Cosmological perturbations in hybrid QC

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Quantum field theory meets gravity
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Motivation

- Canonical quantum cosmology:
  - ⋆ Approach to the quantum representation of spatially homogeneous spacetimes in a controlled way.
  - ⋆ Test the predictive power of quantum gravity techniques/theories in simplified models.

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- Canonical quantum cosmology:
  - ⋆ Approach to the quantum representation of spatially homogeneous spacetimes in a controlled way.
  - ⋆ Test the predictive power of quantum gravity techniques/theories in simplified models.
- Inclusion of inhomogeneities:
  - ⋆ Eventually acquire some insights on the behaviour of inhomogeneous fluctuations in the very Early Universe.
  - ⋆ Development of techniques that might be useful for quantizing general spacetimes coupled to matter.

Motivation

- Canonical quantum cosmology.

- Inclusion of inhomogeneities.

[M. Martín-Benito, L.J. Garay, G.A. Mena Marugán, 2008]

Hybrid quantum cosmology

★ Quantization of homogeneous d.o.f with methods inspired in some theory of quantum gravity.

★ Fock representation of inhomogeneities, restricted by imposing physically desirable properties.

Cosmological perturbations

The cosmology

- FLRW spacetime with flat & compact hypersurfaces.
- Minimally coupled homogeneous scalar field (inflaton).
- Minimally coupled inhomogeneous Dirac field, treated entirely as a perturbation.
- Scalar and tensor perturbations of the metric & inflaton.
- Expansion in spatial (Dirac or Laplace) eigenmodes.
  $l_0\vec{k}/(2\pi) \in \mathbb{Z}^3$
- Canonical formalism: truncation of the action at quadratic order in <u>all</u> the perturbations.

Scalar & tensor perturbations

- Background-dependent canonical transformations lead to:
  - ⋆ Mukhanov-Sasaki scalar gauge-invariants.
  - ⋆ Abelianized perturbative constraints (& momenta).

- Can be completed to be canonical for the entire system by:

$$(a, \pi_a) \longrightarrow (\breve{a}, \pi_{\breve{a}}) = (a - \Delta a, \pi_a - \Delta \pi_a)$$
$$(\phi, \pi_\phi) \longrightarrow (\breve{\phi}, \pi_{\breve{\phi}}) = (\phi - \underbrace{\Delta \phi, \pi_\phi - \Delta \pi_\phi}_{\text{quadratic in perturbations}})$$

- Express the entire Hamiltonian in terms of the new canonical set, keeping the quadratic truncation.

[L. Castelló Gomar, M. Fernández-Méndez, M. Martín-Benito, G.A. Mena Marugán, J. Olmedo, 2014-2015]

Scalar & tensor perturbations

- Hamiltonian: new linear perturbative constraints and zero-mode of the Hamiltonian constraint, which is:

$$[H_{|0} + {}^{s}H_{|2} + {}^{T}H_{|2}](\breve{a}, \pi_{\breve{a}}, \breve{\phi}, \pi_{\breve{\phi}})$$

$$H_{|0} = \frac{1}{2l_0^3 a^3}\left[\pi_\phi^2 - \frac{4\pi G}{3} a^2 \pi_a^2 + 2 l_0^6 a^6 V(\phi)\right] \longrightarrow \text{FLRW constraint}$$

$${}^{s}H_{|2} = \frac{1}{2a}\sum_{\vec{k}\neq 0}\left[\left(k^2 + s^{(s)}\right)|v_{\vec{k}}|^2 + |\pi_{v_{\vec{k}}}|^2\right] \longrightarrow \text{MS Hamiltonian}$$

$${}^{T}H_{|2} = \frac{1}{2a}\sum_{\vec{k}\neq 0,\epsilon}\left[\left(k^2 + s^{(t)}\right)|d_{\vec{k},\epsilon}|^2 + |\pi_{d_{\vec{k},\epsilon}}|^2\right] \longrightarrow \text{Tensor Hamiltonian}$$

- The terms $s^{(s)}$ and $s^{(t)}$ are the background-dependent "masses" for the Mukhanov-Sasaki and tensor perturb.

[F. Benítez Martínez, L. Castelló Gomar, M. Fernández-Méndez, M. Martín-Benito, G.A. Mena Marugán, J. Olmedo, 2014-2016]

Fermionic perturbations

- Contribute to the zero-mode of the Hamiltonian constraint with a quadratic Dirac Hamiltonian $H_D(\breve{a})$.

- At quadratic order, fermions do not contribute to the linearized Hamiltonian & diffeo constraints.

$\Downarrow$

Gauge-invariant perturbations

- Canonical set for homogeneous background+Dirac field:

$$\{(\breve{a},\pi_{\breve{a}}),(\breve{\phi},\pi_{\breve{\phi}}),\underbrace{(x_{\vec{k}},\bar{x}_{\vec{k}}),(y_{\vec{k}},\bar{y}_{\vec{k}})}\}$$

$\downarrow$

Quiral mode coefficients, $l_0\vec{k}/(2\pi)\in\mathbb{Z}^3$

Fermionic perturbations

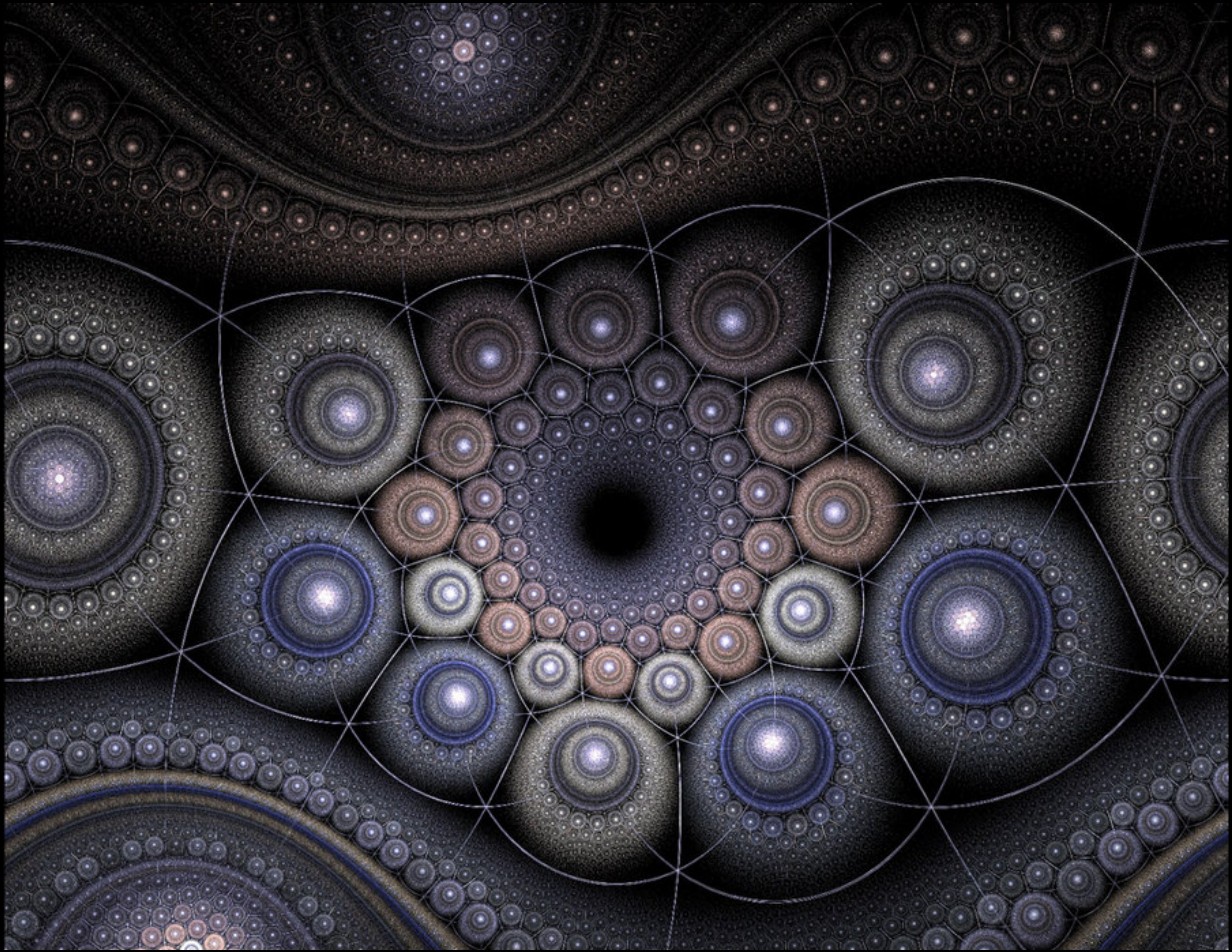
- Motivated by previous works of D'Eath & Halliwell (1987), use an $\breve{a}-$dependent transformation to fermionic variables that are, canonically, annihilation and creationlike.
- In terms of them, H_D is diagonal ($\propto$ number operator).
- Complete the transformation for the entire system:

$$\pi_{\breve{a}} \to \pi_{\tilde{a}} = \pi_{\breve{a}} - \underbrace{\Delta\pi_{\breve{a}}}_{\text{quadratic in fermionic perturbations}}$$

[BEN, M. Martín-Benito, G.A. Mena Marugán, 2017]

- In terms of the new canonical set, fermionic contribution to the Hamiltonian constraint is of the form:

$$H_D(\tilde{a}) + \underbrace{H_I(\tilde{a}, \pi_{\tilde{a}})}_{\text{Interaction (non-diagonal) term}}$$



Hybrid quantization

Hybrid quantization

- QC representation for the perturbatively corrected volume (or scale factor) and its momentum.

- Schrödinger representation for the perturbatively corrected inflaton field and its momentum.

- Fock representations for the MS and tensor perturbations, within a unique privileged equivalence class selected by:

  ⋆ Invariance of the vacuum under the symmetries of the spatial hypersurfaces.

  ⋆ In the context of QFT in curved spacetimes, admits a unitarily implementable quantum field dynamics.

[L. Castelló Gomar, J. Cortez, M. Fernández-Méndez, L. Fonseca,D. Martín de Blas, G.A. Mena Marugán, J. Olmedo, J.M. Velhinho, 2011-2013]

Hybrid quantization

- QC representation for the perturbatively corrected volume (or scale factor) and its momentum.

- Schrödinger representation for the perturbatively corrected inflaton field and its momentum.

- Fock representations for the MS and tensor perturbations.

- Fock representation for the fermionic perturbations characterized by the variables of D'Eath & Halliwell.

  ⋆ Is in the unique equivalence class whose vacuum is invariant under the symmetries of the system & admits a unitarily impementable dynamics in the context of QFT.

[J. Cortez, BEN, M. Martín-Benito, G.A. Mena Marugán, J.M. Velhinho, 2017]

Hybrid quantization: ansatz

- Imposition of perturbative constraints.

- Kinematical Hilbert space $\mathcal{H}_{\mathrm{QC}}^{\mathrm{grav}} \otimes L^2(\mathbb{R}, d\breve{\phi}) \otimes \mathcal{F}_S \otimes \mathcal{F}_T \otimes \mathcal{F}_D$.

- Ansatz for quantum states: $\Gamma(V,\breve{\phi})\psi_S(\mathcal{N}_S,\breve{\phi})\psi_T(\mathcal{N}_T,\breve{\phi})\psi_D(\mathcal{N}_D,\breve{\phi})$

$$-i\partial_{\breve{\phi}}\Gamma(V,\breve{\phi}) = \hat{\tilde{\mathcal{H}}}_0\Gamma(V,\breve{\phi})$$

- Γ "almost-solutions" of the unperturbed cosmology (in a Dirac sense): Solve the constraint up to "small" terms.

- Imposition of zero-mode of the Hamiltonian constraint operator, which couples the background with perturbations.

Schrödinger equations

- Approximations in the Hamiltonian constraint equation:
  - ⋆ Homogeneous geometry operators have relatively small dispersions ➡ Expectation values on Γ w.r.t. $\mathcal{H}_{\mathrm{QC}}^{\mathrm{grav}}$.
  - ⋆ Neglect $-\partial_{\phi}^{2}(\psi_S \psi_T \psi_D)$ in a kind of Born-Oppenheimer approximation, as well as few perturbative terms.

[L. Castelló Gomar, BEN, M. Fernández-Méndez, M. Martín-Benito, G.A. Mena Marugán, J. Olmedo, 2014-2017]

Schrödinger equations

- Approximations in the Hamiltonian constraint equation.

$\Downarrow$

- Set of Schrödinger equations, the inflaton being the time, for each of the partial wave-functions for the perturbations.

- "Dynamics" generated by the Fock representation of the classical Hamiltonians, replacing all their dependence on the homogeneous geometry by expectation values on Γ.

- Besides, each Schrödinger equation contains a multiplicative "backreaction" contribution $C^{(\Gamma)}_{S,T,D}(\breve{\phi})$. [BEN, M. Martín-Benito, G.A. Mena Marugán, 2017]

$\Downarrow$

- Their sum measures how much, in mean value, Γ differs from being an exact solution of the unperturbed cosmology.

Effective equations: tensor & scalar

- Approximations in the Hamiltonian constraint equation:
  - ⋆ Homogeneous geometry operators have relatively small dispersions ➡ Expectation values on Γ w.r.t. $\mathcal{H}_{\mathrm{QC}}^{\mathrm{grav}}$.
- Consider an effective version of the resulting quadratic operator that acts on the tensor & scalar Fock spaces.
- Generates effective hyperbolic equations for the gauge-invariant perturbations, with "dressed" masses.
- QC effective dynamics & choice of initial data (vacuum): Possible predictions on the primordial power spectra.

[L. Castelló Gomar, BEN, M. Fernández-Méndez, M. Martín-Benito, D. Martín de Blas, G.A. Mena Marugán, J. Olmedo, 2014-2018]

Splitting of phase space: Choice of vacua

Consequences of choice of variables

- Many ways of separating the phase space into a homogeneous sector and an inhomogeneous one using canonical transformations that mix them.

- Determines the properties of the resulting quantization.

- In particular, the representation of the Hamiltonian constraint and its UV features strongly depend on the choice.

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- Many ways of separating the phase space into a homogeneous sector and an inhomogeneous one using canonical transformations that mix them.

- Determines the properties of the resulting quantization.

- In particular, the representation of the Hamiltonian constraint and its UV features strongly depend on the choice.

  ★ Ill-defined action of the Fock representation of the MS, tensor, and fermionic Hamiltonian (with D&H variables).

  ★ Backreaction function $C_D^{(\Gamma)}(\breve{\phi})$ can be computed by constructing the unitary operator that implements the Heisenberg dynamics: not absolutely convergent.

[BEN, M. Martín-Benito, G.A. Mena Marugán, S. Prado Loy, T. Thiemann 2017-2019]

New gauge-invariants

- Considering the system as a whole, freedom in:
  - ⋆ Dynamical separation of homogeneous geometry and gauge-invariants via canonical transformations.
  - ⋆ Choice of Fock vacua for the different perturbations, within the hybrid scheme.

- This ambiguity can be encoded in choices of the form:

$$a_{\vec{k}} = f_k(\breve{a}, \pi_{\breve{a}}, \breve{\phi}, \pi_{\breve{\phi}}) v_{\vec{k}} + g_k(\breve{a}, \pi_{\breve{a}}, \breve{\phi}, \pi_{\breve{\phi}}) \bar{\pi}_{v_{\vec{k}}}$$

for the MS annihilationlike variables, with

$$f_k \bar{g}_k - g_k \bar{f}_k = -i$$

and analogous variables for the tensor perturbations.

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- This ambiguity can be encoded in choices of the form:

$$a_{\vec{k}}^{(x,y)} = f_1^k(\breve{a}, \pi_{\breve{a}}, \breve{\phi}, \pi_{\breve{\phi}}) x_{\vec{k}} + f_2^k(\breve{a}, \pi_{\breve{a}}, \breve{\phi}, \pi_{\breve{\phi}}) \bar{y}_{-\vec{k}-2\vec{\tau}},$$

$$\bar{b}_{\vec{k}}^{(x,y)} = g_1^k(\breve{a}, \pi_{\breve{a}}, \breve{\phi}, \pi_{\breve{\phi}}) x_{\vec{k}} + g_2^k(\breve{a}, \pi_{\breve{a}}, \breve{\phi}, \pi_{\breve{\phi}}) \bar{y}_{-\vec{k}-2\vec{\tau}}$$

for, respectively, fermionic annihilationlike variables of particles and creationlike of antiparticles, with

$$f_2^k = e^{iF_2^k}\sqrt{1-\left|f_1^k\right|^2} \quad g_1^k = e^{iJ_k}\bar{f}_2^k, \quad g_2^k = -e^{iJ_k}\bar{f}_1^k$$

New gauge-invariants

- Considering the system as a whole, freedom in:
  - ★ Dynamical separation of homogeneous geometry and gauge-invariants via canonical transformations.
  - ★ Choice of Fock vacua for the different perturbations, within the hybrid scheme.

- This ambiguity can be encoded in choices of annihilation and creationlike variables defined by background-dependent linear transformations.

- Can be completed into a canonical set for the full cosmology:

$$(\breve{a}, \pi_{\breve{a}}) \longrightarrow (\tilde{a}, \pi_{\tilde{a}}), \qquad (\breve{\phi}, \pi_{\breve{\phi}}) \longrightarrow (\tilde{\phi}, \pi_{\tilde{\phi}}),$$

  correcting again the homogeneous variables with known contributions that are quadratic in perturbations.

New Hamiltonians: possibilities

- In terms of the new canonical set, the resulting Mukhanov-Sasaki, tensor and fermionic Hamiltonians are the old ones plus some known corrections.

- These new contributions contain, in general, both diagonal products of annihilation and creationlike variables, and terms responsible for the creation and destruction of pairs.

- The asymptotic behavior, when $k \to \infty$, of the interaction terms is what tells if the quantization of the Hamiltonians is well-defined on the vacuum, with normal ordering.

- In all the cases, f_k, g_k, f_1^k, f_2^k can be chosen so that the higher order powers of k, that prevent the nice definition of the Hamiltonian operators on Fock space, are eliminated.

[BEN, G.A. Mena Marugán, S. Prado Loy, T. Thiemann 2018-2019]

Asymptotic diagonalization

- It is possible to eliminate, order by order in inverse powers of the Fourier scale, all the asymptotic contribution to the interaction terms in the Hamiltonians.

- Example: Mukhanov-Sasaki (analogous tensor) Hamiltonian gets in this way asymptotically diagonalised with

$$k g_k = i f_k \left[1 - \frac{1}{2k^2} \sum_{n=0}^{\infty} \left(\frac{-i}{2k} \right)^n \gamma_n \right], \qquad \gamma_0 = s^{(s)},$$

$$\gamma_{n+1} = a\{H_{|0}, \gamma_n\} + 4s^{(s)} \left[\gamma_{n-1} + \sum_{l=0}^{n-3} \gamma_l \gamma_{n-(l+3)} \right] - \sum_{l=0}^{n-1} \gamma_l \gamma_{n-(l+1)}, \qquad \forall n \geq 0$$

- Similar asymptotic characterisation for fermions.

- In both cases, the first few terms are enough to construct well-defined Hamiltonians (and mentioned backreaction).

[BEN, G.A. Mena Marugán, S. Prado Loy, T. Thiemann 2018-2019]

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- Annihilation and creationlike variables are then “fixed”, up to a phase, since from the canonical commutation relations:

$$2|f_k|^2 = -|h_k|^2[\mathrm{Im}(h_k)]^{-1} \qquad\qquad h_k = g_k^{-1} f_k$$

Asymptotic diagonalization

- It is possible to eliminate, order by order in inverse powers of the Fourier scale, all the asymptotic contribution to the interaction terms in the Hamiltonians.

- Annihilation and creationlike variables (also in the fermionic case) are then asymptotically fixed, up to phases.

- These phases can be chosen by physical considerations:
  - Reduce to a minimum the background dependence extracted from the original perturbations.
  - Positivity of the resulting asymptotic tail of the diagonal Hamiltonians, as a function of the background.

- What about the non-asymptotically large Fourier scales?

Diagonalization: scalar & tensor

- In fact, the interaction terms in the Hamiltonian for each Fourier scale k are completely eliminated iff:

$$k^2 + s^{(s)} + h_k^2 - a\{h_k, H_{|0}\} = 0, \qquad h_k = g_k^{-1} f_k$$

which is a semilinear PDE whose complex solutions satisfy:

$$\mathrm{Im}(h_k)^2 = k^2 + s^{(s)} - \frac{\mathrm{Im}(h_k)''}{2\mathrm{Im}(h_k)} + \frac{3}{4}\left[\frac{\mathrm{Im}(h_k)'}{\mathrm{Im}(h_k)}\right]^2, \quad {}' = a\{., H_{|0}\}$$

- In the linearized context of QFT in curved spacetimes, our asymptotic characterization leads to: the Minkowski vacuum, in the case of constant mass; and the Bunch-Davies vacuum, when the homogeneous background is fixed as the de Sitter solution.

[BEN, G.A. Mena Marugán, T. Thiemann 2019]

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- In the linearized context of QFT in curved spacetimes, our asymptotic characterization leads to standard vacua.

- Way to provide initial conditions for the perturbations, optimally adapted to the structure of the full Hamiltonian?

[BEN, G.A. Mena Marugán, T. Thiemann 2019]

To conclude

- Hybrid approach: description of inhomogeneous cosmologies that either display symmetries, or where the inhomogeneities can be described in a perturbative way over a highly symmetric background.

- Combination of QC and Fock representations: emphasis on the splitting of phase space in homogeneous and inhomogeneous sectors.

- Freedom can be employed, for cosmological perturbations, to give nice properties to the quantization of the Hamiltonian constraint.

- Restricts the dynamical definition of the perturbative d.o.f., as well as their Fock representation: selection of vacua.

- Criteria such as diagonalization, following the asymptotic structure of the Hamiltonian, might be able to completely fix a natural choice.