Scattering Amplitudes: Spinning BH vs Soft Theorems

Alfredo Guevara
(Perimeter Institute)

1903.12419, 1908.11349 with F. Bautista

Quantum Field Theory Meets Gravity, September 26 2019

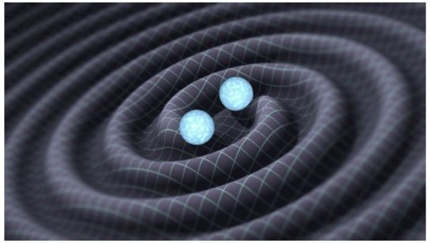
See also - 1705.10262 w. F. Cachazo
- 1706.02314
-1812.06895 , 1906.10071 w A. Ochirov & J. Vines

GW Catalogue (from LIGO Collaboration ‘18)

Event	$m_1/\mathrm{M}_\odot$	$m_2/\mathrm{M}_\odot$	$\mathcal{M}/\mathrm{M}_\odot$	χ_{eff}	$M_{\mathrm{f}}/\mathrm{M}_\odot$	a_{f}	$E_{\mathrm{rad}}/(\mathrm{M}_\odot c^2)$	$\ell_{\mathrm{peak}}/(\mathrm{erg\,s}^{-1})$	d_L/Mpc	z	$\Delta\Omega/\mathrm{deg}^2$
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.6}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.3}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$3.1^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4} \times 10^{56}$	430^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	180
GW151012	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.0}_{-1.1}$	$0.04^{+0.28}_{-0.19}$	$35.7^{+9.9}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	$1.5^{+0.5}_{-0.5}$	$3.2^{+0.8}_{-1.7} \times 10^{56}$	1060^{+540}_{-480}	$0.21^{+0.09}_{-0.09}$	1555
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$8.9^{+0.3}_{-0.3}$	$0.18^{+0.20}_{-0.12}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	$1.0^{+0.1}_{-0.2}$	$3.4^{+0.7}_{-1.7} \times 10^{56}$	440^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$	1033
GW170104	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$21.5^{+2.1}_{-1.7}$	$-0.04^{+0.17}_{-0.20}$	$49.1^{+5.2}_{-3.9}$	$0.66^{+0.08}_{-0.10}$	$2.2^{+0.5}_{-0.5}$	$3.3^{+0.6}_{-0.9} \times 10^{56}$	960^{+430}_{-410}	$0.19^{+0.07}_{-0.08}$	924
GW170608	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$7.9^{+0.2}_{-0.2}$	$0.03^{+0.19}_{-0.07}$	$17.8^{+3.2}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	$0.9^{+0.05}_{-0.1}$	$3.5^{+0.4}_{-1.3} \times 10^{56}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$	396
GW170729	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$35.7^{+6.5}_{-4.7}$	$0.36^{+0.21}_{-0.25}$	$80.3^{+14.6}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	$4.8^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-1.5} \times 10^{56}$	2750^{+1350}_{-1320}	$0.48^{+0.19}_{-0.20}$	1033
GW170809	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$25.0^{+2.1}_{-1.6}$	$0.07^{+0.16}_{-0.16}$	$56.4^{+5.2}_{-3.7}$	$0.70^{+0.08}_{-0.09}$	$2.7^{+0.6}_{-0.6}$	$3.5^{+0.6}_{-0.9} \times 10^{56}$	990^{+320}_{-380}	$0.20^{+0.05}_{-0.07}$	340
GW170814	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$24.2^{+1.4}_{-1.1}$	$0.07^{+0.12}_{-0.11}$	$53.4^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	$2.7^{+0.4}_{-0.3}$	$3.7^{+0.4}_{-0.5} \times 10^{56}$	580^{+160}_{-210}	$0.12^{+0.03}_{-0.04}$	87
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$	$0.00^{+0.02}_{-0.01}$	≤ 2.8	≤ 0.89	≥ 0.04	$\geq 0.1 \times 10^{56}$	40^{+10}_{-10}	$0.01^{+0.00}_{-0.00}$	16
GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$26.7^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$	$59.8^{+4.8}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	$2.7^{+0.5}_{-0.5}$	$3.4^{+0.5}_{-0.7} \times 10^{56}$	1020^{+430}_{-360}	$0.20^{+0.07}_{-0.07}$	39
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$29.3^{+4.2}_{-3.2}$	$0.08^{+0.20}_{-0.22}$	$65.6^{+9.4}_{-6.6}$	$0.71^{+0.08}_{-0.10}$	$3.3^{+0.9}_{-0.8}$	$3.6^{+0.6}_{-0.9} \times 10^{56}$	1850^{+840}_{-840}	$0.34^{+0.13}_{-0.14}$	1651

- **As more precise measurements will take place in LISA, more accurate templates will be needed! =>We can use modern methods in QFT! (for inspiral phase)**

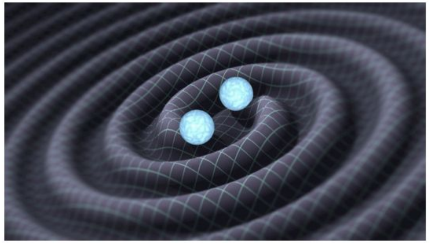
Some problems for (analytic) theorists: (From Bern's talk)



1. Spin.
2. Finite size effects.
3. New physics effects.
4. Radiation.
5. High orders in perturbation theory.

Donoghue '93; Bjerrum-Bohr, Donoghue, Holstein '03; + Plante, Vanhove '15; Neil, Rothstein '13; Cachazo, Guevara '17; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove '18; Plefka, Steinhoff, Wormsbecher '18; Cheung, Rothstein, Solon '18; Bern, Cheung, Roiban, Shen, Solon, Zeng '19 ….

Some problems for (analytic) theorists: (From Bern's talk)



1. **Spin.** Vaidya '14; Guevara '17; Chung, Huang, Kim, Lee '18 ….
2. **Finite size effects.**
3. **New physics effects.**
4. **Radiation.** Goldberger, Ridgway '17; Luna et al. '17; Shen '18; Kosower et al. '18….
5. **High orders in perturbation theory.**

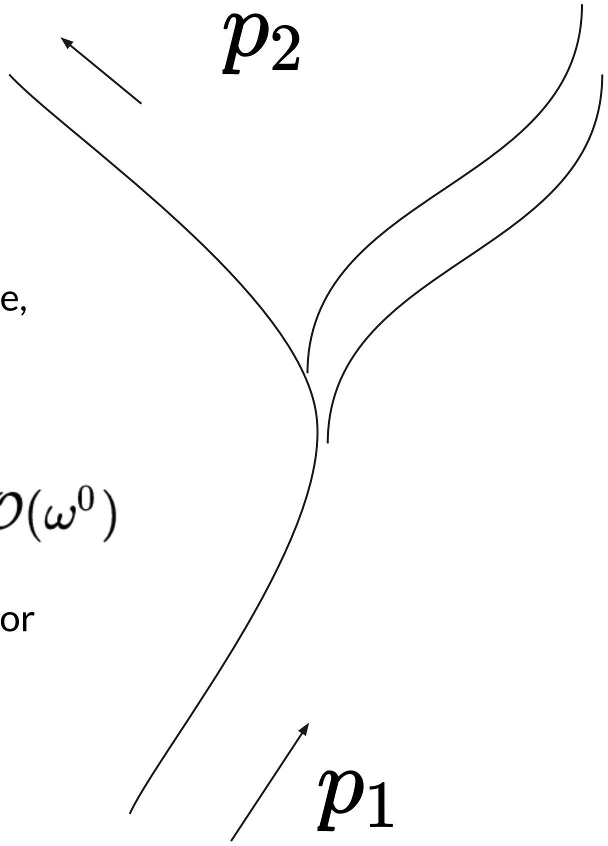
Donoghue '93; Bjerrum-Bohr, Donoghue, Holstein '03; + Plante, Vanhove '15; Neil, Rothstein '13; Cachazo, Guevara '17; Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove '18; Plefka, Steinhoff, Wormsbecher '18; Cheung, Rothstein, Solon '18; Bern, Cheung, Roiban, Shen, Solon, Zeng '19….

Soft Theorem

- Massive body accelerating in a finite interval of time, point particle approximation.
- At $r \to \infty$ we can drop Coulomb modes to get

$$T_{\mu\nu}(k) = \sqrt{8\pi G}\left(\frac{p_{1\mu}p_{1\nu}}{p_1 \cdot k + i\epsilon} - \frac{p_{2\mu}p_{2\nu}}{p_2 \cdot k - i\epsilon}\right) + \mathcal{O}(\omega^0)$$

which corresponds to a classical instance of the Soft Factor
[Weinberg '65, Yennie, Frautschi, Suura '61]
(see also Strominger et al.)



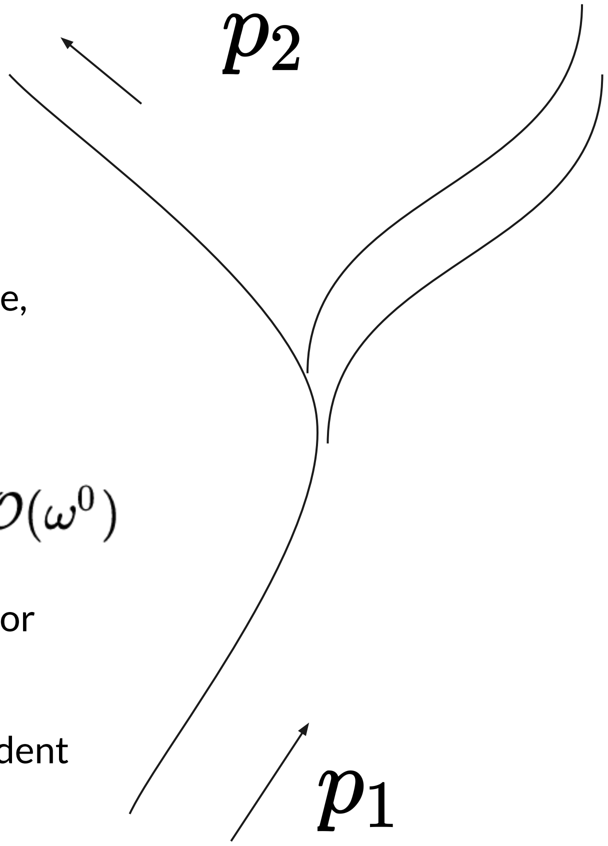
Soft Theorem

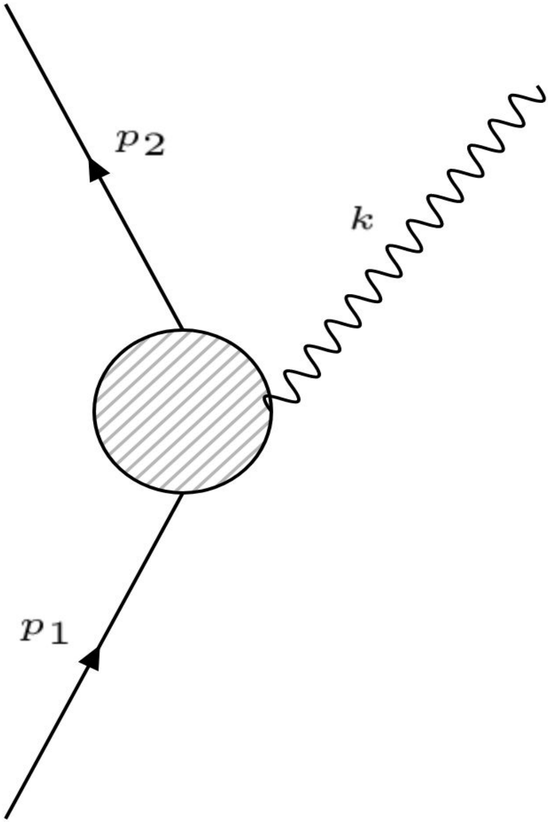
- Massive body accelerating in a finite interval of time, point particle approximation
- At $r \to \infty$ we can drop Coulomb modes to get

$$T_{\mu\nu}(k) = \sqrt{8\pi G}\left(\frac{p_{1\mu}p_{1\nu}}{p_1\cdot k+i\epsilon} - \frac{p_{2\mu}p_{2\nu}}{p_2\cdot k-i\epsilon}\right) + \mathcal{O}(\omega^0)$$

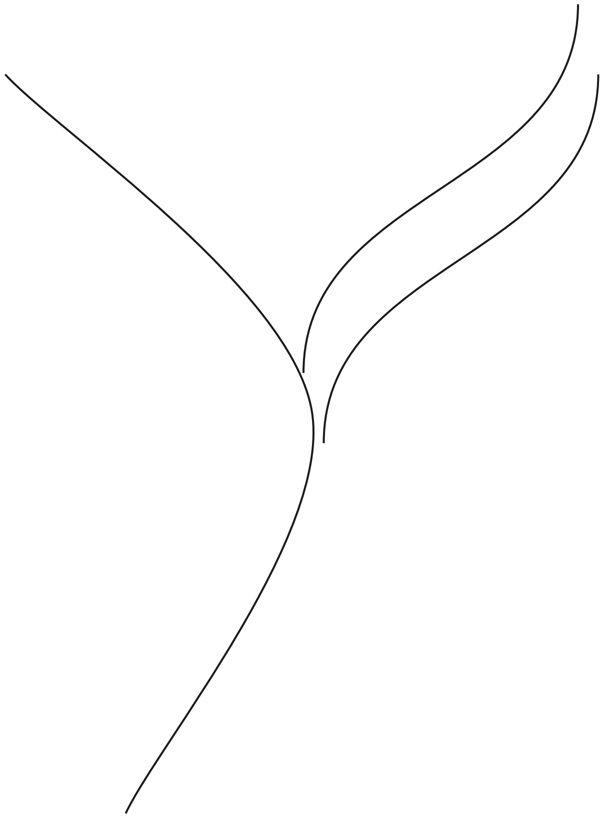
which corresponds to a classical instance of the Soft Factor
 [Weinberg ‘65, Yennie, Frautschi, Suura ‘61]
(see also Strominger et al.)

- Large-wavelength behaviour is **universal**. Independent of the acceleration or internal structure (**spin**)




p_2
k
p_1

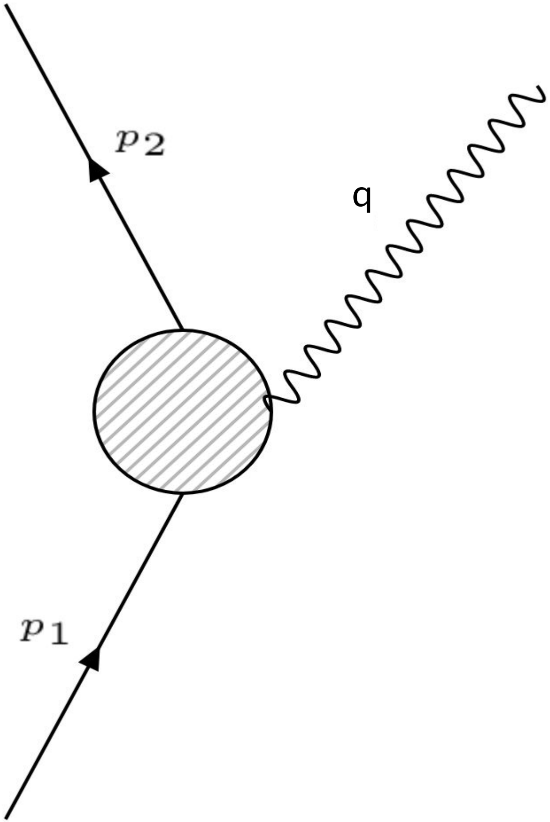
=



$$\left(\frac{p_{1\mu}p_{1\nu}}{p_1\cdot q+i\epsilon}-\frac{p_{2\mu}p_{2\nu}}{p_2\cdot q-i\epsilon}\right)_{p_2=p_1+q} =$$

$$p_{1\mu}p_{1\nu}\,\bar{\delta}(p_1\cdot q) =$$

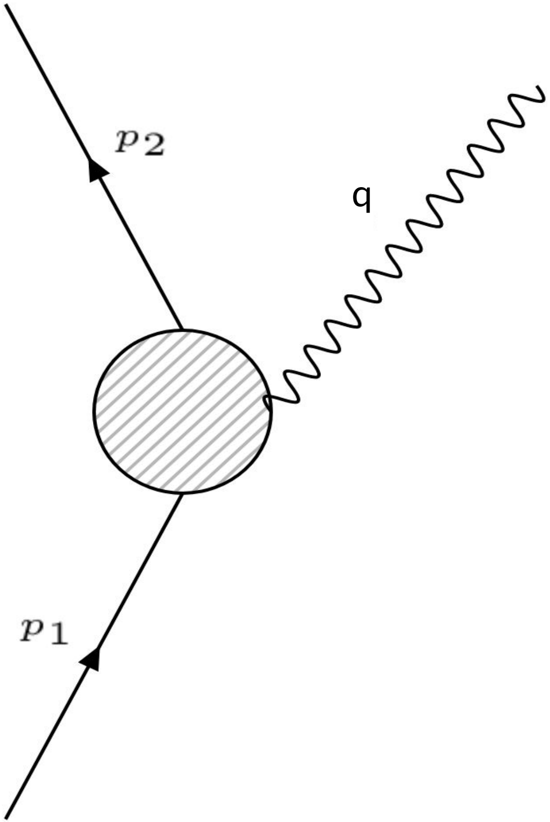
- No support for radiation yet



$$\left(\frac{p_{1\mu}p_{1\nu}}{p_1\cdot q+i\epsilon} - \frac{p_{2\mu}p_{2\nu}}{p_2\cdot q-i\epsilon}\right)_{p_2=p_1+q} =$$

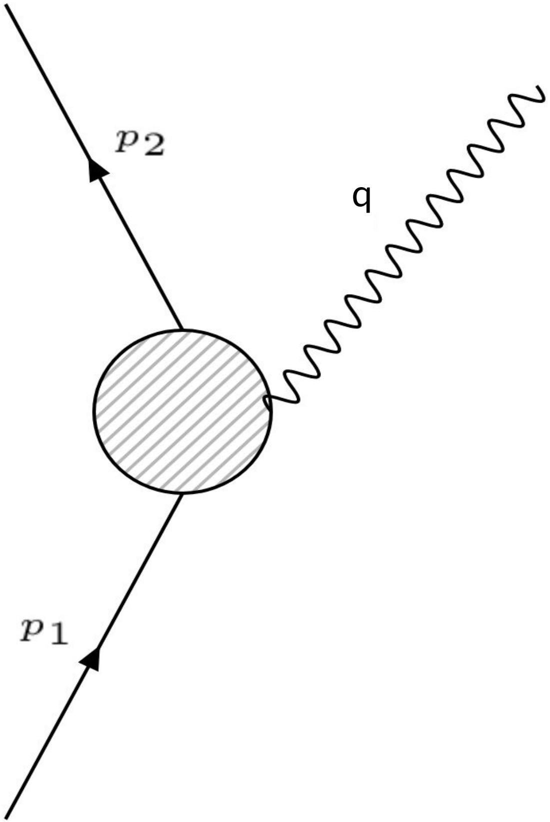
$$p_{1\mu}p_{1\nu}\,\bar{\delta}(p_1\cdot q) =$$

- No support for radiation yet
- Still useful as (complex) building block



$$\left(\frac{p_{1\mu}p_{1\nu}}{p_1 \cdot q + i\epsilon} - \frac{p_{2\mu}p_{2\nu}}{p_2 \cdot q - i\epsilon}\right)_{p_2 = p_1 + q} =$$

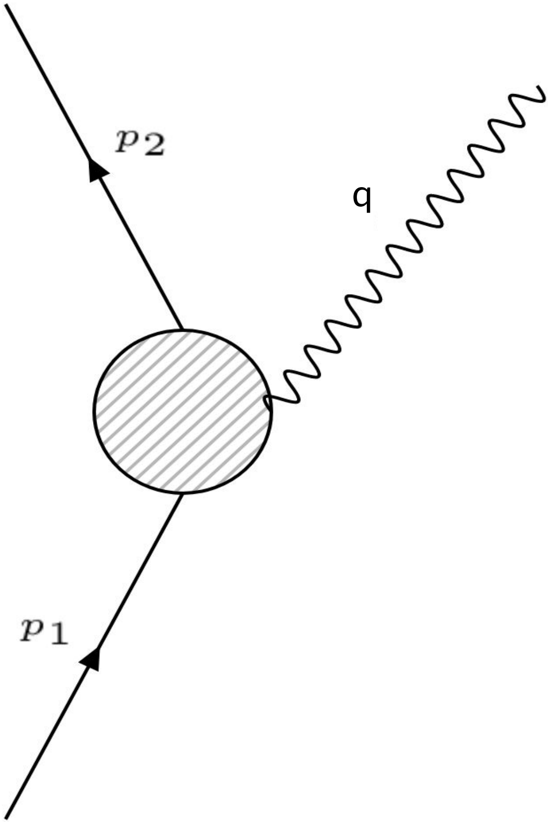
$$p_{1\mu}p_{1\nu}\,\bar{\delta}(p_1 \cdot q) =$$



- No support for radiation yet
- Still useful as (complex) building block
- Unique, fixed by little group

$$\left(\frac{p_{1\mu} p_{1\nu}}{p_1 \cdot q + i\epsilon} - \frac{p_{2\mu} p_{2\nu}}{p_2 \cdot q - i\epsilon} \right)_{p_2 = p_1 + q} =$$

$$p_{1\mu} p_{1\nu} \, \bar{\delta}(p_1 \cdot q) =$$



- No support for radiation yet
- Still useful as (complex) building block
- Unique, fixed by little group
- **Double Copy** of photon vertex...

What about corrections in the graviton frequency q? Weinberg also showed that they are encoded in a multipole expansion, i.e.

Soft Expansion = Multipole Expansion

$$\underset{\sim}{\Gamma}(\mathbf{p}',\mathbf{p}) \longrightarrow e(\mathbf{p}'+\mathbf{p})\,1 + \left(\frac{2im\mu}{J}\right)\underset{\sim}{J}\times(\mathbf{p}'-\mathbf{p}) + \text{cubic} \qquad (2.G.19)$$

$$\Gamma^0(\mathbf{p}',\mathbf{p}) \longrightarrow 2me + \frac{e}{m}\,\mathbf{p}\cdot\mathbf{p}' + Q_{ij}(\mathbf{p}'-\mathbf{p})_i(\mathbf{p}'-\mathbf{p})_j + i\left(\frac{2\mu}{J}-\frac{e}{m}\right)(\mathbf{p}'\times\mathbf{p})\cdot\underset{\sim}{J} + \text{quartic} \qquad (2.G.20)$$

From Brandeis Lectures (1970)

We introduce a covariant multipole expansion for gravitons (or photons) being emitted from a massive spin-s source. Set $\epsilon^{\mu\nu} = \epsilon^{\mu}\epsilon^{\nu}$, $E^{I} = E^{\mu_1\cdots\mu_s}$

$$\epsilon_{\mu\nu}T^{\mu\nu}(k)=(\epsilon\cdot p)^2 E_1^I\left[1+\frac{g_s}{2}\,\frac{k_\mu\epsilon_\nu J^{\mu\nu}}{\epsilon\cdot p}+\omega^{(s)}_{\mu\nu\rho\sigma}\{J^{\mu\nu},J^{\rho\sigma}\}+\ldots\right]^{J}{}_{I}E_{2,J}$$

We introduce a covariant multipole expansion for gravitons (or photons) being emitted from a massive spin-s source. Set $\epsilon^{\mu\nu} = \epsilon^{\mu}\epsilon^{\nu},\ E^{I} = E^{\mu_1\cdots\mu_s}$

$$\epsilon_{\mu\nu}T^{\mu\nu}(k) = (\epsilon\cdot p)^2 E_1^I \left[1 + \frac{g_s}{2}\frac{k_\mu \epsilon_\nu J^{\mu\nu}}{\epsilon\cdot p} + \omega^{(s)}_{\mu\nu\rho\sigma}\{J^{\mu\nu}, J^{\rho\sigma}\} + \ldots\right]^J{}_I E_{2,J}$$

Weinberg’s Soft Factor
(Equivalence principle)

We introduce a covariant multipole expansion for gravitons (or photons) being emitted from a massive spin-s source. Set $\epsilon^{\mu\nu} = \epsilon^{\mu}\epsilon^{\nu},\ E^{I} = E^{\mu_1\cdots\mu_s}$

$$\epsilon_{\mu\nu}T^{\mu\nu}(k)=(\epsilon\cdot p)^2 E_1^I\left[1+\frac{g_s}{2}\frac{k_\mu\epsilon_\nu J^{\mu\nu}}{\epsilon\cdot p}+\omega^{(s)}_{\mu\nu\rho\sigma}\{J^{\mu\nu},J^{\rho\sigma}\}+\ldots\right]^J{}_I E_{2,J}$$

Weinberg's Soft Factor
(Equivalence principle)

g=1/s for QED with spin-s sources
(Belinfante Conjecture)

g=2 for GR **always**
Mathisson–Papapetrou–Dixon '70
Chung, Huang, Kim, Lee '19….

We introduce a covariant multipole expansion for gravitons (or photons) being emitted from a massive spin-s source. Set $\epsilon^{\mu\nu} = \epsilon^{\mu}\epsilon^{\nu}$, $E^{I} = E^{\mu_1 \cdots \mu_s}$

$$\epsilon_{\mu\nu} T^{\mu\nu}(k) = (\epsilon \cdot p)^2 E_1^I \left[1 + \frac{g_s}{2} \frac{k_\mu \epsilon_\nu J^{\mu\nu}}{\epsilon \cdot p} + \omega^{(s)}_{\mu\nu\rho\sigma}\{J^{\mu\nu}, J^{\rho\sigma}\} + \ldots\right]^J{}_I E_{2,J}$$

Weinberg's Soft Factor
(Equivalence principle)

g=1/s for QED with spin-s sources
(Belinfante Conjecture)

g=2 for GR **always**
Mathisson–Papapetrou–Dixon '70
Chung, Huang, Kim, Lee '19….

O(k^2), expected to be the Weyl tensor

Neutron Stars vs Black Holes. Vaidya '14

We introduce a covariant multipole expansion for gravitons (or photons) being emitted from a massive spin-s source. Set $\epsilon^{\mu\nu} = \epsilon^{\mu}\epsilon^{\nu},\ E^{I} = E^{\mu_1\cdots\mu_s}$

$$\epsilon_{\mu\nu}T^{\mu\nu}(k) = (\epsilon\cdot p)^2 E_1^I \left[1 + \frac{g_s}{2}\frac{k_\mu \epsilon_\nu J^{\mu\nu}}{\epsilon\cdot p} + \omega^{(s)}_{\mu\nu\rho\sigma}\{J^{\mu\nu}, J^{\rho\sigma}\} + \ldots\right]^J{}_I E_{2,J}$$

Weinberg's Soft Factor
(Equivalence principle)

g=1/s for QED with spin-s sources
(Belinfante Conjecture)

O(k^2), expected to be the Weyl tensor

Neutron Stars vs Black Holes. Vaidya '14

g=2 for GR **always**
Mathisson–Papapetrou–Dixon '70
Chung, Huang, Kim, Lee '18....

Subleading Soft Theorem
Cachazo, Strominger '14

How do we fix higher multipoles?

- We can use minimal coupling amplitudes for massive higher spins.
  Guevara ‘17; Chung, Huang, Kim, Lee ‘18; Guevara, Ochirov, Vines ‘18 ....

- We can impose **double copy** relations (GR=YM^2), e.g. using the QED dipole J to **fix** GR quadrupole J^2 and so on….
  Guevara, Bautista ‘19; Johansson, Ochirov ‘19 ...

How do we fix higher multipoles?

- We can use minimal coupling amplitudes for massive higher spins.
  Guevara '17; Chung, Huang, Kim, Lee '18; Guevara, Ochirov, Vines '18 ....

- We can impose **double copy** relations (GR=YM^2), e.g. using the QED dipole J to **fix** GR quadrupole J^2 and so on….
  Guevara, Bautista '19; Johansson, Ochirov '19 ...

In both cases we obtain **all** intrinsic multipoles of the linearized Kerr Black Hole encoded in an effective stress-energy tensor in the form of [Vines '17, Arkani-Hamed, Huang, O'Connell '19]

$$\bar{h}^{\text{Kerr}}_{\mu\nu} = u_\mu u_\nu \left(1 - \frac{1}{2!}(a\cdot\partial)^2 + \frac{1}{4!}(a\cdot\partial)^4 - \dots\right)\frac{4Gm}{r}$$
$$+ u_{(\mu}\epsilon_{\nu)\rho\alpha\beta}u^\rho a^\alpha \partial^\beta \left(1 - \frac{1}{3!}(a\cdot\partial)^2 + \dots\right)\frac{4Gm}{r}. \quad (30)$$

How do we fix higher multipoles?

- We can use minimal coupling amplitudes for massive higher spins.
  Guevara ‘17; Chung, Huang, Kim, Lee ‘18; Guevara, Ochirov, Vines ‘18 ....

- We can impose **double copy** relations (GR=YM^2), e.g. using the QED dipole J to **fix** GR quadrupole J^2 and so on….
  Guevara, Bautista ‘19; Johansson, Ochirov ‘19 ...

In both cases we obtain **all** intrinsic multipoles of the linearized Kerr Black Hole encoded in an effective stress-energy tensor in the form of [Vines ‘17, Arkani-Hamed, Huang, O’Connell ‘19]

$$\bar{h}^{\text{Kerr}}_{\mu\nu} = u_\mu u_\nu \left(1 - \frac{1}{2!}(a\cdot\partial)^2 + \frac{1}{4!}(a\cdot\partial)^4 - \dots\right)\frac{4Gm}{r}$$
$$+ u_{(\mu}\epsilon_{\nu)\rho\alpha\beta}u^\rho a^\alpha \partial^\beta \left(1 - \frac{1}{3!}(a\cdot\partial)^2 + \dots\right)\frac{4Gm}{r}. \quad (30)$$

This resembles an **infinite soft theorem** in the spirit of He, Huang, Wen ‘14; Hamada, Shiu ‘18 ; Compère ‘19; ….

Massless Origin and Minimal Coupling

- Minimal coupling massive amplitudes are defined by imposing smooth massless limit, e.g. in the sense of Porrati, Ferrara, Telegdi '92

Soft Factor

$$A(1^h, 2^{-h}, h_3) \sim \left(\frac{\langle 13\rangle\langle 32\rangle}{\langle 12\rangle}\right)^2 \left(\frac{\langle 13\rangle}{\langle 23\rangle}\right)^{2h} \quad \xleftarrow{\text{massless limit}} \quad (\epsilon_3 \cdot p_1)^2 \langle \epsilon_1^s | \exp\left(-\frac{k_{3\mu}\epsilon_{3\nu}}{\epsilon_3 \cdot p_1} J_s^{\mu\nu}\right) | \epsilon_2^s \rangle$$

$$\xrightarrow{\text{“compactification”}}$$

$m = 0$

$m \neq 0$

Massless Origin and Minimal Coupling

- Minimal coupling massive amplitudes are defined by imposing smooth massless limit, e.g. in the sense of Porrati, Ferrara, Telegdi '92

Soft Factor

$$A(1^h, 2^{-h}, h_3) \sim \left(\frac{\langle 13\rangle\langle 32\rangle}{\langle 12\rangle}\right)^2 \left(\frac{\langle 13\rangle}{\langle 23\rangle}\right)^{2h} \quad \xleftarrow{\text{massless limit}} \quad \xrightarrow{\text{"compactification"}} \quad (\epsilon_3 \cdot p_1)^2 \langle \epsilon_1^s | \exp\left(-\frac{k_{3\mu}\epsilon_{3\nu}}{\epsilon_3 \cdot p_1} J_s^{\mu\nu}\right) | \epsilon_2^s \rangle$$

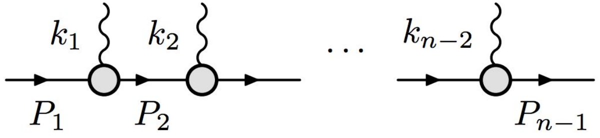
$$m = 0 \qquad\qquad m \neq 0$$

- The massless 3-pt amplitude (fixed by little group) can be 1) rewritten in exponential form and 2) reinterpreted as the **massive** 3-pt amplitudes derived in Arkani-Hamed, Huang, Huang '17

Massless Origin and Minimal Coupling

- At higher multiplicity (n>3) we can apply the compactification to get the minimal coupling amplitudes with one matter line, which is (conjecturally) all we need for the classical 2-body problem. A.G., Bautista ‘19

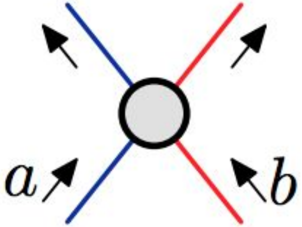
- We also get exponential forms => Exponentiated soft theorems

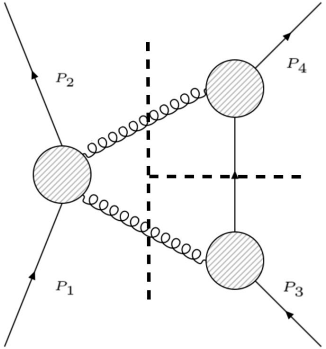


$$= \prod_i (P_i \cdot \epsilon_i)^h \langle \varepsilon_2 | e^{J_{n-2}} \cdots e^{J_1} | \varepsilon_1 \rangle = \prod_i (P_i \cdot \epsilon_i)^h \langle \varepsilon_2 | \tilde{\varepsilon}_2 \rangle.$$

- However, because massless HS particles are inconsistent at n>3 (Cachazo, Benincasa ‘07) this leads to unphysical poles for **spin>2.** A.G., Ochirov, Vines ‘18, Chung, Huang, Kim ‘19…. This is equivalent to state that any physical **spin>2, n>3** amplitude will have 1/m singularities!

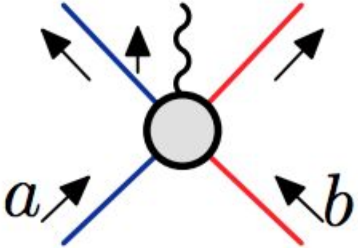
Finally, we can use the higher point amplitudes to extract observables in the 2-body problem! This includes conservative and non-conservative pieces...

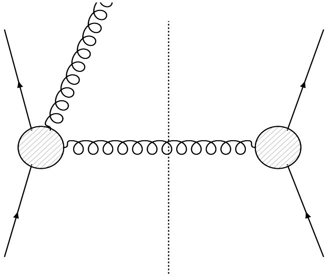
$M_4 =$ 



A.G., F. Cachazo '17

$\lim_{r\to\infty} h_{\mu\nu}(r)$

$M_5 =$ 



A.G., F. Bautista '19

Here a and b represent two different compact objects such as spinning Black Holes. See also Kosower, Maybee, O'connell '19

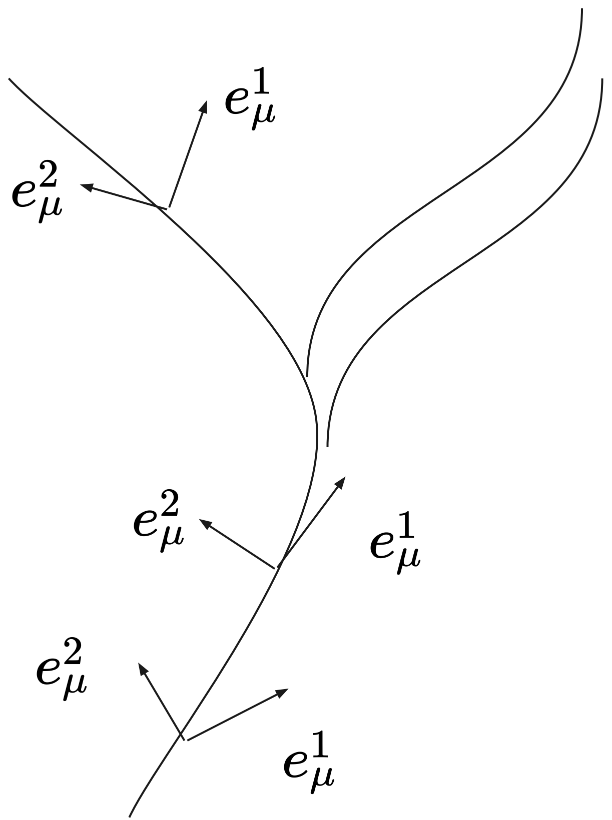
Thank you!

Spin

$$\Omega^{IJ} \equiv \eta^{\mu\nu} e^I_\mu \frac{d}{ds} e^J_\nu$$

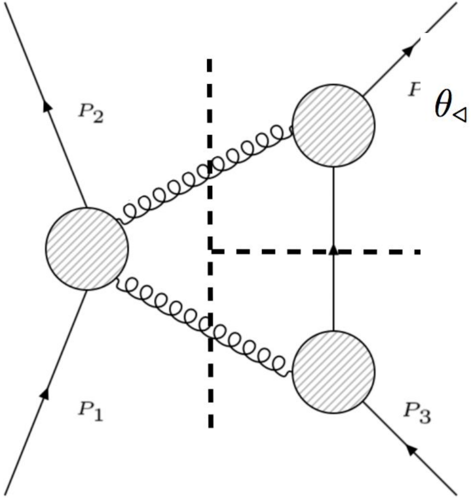
$$S_{pp} = \int ds \Big[-\dot{x}^\mu e^I_\mu p_I + \tfrac{1}{2} S^{IJ} \Omega_{IJ} + \tfrac{1}{2} e \left(p^I p_I - m^2 \right) + e \lambda_I S^{IJ} p_J \Big]$$

$$S_{IJ} e^I_\mu e^J_\mu = \epsilon_{\mu\nu\rho\sigma} p^\rho a^\sigma$$



Choosing the gauge $\epsilon_1^+ \cdot \epsilon_2^- = 0$ the Compton amplitude exponentiates up to s=2

$$A_4^{\mathrm{gr,s}}(p_1, p_2, k_1^+, k_2^-) = A_4^{\mathrm{gr},0} \times \exp\left(\frac{F_{\mu\nu}J^{\mu\nu}}{2\epsilon\cdot p}\right)$$

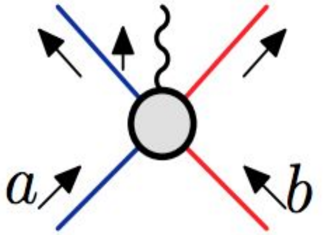


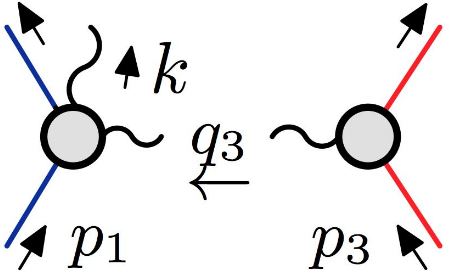
$$\begin{aligned}\theta_\triangleleft &= \pi G^2 E \frac{m_b}{2v^4}\frac{\partial}{\partial b}\int_{\Gamma_{\mathrm{LS}}}\frac{dz}{2\pi i}\frac{(1-vz)^4}{(z^2-1)^{3/2}}\int\frac{d^2\boldsymbol{k}}{2\pi|\boldsymbol{k}|}\exp\left(i\boldsymbol{k}\cdot\left[\boldsymbol{b} - z\hat{\boldsymbol{p}}\times\boldsymbol{a}_b - \frac{z-v}{1-vz}\hat{\boldsymbol{p}}\times\boldsymbol{a}_a\right]\right) \\ &= \pi G^2 E \frac{m_b}{2v^4}\frac{\partial}{\partial b}\int_{\Gamma_{\mathrm{LS}}}\frac{dz}{2\pi i}\frac{(1-vz)^4}{(z^2-1)^{3/2}}\left|b - za_b - \frac{z-v}{1-vz}a_a\right|^{-1}, \qquad (3.\end{aligned}$$

(from Guevara, Ochirov, Vines '18)

- All orders in a_b. Agreement up to a_a^3, **all orders in v.**
- Can be matched to effective (bounded) Hamiltonian.
  Siemonsen, Vines '19

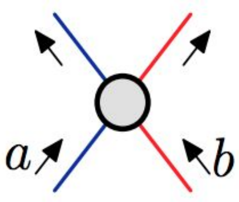
Exponentiated Classical Soft Theorem

$\Bigg($ 

$\Bigg)_{\hbar \to 0} =$ 

$+ \ (1 \leftrightarrow 3)$

$$= \sum_{i=1,3} \mathcal{S}_i \, e^{\eta_i \left(F_p \frac{p_i \cdot k}{F_{iq}} \frac{\partial}{\partial (p_1 \cdot p_3)} + q \cdot k \frac{\partial}{\partial q^2} \right)}$$

$\Bigg($ 

$\Bigg)_{\hbar \to 0}$

The radiation field exponentiates even for scalars! The soft theorem extends to all orders when the classical limit is taken.

It should be possible to derive from this the full multipole expansion of the radiative field at leading order in G.