Non perturbative quantum field theory tests in the LUXE strong field experiment

A. Hartin

UCL

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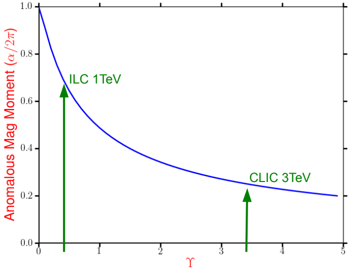
Motivation: polarising the vacuum

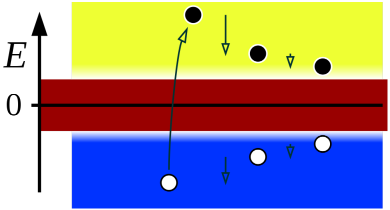
Anomalous magnetic moment

$$\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi} \int_0^\infty \frac{2\pi\, dx}{(1+x)^3} \left(\frac{x}{\Upsilon}\right)^{1/3} \mathrm{Gi}\left(\frac{x}{\Upsilon}\right)^{1/3}$$



- There is a predicted strong field correction to the AMM
- Υ related to the strong field strength
- The quantum vacuum becomes dispersive in the presence of a strong field

Schwinger limit



- Quantum vacuum becomes more dispersive with field strength
- At Schwinger limit quantum vacuum decays into a real pair
- The Schwinger critical field ($E_{\text{cr}} = m_e^2 c^3 / e\hbar = 1.32 \times 10^{18}$ V/m)
- How do we incorporate these vacuum changes into our theories?

Furry Picture - a non perturbative, semi classical QFT

- Separate gauge field into external A_μ^{ext} and quantum A_μ parts

$$\mathcal{L}_{\text{QED}}^{\text{Int}} = \bar\psi(i\partial\!\!\!/ - m)\psi - \frac{1}{4}(F_{\mu\nu})^2 - e\bar\psi(A\!\!\!/^{\text{ext}} + A\!\!\!/)\,\psi$$

$$\mathcal{L}_{\text{QED}}^{\text{FP}} = \bar\psi^{\text{FP}}(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} - \frac{1}{4}(F_{\mu\nu})^2 - e\bar\psi^{\text{FP}} A\!\!\!/\,\psi^{\text{FP}}$$

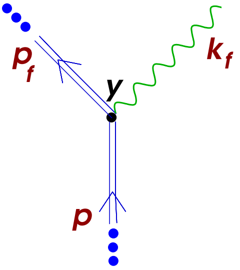
- Euler-Lagrange equation $\rightarrow$ new equations of motion requires exact (w.r.t. A^{ext}) solutions ψ^{FP}

$$(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} = 0$$

- For certain classes of external fields (plane waves, Coloumb fields and combinations) exact solutions exist **[Volkov Z Physik 94 250 (1935), Bagrov and Gitman,** *Exact solutions of relativistic wave equations* **(1990)]**

$$\psi^{\text{FP}} = \mathbf{E}_p\, e^{-ipx}\, u_p, \quad \mathbf{E}_p = \exp\left[-\frac{1}{2(k\cdot p)}\left(eA\!\!\!/^{\text{ext}} k\!\!\!/ + i2e(A^e\cdot p) - ie^2 A^{\text{ext}\,2}\right)\right]$$

Dressed Furry Picture (FP) vertices



- Double fermion lines are Volkov-type solutions
- Volkov E_p functions "dress" the vertex

$$\gamma_\mu^{\mathrm{FP}} = \int d^4x\, \overline{\mathbf{E}_f}(x)\gamma_\mu \mathbf{E}_p(x)\, e^{i(p_f - p + k_f)\cdot x}$$

- Momentum space vertex has contribution nk from external field

$$\gamma_\mu^{\mathrm{FP}}(x) = \sum_{n=-\infty}^{\infty} \int_{-\pi L}^{\pi L} \frac{d\phi}{2\pi L} \exp\left(i\frac{n}{L}[\phi - (kx)]\right)\gamma_\mu^{\mathrm{FP}}(\phi)$$

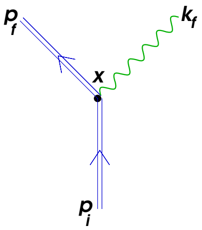
- Fourier transform of circularly polarised field leads to Bessel functions

$$\int_{-\pi}^{\pi} \frac{d\phi_v}{2\pi} \exp\left[ir\phi_v - z\sin\phi_v\right] = \mathrm{J}_r(z)$$

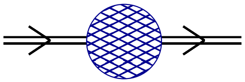
- Transition probabilities built from FP Feynman diagrams/dressed vertices

$$M_{\mathrm{fi}}^{\mathrm{HICS}} = \int \mathrm{d}^4x\ \bar{u}_{\mathrm{fr}}\, \gamma^{\mathrm{FP}}\, u_{\mathrm{is}}\, e^{-i(p_f + k_f - k_i)}, \qquad W = \int \frac{\mathrm{d}\vec{p}_{\mathrm{f}}}{2\epsilon_{\mathrm{f}}} \frac{\mathrm{d}\vec{k}_{\mathrm{f}}}{\omega_{\mathrm{f}}} \left|M_{\mathrm{fi}}^{\mathrm{HICS}}\right|^2$$

Unstable Strong field particles & resonant transitions

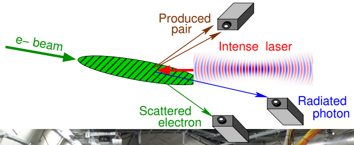


- **Electrons decay in strong field Furry picture**
  - Background field renders vacuum a dispersive medium
  - new effects: Lamb shift, vacuum birefringence, resonant transitions
  - electron has a finite lifetime, Γ and probability of radiation, W



- **Resonant transitions in propagator**
  - required by S-matrix analyticity
  - Optical theorem $W = \text{Im}(\Sigma)$
  - extra propagator poles leading to physically accessible resonances
  - related to energy level structure of vacuum

Similar decay (one photon pair production) and lifetime for photons



$$\xi(=a_0) = \frac{e|\vec{A}|}{m}$$ **intensity parameter**

ξ appears as a mass shift,

$m_* \to m\sqrt{1+\xi}$

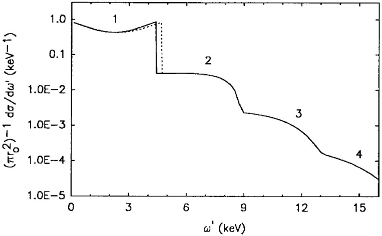
$\xi = 3.7 \times 10^{-19} I$ [W/cm^2] λ [micron]

$\xi \approx 1$ reached with a $\sim$1J, focused optical laser pulse

$$\chi, \Upsilon = \xi \frac{k \cdot p}{m^2}$$ **recoil parameter**

coupling constant conjecture: $\alpha\chi^{2/3}$

$\chi, \Upsilon \approx 1$ Schwinger field in rest frame



Experiment	$\lambda (nm)$	E_{laser}	focus	pulse	E_{e^-} (GeV)	ξ	χ
SLAC E144	527/1053	2 J	50 μm^2	1.88 ps	46.6	0.66	2.7
LUXE	800	3 J	100 μm^2	0.035 ps	17.5	2.64	0.96
FACET II†	527/800/1053	1 J	64 μm^2	0.04 ps	15	2	2.1
ELI-NP	1053	2.2 J	100 μm^2	0.022 ps	0.750	9.25	0.09
AWAKE	527	1 J	64 μm^2	0.04 ps	50	2	5.1

Modelling the strong field - IPW,LCFA,Envelope

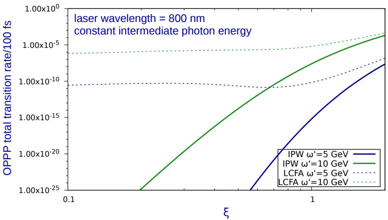
- LUXE strong laser - 800 nm, 35 fs gaussian pulse, linearly or circularly polarised, 10 micron spot, $0.1 < \xi < 5$
- Full SQED calculations in a pulse are very difficult, so approximate
- Locally constant field approx (LCFA) assumes $\xi \gg 1$. Not appropriate for LUXE
- Finite pulse smears out momentum transfer

  Prob $= \text{sinc} N\pi \, (I - F + nk) \, M(I, F, nk)$

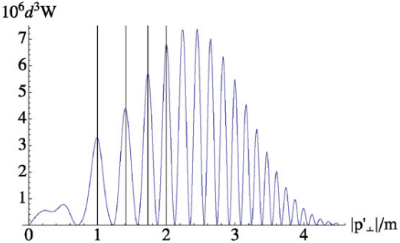
  N is the number of wavelengths in the pulse

  $N \approx 12$ for LUXE, well described by delta comb (discrete external field photons)
- So infinite plane wave (IPW) is suitable. Well understood Volkov solutions can be employed
- In simulation the local intensity of the laser pulse is used as the amplitude of the IPW calculation

LCFA approximation in OPPP



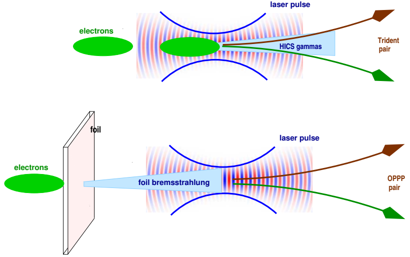
Momentum transfer in a laser pulse



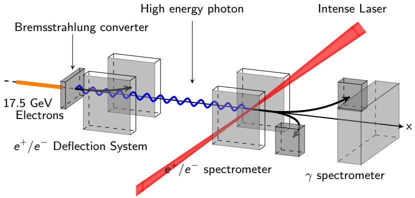
Isolating the main processes at LUXE

- Three main processes at LUXE - HICS gamma production, One photon pair production (OPPP), Trident process (HICS gammas + strong field) pair production
- There are compelling reasons to study the three processes separately
- HICS shows mass shift - strong field leads to increase in electron rest mass
- Trident leads to rare resonance processes, related to dispersive vacuum
- OPPP pair production at ultra high intensity - non-perturbative physics
- PROBLEM: Trident process pair production limited by laser intensity (suppressed already at $\xi \sim 3$)
- SOLUTION: Use foil to convert electrons to gammas upsteam of the strong field IP with high intensity laser further upstream

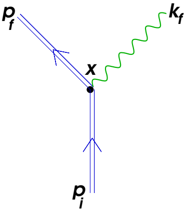
Strong field IPs



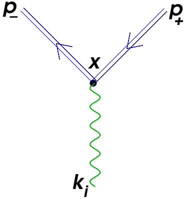
Strong field + foil, experimental setup



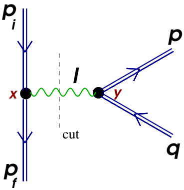
Tour of LUXE processes



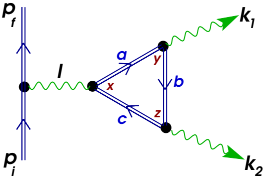
High intensity Compton scatttering (HICS)



One photon pair production (OPPP)



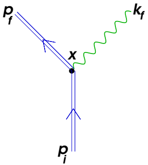
Trident process (one step and two step)



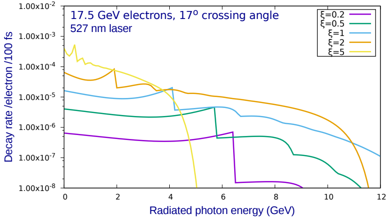
Photon splitting (vacuum birefringence)

HICS - High Intensity Compton scattering

- Created inidicative parameter sets with different laser intensity
- Increasing ξ increases the HICS rate, but suppresses the photon energy (the mass shift)
- Optimise ξ - need energetic enough photons for pair production, trade off between photon rate and photon energy
- The energy of the radiated photons also dependent on initial electron energy
- **TODO: linear polarisation**



HICS for 17 GeV electrons, intensity sweep



$$\Gamma_{\text{HICS}} = -\frac{\alpha m^2}{\epsilon_{\text{i}}} \sum_{n=1}^{\infty} \int_0^{u_n} \frac{du}{(1+u)^2} \left[\mathrm{J}_n^2(z_u) - \frac{\xi^2}{4} \frac{1+(1+u)^2}{1+u} \left(\mathrm{J}_{n+1}^2 + \mathrm{J}_{n\text{-}1}^2 - 2\,\mathrm{J}_n^2 \right) \right]$$

$$z_{\text{u}} \equiv \frac{m^2 \xi \sqrt{1+\xi^2}}{k \cdot p_i} [u(u_n - u)]^{1/2}, \quad u_n \equiv \frac{2(k.p_i)\, n}{m^2(1+\xi^2)}, \quad \xi \equiv \frac{e|A|}{m}$$

HICS - Mass shift

- Decay rate proportional to

$$\sum_n \delta^{(4)}\left[p_\mathrm{i} + k\frac{\xi^3}{2\chi_\mathrm{i}} + nk - p_\mathrm{f} - k\frac{\xi^3}{2\chi_\mathrm{f}} - k_\mathrm{f}\right]$$

- Momentum conservation is a sum over external field photon contributions, nk
- Even for $n=0$ there is an irreducible contribution

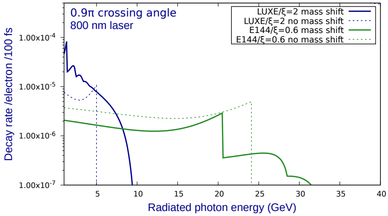
$$p_\mathrm{i} + k\frac{\xi^3}{2\chi_\mathrm{i}} \to p_\mathrm{i}^2 = m^2(1+\xi^2)$$

- Manifests in Compton edge shift

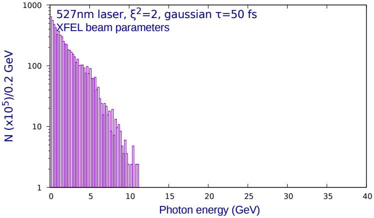
$$\frac{\omega_\mathrm{f}}{\epsilon_\mathrm{i} - \omega_\mathrm{f}} \leq \frac{2nk\cdot p_\mathrm{i}}{m^2(1+\xi^2)} \text{ c.f. } \frac{2nk\cdot p_\mathrm{i}}{m^2}$$

- Significant part of electron energy taken up by electron motion in the field/dispersive vacuum. Less energy available for radiated photon
- Edge shift in comparison to normal Compton scattering
- Real bunch collision smears the edge - recover with cuts?

HICS with mass shift for LUXE and E144



IPstrong monte-carlo, HICS for LUXE/ξ2



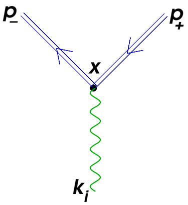
OPPP - one photon pair production

- Assuming that the initial state photon energy is known..
- The produced positron spectra is smooth, maximum at about half the photon energy
- Total OPPP rate is much better with higher laser intensity (and higher photon energy)
  - Pair must be created with the mass shift

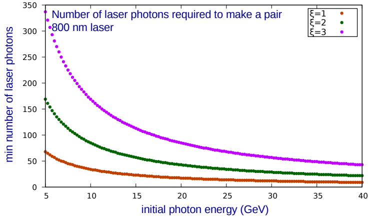
$$n \geq \frac{m^2(1+\xi^2)}{k \cdot k_\mathrm{i}}$$

...so another way to detect mass shift?

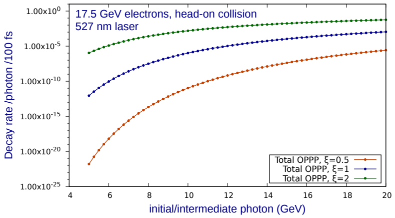
One photon pair production



Minimum laser photons needed for OPPP



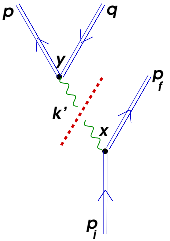
Total OPPP rate for LUXE with different ξ



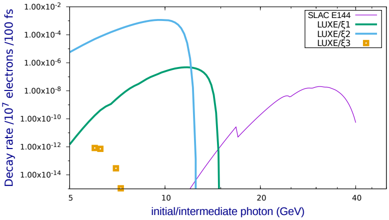
2 step Trident - HICS + OPPP

- This is only one of the ways pairs are produced at LUXE
- $\frac{d\Gamma_{\text{2sTrid}}}{d\omega_f} = \frac{d\Gamma_{\text{HICS}}}{d\omega_f} \times \Gamma_{\text{OPPP}}(\omega_f)$
- Trident rate magnitude orders lower than OPPP. Fortunately, we have a lot of electrons... match the beam to laser spot, given ξ constraints
- Estimate 10^7 interactions per bunch crossing (E144)... total rate VERY ξ dependent
- Total positron rate dependence on ξ will start to decrease somewhere between ξ=1 and ξ=3
- One step trident (virtual photon) will not necessarily be restricted in this way

2 step Trident = HICS + OPPP

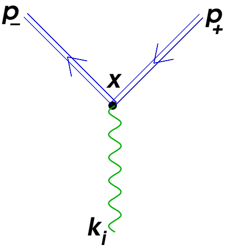


Positron rate for different parameter sets

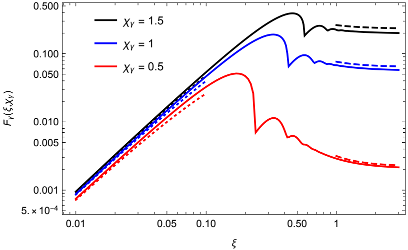


OPPP and Schwinger critical field measurement

One photon pair production (OPPP)



OPPP rate, non perturbative regime



OPPP Rate at constant χ reaches non perturbative asymptote for $\xi \geq 1$

$$\Gamma_{\text{OPPP}} = \frac{\alpha m^2}{2\omega_{\text{i}}} \sum_{s>s_0}^{\infty} \int_1^{v_s} \frac{dv}{v\sqrt{v(v-1)}} \left[J_s^2 + \frac{\xi^2}{2}(2v-1)\left(J_{s+1}^2 + J_{s\text{-}1}^2 - 2\,J_s^2\right) \right] \propto \frac{\alpha m^2}{2\omega_{\text{i}}} \frac{E}{E_c} \exp\left[-\frac{8 m E_c}{3\omega_i E} \right]$$

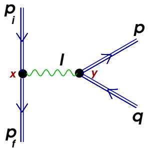
$E_c = \frac{m^2}{e}$, theory. $E_c = ...$ experiment, can now be measured in the lab

The resonant trident spectrum

- **One step trident** has a virtual photon propagating through a dispersive vacuum.
- Additional propagator poles due to contributions from the strong field, $\sum_n (\mathbf{nk})$
- Including lifetime of the unstable state produces quasi-particle resonances
- Resonances manifest as peaks in the positron spectrum

$$M^{\text{Trid}}_{fi} = \sum_{rn} \widetilde{\gamma}_{pq} \int \frac{d^4 l}{(2\pi)^4} \frac{g_{\mu\nu}}{l^2 + i\Gamma} \widetilde{\gamma}_{fi}\, \delta(p_f + l - p_i + rk)\delta(p + q - l + nk)$$

pole condition: $(p_i + q_i + nk)^2 = m^2$



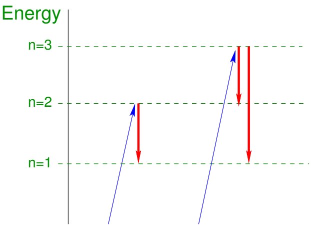
- Resonance conditions in centre of mass frame:

$$n = \frac{\epsilon_q}{\omega} = \frac{\text{positron energy}}{\text{laser photon energy}}$$

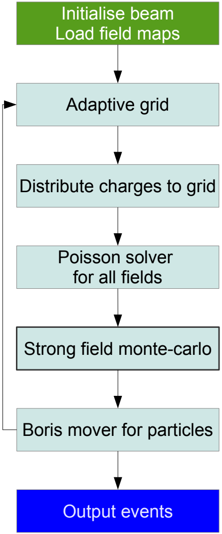
- Resonance width:

$\Gamma \propto \Im \Pi_{\mu\nu} \propto \Gamma_{\text{OPPP}}$, by the Optical theorem

- **Potentially dramatic signal: Resonances imposed over otherwise smooth positron spectrum**

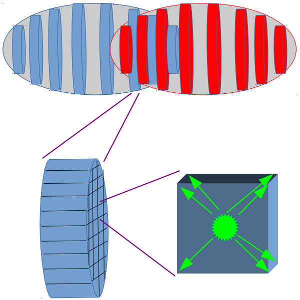


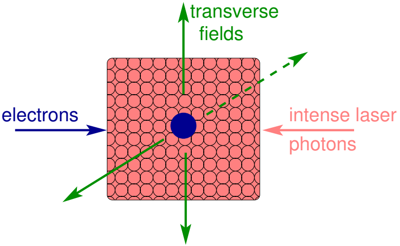
IPstrong - SQED monte carlo, PIC code



- Fortran 2003 with openMPI (Fortran 2008 has inbuilt GPU libraries)
- 3D poisson solver (PSPFFT, MPI)
- HICS photon radiation, resonant higher order SCS and Trident by strong field QFT monte carlo
- Internally generated bunches or externally loaded
- Cross-check with
  - CAIN (KEK, strong field QFT, no FEL)
- e-/laser, e+e-, higher order interactions, FEL
- new processes can be added as modules

Charge bunch/laser/undulator interaction





- Interacting bunches divided into overlapping transverse slices
- Slices divided into voxels
- Charges within each voxel distributed to voxel vertices

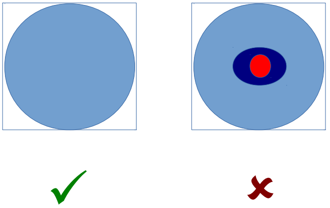
Solve for the potential $\Phi(x)$ from the charge density $S(x)$ via FFTW

$$\nabla^2\Phi(x) = S(x)$$

- Get the field strength at each macroparticle
- longitudinal electrons & photons
- transverse electric/magnetic fields
- ponderomotive force at cell edges
- electron momentum & position via leapfrog method
- Lorentz invariant particle pusher

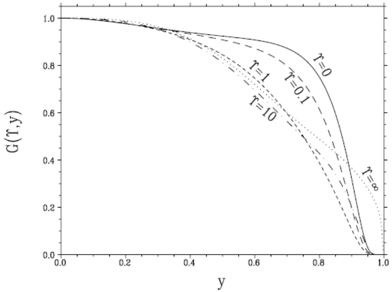
Monte-Carlo implementation

Probability distribution



- based on the Acceptance-Rejection method
- requires a ”flat probability distribution”
- need to transform the transition probability

$$dW = F(\Upsilon, \omega_f) d\omega_f \;\rightarrow\; dn_\gamma = p_0(\Upsilon, \Delta t) G(\Upsilon, y) dy, \;\omega_f = f(y)$$



- Generate 2 rand nos. $0 \leq p, y \leq 1$
- If $p \geq p_0$ or $y \geq p_0 G(\Upsilon, y)$ reject
- Otherwise, generate event $\omega_f = f(y)$
- radiated photon boosted to within a forward cone of angle $1/\gamma$

State of play

- **IPstrong SQED PIC code**
  - HICS and OPPP (2 step trident) fully implemented)
  - One step trident to be implemented
  - Linear polarisation to be implemented
  - Publish and release code - 3rd quarter 2019

- **Monte carlo datasets**
  - BPPP : 5m and 12m foil to IP, 10^4 bunch crossings
  - 2step Trident : 10^4 bunch crossings

/afs/desy.de/user/h/hartin/public/IPstrong

Papers

- ”Strong field QED in lepton colliders and electron/laser interactions”
  A. Hartin
  Int. J. Mod. Phys. A 33, no. 13, 1830011 (2018), arXiv:1804.02934

- ”Measuring the boiling point of the vacuum of quantum electrodynamics”
  A. Hartin, A. Ringwald and N. Tapia.
  PRD (accepted), arXiv:1807.10670, DESY-18-128