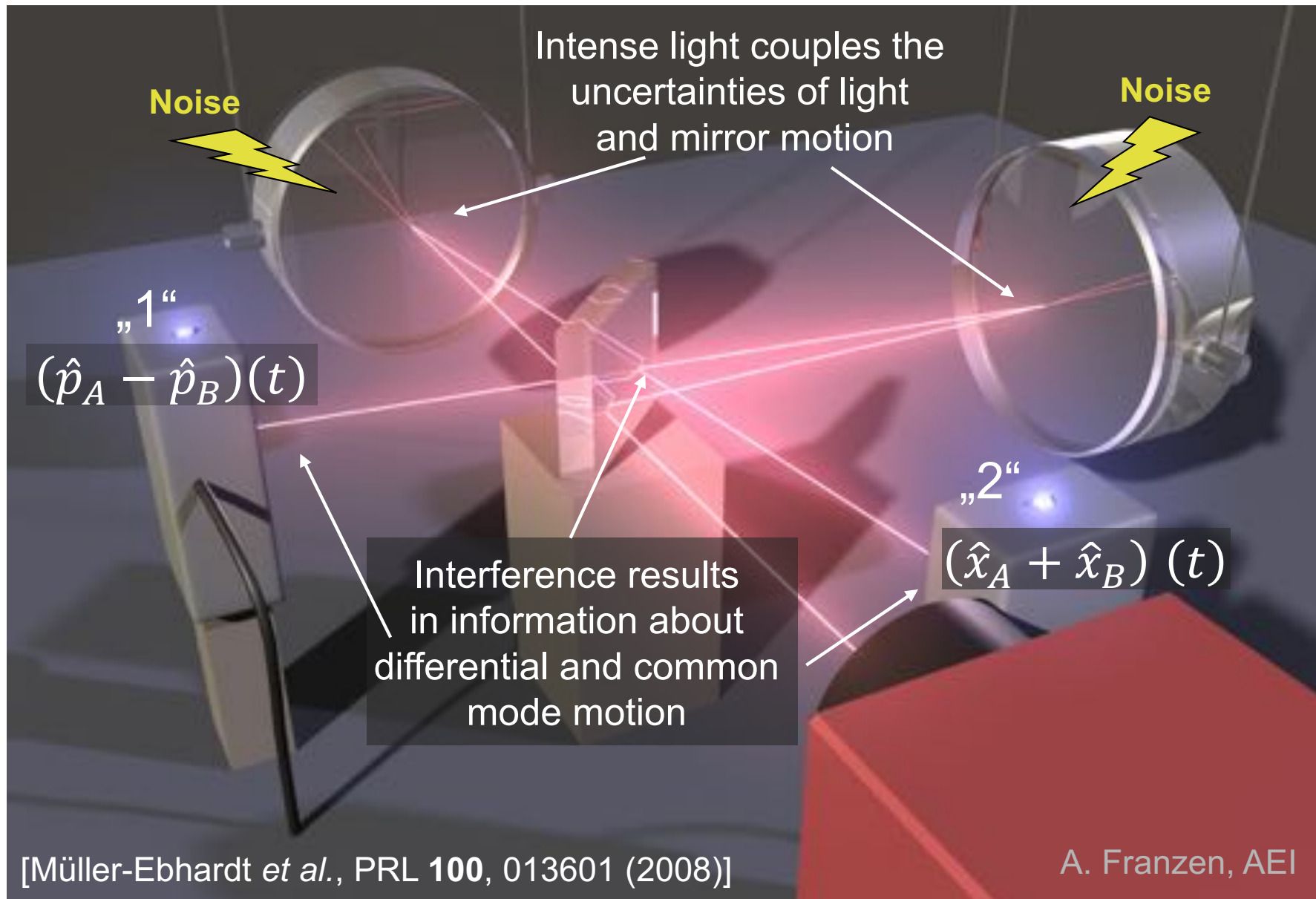
Adding non-relativistic
Newtonian theory of
gravity (NG) to quantum
mechanics (QM),

Motivation: Building a potential
quantum mechanical probe
system for NQG.

[The “Bronstein Cube”]

[L. Diósi, Phys. Lett. **A120**, 377 (1987);
Phys. Rev. A **40**, 1165 (1989); J. Phys. A **40**, 2989 (2007)]
[R. Penrose, Gen. Rel. Grav. **28**, 581 (1996)]

The concept of 1 dim mirror unification – in short



Entanglement of reflected light and mirror motion

Radiation pressure entangles mechanics and light

[Bose, Jacobs, Knight, Phys. Rev. A **56**, 4175 (1997)]

[Mancini, Manko, Tombesi, Phys. Rev. A **55**, 3042 (1997)]

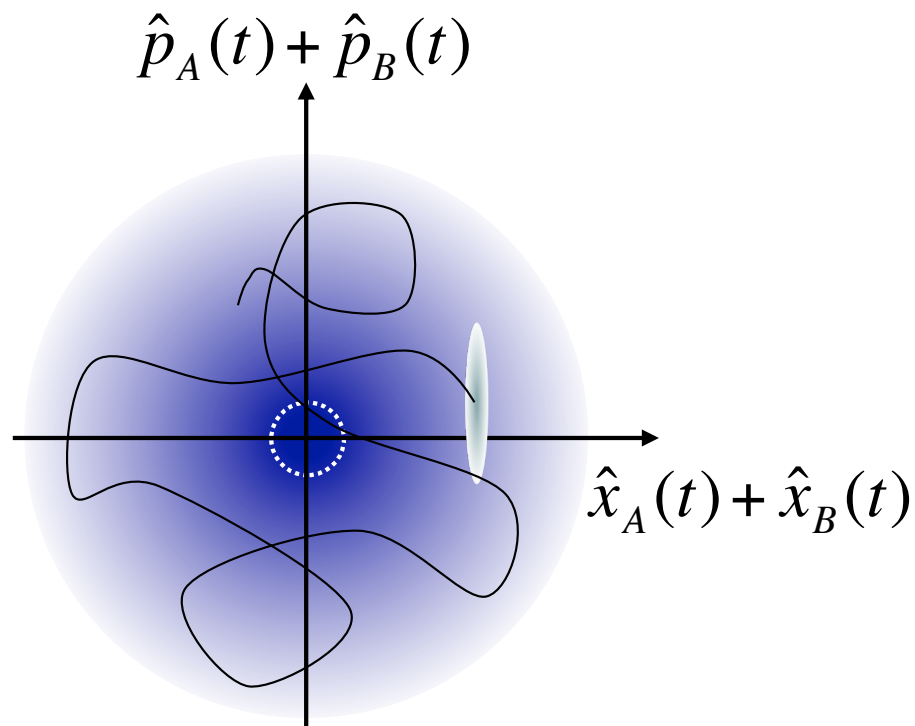
[Vitali, Aspelmeyer *et al.*, Phys. Rev. Lett. **98**, 030405 (2007)]

Light/mirror entanglement needs to be swapped

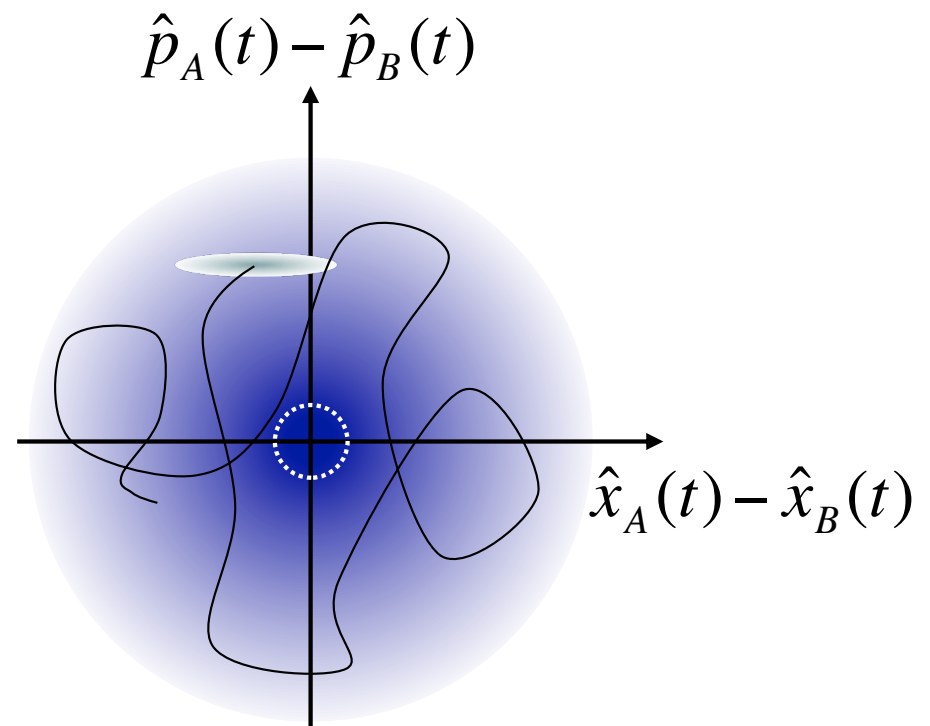
[Pirandola *et al.*, Phys. Rev. Lett. **97**, 150403 (2006)]

Known information (after Kalman filtering)

Detector “1”:
Common mode of motion
is in thermal state



Detector “2”:
Differential mode of motion
is in thermal state



Random walk in phase space.
The ellipses correspond to almost pure squeezed states.

[RS, PRA **92**, 012126 (2015)]

Quantum-state preparation and macroscopic entanglement in gravitational-wave detectors

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 Karsten Danzmann,¹ and Yanbei Chen^{2,3}

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³*Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Am Mühlenberg 1, 14476 Potsdam, Germany*

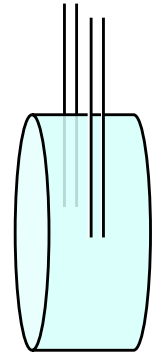
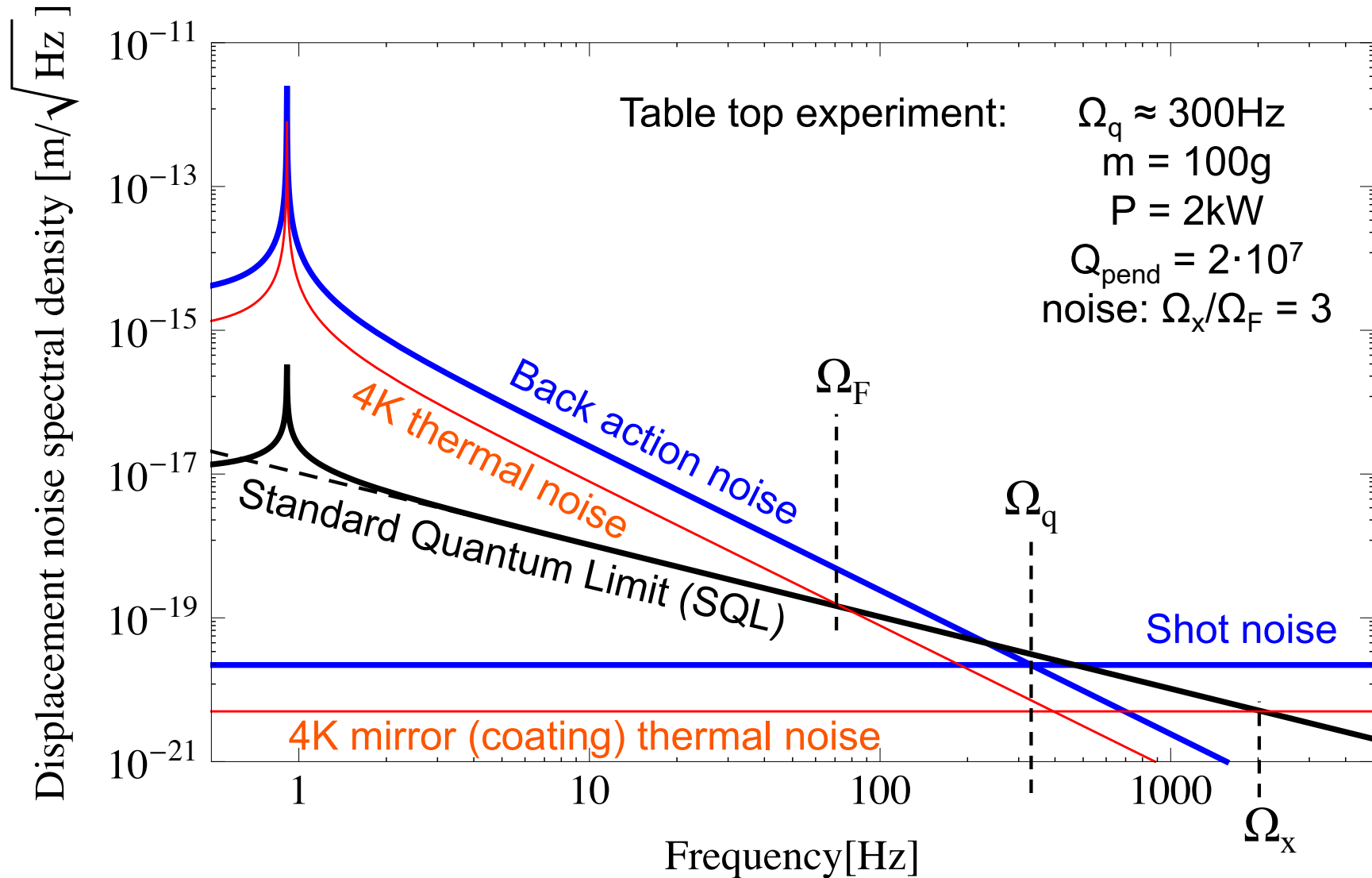
(Received 28 February 2009; published 2 October 2009)

Long-baseline laser-interferometer gravitational-wave (GW) detectors are operating at a factor of ~ 10 (in amplitude) above the standard quantum limit (SQL) within a broad frequency band (in the sense that $\Delta f \sim f$). Such a low-noise budget has already allowed the creation of a controlled 2.7 kg macroscopic oscillator with an effective eigenfrequency of 150 Hz and an occupation number of ~ 200 . This result, along with the prospect for further improvements, heralds the possibility of experimentally probing macroscopic quantum mechanics (MQM)—quantum mechanical behavior of objects in the realm of everyday experience—using GW detectors. In this paper, we provide the mathematical foundation for the first step of a MQM experiment: the preparation of a macroscopic test mass into a nearly minimum-Heisenberg-limited Gaussian quantum state, which is possible if the interferometer's classical noise beats the SQL in a broad frequency band. Our formalism, based on Wiener filtering, allows a straightforward conversion from the noise budget of a laser interferometer, in terms of noise spectra, into the strategy for quantum-state preparation and the quality of the prepared state. Using this formalism, we consider how Gaussian entanglement can be built among two macroscopic test masses and the performance of the planned Advanced LIGO interferometers in quantum-state preparation.

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PACS number(s): 42.50.Dv, 42.50.Xa, 42.50.Lc, 03.65.Ta

Proposal for a table-top experiment at Ω_q



[RS, PRA **92**, 012126 (2015)]

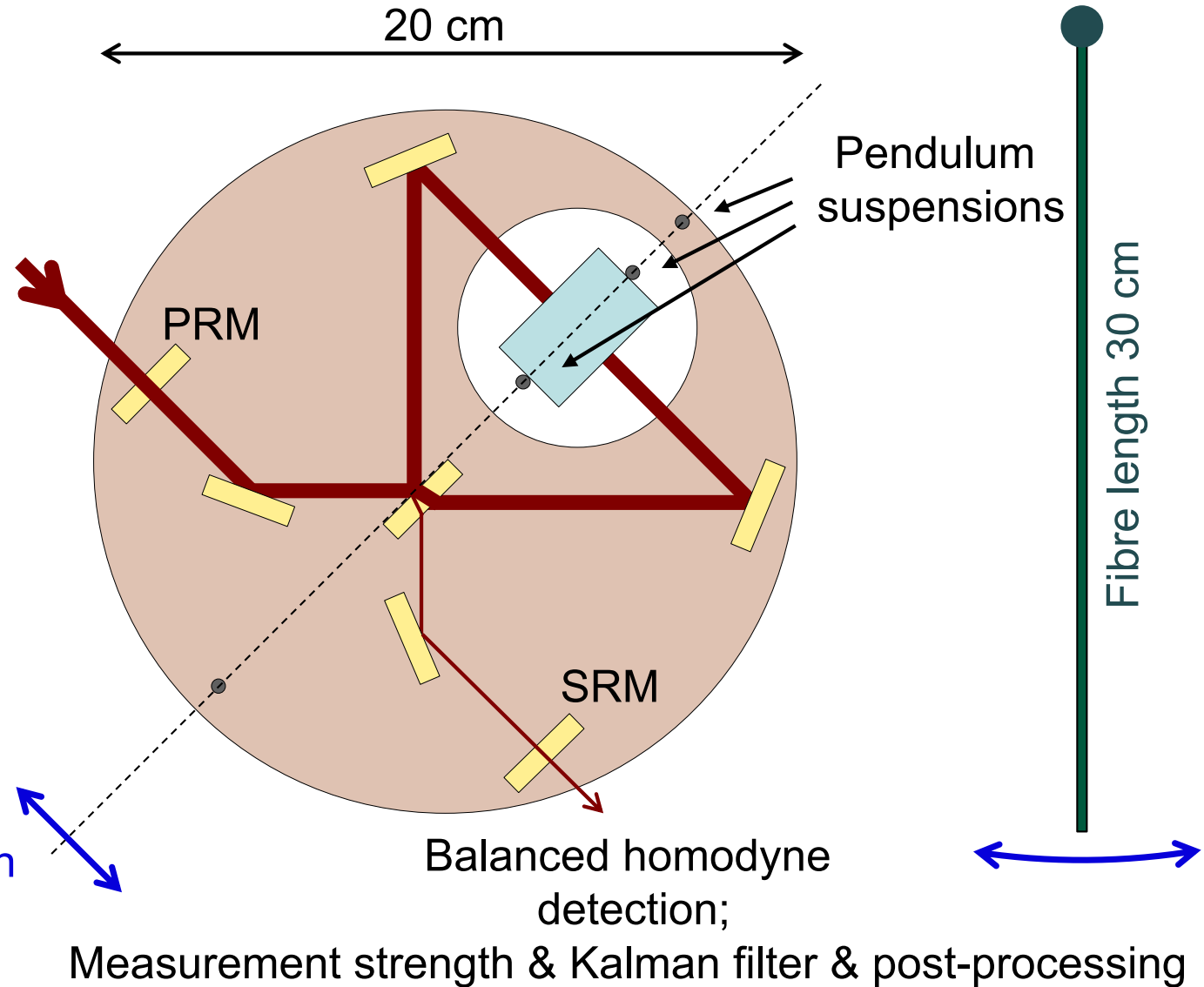
Single mirror pure-state preparation

$m^* = 100 \text{ g}$
 $T = 4 \text{ K}$
 $P = 4 \text{ kW}$

Beam waist located in the centre of test mass

Beam radius on test mass: $> 1 \text{ mm}$

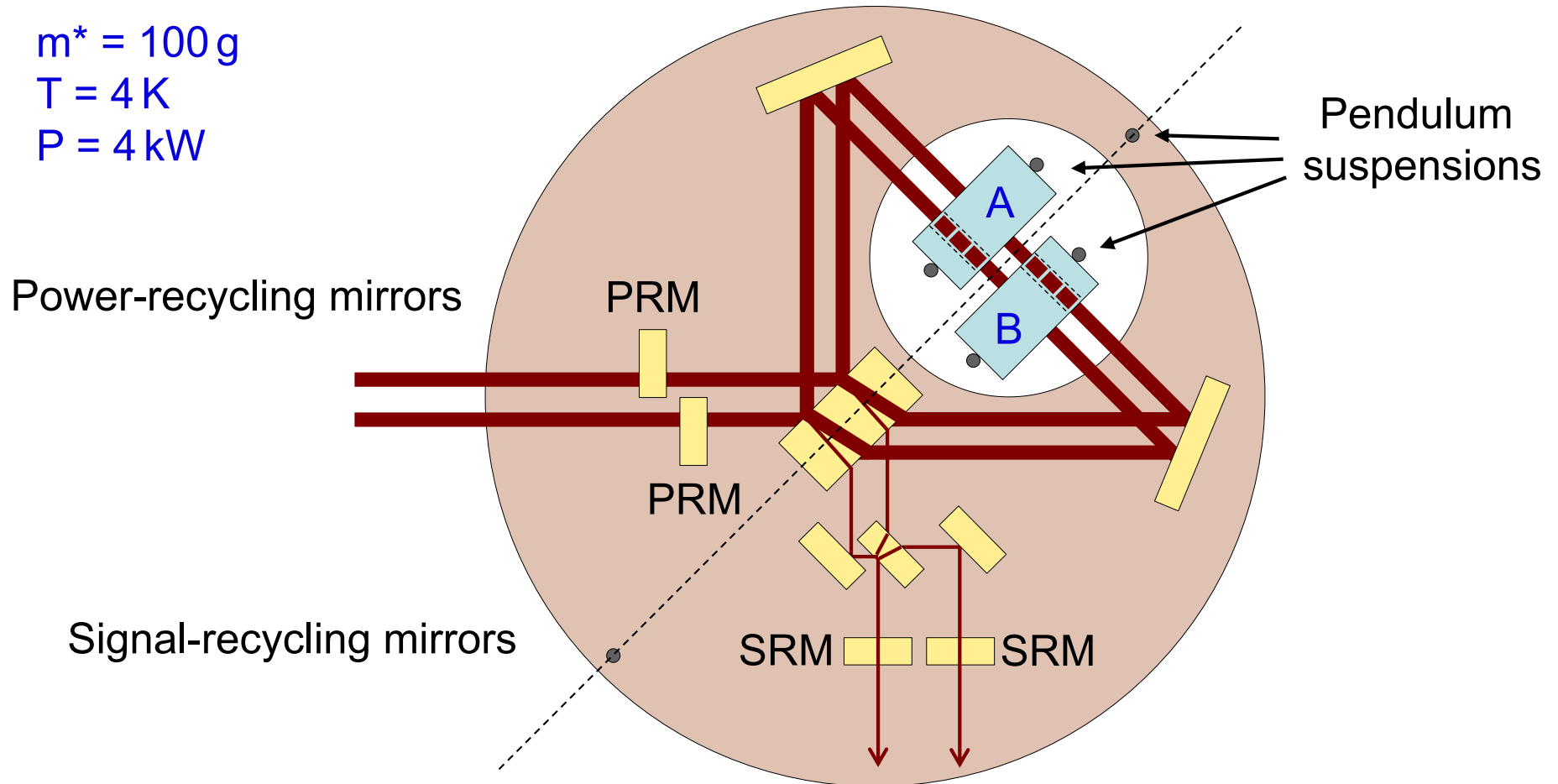
Test mass:
bi-convex; $R = 1 \text{ km}$



Two mirror entangled-state preparation



$m^* = 100 \text{ g}$
 $T = 4 \text{ K}$
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[RS, PRA **92**, 012126 (2015)]

Pendulum thermal phonon occupation number

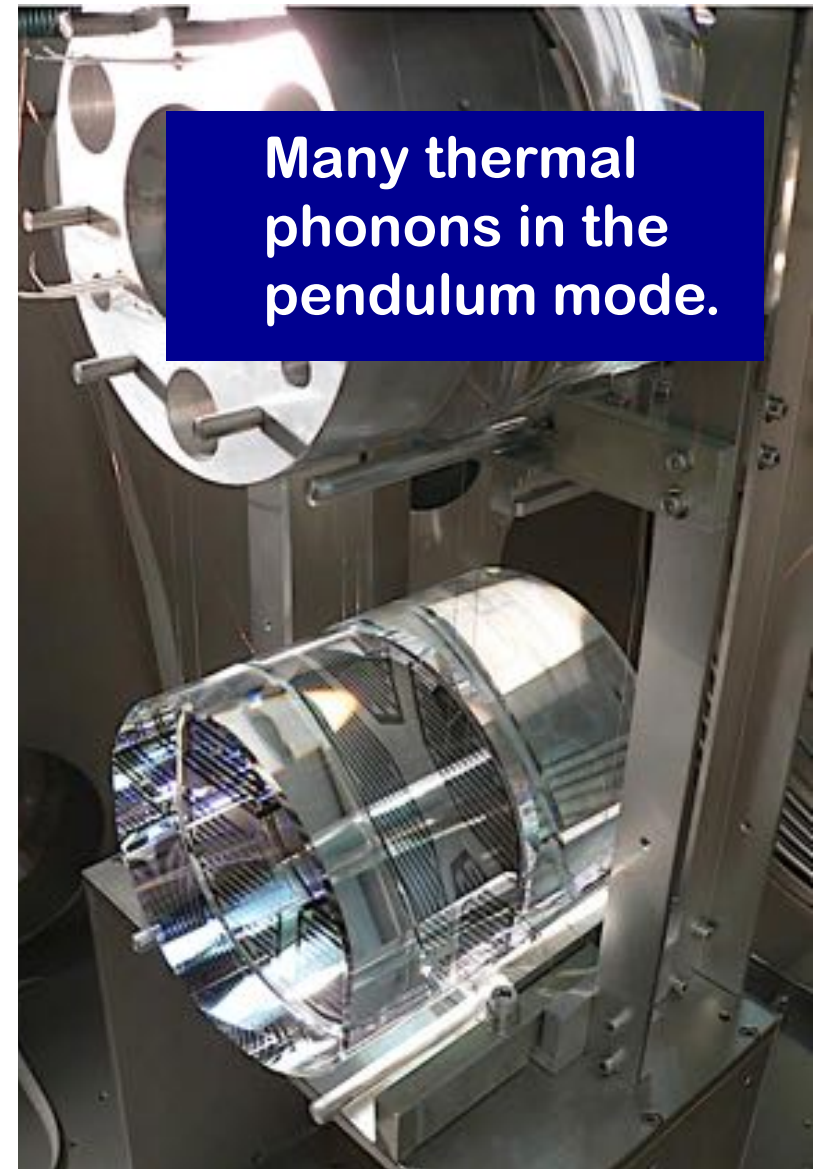
$$\bar{n} = \frac{1}{\frac{\hbar\Omega}{k_B T} - 1} \quad \text{Bose-Einstein statistic}$$

Pendulum with period 1s:

$$\Omega = 2\pi \text{ Hz}$$

$$n < 1 \rightarrow T < 4 \cdot 10^{-10} \text{ K}$$

- Such a low temperature is not at all achievable for a pendulum.
- The pendulum mode can not be prepared close to the quantum ground state.



(i) Consider (pure) coherent states instead of $|0\rangle$

Step (i): Without changing the average occupation number, the coupling rate to the thermal bath is reduced for a larger pendulum Q-factor Q_m .

Condition for the observation of pure coherent (displaced) states:

$$\frac{hf_m Q_m}{k_B T} \geq 1$$

$$\Rightarrow Q_m \cdot f_m \geq \sim 6 \times 10^{12} \left(\frac{T}{300 \text{ K}} \right)$$

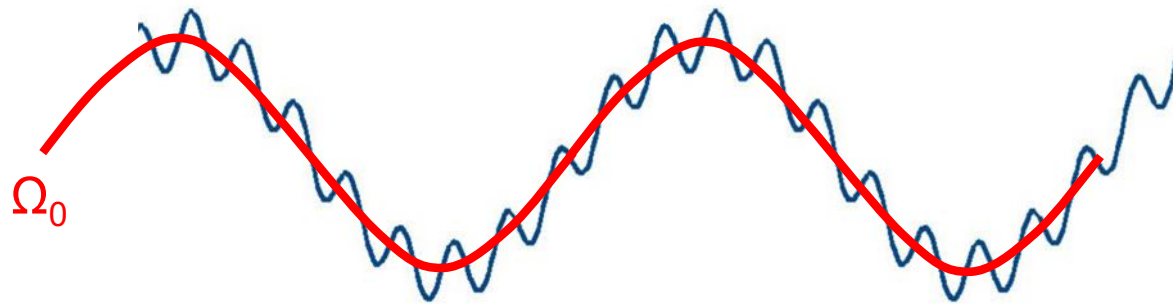
Pendulum Q-Factors larger than 10^6 have been achieved.

Example: $Q_m = 10^6$ at $f_m = 1$ Hz still requires $T < 0,05$ mK.
Also this temperature is not achievable in an experiment.

(ii) “Overtone” mechanical mode

Pendulum motion:

Overtone ($\Omega_q \gg \Omega_0$)



How does the measurement define this mechanical mode of motion?

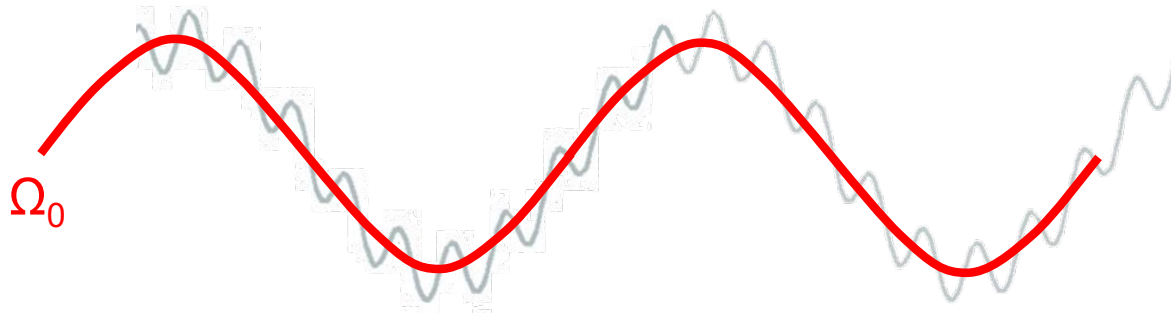
By band-path filtering the photo-electric voltage of an optical readout to $\Omega_q \pm \Delta\Omega_q/2$! We call Ω_q “the *measurement frequency*”.

Why should a pendulum (or even a free mass) have such a superimposed motion?

It should not – apart from off-resonant thermal noise. This mechanical modulation mode should be close to a vacuum state! Its ground state uncertainty should be visible at rather high temperatures.

(ii) “Overtone” mechanical mode

Pendulum motion:
Overtone ($\Omega_q \gg \Omega_0$)



Zero-point fluctuation of this “measurement-defined” mechanical mode:

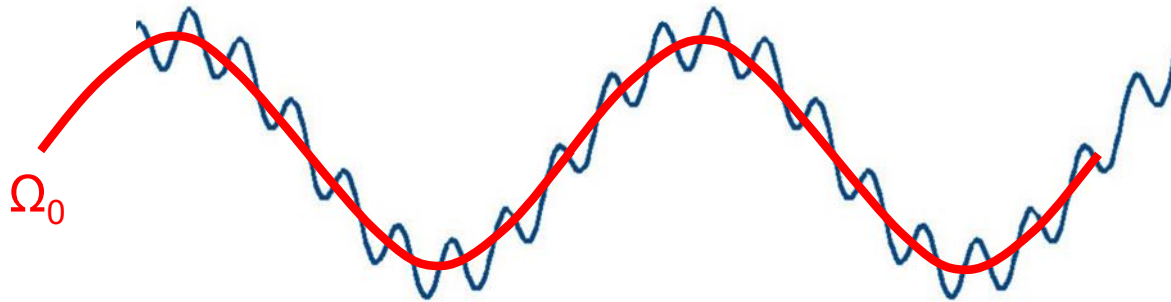
$$\Delta \hat{x}_{\text{zpf}} = \sqrt{\hbar / (2m\Omega_q)}$$

$$\Delta \hat{p}_{\text{zpf}} = \sqrt{\hbar m\Omega_q / 2}$$

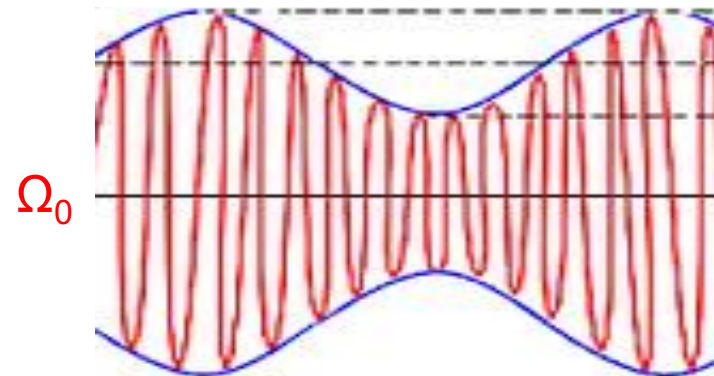
In our experiment: $\Omega_q \approx 300 \text{ Hz}$.

Overtone mechanical mode \leftrightarrow optical AM/FM

Pendulum motion:
Overtone ($\Omega_q \gg \Omega_0$)

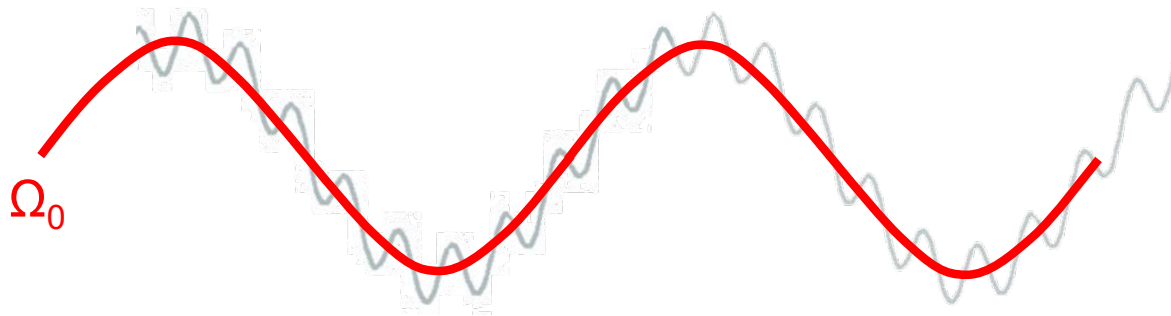


Optical light:
Low frequency amplitude modulation (AM)
($\Omega \ll \Omega_0$)



(ii) “Overtone” mechanical mode

Pendulum motion:
Overtone ($\Omega_q \gg \Omega_0$)



Zero-point fluctuation of this “measurement-defined” mechanical mode:

Bsp.: $Q_m = 10^6$, $\Omega_q = 2\pi \cdot 300 \text{ Hz}$

$$Q_m \cdot f_q \geq \sim 6 \times 10^{12} \left(\frac{T}{300 \text{ K}} \right)$$

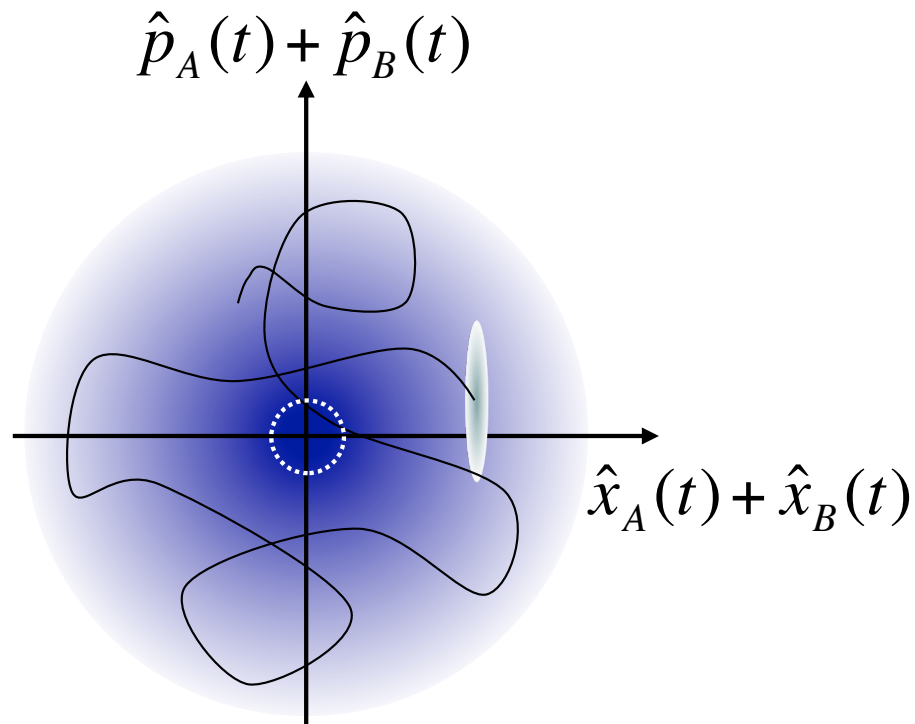
$$\Rightarrow T = 15 \text{ mK}$$

This temperature is almost achievable for a pendulum.

(iii) Precise fast monitoring: Random-walk tracking

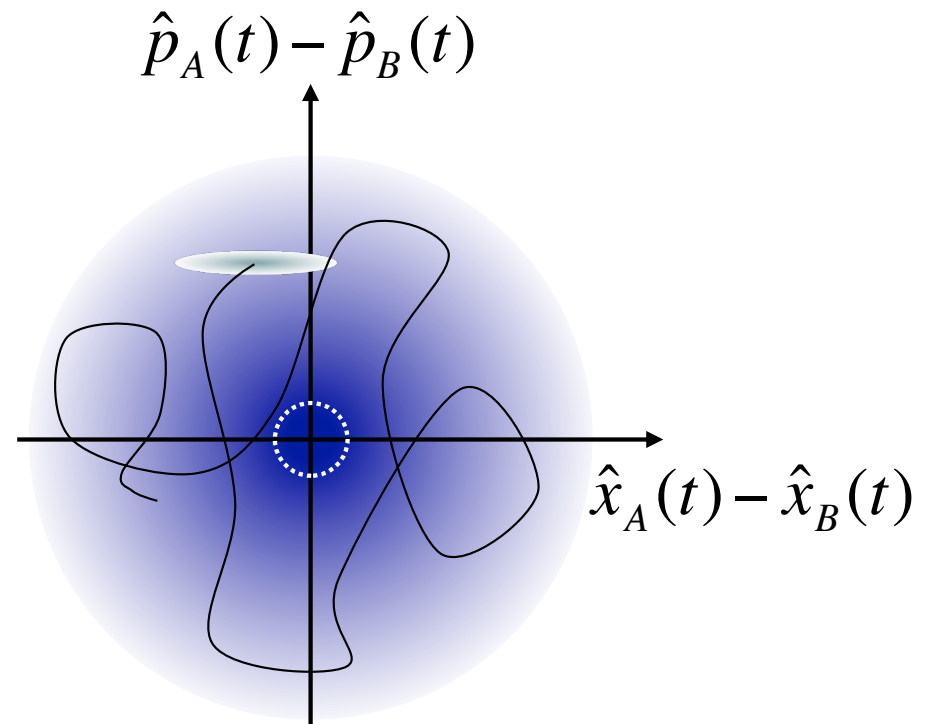
Detector “1”:

Common mode of motion is in thermal state



Detector “2”:

Differential mode of motion is in thermal state



Random walk in phase space.

The ellipses correspond to almost pure squeezed states.

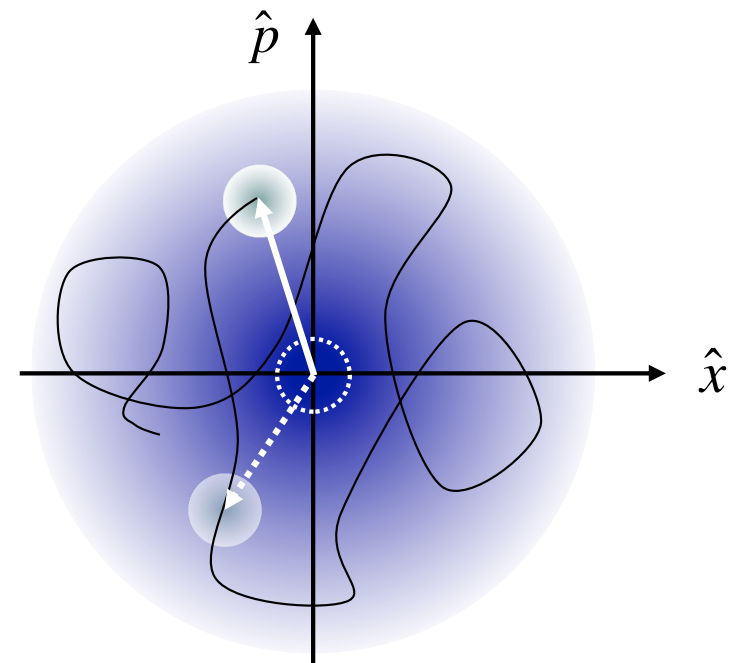
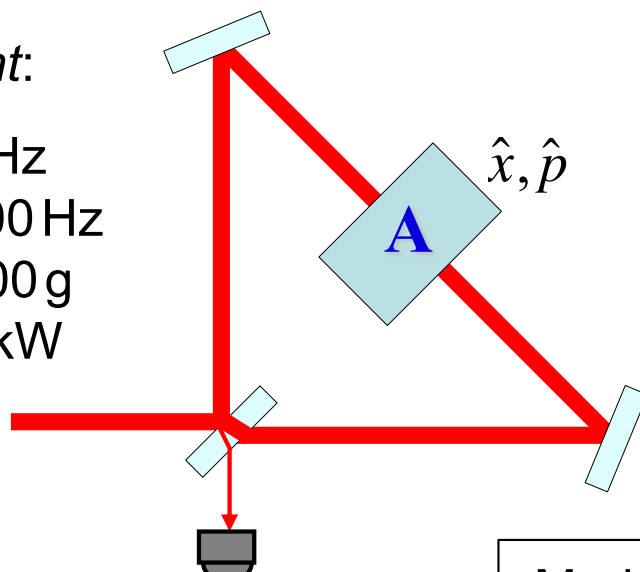
[RS, PRA **92**, 012126 (2015)]

Experiment with a *single* mirror

Short measurements with high precision require intense light.
 With a specific measurement technique and a model description of the experiment (Kalman filter) provides almost pure coherent states changing with time.

Experiment:

$\Omega_{\text{res}} \approx 1 \text{ Hz}$
 $f_q \approx 300 \text{ Hz}$
 $m = 100 \text{ g}$
 $P = 1 \text{ kW}$



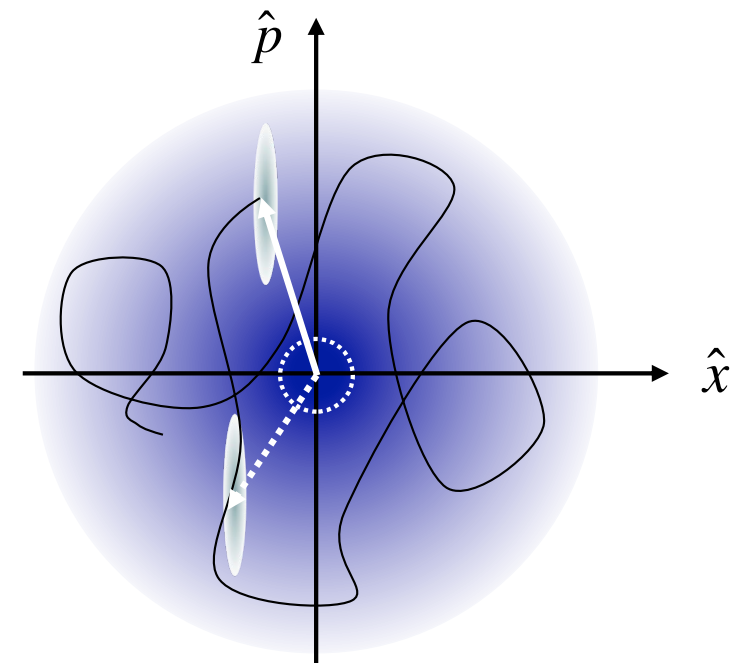
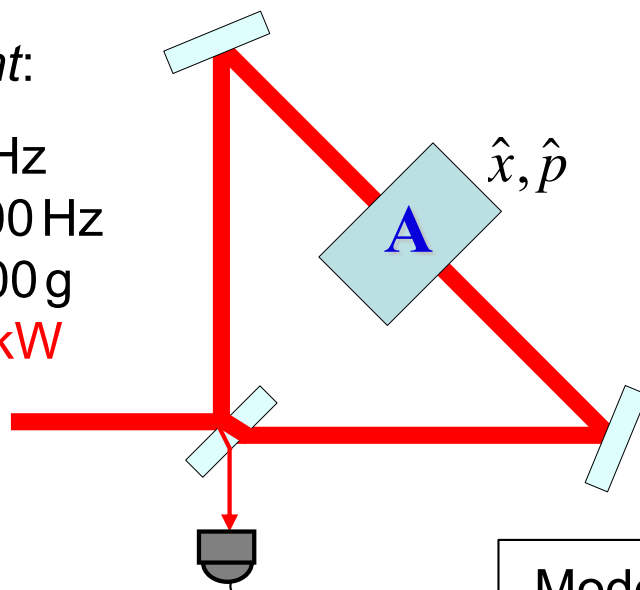
$\hat{X} \rightarrow$ Model of experiments (Kalman-Filter) $\rightarrow \hat{x}, \hat{p}(t) \rightarrow |\alpha\rangle(t)$.

Experiment with a *single* mirror

Even more intense light provides almost pure *squeezed* states changing with time. Due to higher measurement strength, the position information is more certain.

Experiment:

$\Omega_{\text{res}} \approx 1 \text{ Hz}$
 $f_q \approx 300 \text{ Hz}$
 $m = 100 \text{ g}$
 $P = 4 \text{ kW}$



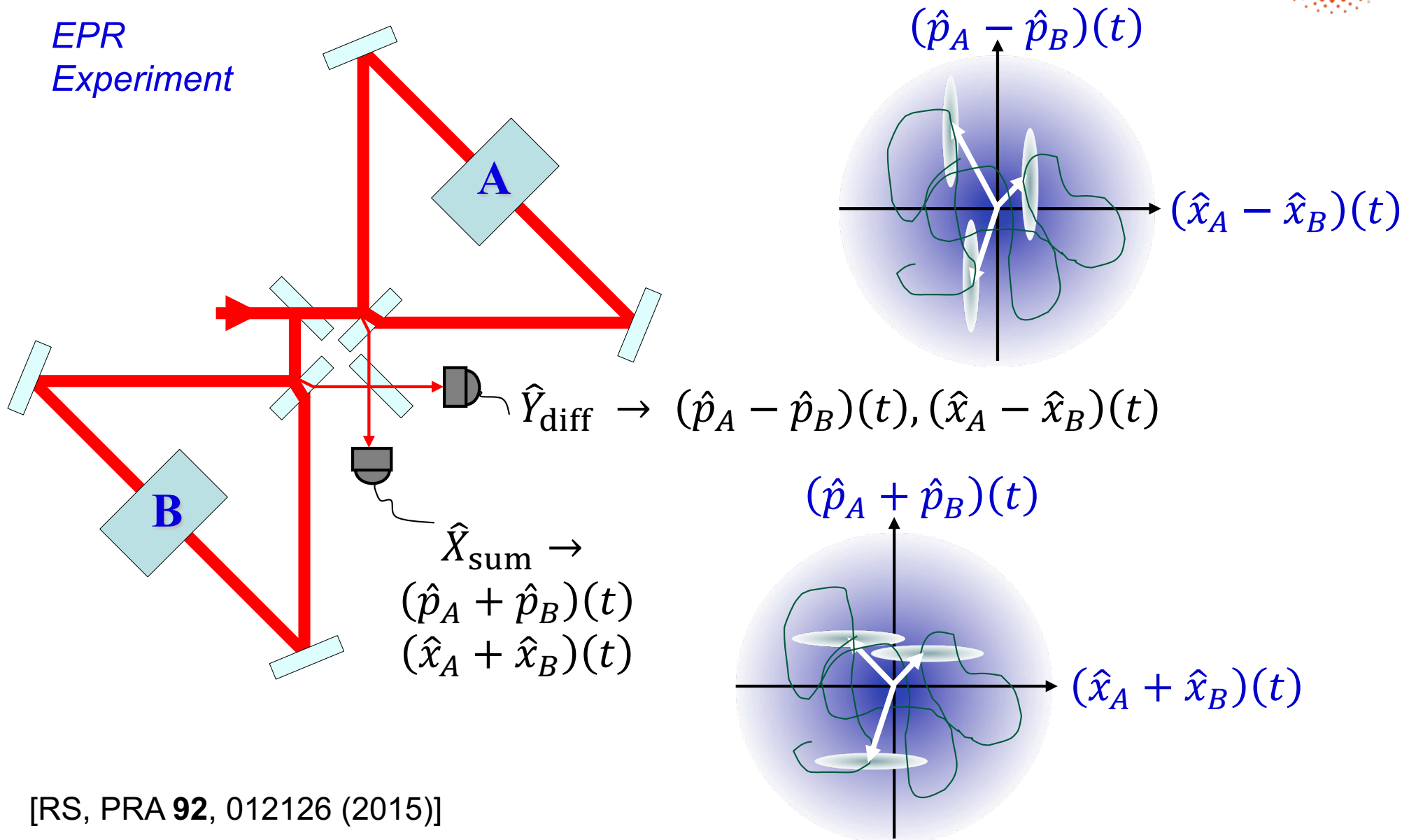
$\hat{X} \rightarrow$

Model of experiments (Kalman-Filter)

$\rightarrow \hat{x}, \hat{p}(t) \rightarrow |\theta, r\rangle(t).$

Two mirror entangled-state preparation

EPR
Experiment

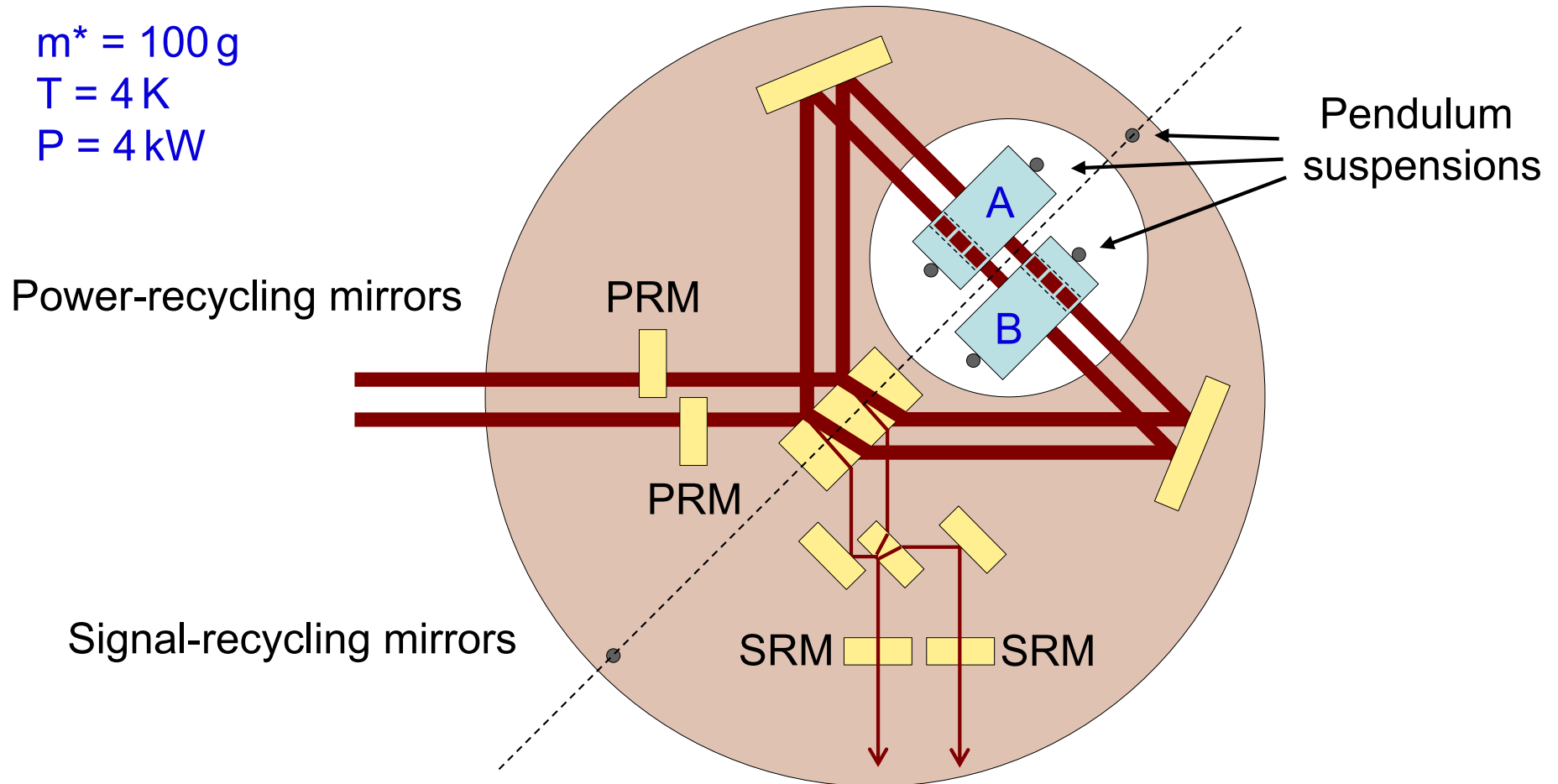


[RS, PRA 92, 012126 (2015)]

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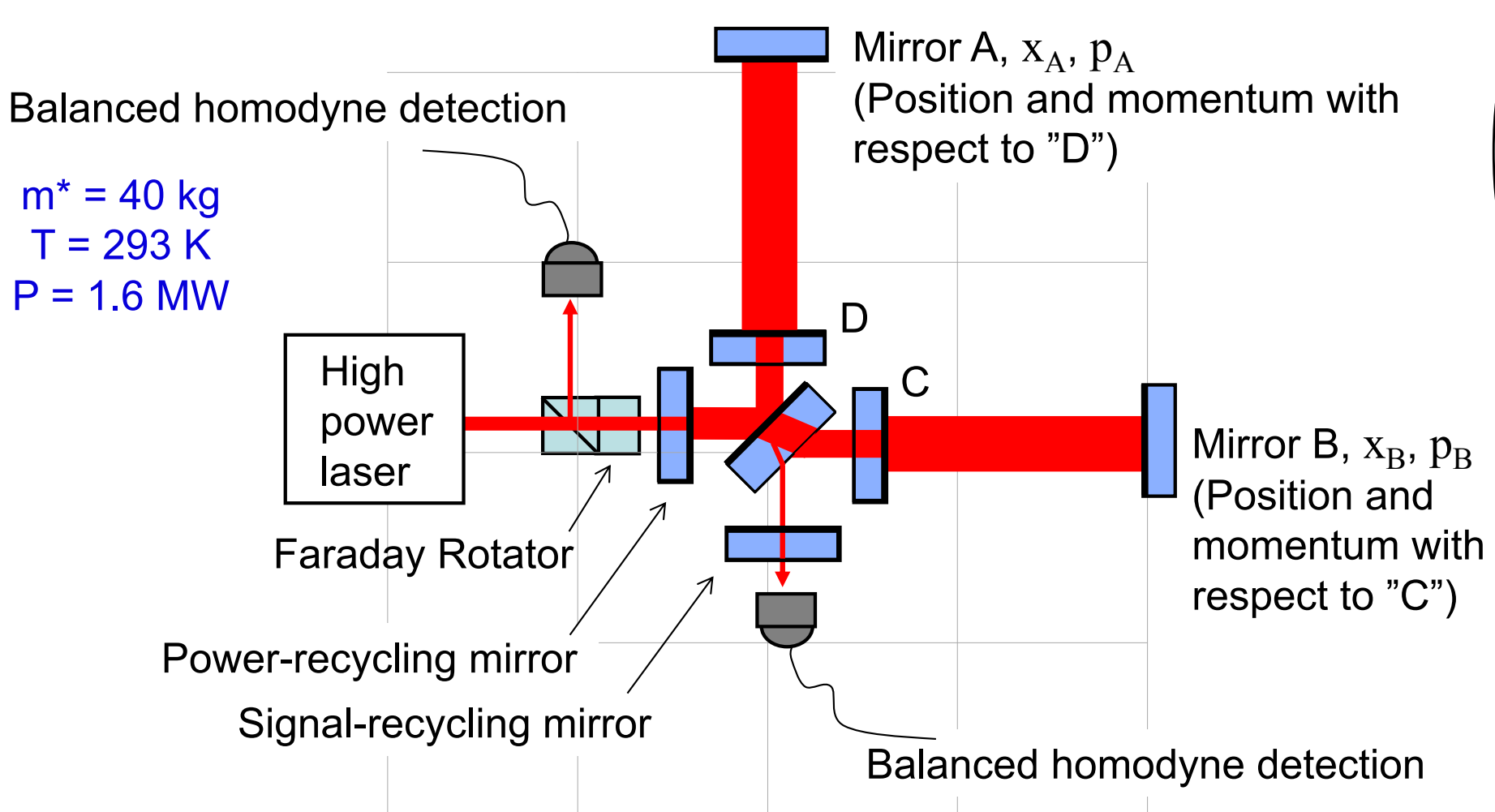
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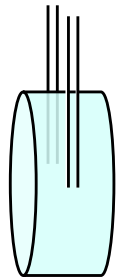
Entanglement in Gravitational-Wave Detectors?



$m^* = 40 \text{ kg}$
 $T = 293 \text{ K}$
 $P = 1.6 \text{ MW}$

EPR entanglement: $\Delta(x_A - x_B) \rightarrow 0$ together with $\Delta(p_A + p_B) \rightarrow 0$

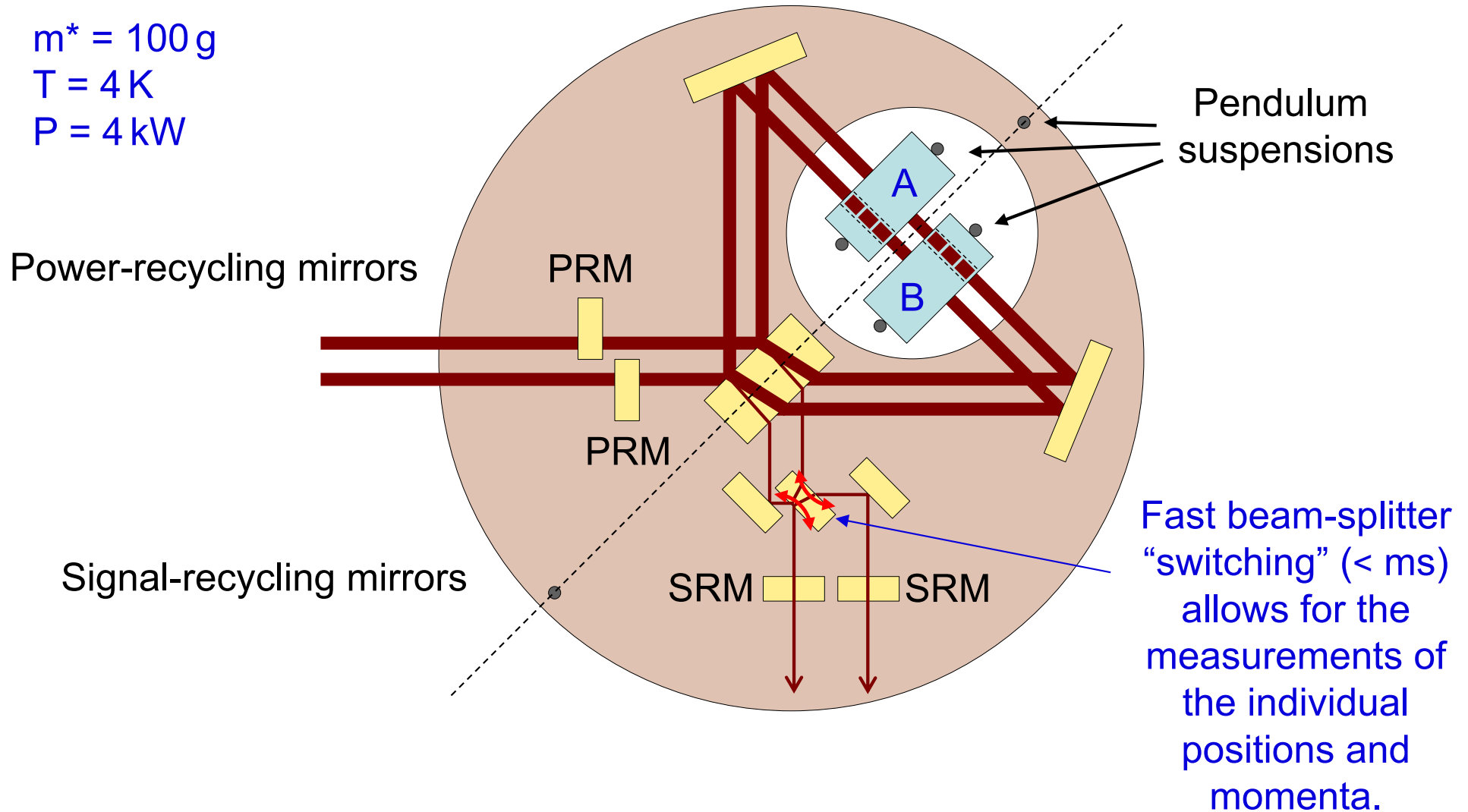
Müller-Ebhardt *et al.*, PRA **80**, 043802 (2009)



Verification of generated EPR Entanglement



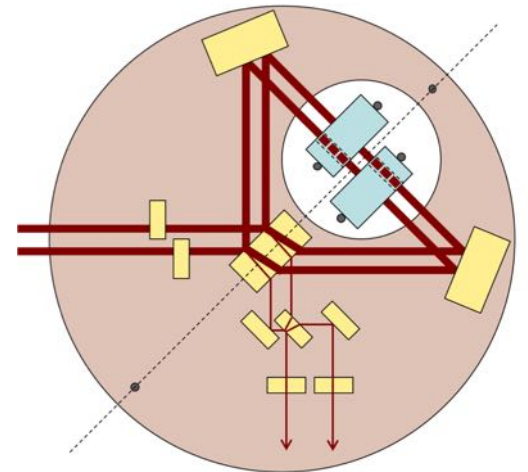
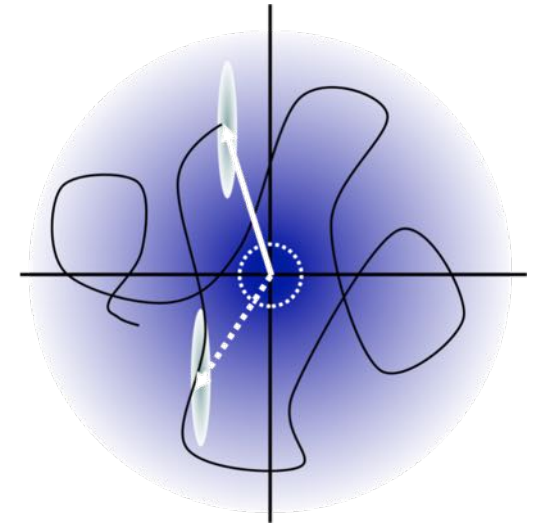
$m^* = 100 \text{ g}$
 $T = 4 \text{ K}$
 $P = 4 \text{ kW}$



[RS, PRA **92**, 012126 (2015)]

Summary

- The EPR gedanken experiment leads to the definition: “If a *single quantum* object decays the decay products allow for the observation of entanglement.”
- The Einstein-Podolsky-Rosen gedanken experiment can be realised with massive objects.
- In principle, entangled states of two 40kg mirrors being at $T = 293\text{K}$ can be generated in GW detectors, once the standard quantum limit (SQL) is reached.
- My ERC project ‘MassQ’ aims for the table-top generation of EPR-entangled motion of two mirrors of 100g.



<http://photon.physnet.uni-hamburg.de/de/ilp/schnabel/>

