Novel signatures of additional Higgs bosons at the LHC

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NEW PHYSICS IN THE HIGGS SECTOR?



New physics in the Higgs sector?



Possible BSM effects:

(*I*) Modifications of 125 GeV Higgs boson properties (couplings, decay rates, *CP*);

(II) Presence of additional(neutral/charged) scalar bosons;

(III) Presence of other new particles (e.g. SUSY particles) interacting with the Higgs boson.

⇒ Higgs sector is an exciting place to look for new physics!

 $\cdot\,$ 125 ${\rm GeV}$ Higgs boson: measurements consistent with SM hypothesis.

<u> </u>		1 1 1
ATLAS Preliminary Total	Stat Syst	. 🔳 SM
Vs = 13 TeV, 24.5 - 79.8 fb' m = 125.09 GeV /v / z 2.5		
p _{ost} = 71%	Total Stat	Syst
aoE xx 📥	0.96 +0.14 (+0.1	+0.09
00F ZZ	1.04 +0.16 (+0.1	4. +0.05)
ooF WW	1.08 +0.19(+0.1	1. +0.15)
QOF TT H	0.96 +0.56 (+0.30	+0.46
ggF comb.	1.04 ±0.00 (±0.0	7, +0.07
VBF yy	1.39 +0.40 (+0.8	+0.26
VBF ZZ	2.68 +0.98 (+0.94	+0.27
VBF WW H	0.59 +0.38 (+0.28	. ±0.21)
VBF TT I	1.16 +0.58 (+0.4	(, +0.40 -0.35)
VBF bb	3.01 +1.67 (+1.60	+0.39
VBF comb.	1.21 +0.24 (+0.11	(, ^{+0.16} -0.13
VH YY	1.09 +0.58 (+0.50	+0.25
VH ZZ	0.68 +1.20 (+1.11	+0.10
VH bb	1.19 +0.27 (+0.1	+0.20 -0.16
VH comb.	1.15 +0.24 -0.22 (±0.1	e, <u>+0.17</u>)
ttH+tH γγ 📻	1.10 +0.41 (+0.38	-0.14)
ttH+tH VV	1.50 +0.50 (+0.40	-0.38)
ttH+tH tt +	1.38 +1.13 (+0.8	-0.59)
ttH+tH bb	0.79 +0.50 (±0.2	e, ±0.52)
ttH+tH comb.	1.21 TOUR (±0.1	70.16)
-2 0 2 4	6	8
Parameter normali	zed to SM	value

- \cdot 125 GeV Higgs boson: measurements consistent with SM hypothesis.
- Searches for additional Higgs bosons: only limits, limits, limits, ...



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In addition: No signals in other new physics searches (SUSY, Dark Matter, ...), stringent limits on EDMs, ...

Main LHC search channels for neutral and charged scalars:

$$\begin{array}{ll} pp \rightarrow \phi \rightarrow \tau^{+}\tau^{-} & pp \rightarrow tb\phi^{\pm}, \phi^{\pm} \rightarrow \tau\nu \\ pp \rightarrow \phi \rightarrow t\overline{t} & pp \rightarrow tb\phi^{\pm}, \phi^{\pm} \rightarrow tb \\ pp \rightarrow \phi \rightarrow VV \ (V = W^{\pm}, Z) & pp \rightarrow qq\phi^{\pm}, \phi^{\pm} \rightarrow W^{\pm}Z \\ pp \rightarrow \phi \rightarrow \gamma\gamma \\ pp \rightarrow Z\phi \rightarrow Z(\text{invisible}) \\ pp \rightarrow \phi \rightarrow h_{125}h_{125} \\ pp \rightarrow h_{125} \rightarrow \phi\phi \\ pp \rightarrow \phi_{2} \rightarrow \phi_{1}Z \end{array}$$

 \Rightarrow important implications for BSM theories.

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Are there any (well-motivated) signatures missing? \Rightarrow Yes! E.g.:

- 1. $pp \rightarrow \phi_i(+X), \phi_i \rightarrow \phi_j \phi_k$
- 2. $pp \rightarrow \phi^{\pm}(+X), \phi^{\pm} \rightarrow W^{\pm}\phi$
- 3. $pp \rightarrow \phi \rightarrow \text{new particles}$

- ightarrow Two Real Scalar Singlet extension
 - ightarrow Two Higgs Doublet Model
 - \rightarrow MSSM (backup slides)

Additional scalar boson(s) only couple to SM gauge bosons and fermions if singlet field(s) mix with SM Higgs field. E.g., for a real singlet field s,

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h_{\rm SM} \\ s \end{pmatrix}$$

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Couplings of h_a to SM particles

- rescaled universally by κ_a ,
- orthogonality of mixing matrix: \Rightarrow sum rule $\sum_{a} \kappa_{a}^{2} = 1$.

Multi-scalar couplings

- originate from scalar potential parameters,
- can be very different to SM-like Higgs self-coupling.

THE TWO-REAL-SINGLET-MODEL (TRSM)

Scalar potential $(\Phi: SU(2)_L \text{ doublet, } S, X: SU(2)_L \text{ singlets})$

$$\mathcal{V} = \mu_{\Phi}^2 \Phi^{\dagger} \Phi + \mu_S^2 S^2 + \mu_X^2 X^2 + \lambda_{\Phi} (\Phi^{\dagger} \Phi)^2 + \lambda_S S^4 + +\lambda_X X^4 + \lambda_{\Phi S} \Phi^{\dagger} \Phi S^2 + \lambda_{\Phi X} \Phi^{\dagger} \Phi X^2 + \lambda_{S X} S^2 X^2.$$

Imposed $\mathbb{Z}_2 \times \mathbb{Z}'_2$ symmetry, which is spontaneously broken by singlet vevs.

 \Rightarrow three *CP*-even neutral Higgs bosons: h_1, h_2, h_3 Two interesting cases:

Case (a): $\langle S \rangle \neq 0, \langle X \rangle = 0 \Rightarrow X$ is DM candidate; **Case (b):** $\langle S \rangle \neq 0, \langle X \rangle \neq 0 \Rightarrow$ all scalar fields mix.

[Robens, TS, Wittbrodt 1908.08554]

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 \rightarrow Model parameters: 3 masses, 3 mixing angles, 3 vevs.

[Robens, TS, Wittbrodt 1908.08554]

HIGGS-TO-HIGGS DECAY SIGNATURES ($\langle S \rangle \neq 0$, $\langle X \rangle \neq 0$)

Rich phenomenology of $h_i \rightarrow h_j h_k$ **decays:** [Robens, TS, Wittbrodt 1908.08554]

- h_1 , h_2 or h_3 could be SM-like Higgs boson at 125 GeV,
- possible cascade decays \Rightarrow 3- and 4-Higgs boson final states!
- Lightest scalar *h*₁ decays identically as a SM Higgs boson at *M*₁.



HIGGS-TO-HIGGS DECAY SIGNATURES ($\langle S \rangle \neq 0$, $\langle X \rangle \neq 0$)

Rich phenomenology of $h_i \rightarrow h_j h_k$ decays:

ha

[Robens, TS, Wittbrodt 1908.08554]

benchmark plane	h ₁₂₅ candidate	target signature	possible successive decays
BP1	h ₃	$h_{125} \rightarrow h_1 h_2$	$h_2 \rightarrow h_1 h_1$ if $M_2 > 2 M_1$
BP2	h_2	$h_3 \rightarrow h_1 h_{125}$	-
BP3	h_1	$h_3 \rightarrow h_{125}h_2$	$h_2 ightarrow h_{125} h_{125}$ if $M_2 > 250 \ { m GeV}$
BP4	h ₃	$h_2 \rightarrow h_1 h_1$	
BP5	h_2	$h_3 \rightarrow h_1 h_1$	-
BP6	h_1	$h_3 \rightarrow h_2 h_2$	$h_2 ightarrow h_{125} h_{125}$ if $M_2 > 250~{ m GeV}$

BENCHMARK PLANE 1: $h_3 \rightarrow h_1 h_2$ (with $h_3 = h_{125}$)

- + Two light scalars with unknown masses $M_1, M_2 < 125~{\rm GeV},$
- Production of h_3 identical to SM Higgs: $\sigma(pp \rightarrow h_3) \simeq \sigma(pp \rightarrow h_{SM}) \sim 50 \text{ pb.}$



Idea: use invariant masses to (recursively) look for resonances in the spectrum.

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BENCHMARK PLANE 3: $h_3 \rightarrow h_1 h_2$ (with $h_1 = h_{125}$)

- $\cdot\,$ Two heavy scalars with unknown masses 125 ${\rm GeV} < M_2 < M_3,$
- Production of h_3 with signal strength $\mu \simeq 0.057$, decay rate BR $(h_3 \rightarrow h_1h_2) \sim (35 - 50)$ %.





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BENCHMARK PLANE 6: $h_3 \rightarrow h_2 h_2$ (with $h_1 = h_{125}$)

- \cdot Two heavy scalars with unknown masses 125 GeV < $M_2 < M_3$,
- Production of h_3 with signal strength $\mu \simeq 0.06$, decay rate BR $(h_3 \rightarrow h_2h_2) \sim (70 - 80)$ %.



· if 150 ≤ M_2 ≤ 250 GeV ⇒ $W^+W^-W^+W^-$ final state seems most promising,

 \rightarrow first $h_3 \rightarrow h_2 h_2$ search: [ATLAS, 1811.11028]

• if $M_2 > 250 \text{ GeV} \Rightarrow \text{spectacular } h_{125}h_{125}h_{125}h_{125} \text{ signature! } [\text{rate} \leq \mathcal{O}(10 \text{ fb})]$

Extrapolation of $pp \rightarrow H \rightarrow h_{125}h_{125} \rightarrow b\bar{b}b\bar{b}$ search

Consider $gg \rightarrow S_1 \rightarrow S_2S_2 \rightarrow b\bar{b}b\bar{b}$.

Extrapolation of current CMS search:

• modify $m_{b\bar{b}}$ criteria (for the case $M_{S_2} \neq$ 125 GeV).

For increasing M_{S_2} :

- QCD multi-b-jet background contamination in signal region (slowly) decreases,
- Signal rate (quickly) decreases with $BR(H_2 \rightarrow b\bar{b}).$

 \Rightarrow Relevant for $M_2 \lesssim 150~{
m GeV}$ (in TRSM).

[Barducci, Mimasu, No, Vernieri, Zurita, '19]



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HUNTING FOR A CHARGED HIGGS BOSON

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• Flavor observables \Rightarrow constraints in $(M_{\mu\pm}, \tan\beta)$ plane.



[Arbey, Mahmoudi Stål, TS '17]

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- Flavor observables \Rightarrow constraints in $(M_{H^{\pm}}, \tan \beta)$ plane.
- Higgs measurements prefer approximate alignment limit:

 $g_{hVV} \propto \sin(\beta - \alpha) \approx 1$, if light Higgs boson *h* at 125 GeV, $g_{HVV} \propto \cos(\beta - \alpha) \approx 1$, if heavy Higgs boson *H* at 125 GeV.



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Important fact (in 2HDM):

In the alignment limit the $H^{\pm}W^{\mp}\phi$ coupling is maximized [ϕ denotes the non-SM-like CP-even Higgs boson]:

$$g_{H^{\pm}W^{\mp}h} \propto \cos(\beta - \alpha), \quad g_{H^{\pm}W^{\mp}H} \propto \sin(\beta - \alpha)$$

 \Rightarrow If kinematically allowed: sizable $H^{\pm} \rightarrow W^{\pm} \phi$ decay rates!



 ϕ mostly decays to $b\bar{b}$, $\tau^+\tau^-$, or to WW^{*}, ZZ^{*}, $\gamma\gamma$ (if fermiophobic limit).

Current LHC H^{\pm} searches mostly focus on fermionic final states ($\tau \nu_{\tau}$, tb).

 \Rightarrow become insensitive if bosonic H^{\pm} decay modes dominate.



[TS 1908.10900], [TS, Wittbrodt (in progress)]



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Production process	Higgs decay processes	Final state particles		
$pp ightarrow H^{\pm}tb$	$H^{\pm} \rightarrow W^{\pm} \phi, \phi \rightarrow \begin{cases} bb \\ \tau \tau \\ WW \\ ZZ \\ \gamma \gamma \end{cases}$	$tbW^{\pm} + \begin{bmatrix} bb\\ \tau\tau\\ WW\\ ZZ\\ \gamma\gamma \end{bmatrix}$		
$pp ightarrow H^{\pm} \phi$	$H^{\pm} \rightarrow W^{\pm} \phi, \phi \rightarrow \begin{cases} bb \\ \tau \tau \\ WW \\ ZZ \\ \gamma \gamma \end{cases}$	$W^{\pm} + \left[\begin{array}{c} bb\\ \tau\tau\\ WW\\ ZZ\\ \gamma\gamma\end{array}\right] \oplus \left[\begin{array}{c} bb\\ \tau\tau\\ WW\\ ZZ\\ \gamma\gamma\end{array}\right]$		
$pp ightarrow H^{\pm}W^{\mp}$	$H^{\pm} \rightarrow W^{\pm} \phi, \phi \rightarrow \begin{cases} bb \\ \tau \tau \\ WW \\ ZZ \\ \gamma \gamma \end{cases}$	$W^{\pm}W^{\mp} + \begin{bmatrix} bb\\ \tau\tau\\ WW\\ ZZ\\ \gamma\gamma \end{bmatrix}$		
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$pp ightarrow H^{\pm} \phi$	$ \mathbf{H}^{\pm} \to \mathbf{W}^{\pm} \phi, \phi \to \begin{cases} \mathbf{b} \mathbf{b} \\ \tau \tau \\ WW \\ ZZ \\ \gamma \gamma \end{cases} $	$\mathbf{W}^{\pm} + \begin{bmatrix} \mathbf{b}\mathbf{b} \\ \tau\tau \\ WW \\ ZZ \\ 2\gamma \\ \gamma\gamma \end{bmatrix} \oplus \begin{bmatrix} \mathbf{b}\mathbf{b} \\ \tau\pi \\ WW \\ ZZ \\ \gamma\gamma \\ \gamma\gamma \end{bmatrix}$		
DESY-CMS efforts (D. Brunner, D. Krücker, I. Melzer-Pellmann).				
$pp \rightarrow H^{\pm}W^{\mp}$	$H^{\pm} \to W^{\pm} \phi, \phi \to \begin{cases} \tau \tau \\ WW \\ ZZ \\ \gamma \gamma \end{cases}$	$W^{\pm}W^{\mp} + \begin{bmatrix} \tilde{\tau}\tau\\WW\\ZZ\\\gamma\gamma\end{bmatrix}$		
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	(bb	<u>г bb</u> - <u>г bb</u>	
\Rightarrow Many new experimental opportunities for upcoming LHC Run(s)!			
		$\begin{bmatrix} 22 \\ \gamma\gamma \end{bmatrix} \begin{bmatrix} 22 \\ \gamma\gamma \end{bmatrix}$	

Heavy Higgs boson H at 125 GeV?

Can be realized in all 2HDM Types, with $\cos(\beta - \alpha) \approx 1$ (alignment limit), and light Higgs boson *h* with $g_{hVV} \approx 0$, and $M_h \in [M_H/2, 115]$ GeV.

Question:

Will we ever be able to tell whether h or H is at 125 GeV?

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Even in the alignment limit, $\cos(\beta - \alpha) \rightarrow 1$, charged Higgs effects on the Higgs rates do not decouple:

$$g_{HH^+H^-} \xrightarrow{c_{\beta-\alpha} \to 1} -\frac{1}{v} \left(M_H^2 + 2M_{H^+}^2 - 2\overline{m}^2 \right) \xrightarrow{M_{H^+} \gg M_H} -\frac{2M_{H^+}^2}{v}, \qquad H^{\pm} \xrightarrow{H^{\pm}} H^{\pm} \xrightarrow{H^{\pm}} H^{\pm}$$

because $\overline{m}^2 \equiv 2m_{12}^2 / \sin(2\beta) \lesssim \mathcal{O}(v^2)$
imposed by unitarity and stability
conditions.

\Rightarrow suppression of the $H \rightarrow \gamma \gamma$ rate!

[Bernon, Gunion, Haber, Jiang, Kraml '15].

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Higgs-to-Higgs decays can appear at large rates in simple BSM models.

 \Rightarrow Dedicated LHC searches for these signatures are highly motivated!

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We provided six scenarios in the two-real-singlet model as benchmarks for

- searches that generalize existing $H \rightarrow h_{125}h_{125}$ and $h_{125} \rightarrow hh$ searches to masses $\neq 125 \text{ GeV}$ and/or un-identical Higgs bosons in final state,
- novel searches for three- or four-Higgs final states that originate from cascade decays.

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Thank you very much for your attention!

Backup Slides

THE MINIMAL SUPERSYMMETRIC STANDARD MODEL (MSSM)

SUSY: Hypothetical space-time symmetry relating fermions & bosons.

 \Rightarrow Introduce *superpartners* for every SM field.



- \cdot SUSY cannot be exact. Expect SUSY masses $\gtrsim \mathcal{O}(1~{
 m TeV});$
- Neutral/charged EW gauginos and Higgsinos \xrightarrow{mix} neutralinos/charginos.

MSSM HIGGS SECTOR

The tree-level MSSM Higgs sector is a 2HDM of Type II with

$$\begin{split} \lambda_1 &= \lambda_2 = \frac{1}{4} (g^2 + g'^2), \qquad \lambda_3 = \frac{1}{4} (g^2 - g'^2), \\ \lambda_4 &= -\frac{1}{2} g^2, \qquad \qquad \lambda_5 = \lambda_6 = \lambda_7 = 0. \end{split}$$

It is described by only two parameters: M_A , tan β

Predicted tree-level mass spectrum:

$$\begin{split} M_{h,H}^2 &= \frac{1}{2} \left[M_A^2 + M_Z^2 \mp \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta} \right] \quad \Rightarrow M_h^{\text{tree}} \le M_Z \; ! \\ M_{H^\pm}^2 &= M_A^2 + M_W^2 \end{split}$$

(SM-like) Higgs mass M_h receives large radiative corrections:

$$(\Delta M_h^2)_{1L}^{t,\tilde{t}} \approx \frac{3m_t^4}{2\pi^2 v^2} \left[\log\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2} \left(1 - \frac{X_t^2}{12M_S^2}\right) \right] \qquad \begin{array}{l} (M_A \gg M_Z, \tan\beta \gg 1) \\ X_t = A_t - \mu/\tan\beta, \\ M_S = \sqrt{M_{\tilde{t}_1}M_{\tilde{t}_2}}. \end{array}$$

 \Rightarrow with SUSY particles at TeV-scale we can get $M_h \lesssim 135 \text{ GeV}!$

[Bagnaschi, Bahl, Fuchs, Hahn, Heinemeyer, Liebler, Patel, Slavich, TS, Wagner, Weiglein]

6 scenarios with fixed scale $M_S \sim \mathcal{O}(\text{TeV})$, 2 scenarios with variable M_S .

M ¹²⁵ _h	"standard" scenario, all SUSY masses $\gtrsim 1{\rm TeV}$		
$M_h^{125}(ilde{ au})$	light staus: sizable effect on $h\to\gamma\gamma$ at large $\tan\beta$		
$M_h^{125}(ilde{\chi})$	light EW-inos: new decay channels for heavy Higgs bosons		
M_h^{125} (alignment)	h couplings very SM-like even at low M _A values		
M _H ¹²⁵	heavier MSSM Higgs boson H is SM-like at \sim 125 ${ m GeV}$		
M ¹²⁵ (CPV)	interference effects suppress heavy Higgs rate in $ au^+ au^-$ channel		
	[Bagnaschi et al. 1808.07542]		
M ¹²⁵ h,EFT	"standard" scenario for the low $ aneta$ region		
$M_{h, ext{EFT}}^{125}(ilde{\chi})$	light EW-ino scenario for the low $ aneta$ region		
,	[Bahl, Liebler, TS 1901.05933]		

(effort within the LHC Higgs Cross Section Working Group)

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M_h^{125} BENCHMARK SCENARIO



- + Assumption: all SUSY particle masses are $\gtrsim 1\,{\rm TeV}.$
- Higgs rates & limits \Rightarrow H, A and H[±] expected to be heavy (mass \gtrsim 600 GeV).

$M_h^{125}(\tilde{\chi})$ benchmark scenario: light neutralinos & charginos



- Light neutralinos and charginos with masses \sim (100 250) GeV.
- Impact of H/A $\to \tau^+\tau^-$ search limit on parameter space weakened due to additional H/A decay modes.

${\cal M}_h^{ m 125}(ilde{\chi})$ benchmark scenario: light neutralinos & charginos



- $\cdot\,$ Light neutralinos and charginos with masses \sim (100 250) GeV.
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- Lifting EW-ino mass spectrum by +100 GeV does not alter heavy Higgs decays significantly (in relevant parameter region).

${\cal M}_h^{ m 125}(ilde{\chi})$ benchmark scenario: light neutralinos & charginos



- Impact of $H/A \rightarrow \tau^+ \tau^-$ search limit on parameter space weakened due to additional H/A decay modes.
- Lifting EW-ino mass spectrum by \pm 100 GeV does not alter heavy Higgs decays significantly (in relevant parameter region).

RE-OPENING THE LOW tan β **REGION**

Standard scenarios: $M_h < 122 \text{ GeV}$ for $\tan \beta \lesssim 6$, because $M_S \sim (1-2) \text{ TeV}$.

Allow lower tan β values by tuning $M_h = 125$ GeV at every point:

 $\mathcal{O}(\mathrm{TeV}) \lesssim M_{S} \lesssim 10^{16} \mathrm{~GeV}.$

Employ an effective field theory (EFT) calculation with a low-energy 2HDM(plus electroweakinos and/or gluinos).[Bahl, Hollik 1805.00867]

State-of-the-art calculation implemented in (yet unpublished) **FeynHiggs** version.

$$M_h^{125}$$
 scenario $\longrightarrow M_{h,EFT}^{125}$ scenario
 $M_h^{125}(\tilde{\chi})$ scenario $\longrightarrow M_{h,EFT}^{125}(\tilde{\chi})$ scenario

Similar (older) scenarios:

hMSSM [Djouadi et al. 1307.5205], low-tan β -high scenario [LHCHXSWG-2015-002].







Important search channels: $H \rightarrow hh$ and $H/A \rightarrow t\bar{t}$.

Recent discussion of $H/A \rightarrow t\bar{t}$ signal+BG interference effects and discovery prospects: [Djouadi, Ellis, Popov, Quevillon 1901.03417]

$M_{h,\mathrm{EFT}}^{\mathrm{125}}(\tilde{\chi})$ SCENARIO



+ Light electroweakinos lead to upward shift of M_h by $\sim 1.5~{\rm GeV}.$

 \Rightarrow Slightly lower M_S values required as in $M_{h,\text{EFT}}^{125}$ scenario.

$M_{h, { m EFT}}^{ m 125}(ilde{\chi})$ SCENARIO



- Light electroweakinos lead to upward shift of M_h by ~ 1.5 GeV. \Rightarrow Slightly lower M_s values required as in $M_{h, FFT}^{125}$ scenario.
- Light charginos lead to $h \rightarrow \gamma \gamma$ enhancement at low tan β . \Rightarrow very low tan β values are constrained by LHC Higgs signal rates.

$M_{h,\text{EFT}}^{125}(\tilde{\chi})$ scenario: heavy Higgs decays



For $M_A \gtrsim 400 \text{ GeV}$, heavy-Higgs-to-electroweakino decays are dominant.

As in the standard $M_h^{125}(\tilde{\chi})$ scenario:

Cascade decays preferred, leading to multi-W/Z-boson+ $\not{\not{E}}_T$ signatures.

 \Rightarrow Dedicated experimental analyses of $H/A \rightarrow \tilde{\chi}\tilde{\chi}$ decays are well-motivated!

PUBLIC TOOLS FOR TESTING BSM MODELS WITH HIGGS RESULTS

HiggsBounds

Tests BSM Higgs sectors against exclusion limits from LEP, Tevatron and LHC Higgs searches

HiggsSignals

Tests BSM Higgs sectors against LHC (& Tevatron) Higgs signal rate and mass measurements

 \Rightarrow excluded/allowed at 95% C.L.



[Bechtle, Heinemeyer, Klingl, TS, Weiglein, Wittbrodt]

Available at http://higgsbounds.hepforge.org.

$\Rightarrow \chi^2$ (sep. for rates and mass)

IMPACT OF HIGGS RATE MEASUREMENTS AT THE LHC

Singlet model:

(assume heavier Higgs at 125 GeV)





 \Rightarrow Light Higgs h_S must have very reduced couplings $g/g_{SM} = \cos \alpha \lesssim 0.26$. Note: further constraints arise from LEP Higgs searches.

HL-LHC PROSPECTS ON INVISIBLE HIGGS DECAYS



See Section 6 in [Cepeda et al., 1902.00134].

SM-LIKE HIGGS BOSON DECAY RATES



BENCHMARK SCENARIO 2: $h_3 \rightarrow h_1 h_2$ (with $h_2 = h_{125}$)

- $\cdot\,$ One light and one heavy scalar with unknown masses $M_1 < 125~{\rm GeV} < M_3,$
- Production of h_3 with signal strength $\mu \simeq 0.04$.



Idea: generalize existing $H \rightarrow h_{125}h_{125}$ searches to this case (unknown M_1).

BENCHMARK SCENARIOS 4 & 5: $h_{\text{NON-SM}} \rightarrow h_1 h_1$ (with $h_1 \neq h_{125}$)

- (Symmetric) decays to h_1h_1 not involving the SM-like Higgs boson,
- Production of non-SM heavier scalar with signal strength $\mu \simeq 0.06$, decay rates BR($h_{non-SM} \rightarrow h_1h_1$) ~ (70 – 100)%.



Extrapolation of $pp \rightarrow H \rightarrow h_{125}h_{125} \rightarrow b\bar{b}b\bar{b}$ search

[Barducci, Mimasu, No, Vernieri, Zurita, '19]



Complementarity with LHC searches for $pp \rightarrow S_1 \rightarrow SM$ particles.

CP-conserving Two Higgs Doublet Model (2HDM)

2 complex $SU(2)_L$ doublets \Rightarrow 5 Higgs states h, H, A, H^{\pm}

Higgs potential (general basis): $(\Phi_1, \Phi_2: SU(2)_L \text{ doublets})$

$$\begin{aligned} \mathcal{V} &= m_{11}^2 \Phi_1^{\dagger} \Phi_2 + m_{22}^2 \Phi_2^{\dagger} \Phi_2 - [m_{12}^2 \Phi_1^{\dagger} \Phi_2 + \text{h.c.}] + \frac{1}{2} \lambda_1 (\Phi_1^{\dagger} \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^{\dagger} \Phi_2)^2 \\ &+ \lambda_3 (\Phi_1^{\dagger} \Phi_1) (\Phi_2^{\dagger} \Phi_2) + \lambda_4 (\Phi_1^{\dagger} \Phi_2) (\Phi_2^{\dagger} \Phi_1) + [\frac{1}{2} \lambda_5 (\Phi_1^{\dagger} \Phi_2)^2 + \text{h.c.}] \end{aligned}$$

$$\mathbb{Z}_2$$
 symmetry $(\Phi_1 \rightarrow +\Phi_1, \Phi_2 \rightarrow -\Phi_2)$ is softly broken if $m_{12}^2 \neq 0$.

Assuming CP conservation, we can choose all parameters $\in \mathbb{R}$.

CP-conserving Two Higgs Doublet Model (2HDM)

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Extending the \mathbb{Z}_2 to the fermion sector suppresses tree-level FCNCs:

2HDM	u	d	l
Type I	Φ ₂	Φ ₂	Φ ₂
Type II	Φ ₂	Φ ₁	Φ ₁
Type III	Φ ₂	Φ ₁	Φ ₂
Type IV	Φ ₂	Φ ₂	Φ_1

Two parameters govern the tree-level couplings: $\tan \theta = v_0/v_0$

$$\begin{pmatrix} \sqrt{2}\operatorname{Re}(\Phi_2) - v_2 \\ \sqrt{2}\operatorname{Re}(\Phi_1) - v_1 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}$$

Higgs-vector boson couplings:

hVV: $\sin(\beta - \alpha)$, HVV: $\cos(\beta - \alpha)$, AVV: 0.

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Extending the \mathbb{Z}_2 to the	he fermion s	ector suppresses	tree-level FCNCs:
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2HDM	u u	d	l	coupling	Type I	Type II
Type I Type II Type III Type IV	$\begin{vmatrix} \Phi_2 \\ \Phi_2 \\ \Phi_2 \\ \Phi_2 \\ \Phi_2 \\ \Phi_2 \end{vmatrix}$	$\begin{array}{c} \Phi_2 \\ \Phi_1 \\ \Phi_1 \\ \Phi_2 \end{array}$	$\begin{array}{c} \Phi_2 \\ \Phi_1 \\ \Phi_2 \\ \Phi_1 \end{array}$	huu hdd,hll Huu Hdd,Hll Auu Add,All	$ \begin{vmatrix} \cos \alpha / \sin \beta \\ \cos \alpha / \sin \beta \\ \sin \alpha / \sin \beta \\ \sin \alpha / \sin \beta \\ - \cot \beta \\ \cot \beta \end{vmatrix} $	$\begin{array}{l} \cos\alpha/\sin\beta\\ -\sin\alpha/\cos\beta\\ \sin\alpha/\sin\beta\\ \cos\alpha/\cos\beta\\ -\cot\beta\\ -\cot\beta\\ -\tan\beta \end{array}$