

Fundamental Aspects of Asymptotic Safety

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L. Bosma, B. Knorr, F.S., Phys. Rev. Lett. 123 (2019)101301 (Editor's suggestion)

B. Knorr, C. Ripken, F.S., arXiv:1907.02903

DESY Theory Workshop

Hamburg, September 26th, 2019

Outline

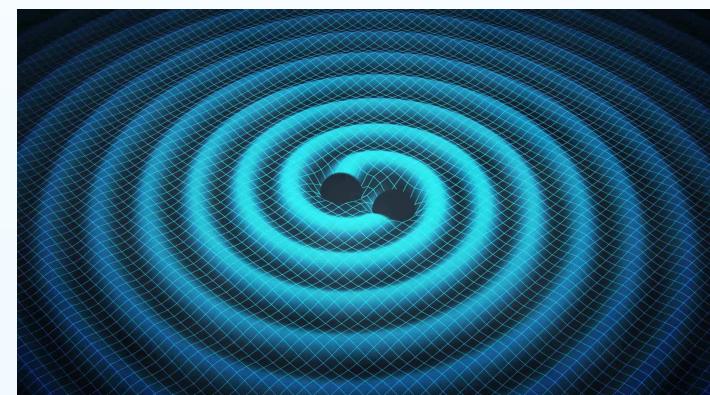
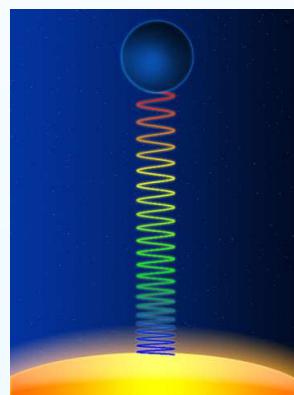
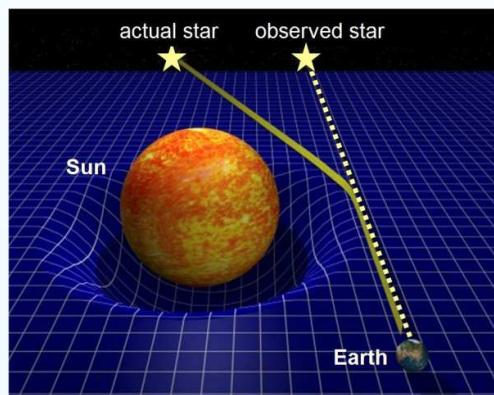
- Introduction and Motivation
- Form factors – structural aspects and computational remarks
- Results: gravitational propagator for transverse-traceless modes
- Application: Quantum corrections to the gravitational potential
- Summary and Outlook

General Relativity

Einstein's equations

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu}}_{\text{dynamics of spacetime}} = \underbrace{8\pi G_N T_{\mu\nu}}_{\text{matter}}$$

experimentally well-tested in the weak gravity regime

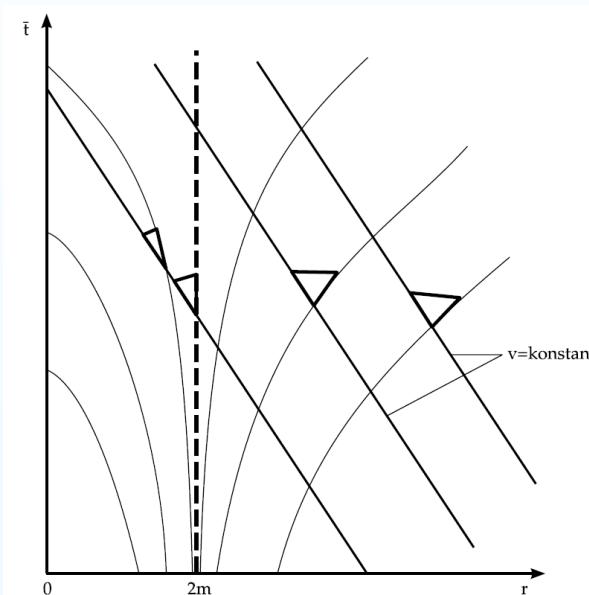


- bending of light rays in the gravitational field
- gravitational redshift
- detection of gravitational waves

Black Holes and Spacetime Singularities

Schwarzschild solution

$$ds^2 = - \left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$



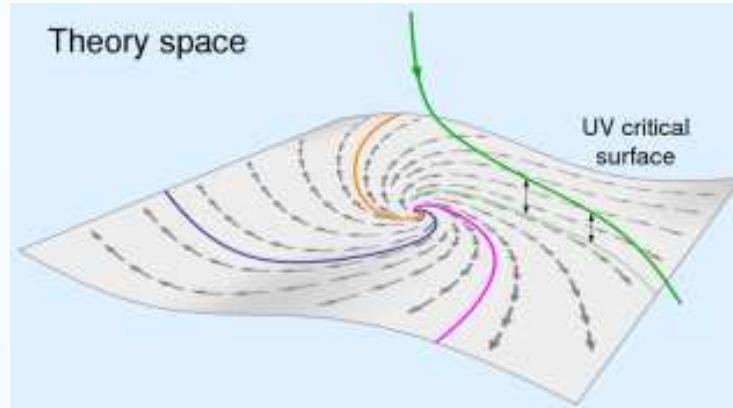
- curvature singularity at $r = 0$

basic objective of quantum gravity

understand gravity under extreme conditions - singularities

Quantum Gravity from the Reuter fixed points

gravity at high energy is controlled by non-Gaussian RG fixed point



[scholarpedia '13]

- 2 classes of RG trajectories:
 - relevant = end at fixed point in UV
 - irrelevant = go somewhere else...
- theory ending at the fixed point is free of unphysical UV divergences
- predictive power:
 - number of relevant directions \iff free parameters (experimental input)

Wetterich Equation for Gravity

C. Wetterich, Phys. Lett. **B301** (1993) 90

T. Morris, Int. J. Mod. Phys. A9 (1994) 24110

M. Reuter, Phys. Rev. D **57** (1998) 971

central idea: integrate out quantum fluctuations shell-by-shell in momentum-space

implementation: flow equation for effective average action Γ_k :

$$\partial_t \Gamma_k[h_{\mu\nu}; \bar{g}_{\mu\nu}] = \frac{1}{2} \text{Tr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \partial_t \mathcal{R}_k \right]$$

- depends on two arguments $h_{\mu\nu}, \bar{g}_{\mu\nu}$
- effective action Γ is recovered for $k = 0$
- non-trivial fixed points for theories involving gravity:
 - well-established for 4-dimensional gravity
 - gravity-matter systems:
 - fixed points with more predictive power than the standard model

Open Questions

- exact matter content supporting asymptotic safety?
- number of free parameters?
- low-energy physics compatible with observations?
- degrees of freedom associated with the NGFP?
- unitarity?

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Investigation method:

- derivative expansion of Γ_k
- derivative expansion fails

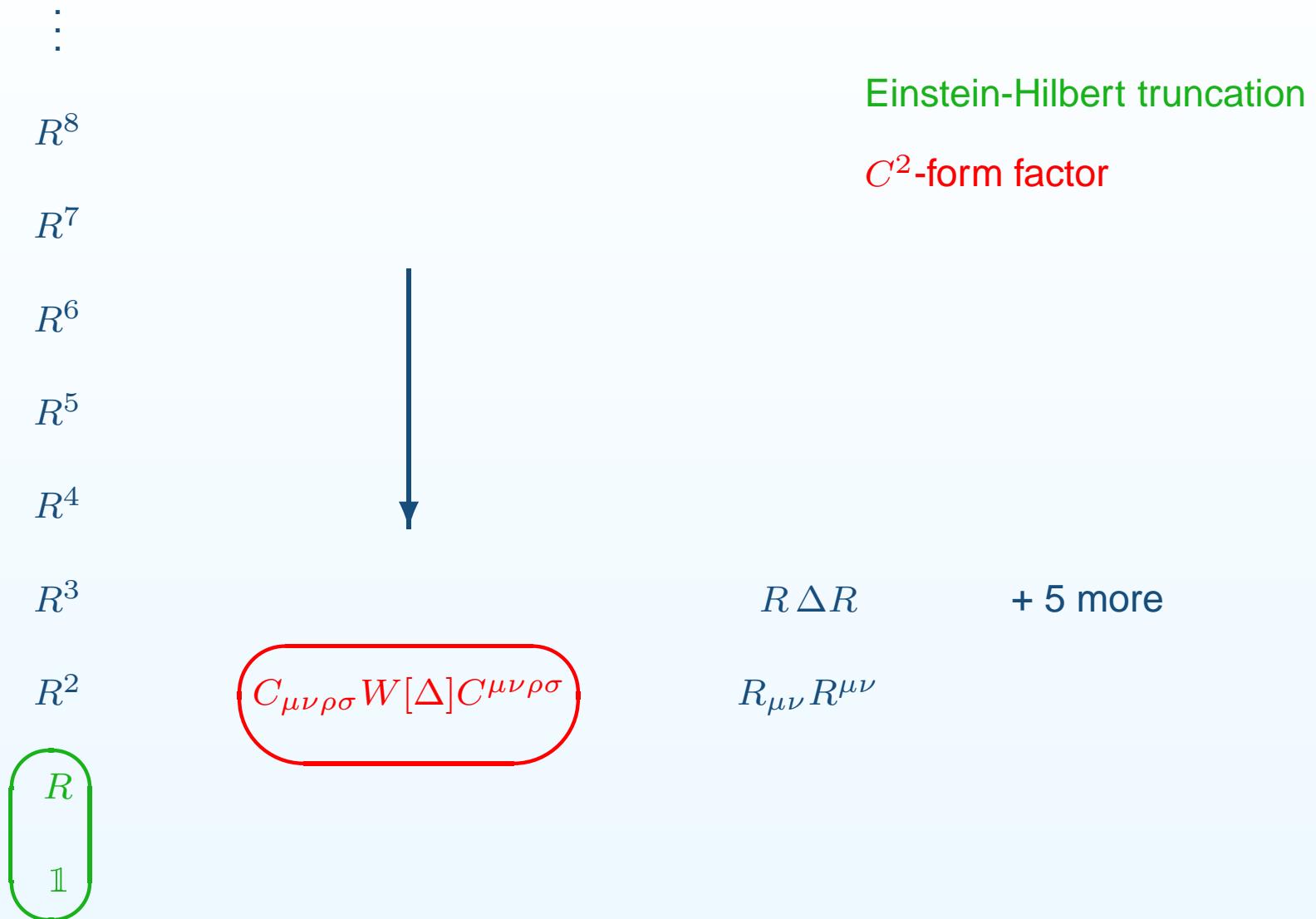
Derivative expansion of $\Gamma_k^{\text{grav}}[g]$

\vdots	\vdots	
R^8	$C_{\mu\nu\rho\sigma}\Delta^6C^{\mu\nu\rho\sigma}$	Einstein-Hilbert truncation
R^7	$C_{\mu\nu\rho\sigma}\Delta^5C^{\mu\nu\rho\sigma}$	
R^6	$C_{\mu\nu\rho\sigma}\Delta^4C^{\mu\nu\rho\sigma}$	
R^5	$C_{\mu\nu\rho\sigma}\Delta^3C^{\mu\nu\rho\sigma}$	
R^4	$C_{\mu\nu\rho\sigma}\Delta^2C^{\mu\nu\rho\sigma}$	
R^3	$C_{\mu\nu\rho\sigma}\Delta C^{\mu\nu\rho\sigma}$	$R \Delta R$ + 5 more
R^2	$C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$	$R_{\mu\nu}R^{\mu\nu}$
		

Derivative expansion of $\Gamma_k^{\text{grav}}[g]$

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R^7	$C_{\mu\nu\rho\sigma}\Delta^5 C^{\mu\nu\rho\sigma}$	$C\square^n C$ -truncation
R^6	$C_{\mu\nu\rho\sigma}\Delta^4 C^{\mu\nu\rho\sigma}$	
R^5	$C_{\mu\nu\rho\sigma}\Delta^3 C^{\mu\nu\rho\sigma}$	
R^4	$C_{\mu\nu\rho\sigma}\Delta^2 C^{\mu\nu\rho\sigma}$	
R^3	$C_{\mu\nu\rho\sigma}\Delta C^{\mu\nu\rho\sigma}$	$R \Delta R$ + 5 more
R^2	$C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$	$R_{\mu\nu} R^{\mu\nu}$
R		
1		

Derivative expansion of $\Gamma_k^{\text{grav}}[g]$



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derivative expansion \implies curvature expansion keeping derivatives

A. Codello and O. Zanusso, Math. Phys. **54** (2013) 013513

S. A. Franchino-Viñas, T. de Paula Netto, I. L. Shapiro and O. Zanusso, Phys. Lett. B **790** (2019) 229

Form Factors

Diffeomorphism-invariant Form Factors for Gravity

form factors \iff momentum-dependent interactions in the effective action

2 form factors at second order in the curvature:

$$\Gamma_k^C[g] = \frac{1}{16\pi G_k} \int d^d x \sqrt{g} C_{\mu\nu\rho\sigma} W_k^C(\Delta) C^{\mu\nu\rho\sigma},$$

$$\Gamma_k^R[g] = \frac{1}{16\pi G_k} \int d^d x \sqrt{g} R W_k^R(\Delta) R,$$

determine gravitational propagators in flat space

- working assumption: $W_k(\Delta)$ has representation as Laplace transform
 - allows to eliminate a third structure function via

$$\int d^d x \sqrt{g} [R^{\rho\sigma\mu\nu} \Delta^n R_{\rho\sigma\mu\nu} - 4R^{\mu\nu} \Delta^n R_{\mu\nu} + R \Delta^n R] = \mathcal{O}(R^3), n \geq 1$$

Flow Equations for Form Factors

approximation of Γ_k	structure of RG flow	fixed points
finite number of \mathcal{O}_i	ODEs	algebraic
field-dependent functions $f(R_1, \dots, R_n; t)$	PDEs ($n + 1$ var.)	PDEs (n var.)
momentum-dependent form factors $f(p_1, \dots, p_n; t)$	IDEs ($n + 1$ var.)	IDEs (n var.)

- ordinary differential equation (ODE)
- partial differential equation (PDE)
- integro-differential equation (IDE)

Computing Form Factors

technical compendium: B. Knorr, C. Ripken and F.S., arXiv:1907.02903

The C^2 -Form Factor

ansatz

$$\Gamma_k^{\text{grav}} = \frac{1}{16\pi G_k} \int d^4x \sqrt{g} \left[2\Lambda_k - R + C_{\mu\nu\rho\sigma} W_k(\Delta) C^{\mu\nu\rho\sigma} \right]$$

- $W_k(\Delta)$ gives corrections to transverse-traceless propagator

$$\mathcal{G}^{\text{TT}}(q^2) \propto \left(\underbrace{q^2}_{\text{Einstein-Hilbert}} + \underbrace{2(q^2)^2 W_k(q^2)}_{\text{form factor}} \right)^{-1}$$

computing the flow:

- G_k and Λ_k from the Einstein-Hilbert truncation
 - neglects back-reaction of form factor in the TT-propagator
- flow of $W_k(\Delta/k^2)$ in the conformally reduced approximation

$$g_{\mu\nu} = \left(1 + \frac{1}{4}h\right) \hat{g}_{\mu\nu}$$

Flow Equation for the Form Factor

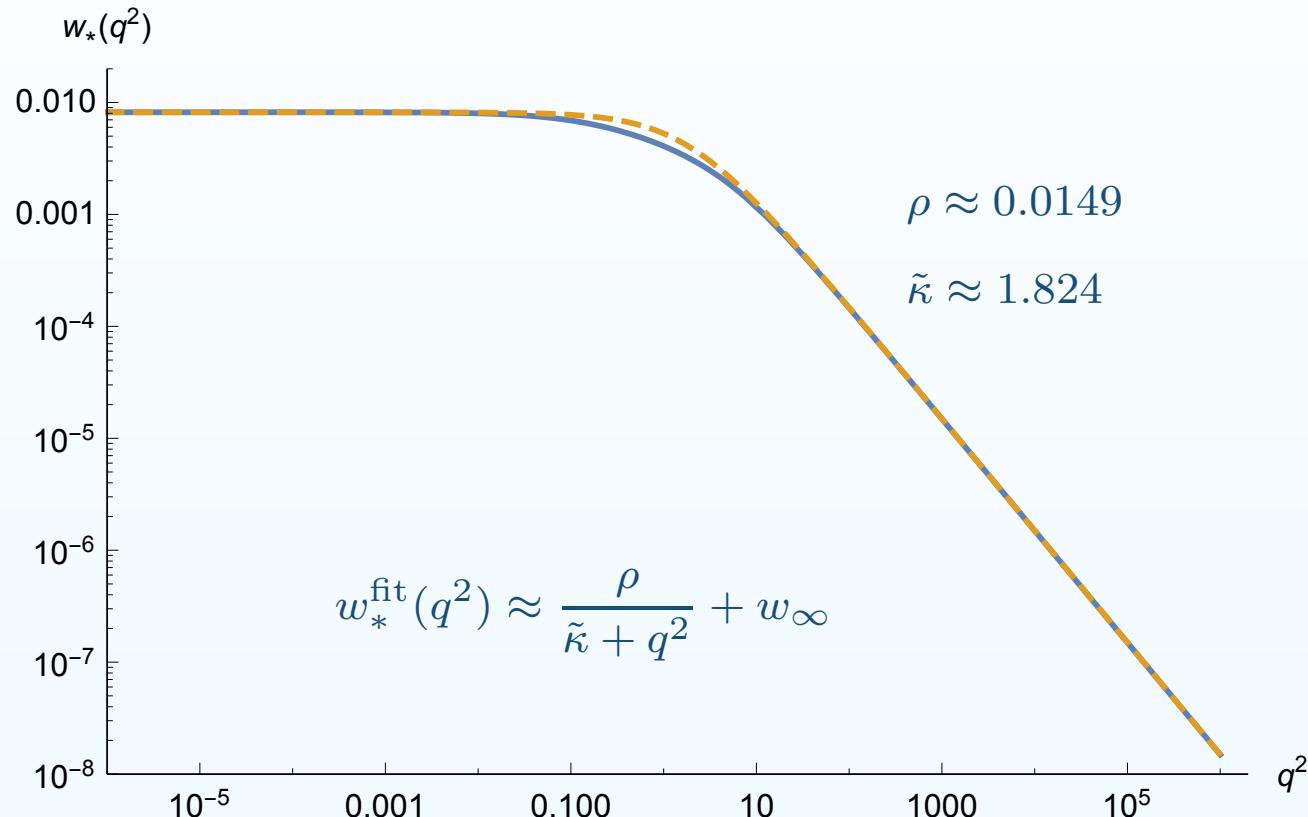
dimensionless form factor: $w(q^2) \equiv k^{-2} W_k(\Delta/k^2)$

$$\begin{aligned}
k \partial_k w(q^2) = & (2 + \eta_N)w(q^2) + 2q^2 w'(q^2) \\
& + \frac{g}{24\pi} \int_0^{\frac{1}{4}} du (1 - 4u)^{\frac{3}{2}} \frac{(2 - \eta_N)R(uq^2) - 2uq^2 R'(uq^2)}{uq^2 + R(uq^2) + \mu} \\
& + \frac{16g}{3\pi^2} \int_0^\infty dp \int_{-1}^1 dx p^3 \sqrt{1 - x^2} \frac{(2 - \eta_N)R(p^2) - 2p^2 R'(p^2)}{(p^2 + R(p^2) + \mu)^2} \\
& \left[\frac{1}{8} (w(p^2 + 2pqx + q^2) - w(q^2)) \right. \\
& + \frac{2q^4 + 4(q^2 - p^2)(pqx) + p^2 q^2 (7 - 6x^2)}{16(p^2 + 2pqx)^2} (w(p^2 + 2pqx + q^2) - w(q^2)) \\
& \left. + \frac{3p^4 - 2q^4 + 22p^2(pqx) - 5p^2 q^2 (1 - 6x^2)}{16(p^2 + 2pqx)} w'(q^2) \right].
\end{aligned}$$

- inhomogeneous term \implies form factor is induced
- integro-differential equation requires knowing $w(x)$ on positive real axis
- linear equation ($w(q^2)$ does not enter the conformal propagator)

The Form Factor

solving fixed point equation with pseudo-spectral methods:



- w_∞ undetermined constant (lifted in full computation)
- expansion: $w_*^{\text{fit}}(q^2)$ is an infinite power series in q^2
⇒ avoids Ostrogradski instability

Remark on the vertex expansion

P. Dona, A. Eichhorn and R. Percacci, Phys. Rev. D **89** (2014) 084035

N. Christiansen, B. Knorr, J. Meibohm, J. M. Pawłowski, M. Reichert, Phys. Rev. D **92** (2015) 121501

A. Eichhorn, P. Labus, J. M. Pawłowski, M. Reichert, SciPost Phys. **5** (2018) 031

idea: expand Γ_k in fluctuations around flat Minkowski space

vertex-expansion for gravity coupled to scalar matter:

- 1 form factor for the kinetic term

$$\Gamma_k^{\text{s,kin}} = \frac{1}{2} \int d^d x \phi f_k^{(\phi\phi)} (-\partial^2) \phi$$

- 4 form factors at $\mathcal{O}(h)$:

$$\Gamma_k|_{h\phi\phi} = \int d^d x \left[f_{(\bar{g})}^{(h\phi\phi)} \delta^{\mu\nu} + f_{(11)}^{(h\phi\phi)} \partial_1^\mu \partial_1^\nu + f_{(22)}^{(h\phi\phi)} \partial_2^\mu \partial_2^\nu + f_{(12)}^{(h\phi\phi)} \partial_1^\mu \partial_2^\nu \right] h_{\mu\nu} \phi\phi.$$

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Conceptually:

- vertex expansions decouple form factors \implies chose closure of FRGE
- diffeomorphism invariant form factors \implies generate closure of FRGE

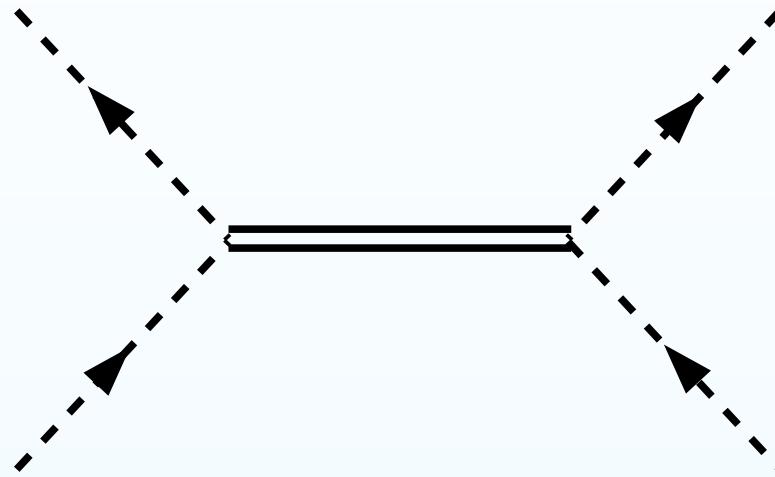
Application

The Newtonian Gravitational Potential

Newtonian Gravitational Potential from Field Theory

J. F. Donoghue, Phys. Rev. D50 (1994) 3874

non-relativistic graviton-mediated interaction of two scalar fields: (masses m_1, m_2):



$$V(\mathbf{r}) = -\frac{1}{2m_1} \frac{1}{2m_2} \int \frac{d^3\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \mathcal{M} = -\frac{G m_1 m_2}{\mathbf{r}}$$

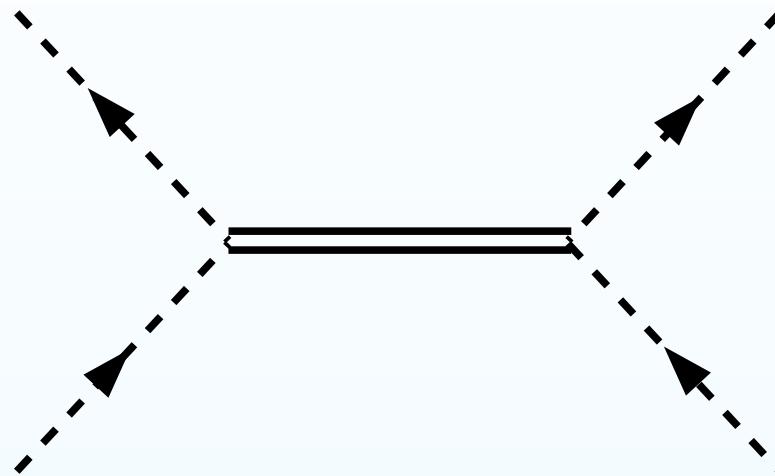
classical scattering amplitude ($q = (0, \mathbf{q})$)

$$\mathcal{M} = 16\pi G m_1^2 m_2^2 \mathcal{G}(\mathbf{q}^2) \quad , \quad \mathcal{G}_{\text{classical}}(\mathbf{q}^2) = \underbrace{\mathbf{q}^{-2}}_{\text{Einstein--Hilbert}}$$

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effective field theory corrections:

$$V(\mathbf{r}) = -\frac{G m_1 m_2}{\mathbf{r}} \left[1 + a \frac{G(m_1 + m_2)}{\mathbf{r} c^2} + b \frac{G \hbar}{\mathbf{r}^2 c^3} \right]$$

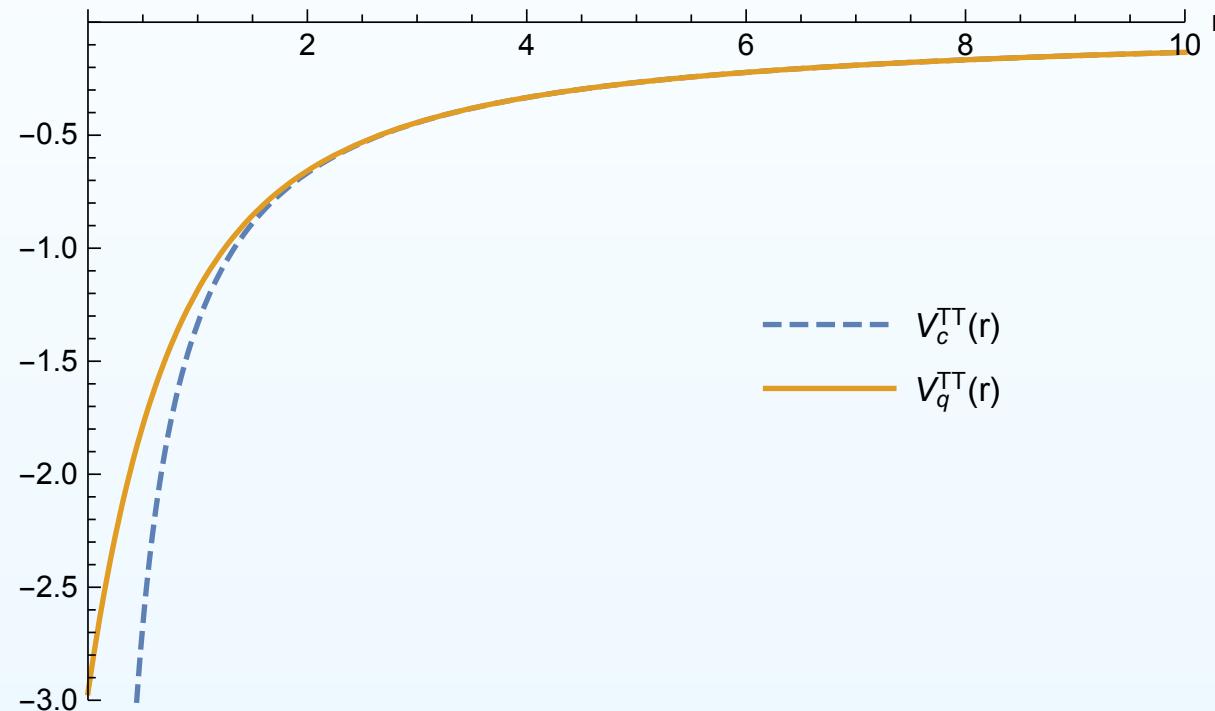
with

$$a = -1, \quad b = -\frac{127}{30\pi^2}$$

Newtonian Potential from Asymptotic Safety

Strategy:

- restrict scattering amplitude to transverse-traceless contribution
- replace $\mathcal{G}_{\text{classical}}^{\text{TT}}(\mathbf{q}^2) \Rightarrow \mathcal{G}_{\text{non-perturbative}}^{\text{TT}}(\mathbf{q}^2)$



$V_{\text{quantum}}^{\text{TT}}(\mathbf{r})$ remains finite as $\mathbf{r} \rightarrow 0$

Summary and Outlook

Summary . . .

form factors:

- highly relevant for the dynamics
- gravity: Newtonian potential rendered finite
 - propagator avoids the Ostrogradski-instabilities
- gravity-matter: scalar propagator agrees with vertex-expansion results

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asymptotically safe gravity + standard model matter:

- UV-completion for scenarios with no new physics below Planck scale
- predictions for standard model parameters

A. Eichhorn, Front. Astron. Space Sci. **5** (2019) 47
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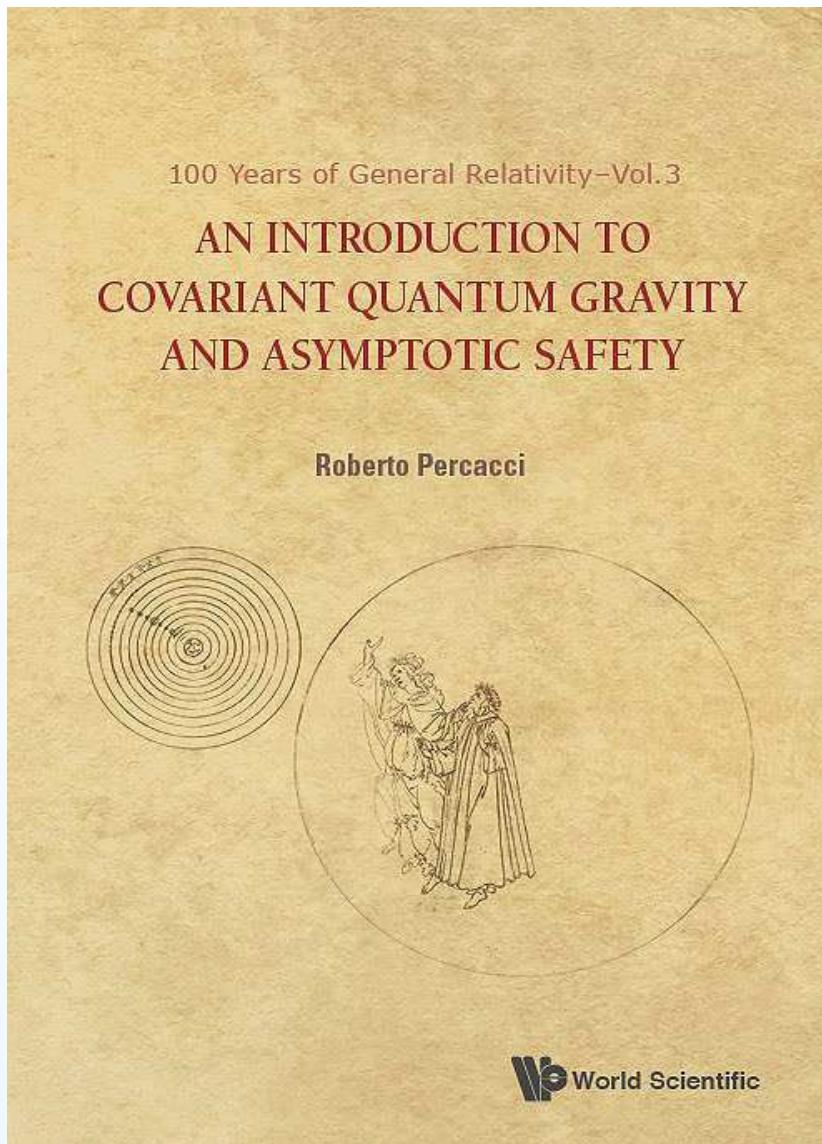
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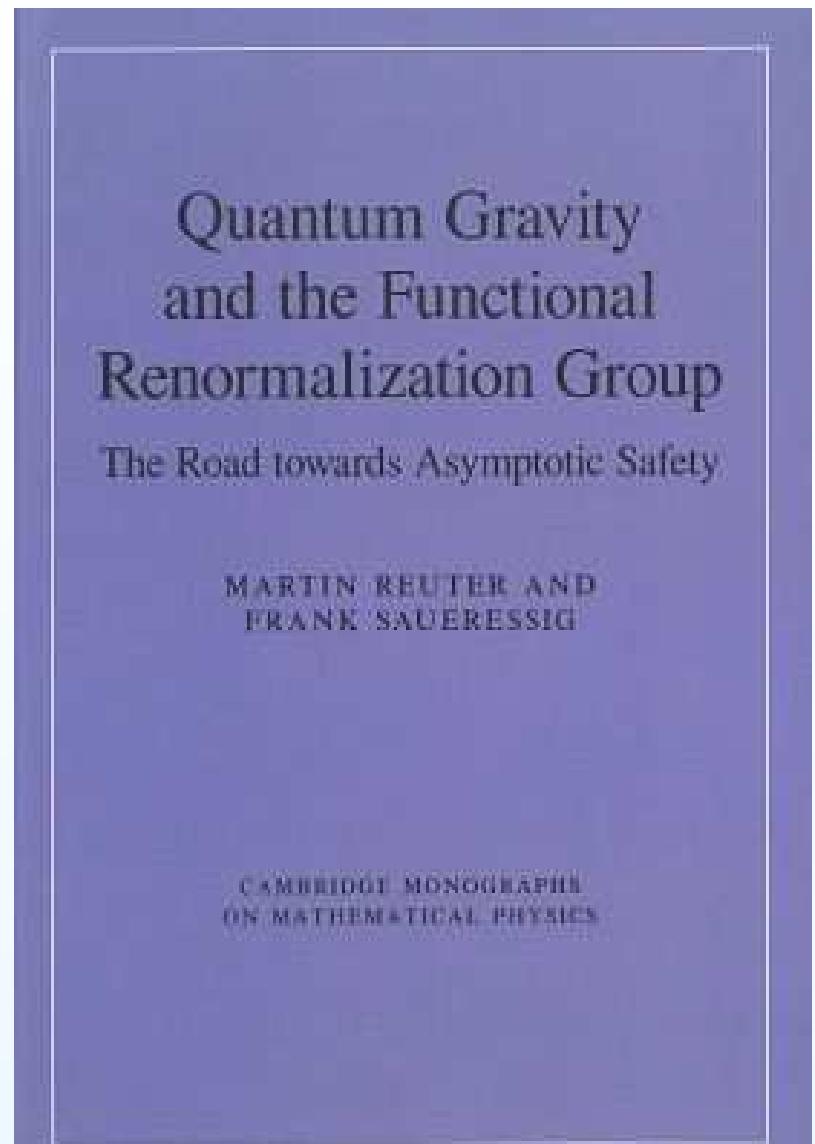
still a lot to do

prospects warrant this investment

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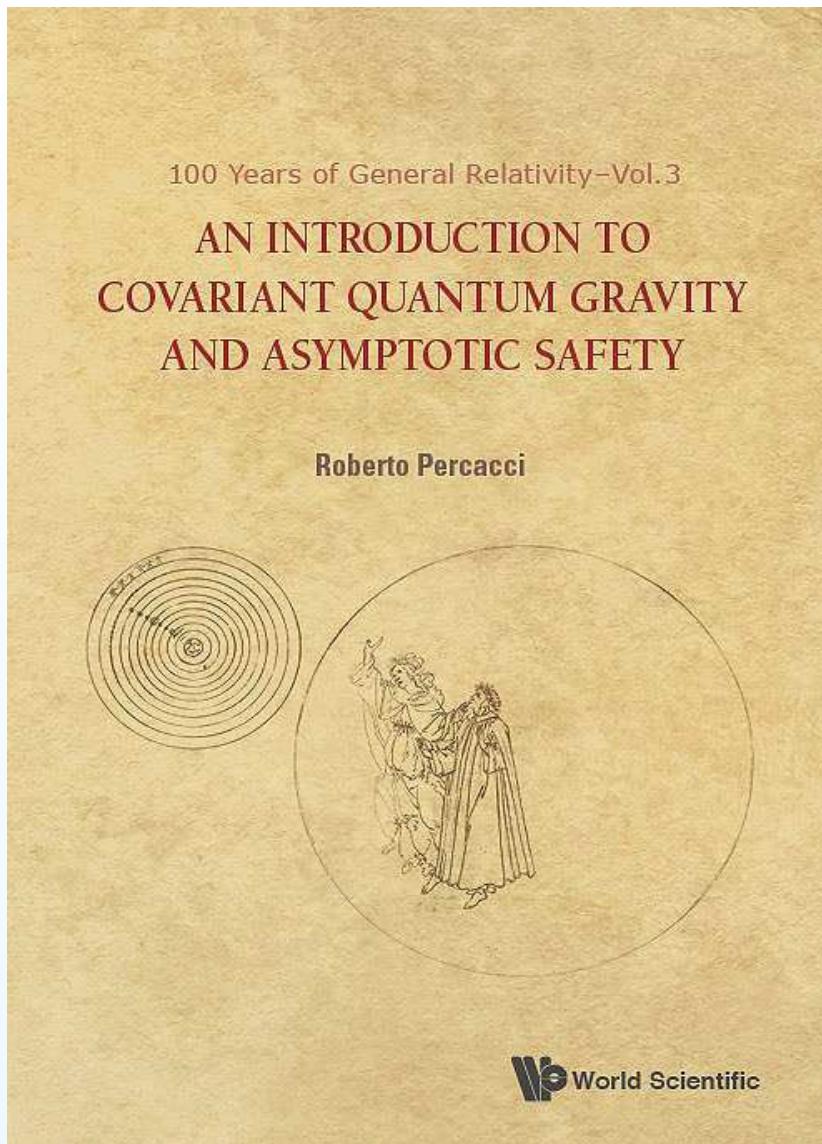


Percacci '17

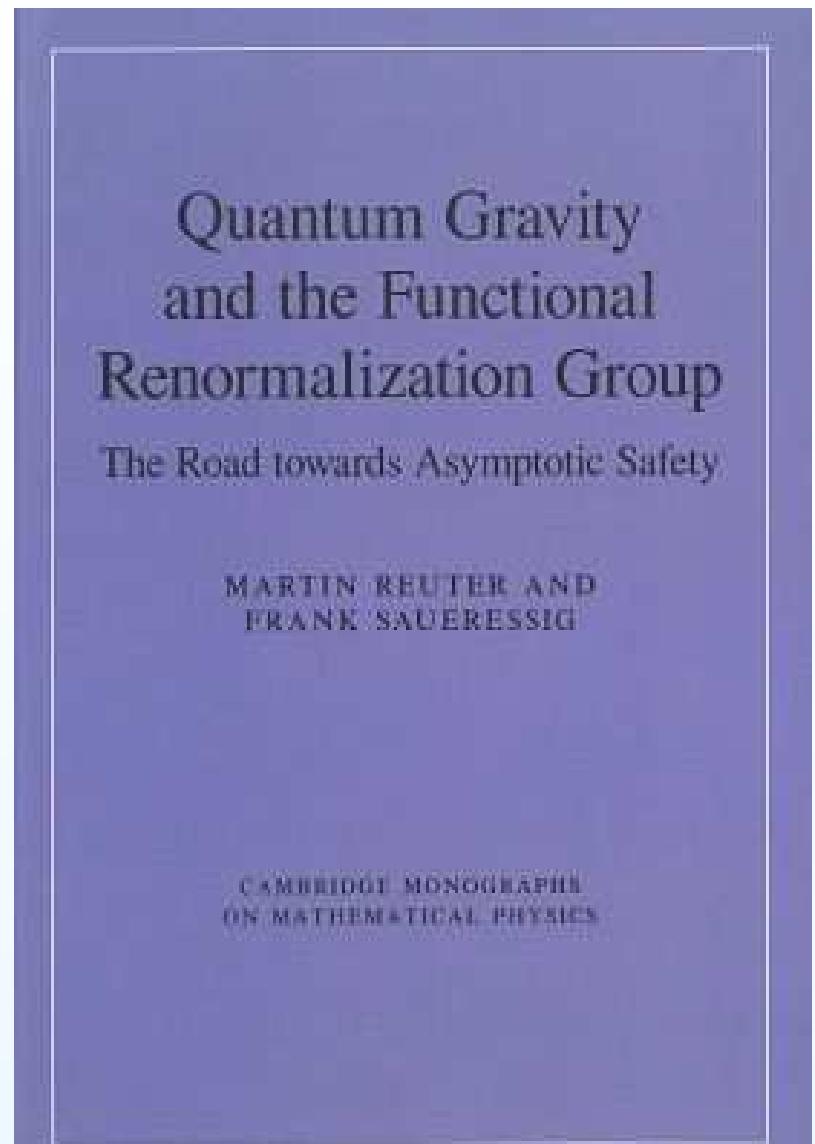


Reuter, Saueressig '19

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Percacci '17



Reuter, Saueressig '19