

The relativistic binary problem

theoretical challenges

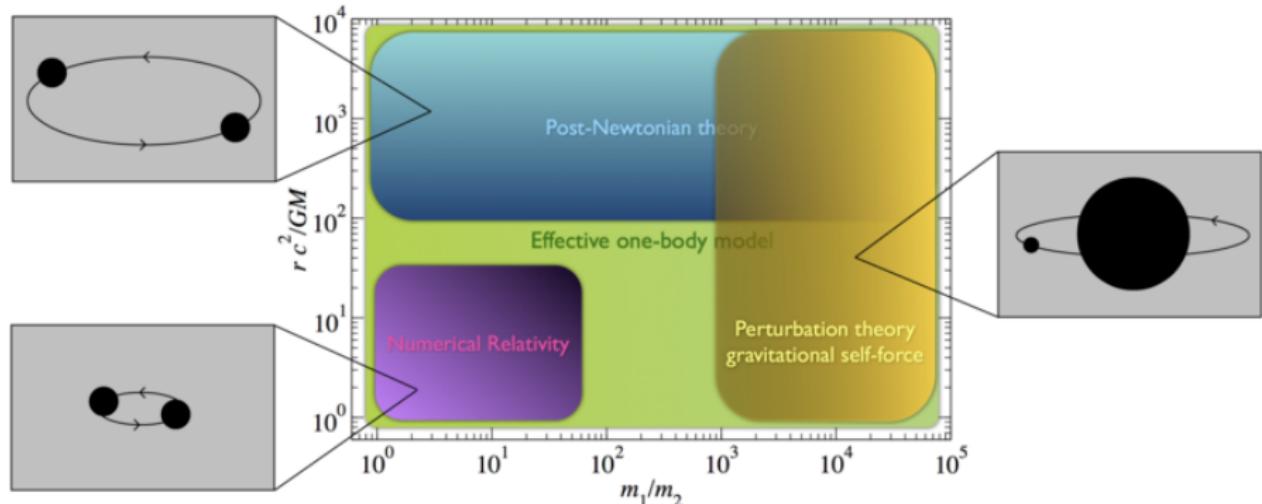
Jan Steinhoff



Max-Planck-Institute for Gravitational Physics (Albert-Einstein-Institute), Potsdam-Golm, Germany

DESY Theory Workshop “Quantum field theory meets gravity”
September 27th, 2019

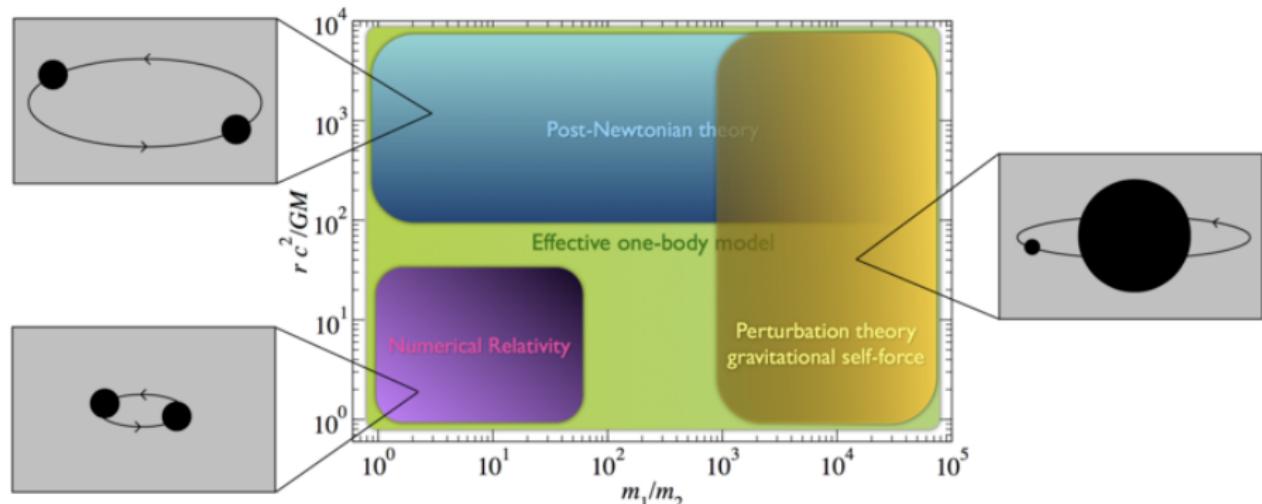
Approaches to the relativistic binary problem



Missing third dimension: eccentricity!
→ scattering, post-Minkowskian approximation

image credit: A. Buonanno, B.S. Sathyaprakash

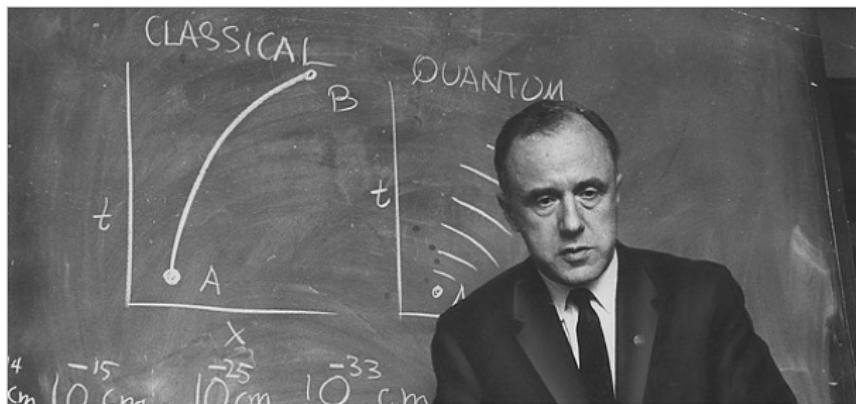
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Main message



J. A. Wheeler

nytimes.com

Look at the relativistic binary problem from a quantum perspective!

Hamilton-Jacobi theory very useful

[Misner, Thorne, Wheeler]

More modern approach: effective field theory (EFT)!

[Goldberger, Rothstein, PRD 73 (2006) 104029]

see talk by R. Porto

Results for the post-Newtonian potential

conservative part of the motion of the binary

post-Newtonian (PN) approximation: expansion around $\frac{1}{c} \rightarrow 0$ (Newton)

order	c^0 N	c^{-1} 1PN	c^{-2}	c^{-3}	c^{-4} 2PN	c^{-5}	c^{-6} 3PN	c^{-7}	c^{-8} 4PN
non spin	✓		✓		✓		✓		✓
spin-orbit				✓		✓		✓	
Spin ²					✓		✓		✓
Spin ³								✓	
Spin ⁴									✓
⋮								⋮	

Work by many people ("just" for the spin sector): Barker, Blanchet, Bohé, Buonanno, O'Connell, Damour, D'Eath, Faye, Hartle, Hartung, Hergt, Jaranowski, Marsat, Levi, Ohashi, Owen, Perrodin, Poisson, Porter, Porto, Rothstein, Schäfer, Steinhoff, Tagoshi, Thorne, Tulczyjew, Vaidya

Code for the spin part using EFT: M. Levi, JS, CQG **34** (2017), 244001

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Possible resummation: along diagonal

~ naked (st)ring singularities

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Summing spin to infinity (leading PN order)

J. Vines, JS, PRD **97** (2018), 064010

Start from an effective point-particle action for black-holes (BHs):
Infinite number of higher dimensional couplings, one for each multipole

$$(\text{mass } \ell\text{-pole}) + i(\text{current } \ell\text{-pole}) = \text{mass } (ia)^\ell, \quad a = \frac{\text{spin}}{\text{mass}}$$

Still, in the leading-order Hamiltonian, the S^∞ series can be resummed:

$$H = \frac{\vec{P}^2}{2\mu} - \mu U + 4\vec{P} \cdot \vec{A} + \frac{1}{2}\vec{P} \times \left[\frac{\vec{S}_1}{m_1^2} + \frac{\vec{S}_2}{m_2^2} \right] \cdot \vec{\nabla}_\mu U$$

where $M = m_1 + m_2$, $\mu = M_1 m_2 / M$,

$$\vec{a}_0 = \vec{a}_1 + \vec{a}_2, \quad \vec{a}_i = \vec{S}_i / m_i$$

$$U = \frac{Mr}{r^2 + a_0^2 \cos^2 \theta}, \quad \vec{A} = -\frac{U}{2} \frac{\vec{R} \times \vec{a}_0}{r^2 + a_0^2}$$

Linearized Kerr metric!

~ Test-mass motion in Kerr metric

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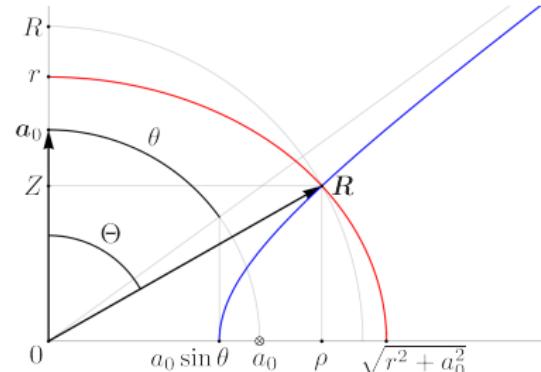
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oblate-spheroidal coord. r, θ

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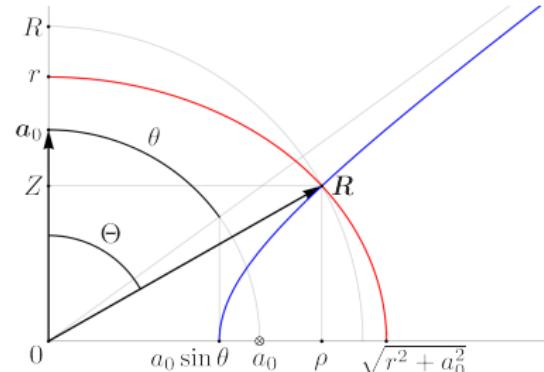
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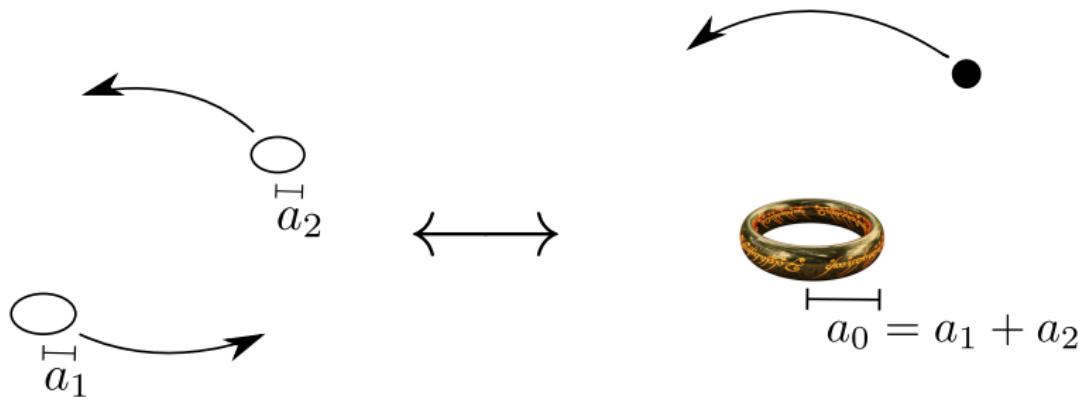


oblate-spheroidal coord. r, θ

Summing spin to infinity

Visualization of the result:

[J. Vines, JS, PRD **97** (2018), 064010]



Parallels to the Effective-One-Body (EOB) approach!

Gauge invariant quantities simplify: binding energy and radiation modes

[N. Siemonsen, JS, J. Vines, PRD **97** (2018), 124046]

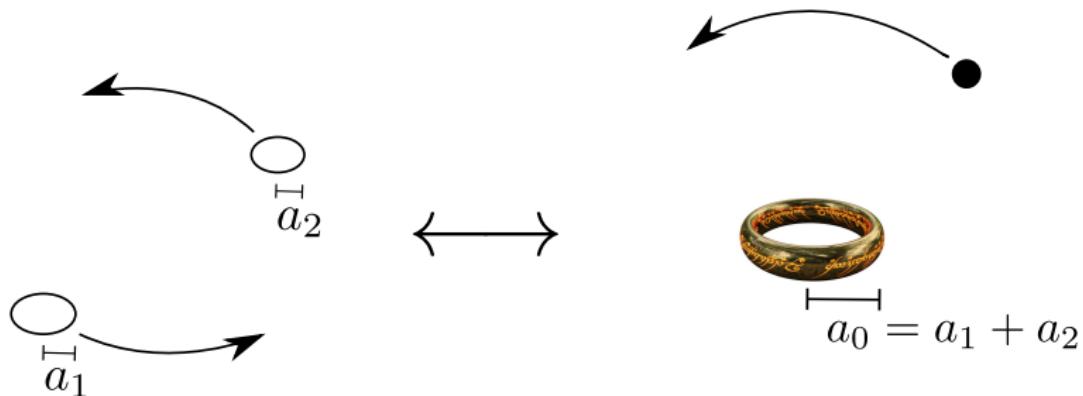
Extension to 1st post-Minkowskian (PM) Hamiltonian and scattering angle

[J. Vines, CQG **35** (2018), 084002]

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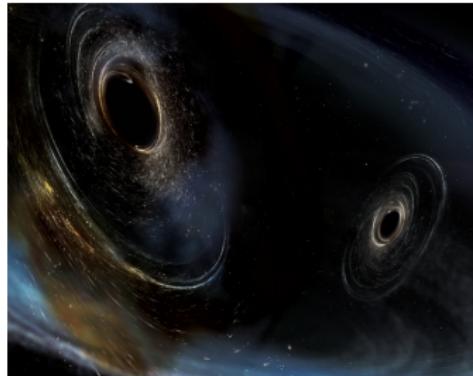
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Extension to 1st post-Minkowskian (PM) Hamiltonian and **scattering angle**

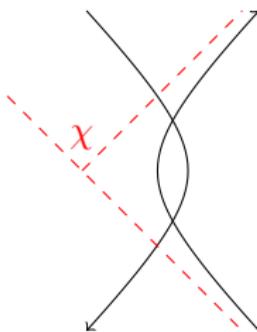
[J. Vines, CQG **35** (2018), 084002]

Scattering black holes



dancing duo of black holes

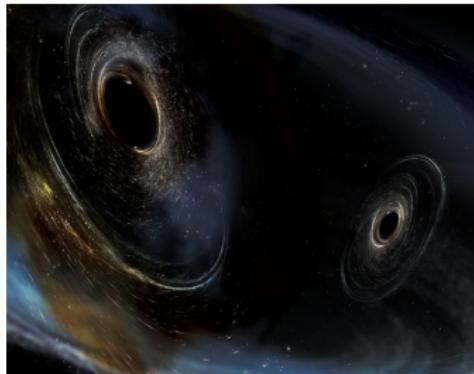
www.ligo.caltech.edu



- Classical scattering: scattering angle χ (more for spinning BHs)
- Quantum analog: scattering amplitude
- BHs \sim “minimally coupled” higher-spin massive particles ?

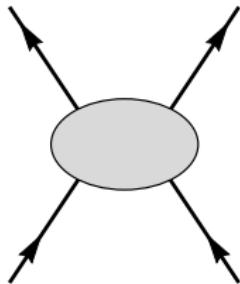
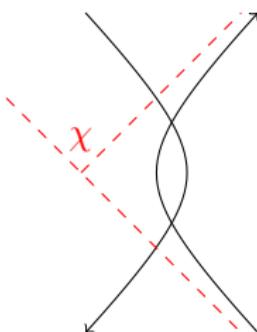
Vaidya (2015); Guevara, Ochirov, Vines (2018); Chung, Huang, Kim, Lee (2019);
Guevara, Ochirov, Vines (2019); Siemonsen, Vines (2019); Arkani-Hamed, Huang, O'Connell (2019);
Bautista, Guevara (2019); Guevara (2019); Arkani-Hamed, Huang, Huang (2017);

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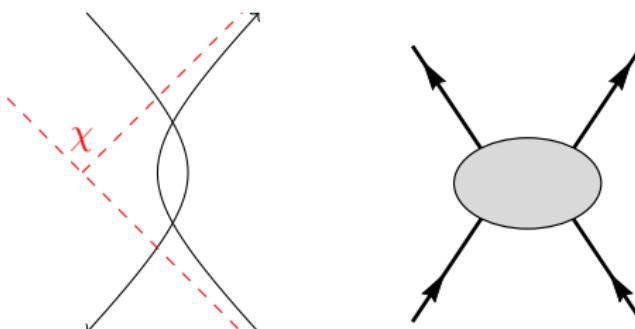
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2PM scattering angle of (aligned-)spinning BHs

J. Vines, JS, A. Buonanno, PRD 99 (2019) 064054

- Split the scattering angle χ analogous to the amplitude:

$$\begin{aligned}\chi &\sim \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \mathcal{O}(G^3, \hbar) \\ &\sim E [f(a_1, a_2) + m_2 f_{\triangle}(a_1, a_2) + m_1 f_{\triangleright}(a_1, a_2)] + \mathcal{O}(G^3, \hbar, a^3)\end{aligned}$$

The three diagrams are Feynman-like diagrams representing different contributions to the scattering angle. Diagram 1 shows two particles with momenta a_1 and a_2 interacting via a central vertex. Diagram 2 shows a similar interaction but with a different internal loop configuration. Diagram 3 shows a different interaction where one particle's path is deflected by the other.

- The $f_{...}$ depend impact parameter, velocity, a_1 , and a_2 , but not of m_1 , m_2 !
- Can take probe (test-BH) limit: $(m_1 \rightarrow 0, E \rightarrow m_2, \chi \rightarrow \chi_t)$

$$\chi_t(m_2, a_1, a_2) = m_2 [f(a_1, a_2) + m_2 f_{\triangle}(a_1, a_2)]$$

- Using symmetry $f_{\triangleright}(a_1, a_2) = f_{\triangle}(a_2, a_1)$, we can get back to χ !

$$\chi = \frac{E}{M} \left[\frac{m_1}{M} \chi_t(M, a_1, a_2) + \frac{m_2}{M} \chi_t(M, a_2, a_1) \right], \quad M = m_1 + m_2$$

- Why? χ_t is rather simple to obtain. We have exact BH solutions!
- Can be generalized beyond the probe limit \rightarrow 4PM result in reach!

[D. Bini, T. Damour, A. Geralico, arXiv:1909.02375]

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The diagrams show three contributions to the scattering angle. Diagram 1 shows two particles with spin axes aligned along the direction of motion. Diagram 2 shows one particle with spin aligned along the direction of motion, and the other with spin perpendicular. Diagram 3 shows both particles with spins perpendicular to the direction of motion.

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The diagrams are Feynman-like: 1) Two vertices connected by a wavy line, with two external lines meeting at each vertex. 2) Three vertices connected by a wavy line, with two external lines meeting at each vertex. 3) Three vertices connected by a wavy line, with one external line meeting at each vertex.

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Gravitating binaries from the (classical) double copy

J. Plefka, JS, W. Wormsbecher, PRD **99**, 024021 (2019); J. Plefka, C. Shi, JS, T. Wang (2019)

Simplified classical limit (compared to amplitudes) by using [classical sources?](#)

Dynamical color charge $c^a(\tau) = \psi^\dagger T^a \psi$ moving on an worldline $x^\mu(\tau)$:

$$S_{\text{cl quark}} = - \int [m d\tau - \psi^\dagger i D_\mu \psi dx^\mu]$$
$$\equiv - \int d\tau \left[p_\mu u^\mu - i \psi^\dagger \dot{\psi} - \frac{\lambda}{2} \left(p^2 - m^2 + 2g p_\mu A_a^\mu c^a + g^2 A_a^\mu A_{b\mu} c^a c^b \right) \right]$$

Auxiliary field ψ minimally coupled, $D_\mu = \partial_\mu - ig A_\mu^a T^a$, $[T^a, T^b] = i f^{abc} T^c$

Take 2 “quarks”, integrate out A^μ & Bern-Carrasco-Johansson double copy:

$$e^{iS_{\text{eff, YM}}} \sim \sum_{i \in \text{cubic}} \int \frac{C_i N_i}{D_i} \quad \Rightarrow \quad e^{iS_{\text{eff, gravity}}} \sim \sum_{i \in \text{cubic}} \int \frac{N_i N_i}{D_i}$$

C_i : color structure N_i : kinematic numerator D_i : propagators

$c_i \pm c_j \pm c_k = 0 \leftrightarrow n_i \pm n_j \pm n_k = 0$ color-kinematics duality

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Gravitating binaries from the (classical) double copy II

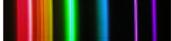
Results:

- $S_{\text{eff,gravity}}$ is correct at NLO.
[J. Plefka, JS, W. Wormsbecher, PRD **99**, 024021 (2019)]
- Disagreement with known results in scalar-tensor theory at NNLO
→ **breakdown** of the proposed double copy
[J. Plefka, C. Shi, JS, T. Wang, arXiv:1906.05875]

Further literature:

- Balachandran et al, PRD **15** (1977) 2308
R. Monteiro, D. O'Connell, and C. D. White, JHEP12, 056 (2014)
A. Luna, et al. JHEP06, 023 (2016)
W. D. Goldberger, A. K. Ridgway, PRD **95** (2017) 125010; PRD **97** (2018) 085019
C. H. Shen, JHEP11, 162(2018)
A. Luna, I. Nicholson, D. O'Connell, C. D. White, JHEP03, 044 (2018)
H. Johansson, A. Ochirov, JHEP 1909 (2019) 040
...

Theoretical challenges

- Make sense of amplitudes involving SM particles + BHs + gravitons
→ include **BH absorption** (+decay?)
- Radiation modes from amplitudes (at high orders)
- **Tidal effects**, oscillation modes
→ spectroscopy 
- QFT methods for self-force/small mass ratio approximation
→ **all orders in the coupling**: “nonperturbative” !
[C. R. Galley, B. L. Hu, PRD **79** (2009)]
- The classical “piece” of an amplitude
recent papers by: Bjerrum-Bohr, Cheung, O’Connell, Damgaard, Festuccia, Guevara, Kosower, Maybee, Ochirov, Planté, Rothstein, Solon, Vanhove, Vines, ...
- Skipping ugly gauge dependent potentials for waveform models?
scattering ↔ energy levels ↔ waveform model ?

Standard Model of Elementary Particles

