Entanglement in CFTs with Discrete Gauge Symmetry and Bulk Geometry Reconstruction in AdS_3

Marius Gerbershagen

Quantum field theory meets gravity

September, 25 2019

[arXiv:1909.xxxxx]

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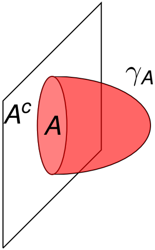
Motivation

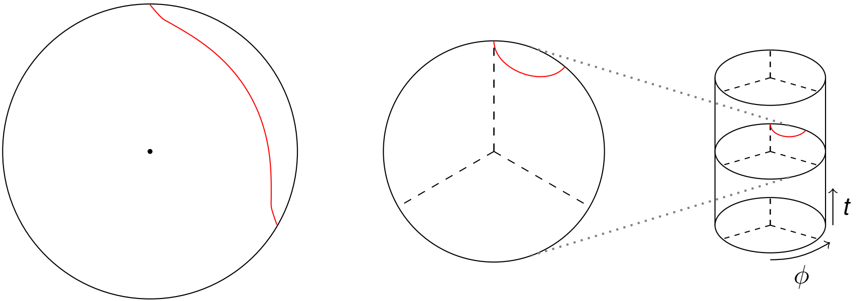
- Main question: How is the bulk geometry encoded in the boundary field theory?
- Ryu, Takayanagi [arXiv:hep-th/0603001]:



$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{A^c}, \qquad \rho_A = \mathrm{Tr}_{\mathcal{H}_{A^c}}(\rho)$$

$$S_A = -\mathrm{Tr}_{\mathcal{H}_A}(\rho_A \log \rho_A) = \frac{\mathrm{Area}(\gamma_A)}{4G_N}$$

Conical defects in AdS_3



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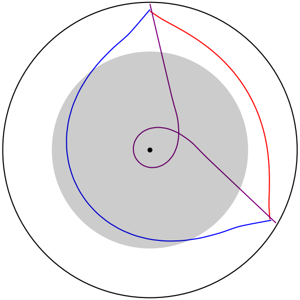

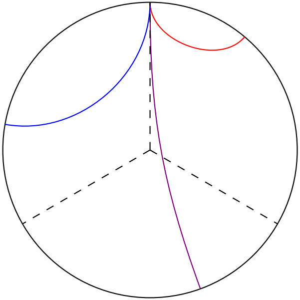

Dual field theory

- $\text{CFT}^N/\mathbb{Z}_N$ orbifold, set of fields X^i ($i = 0, ..., N-1$)
- Hilbert space

$$\mathcal{H}_{\text{tot}} = \bigoplus_{k=0}^{N-1} \mathcal{H}_k$$

with $\mathcal{H}_k$ twisted sector: $X^i(\phi + 2\pi) = X^{i+k}(\phi)$

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- $\mathcal{H}_{\mathrm{tot}}$ contains only $\mathbb{Z}_N$ invariant states
- Example: $N = 2$

$$\mathcal{H}_k = \left\{ \frac{1}{\sqrt{2}} \left(|XY\rangle_k + |YX\rangle_k \right) \right\}$$

$\mathbb{Z}_N$ invariant Hilbert space

- Concentrate on single $\mathcal{H} := \mathcal{H}_1$ factor in the following
- $\mathcal{H}$ does not decompose into tensor factors,

$$\mathcal{H} \neq \mathcal{H}_A \otimes \mathcal{H}_{A^c}$$

  due to the $\mathbb{Z}_N$ gauge symmetry
- Analogous problem: 2 indistinguishable qubits

  Hilbert space spanned by

$$|00\rangle, \quad |11\rangle, \quad |01\rangle + |10\rangle$$

Defining a reduced density matrix

Two ways of dealing with the problem:

1. Embed the density matrix into a larger Hilbert space $\tilde{\mathcal{H}}$ containing unphysical non-$\mathbb{Z}_N$ invariant states
   Example: $N = 2$

$$\tilde{\mathcal{H}} = \{|XY\rangle\} = \tilde{\mathcal{H}}_A \otimes \tilde{\mathcal{H}}_{A^c}$$

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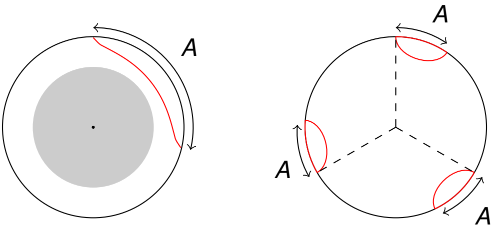


$\Rightarrow$ reproduces Ryu-Takayanagi [arXiv:1406.5859]

Defining a reduced density matrix

2. Algebraic approach [arXiv:1607.03901]; Ohya, Petz 1993:
Consider algebra M_A of operators acting locally in A

M_A defines a unique density matrix $\rho_{M_A} \in M_A$ by

$$\mathrm{Tr}_{\mathcal{H}}(\rho\mathcal{O}) = \mathrm{Tr}_{\mathcal{H}}(\rho_{M_A}\mathcal{O}) \quad \forall\mathcal{O} \in M_A$$

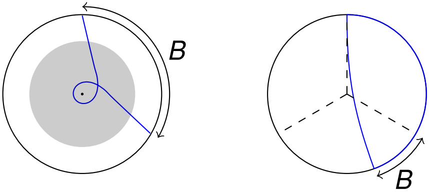
von Neumann entropy of ρ_{M_A} is a measure of entanglement between A and A^c,

$$S_A \sim \mathrm{Tr}_{\mathcal{H}}(\rho_{M_A} \log \rho_{M_A})$$

Result: both approaches agree

Entwinement

- "Entwinement" = Entanglement between non-spatially organized degrees of freedom [arXiv:1406.5859]
- Simply speaking: trace out different fields over different subregions



Example: $N = 3$, $\tilde{\mathcal{H}} = \{|XYZ\rangle\}$

$$\rho_B = \mathrm{Tr}_{\{|Y_{B^c}Z\rangle\}}(\rho)$$

Defining a reduced density matrix

1. Embedding in enlarged Hilbert space: reproduces length of non-minimal geodesic
2. ~~Algebraic approach~~: No algebra of operators associated to the non-spatially organized degrees of freedom

   $\rightarrow$ set of operators M_B only forms a linear subspace [arXiv:1806.02871]

   But: can define an entropy associated with M_B

Entropies associated to linear subspaces

In general: projective measurements with Hermitean operators

$$\mathcal{O} = \sum_j o_j \mathcal{O}_j \ , \quad \mathcal{O}_j = |\chi_j\rangle\langle\chi_j|$$

increase the von Neumann entropy

$$S(p_{\mathcal{O},j}) \geq S(\rho) \quad \text{with } p_{\mathcal{O},j} = \mathrm{Tr}(\rho\mathcal{O}_j) \tag{1}$$

The linear subspace M_B is spanned by basis operators

$$\mathcal{O}_j = P_{\mathbb{Z}_N}(|\chi_j\rangle\langle\chi_j| \otimes \mathbb{1})P_{\mathbb{Z}_N}$$

Proposed definition of Entwinement

$$S(\rho, M_B) = \min S(p_{\mathcal{O},j})$$

Result: Again, both approaches agree.

Conclusions

- Generalized notions of entanglement reproduce naive calculations using unphysical states
- There exists a well defined quantum information quantity dual to the length of a non-minimal geodesic
- The full bulk geometry of the conical defect including the entanglement shadow is reconstructible from generalized boundary entanglement (for example using integral geometry techniques [arXiv:1505.05515])
- It is likely that entwinement can also probe features of the BTZ black hole beyond ordinary entanglement entropy (for the massless black hole case, see [arXiv:1609.03991])