Microscopic origin of the Bekenstein-Hawking entropy of supersymmetric AdS_5 black holes

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DESY THEORY WORKSHOP

Quantum field theory meets gravity

DESY, Hamburg 24 – 27 September, 2019

Outline

Supersymmetric black holes

One of the most remarkable achievements of string theory is the microscopic explanation of the Bekenstein-Hawking entropy of a class of supersymmetric black holes [Sen;Strominger,Vafa]

- Supersymmetric black holes have zero temperature but carry a non-zero Bekenstein-Hawking entropy, given by $S_{BH} = \frac{1}{4}A$
- It is natural to suspect that some extremization principle should govern supersymmetric black holes: there should by a supersymmetric black hole thermodynamics
- For supersymmetric black holes with AdS_{d+1} asymptotics the AdS/CFT correspondence gives a framework for performing a concrete quantum mechanical (microscopic) calculation of the entropy

Supersymmetric black holes in AdS

- The black hole can be thought of as a (quantum) statistical ensemble of a dual SCFT, placed on a background d-dimensional geometry $\mathcal{M}$ dictated by the asymptotic conformal boundary
- In this picture the Bekenstein-Hawking entropy arises from the degeneracies of states of the dual SCFT, which may be computed by appropriate partition functions characterising these theories
- Concretely, the rules of the AdS/CFT correspondence imply that the supersymmetric partition function $Z_{\mathcal{M}}$ of these theories is identified with minus the exponential of the on-shell gravity action $\mathrm{e}^{-I_{gravity}}$
- In this talk I will discuss the microscopic origin of the BH entropy of a class of supersymmetric black holes in AdS_5

Supersymmetric black holes in AdS_5

- The problem of the microscopic explanation of the entropy of susy AdS_5 black holes has been around since 2004 [Gutowski,Reall]
- The solution has electric charge $Q \neq 0$, angular momentum $J \neq 0$, entropy $S \neq 0$, but zero temperature $T = 0$
- It was obtained in 5d minimal gauged supergravity, so it can be uplifted to solutions of ten dimensional type IIB supergravity of the type black-hole$_5 \times$Sasaki-Einstein$_5$
- They must be AdS/CFT-dual to an ensemble of states in an $\mathcal{N} = 1$ SCFT dual to the solutions $AdS_5 \times$Sasaki-Einstein$_5$
- We can pick Sasaki-Einstein$_5$ $= S^5$, but we don't have to $\rightarrow$ we should not rely on properties of $\mathcal{N} = 4$ SYM to discuss these BH's

Previous attempts

- The first attempt to explain the entropy microscopically was made by [Kinney,Maldacena,Minwalla,Raju] in 2005 by considering various types of "partition functions" counting states of $\mathcal{N} = 4$ SYM on $S^3 \times (\text{time})$
- They were on the right track and the problem could have been solved in 2005 – but it wasn't
- In particular, they defined an "index" of the type $\mathcal{I} = Tr(-1)^F e^{2\pi i y_i q_i}$ where y_i are four independent (compatible with susy) fugacities for $U(1)^3 \in SO(6)$ R-symmetry + one angular momentum
- They expected that for large N, $\log \mathcal{I} = \mathcal{O}(N^2)$, so that the exponential growth of states should have accounted for $S_{BH} = \mathcal{O}(N^2)$
- Instead, they concluded that

$$\lim_{N\to\infty} \log \mathcal{I} = \mathcal{O}(1) \neq \mathcal{O}(N^2) = \frac{A}{4} = S_{BH}$$

Previous attempts

- Potential explanation: cancellations between fermions and bosons?
- Perhaps one should consider different types of partition functions?
- Various attempts over the years didn't work...
- There wasn't enough confidence in the proposal made by [Kinney,Maldacena,Minwalla,Raju] in 2005
- Key new ingredients that led to our present understanding
  1. Localization of supersymmetric field theories in curved backgrounds
  2. Several precision tests of the AdS/CFT correspondence, with gravity duals as in (1)

A four-parameter family of AdS_5 black hole solutions

- The solution of [Gutowski,Reall] is characteirsed by one parameter
- To understand its properties it is convenient to consider a more general solution [Chong,Cvetic,Lu,Pope] which has four parameters (m, a, b, q)
- These parameterize the four independent physical variables (E, J_1, J_2, Q): mass, two angular momenta, electric charge
- The non-zero temperature $T(m, a, b, q) \equiv 1/\beta$ and the non-zero entropy $S(m, a, b, q) \equiv \frac{1}{4}\mathrm{Area}$ are also functions of these variables
- If $m, a, b, q \in \mathbb{R}$ everything is real and these solutions describe physical AdS_5 black holes with a Schwarchild-like horizon
- From the gauge field A and the Killing vector V vanishing at the horizon we can define "chemical potentials" $(\Omega_1, \Omega_2, \Phi)$

Laws of non supersymmetric black hole thermodynamics

- Chemical potentials

$$V = \frac{\partial}{\partial t} + \Omega_1 \frac{\partial}{\partial \phi} + \Omega_2 \frac{\partial}{\partial \psi} \qquad \Phi = \iota_V A|_{r_+} - \iota_V A|_\infty$$

- First law of thermodynamics

$$dE = TdS + \Omega_1 \, dJ_1 + \Omega_2 \, dJ_2 + \Phi \, dQ$$

- Quantum statistical relation

$$I = \beta E - S - \beta \Omega_1 J_1 - \beta \Omega_2 J_2 - \beta \Phi Q$$

where $I(m, a, b, q)$ is the on-shell gravitational action

$$I(\beta, \Omega_1, \Omega_2, \Phi) = -\log Z(\beta, \Omega_1, \Omega_2, \Phi)$$

- Z is the grand-canonical partition function and we have

$$E = \frac{\partial I}{\partial \beta} \qquad J_1 = -\frac{1}{\beta}\frac{\partial I}{\partial \Omega_1} \qquad J_2 = -\frac{1}{\beta}\frac{\partial I}{\partial \Omega_2} \qquad Q = -\frac{1}{\beta}\frac{\partial I}{\partial \Phi}$$

The supersymmetric AdS_5 black hole limit

- The entropy $S(E, J_1, J_2, Q) = -\log Z_{micro}$ is the Legendre transform of $I(\beta, \Omega_1, \Omega_2, \Phi)$

- The thermodynamics of the finite-T non-supersymmetric black holes is under control. The idea is to obtain a supersymmetric version from this

- However the zero temperature limit is a bit subtle: $\beta \to \infty$

- The key observation is that in order to obtain the supersymmetric black holes two relations must be imposed, reducing the number of independent parameters to two:

  1. Supersymmetry (exist Killing spinors)
  2. "Extremality" (outer and inner horizon coalesce)

- This explains why the supersymmetric black holes are characterised by 2 independent parameters, instead of 3, namely (E, J_1, J_2, Q) subject to the "BPS relation" (imposed by supersymmetry alone)

$$E^* = \Omega_1^* J_1^* + \Omega_2^* J_2^* + \Phi^* Q^*$$

Detuning the supersymmetric AdS_5 black hole limit

- After these two conditions have been imposed, the charges satisfy an additional relation

$$(Q^*)^3 + 2\pi J_1^* J_2^* = \left(3Q^* + \frac{\pi}{2}\right)\left(3\,(Q^*)^2 - \pi(J_1^* + J_2^*)\right)$$

and the entropy can be written as (subject to the above)

$$S^* = \pi\sqrt{3\,(Q^*)^2 - \pi\left(J_1^* + J_2^*\right)}$$

- In [Cabo-Bizet,Cassani,DM,Murthy] we proposed to study the problem without imposing this condition – which is not natural from the field theory perspective – but imposing only supersymmetry
- This gives a three-parameter family of supersymmetric solutions which are necessarily complex!

Supersymmetric AdS_5 solutions with complex fugacities

- While this at first looks like an unphysical solution, it turns out that complex fugacities are a key ingredient to make contact with the field theory dual description

- Since we have imposed one condition, there are only three independent fugacities, and this can be expressed by the simple constraint

$$\beta\,(1 + \Omega_1 + \Omega_2 - 2\Phi) = 2\pi i$$

- In the gauge in which $\iota_V A = 0$ at the horizon, this condition is equivalent to the Killing spinors being anti-periodic along the compactified time-circle, which is necessary since this is contractible

- The on-shell action I takes a remarkably simple form when expressed in terms of a new set of variables defined as

$$\omega_1 \equiv \beta(\Omega_1 - 1) \qquad \omega_2 \equiv \beta(\Omega_2 - 1) \qquad \varphi \equiv \beta(\Phi - \tfrac{3}{2})$$

On-shell action and supersymmetric thermodynamics

$$I = \frac{2\pi}{27G_5}\frac{\varphi^3}{\omega_1\omega_2}$$

where the complex fugacities satisfy the constraint $\omega_1 + \omega_2 - 2\varphi = 2\pi i$

- I can be re-written in terms of the fugacities and the charges as

$$I = -S - \omega_1 J_1 - \omega_2 J_2 - \varphi\, Q$$

  this is a supersymmetric quantum statistical relation that encodes the "supersymmetric thermodynamics" of the black hole

- Note that $I(\omega_1, \omega_2, \varphi)$ manifestly does not depend on β and in fact it has the same form in the "extremal limit" – when the physical black hole solution is recovered ($\beta \to \infty$ in this limit)

The extremization principle of AdS_5 black holes

- The previous slide can be rephrased by saying that the entropy $S(J_1, J_2, Q)$ is the Legendre transform of $I(\omega_1, \omega_2, \varphi)$, subject to the constraint $\omega_1 + \omega_2 - 2\varphi = 2\pi i$
- This holds all along the complex supersymmetric solution and it remains true in the extremal limit, where the black hole is recovered
- This limit may equivalently be characterized by the requirement that the charges J_1, J_2, Q and the entropy S are real (while evidently the fugacities and the on-shell action I are never real)
- This is a first principle derivation – not based on any conjecture or field theory input – of the entropy function characterizing the supersymmetric AdS_5 black holes

Dual field theory interpretation of the entropy function

- If we believe in the AdS/CFT conjecture, these results should be derived from the large N limit of a field theory partition function
- Namely, there must be a supersymmetric partition function of the dual $\mathcal{N} = 1$ SCFT, such that

$$I(\omega_1, \omega_2, \varphi) = \lim_{N \to \infty} - \log Z_{SCFT}(\omega_1, \omega_2, \varphi)$$

- What is the correct partition function?
- Using AdS/CFT, we can deduce this from the geometry of the boundary of our complex supergravity solutions
- The bulk supergravity solution fixes the rigid background on which to perform the localization computation of the boundary field theory

Entropy function from a field theory computation

- The 4d metric and gauge field of the conformal supergravity background are read off from the asymptotics of the bulk 5d supergravity solution and read

$$ds^2 = d\tau^2 + d\theta^2 + \sin^2\theta\,(d\phi_1 - i\,\Omega_1 d\tau)^2 + \cos^2\theta\,(d\phi_2 - i\,\Omega_2 d\tau)^2$$

with $\tau \sim \tau + \beta \qquad \phi_1 \sim \phi_1 + 2\pi \qquad \phi_2 \sim \phi_2 + 2\pi$

$$A^{cs} = i\Phi d\tau$$

- Correct field theory computation: perform supersymmetric localization of $\mathcal{N} = 1$ SCFT on this background

Entropy function from a field theory computation

- Technically it is convenient to re-phrase this rigid background in the formalism of new minimal supergravity

$$A^{nm} = i(\Phi - \tfrac{2}{3})d\tau \qquad V^{nm} = -id\tau$$

$$A^{cs} = A^{nm} - \tfrac{3}{2}V^{nm} = i\Phi d\tau$$

- Plugging this background into the new minimal Killing spinor equation we find that the Killing spinor is proportional to

$$\epsilon \sim e^{\frac{\tau}{\beta}(\omega_1+\omega_2-2\varphi)} \Rightarrow \qquad \omega_1 + \omega_2 - 2\varphi = 2\pi i n \qquad n \in \mathbb{Z}$$

n even $\Rightarrow$ periodic spinor

n odd $\Rightarrow$ anti-periodic spinor

- The bulk solution fixes $n = \pm 1$, which is consistent with the fact that in the bulk the τ-circle shrinks smoothly in the interior

Result of the localization computation

- The localization computation is a generalisation of that performed in [Assel,Cassani,DM] for a (real) background with $S^1 \times S^3$ topology (Hopf surfaces)
- The result is that $\mathcal{I}(\omega_1, \omega_2, \varphi)$ is an n-dependent slight generalisation of the usual superconformal index, namely as a trace it reads

$$\mathcal{I}(\omega_1, \omega_2; \varphi) = \mathrm{Tr} e^{\pi i F(n+1)} e^{\omega_1 J_1 + \omega_2 J_2 + \varphi Q}$$

which after using the constraint $\omega_1 + \omega_2 - 2\varphi = 2\pi i n$ reads

$$\mathcal{I}(\omega_1, \omega_2; n) = \mathrm{Tr} e^{\pi i F(n+1) - \pi i n Q} e^{\omega_1 (J_1 + \frac{1}{2} Q) + \omega_2 (J_2 + \frac{1}{2} Q)}$$

Relation to the usual superconformal index

- For $n = 0$ this reduces to the familiar expression

$$\mathcal{I}(\omega_1, \omega_2; 0) = \mathrm{Tr}(-1)^F e^{\omega_1(J_1+\frac{1}{2}Q)+\omega_2(J_2+\frac{1}{2}Q)}$$

while for $n = \pm 1$ it can be written as

$$\mathcal{I}(\omega_1, \omega_2; \pm 1) = \mathrm{Tr}(-1)^Q\, e^{\omega_1(J_1+\frac{1}{2}Q)+\omega_2(J_2+\frac{1}{2}Q)}$$

- Effectively replace $(-1)^F \to (-1)^Q$ in the usual index
- Notice that while it is true that

$$\mathcal{I}(\omega_1, \omega_2; n) = \mathcal{I}(\omega_1 - 2\pi i n, \omega_2; 0) = \mathcal{I}(\omega_1, \omega_2 - 2\pi i n; 0)$$

it is not true that all the different "n-indices" are equivalent

Conclusion of the analysis

- This is clear since for $\mathcal{N} = 1$ SCFTs, Q has eigenvalues that are generically irrational numbers
- This is an important point: in the grand-canonical ensamble, the fugacities are held fixed, and therefore we can chose the value of n
- The gravity computation in the (regularised) black hole background instructs us to pick $n = \pm 1$, and *not* something else
- Conclusion: the AdS/CFT correspondence leads us to consider the large N limit of the $\mathcal{I}(\omega_1, \omega_2; n)$ index and predicts that

$$-\log \mathcal{I}(\omega_1, \omega_2; n = 1) \longrightarrow_{N\to\infty} I(\omega_1, \omega_2, \varphi) = \mathrm{c}\frac{16}{27}\frac{\varphi^3}{\omega_1\omega_2}$$

with $\omega_1 + \omega_2 - 2\varphi = 2\pi i$

- The entropy follows from this by taking the Legendre transform to pass to the microcanonical ensamble

Entropy from the large N limit of the index

- This was confirmed by the analysis of [Benini,Milan] for $\mathcal{N} = 4$ SYM in the large N limit, using an approach based of BAE's (and re-derived using a different approach in [Cabo-Bizet,Murthy])
- A limit that is technically simpler to perform is that of small fugacities, $|\omega_i| \to 0$, which is referred to as Cardy-like limit
- Various papers have studied this limit, for $\mathcal{N} = 4$ SYM, as well as more general $\mathcal{N} = 1$ SCFTs, exactly reproducing the entropy function that we derived in gravity
- More recently, [González-Lezcano,Pando-Zayas], [Lanir,Nedelin,Sela] have extended the BEA approach to studying the large N limit of $\mathcal{N} = 1$ SCFTs, again finding perfect agreement

For more details I refer you to the talk of Francesco Benini

Conclusions

- Despite the successes of string theory in explaining microscopically the entropy of some black holes, supersymmetric black holes in AdS have been somewhat harder to attack
- Surprisingly, one of the most puzzling set up has been that of AdS_5 black holes, that is AdS/CFT dual to ensembles of states in $\mathcal{N} = 4$ SYM or other well-understood four-dimensional $\mathcal{N} = 1$ SCFTs
- In our work we have obtained a detailed understanding of the supersymmetric AdS_5 black hole "thermodynamics", in particular deriving an entropy function and its associated extremization principle
- Moreover, following the rules of the AdS/CFT correspondence, we have computed the appropriate field theory partition function performing a localization computation
- The large N limit, as well as a Cardy-like limit of this, exactly matches the results of the gravity computation, finally providing a microscopic derivation of the entropy of the AdS_5 black holes