Towards a unitary and renormalizable quantum theory of gravity

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General Relativity

General Relativity is a classical field theory for the metric field $g_{\mu\nu}(X)$

$$S_{\mathrm{EH}}[g] = \frac{c^4}{16\,\pi G_{\mathrm{N}}} \int \mathrm{d}^4 X \, \sqrt{-g} \, \Big(R - 2\Lambda \Big)$$

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Riemann curvature tensor & Christoffel symbol

$$\begin{aligned} R^{\rho}{}_{\mu\sigma\nu} &= \partial_\sigma \Gamma^{\rho}_{\mu\nu} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\lambda}_{\mu\nu} \Gamma^{\rho}_{\lambda\sigma} - \Gamma^{\lambda}_{\mu\sigma} \Gamma^{\rho}_{\lambda\nu}, \\ \Gamma^{\rho}_{\mu\nu} &= \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right), \quad \partial_\mu = \frac{\partial}{\partial X^\mu}, \\ R_{\mu\nu} &= R^{\rho}{}_{\mu\rho\nu}, \quad R = g^{\mu\nu} R_{\mu\nu} \end{aligned}$$

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\end{aligned}$$

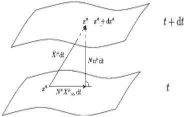
Gravitational field equations (system of ten coupled 2^{nd} order nonlinear PDE's)

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu}}_{\text{geometry}} = \frac{8\pi G_{\mathrm{N}}}{c^4} \underbrace{T_{\mu\nu}}_{\text{matter}}$$

"Spacetime tells matter how to move –
matter tells spacetime how to curve" (J. A. Wheeler)

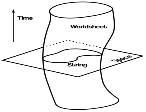
Approaches to quantum gravity

Quantum geometrodynamics



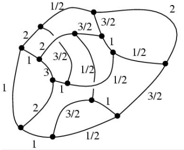
from: Gen. Rel. Grav. 41 (2009)

String theory



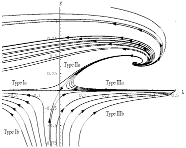
credit: Riccardo Antonelli

Loop quantum gravity



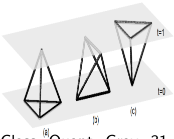
credit: John Baez

Asymptotic safety



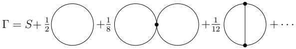
from: Phys. Rev. D 65 (2002) 065016

Causal dynamical triangulations



from: Class. Quant. Grav. 31 (2014)

Perturbative quantum gravity



Perturbative quantization of Einstein gravity

Perturbative QFT: same formalism for gravity and "matter" fields $\rightarrow$ unification

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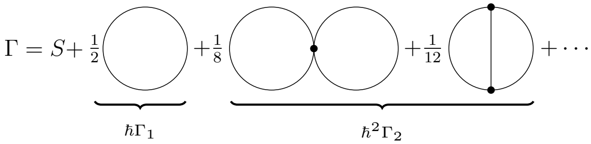
Quantum effective action $\Gamma = S + \sum_{L=1}^{\infty} \hbar^L \Gamma_L$



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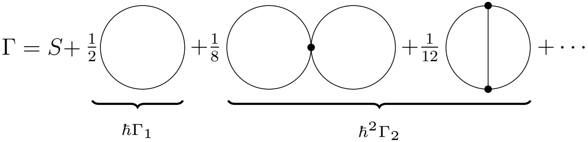

Loop integrals lead to ultraviolet divergences: coupling $G_N \overset{c=\hbar=1}{\sim} M_{\mathrm{P}}^{-2}$

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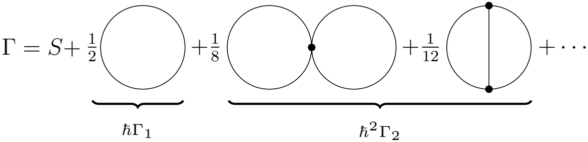

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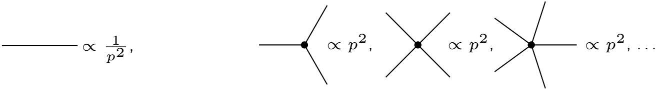
Power counting in GR: propagators $\sim p^{-2}$, vertices $\sim p^2$



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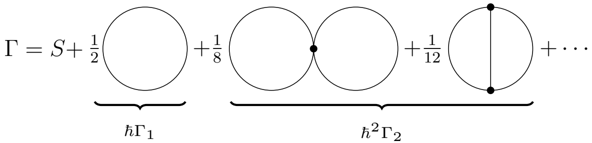

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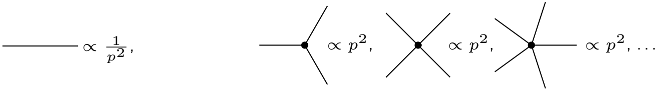

Integral $\sim \int \left(\mathrm{d}^4 p\right)^L \frac{1}{\left(p^2\right)^I} \left(p^2\right)^V$, topological relation: $L = I - (V - 1)$,

$D_{\mathrm{div}}^{\mathrm{GR}} = 4\,L - 2\,I + 2\,V = 2\,(L + 1)$ grows with increasing loop order

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$$\Gamma_1^{\mathrm{div}} = \int \mathrm{d}^4X\sqrt{-g}\left[\mathfrak{g}_1^{\mathrm{div}} R^2 + \mathfrak{g}_2^{\mathrm{div}} R_{\mu\nu}R^{\mu\nu} + \mathfrak{g}_3^{\mathrm{div}} R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right]$$

$$\Gamma_2^{\mathrm{div}} = \frac{1}{M_{\mathrm{P}}^2}\int \mathrm{d}^4X\sqrt{-g}\left[\mathfrak{g}_4^{\mathrm{div}} R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\alpha\beta}R_{\alpha\beta}{}^{\mu\nu}\right]$$

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Need to introduce new interactions R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, ...
in original action S_{EH} – each with a new free parameter $\mathfrak{g}_1$, $\mathfrak{g}_2$, $\mathfrak{g}_3$,...

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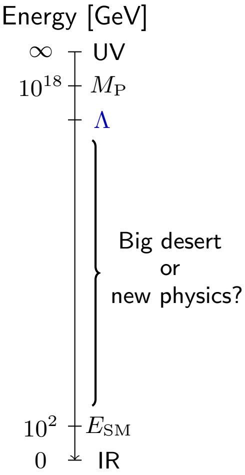
Quantum Einstein gravity perturbatively non-renormalizable

['t Hooft, Veltman (1974), Goroff, Sagnotti (1986), van de Ven (1992)]
[Bern, Cheung, Chi, Davies, Dixon, Nohle (2015)]

Quantum gravity as an effective field theory



Effective field theory (EFT) = low energy approximation to (potentially unknown) fundamental theory in the UV

[Wilson (1971), Weinberg (1979), Donoghue (1995),...]

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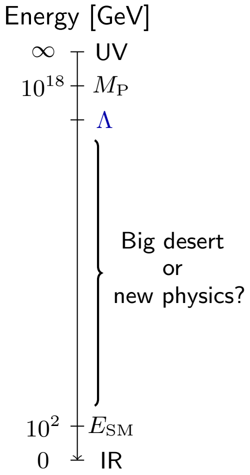

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In GR the natural cutoff scale is $\Lambda = M_{\mathrm{P}} \simeq 10^{18}$ GeV

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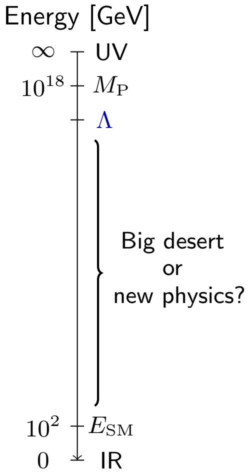

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Gravity as EFT: agnostic about UV degrees of freedom
Parametrize ignorance by inclusion of correction terms

$$\mathfrak{g}_1 R^2, \quad \mathfrak{g}_2 \frac{R\,\nabla^2 R}{\Lambda^2}, \quad \mathfrak{g}_3 \frac{R^3}{\Lambda^2}, \quad \mathfrak{g}_4 \frac{R\,\nabla^4 R}{\Lambda^4}, \quad \dots$$

Accuracy set by order of the expansion (if $\mathfrak{g}_i = \mathcal{O}(1)$)

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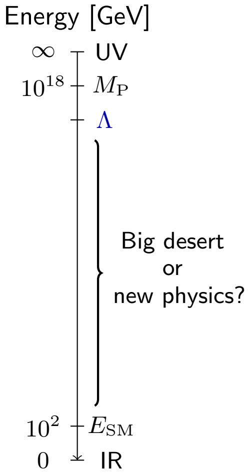

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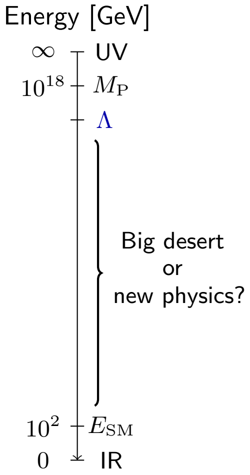

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Asymptotic safety might provide a non-perturbative UV-completion of gravity
see talks by D. Litim and F. Saueressig

Modified gravity: Renormalizability vs. unitarity

$f(R)$ models relevant in inflationary cosmology, [Starobinsky 1980], one-loop divergences known on generic background [Ruf, CS 2018] but non-renormalizable

$$S = \int \mathrm{d}^4 X \sqrt{-g} f(R)$$

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Quadratic gravity perturbatively renormalizable and asymptotically free

[Utiyama, DeWitt (1962), Stelle (1977), Fradkin, Tseytlin (1982), Avramidi, Barvinsky (1985)]

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Higher derivatives $R^2 \sim \partial^4 g$ lead to improved UV behaviour ...

$$\mathcal{P} = \frac{M_{\mathrm{P}}^2}{p^2(p^2 - M_{\mathrm{P}}^2)} = \frac{1}{p^2} - \frac{1}{p^2 - M_{\mathrm{P}}^2}$$

... but higher time derivatives also lead to new particles in the spectrum: healthy spin-zero scalar and massive spin-two ghost $\rightarrow$ violation of unitarity

[Stelle (1977), Hawking (1985)]

Hořava gravity

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Basic idea: Lorentz invariance broken in the UV but emergent in the IR!?

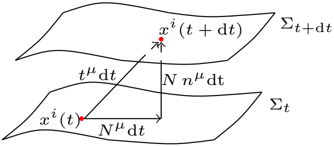
[Hořava (2009)]

$$\mathcal{P} = \frac{1}{\omega^2 - k^2 - G\,(k^2)^z} \simeq \begin{cases} \text{IR: } \frac{1}{\omega^2 - k^2} = \frac{1}{p^2} & \text{Lorentz invariance restored} \\ \text{UV: } \frac{1}{\omega^2 - G\,(k^2)^z} & \text{anisotropic scaling parameter } z \\ & \text{critical scaling: } z = d \end{cases}$$

Geometric setting: foliation of spacetime in GR

Arnowitt-Deser-Misner: foliation of spacetime into spatial hypersurfaces Σ_t

$$\mathrm{d}s^2 = g_{\mu\nu}(X)\mathrm{d}X^\mu \mathrm{d}X^\nu = N^2\mathrm{d}t^2 - \gamma_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t)$$



$N(t, x^i)$: lapse function

$N^i(t, x^i)$: shift vector

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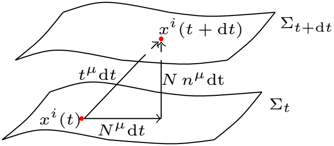

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Extrinsic curvature: $K_{ij} = \frac{1}{2\,N}\left(\partial_t \gamma_{ij} - \nabla_i N_j - \nabla_j N_i\right)$

Gauss-Codazzi: $R^{(D)}(g) = K_{ij}K^{ij} - K^2 + R^{(d)}(\gamma)$

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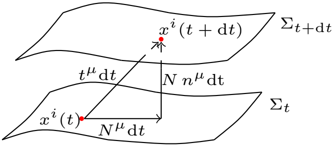

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$$S_{\mathrm{EH}} = \frac{M_{\mathrm{P}}^2}{2}\int \mathrm{d}t\, \mathrm{d}^3x\, N\, \gamma^{1/2}\Big(\underbrace{K_{ij}K^{ij} - K^2}_{\text{“kinetic term”}} + \underbrace{R(\gamma)}_{\text{“potential”}}\Big)$$

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Fundamental Lorentz invariance broken: anisotropic scaling

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$$\text{FDiff}(\mathcal{M}):\; x^i \to \tilde{x}^i(\vec{x}, t), \quad t \to \tilde{t}(t)$$

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Projectable Hořava gravity in $D = d + 1$ for critical scaling $z = d$

$$S_{\text{HG}} = \frac{1}{2\,G} \int \mathrm{d}t\mathrm{d}^d x\, \gamma^{1/2}\, N \Big(\underbrace{K_{ij} K^{ij} - \lambda\, K^2}_{\text{"kinetic term"}} - \underbrace{\mathcal{V}^{(d)}}_{\text{"potential"}} \Big)$$

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Less symmetry FDiff($\mathcal{M}$) vs. Diff($\mathcal{M}$) allows for more structure

$$\mathcal{V}^{(d=2)} = 2\Lambda + \mu R^2$$

$$\mathcal{V}^{(d=3)} = 2\Lambda - \eta R + \mu_1 R^2 + \mu_2\, R_{ij}R^{ij} + \nu_1 R^3 + \nu_2 R R_{ij} R^{ij}$$
$$+ \nu_3 R^i{}_j R^j{}_k R^k{}_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk}$$

Particle spectrum and phenomenology of Hořava gravity

Two "versions" of Hořava gravity:

i.) "Non-projectable": $N(\mathbf{x}, t)$, includes invariants with $a_i = \partial_i \ln N$

ii.) "Projectable": $N(t)$ global time slicing, gauge $N = 1$

Particle spectrum and phenomenology of Hořava gravity

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“healthy extension”: add relevant $a^i a_i$ operator (non-projectable version)
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Strongest observational constraints from PPN and speed of gravitational waves
Non-projectable model still phenomenological viable
[Gümrükçüoğlu, Saravani, Sotiriou (2018)]

Renormalizability of Hořava gravity

1. Absence of spurious non-local divergences $\leftrightarrow$ regular propagators:
   Requires non-local gauge fixing [Barvinsky, Blas, Herrero-Valea, Sibiryakov, CS (2016)]

$$\mathcal{P} \propto [A\,\omega^2 - B\,k^{2d}]^{-1}, \quad \alpha, \beta > 0$$

2. Finite set of counterterms $\leftrightarrow$ power counting:
   Order-by-order subtraction works as in relativistic case [Anselmi (2009)]

$$D_{\rm HG}^{\rm div} = 2\,d - d\,T - X - (d-1)\,l_N\,, \quad D_{\rm HG}^{\rm div} < 0 \text{ diagram convergent}$$

3. Gauge invariance of counterterms $\leftrightarrow$ manifest FDiff covariant formulation:
   BF method+BRST formalism [Barvinsky, Blas, Herrero-Valea, Sibiryakov, CS (2018)]

$$D_t = \frac{1}{N}\left(\partial_t - \mathcal{L}_{\vec{N}}\right), \quad \nabla_i = \partial_i + \Gamma_i(\gamma),$$
$$K_{ij} = 2D_t\gamma_{ij}, \quad R_{ijkl}, \quad a_i = \nabla_i \ln N$$

Projectable Hořava gravity is perturbatively renormalizable (for any D)

[Barvinsky, Blas, Herrero-Valea, Sibiryakov, CS (2016)]

One-loop divergences of $2+1$ projectable Hořava gravity

In addition to renormalizability: need to know RG flow for UV complete theory

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Focus on $D = 2+1$: only scalar mode present (no spin-2 as R topological)

$$S_{\mathrm{HG}} = \frac{1}{2\,G} \int \mathrm{d}t\mathrm{d}^2x\,\gamma^{1/2}\,N\Big\{K_{ij}K^{ij} - \lambda\,K^2 - \mu R^2\Big\}$$

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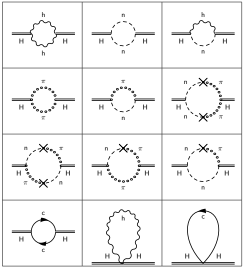
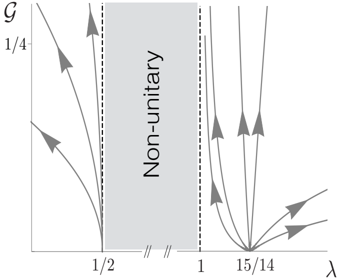


Explicit one-loop calculation required

One-loop renormalization of G, λ and μ via BF method (only two-point functions)

Only λ and combination $\mathcal{G} = G/\sqrt{\mu}$ are essential couplings – inessential couplings can be changed by field redefinitions

RG flow of $2+1$ projectable Hořava gravity



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Two fixed points at $(1/2, 0)$ and $(15/14, 0)$

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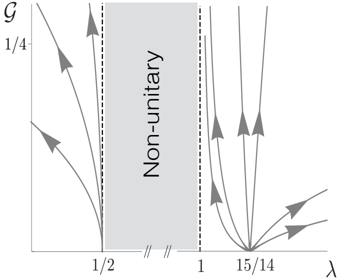

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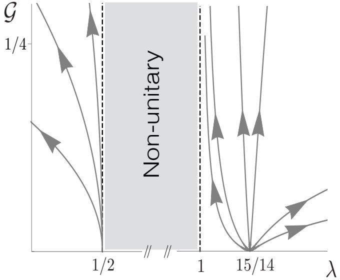

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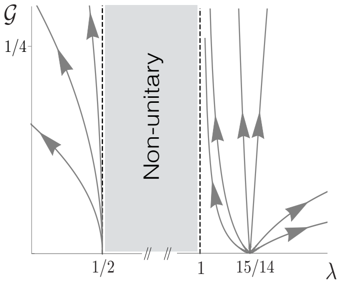

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Results for one-loop beta functions of kinetic couplings in $D = 3+1$ projectable Hořava gravity [Barvinksy, Herrero-Valea, Sibiryakov (2019)]

Conclusions

Projectable Hořava gravity is unitary and renormalizable (for any D)

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Extension of heat-kernel techniques to anisotropic Lifshitz theories

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Full one-loop RG flow of essential couplings in $D = 3 + 1$ required

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[Barvinsky, Blas, Herrero-Valea, Sibiryakov, CS (2016)]

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Thank you!

RG flow might dynamically recover "relativistic value" $\lambda \to 1$ at low energies
[Barvinsky, Blas, Herrero-Valea, Sibiryakov, CS (2017)]

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Backup slide: Non-local gauge fixing

Modified relativistic (harmonic) gauge condition $[F^i] = 2d-1$:

$$\mathcal{L}_{\rm gf} = F^i \mathcal{O}_{ij} F^j \qquad F^i = \dot{n}^i + \tfrac{1}{2\sigma}[\mathcal{O}^{-1}]^{ij}\partial_k h^j{}_k - \tfrac{\lambda}{2\sigma}[\mathcal{O}^{-1}]^{ij}\partial_j h$$

Anisotropic scaling requires non-local operator: $[\mathcal{O}_{ij}] = -2(d-1)$

$$\mathcal{O}_{ij} = -\Delta^{-(d-2)}\left[\delta_{ij}\Delta + \xi\partial_i\partial_j\right]^{-1}, \qquad \xi \neq -1$$

Non-local terms only remain in shift-shift sector (here $d=2$).

$$\begin{aligned}\mathcal{L}^{d=2}_{(2)} + \mathcal{L}_{\rm gf} = \frac{1}{2\kappa^2}\Bigg[&\frac{\dot{h}_{ij}^2}{4} - \frac{\lambda\dot{h}^2}{4} - \frac{1}{4\sigma}\partial^i h_{ij}\Delta\partial^k h_{ik} + \left(\mu + \frac{\xi}{4\sigma}\right)(\partial^i\partial^j h_{ij})^2 \\ &- \left(2\mu + \frac{\lambda(1+\xi)}{2\sigma}\right)\Delta h\partial^i\partial^j h_{ij} + \left(\mu + \frac{\lambda^2(1+\xi)}{4\sigma}\right)(\Delta h)^2 \\ &- \sigma\,\dot{n}^i\left[\delta_{ij}\Delta + \xi\partial_i\partial_j\right]^{-1}\dot{n}^j + \frac{(\partial_i n^j)^2}{2} + \left(\frac{1}{2} - \lambda\right)(\partial_i n^i)^2\Bigg]\end{aligned}$$

Can be localized by “integrating in” the auxiliary “Nakanishi-Lautrup” field π

$$\sigma\,(D_t n^i)\,\mathcal{O}_{ij}\,(D_t n_j) \mapsto \frac{1}{2\sigma}\pi_i\left[\mathcal{O}^{-1}\right]^{ij}\pi_j - i\,\pi_i(D_t n^i)$$

Backup slide: Integral convergence and regular propagators

Individual integrals over frequency or momentum can diverge despite $D_{\mathrm{div}} < 0$

$$I = \int \mathrm{d}\omega_{(1)}\, \mathrm{d}^d k_{(1)} \underbrace{\int \prod_{l=2}^{L} [\mathrm{d}\omega_{(l)} \mathrm{d}^d k_{(l)}]\, f(\{\omega_{(l)}\}, \{k_{(l)}\}),}_{= \tilde{f}(\omega_{(1)}, k_{(1)}) \times \text{ convergent, } [\tilde{f}] = D_{\mathrm{div}} - 2d} \quad [I] = D_{\mathrm{div}} < 0$$

$$f(\omega_{(1)}, k_{(1)}) = \omega_{(1)}^{-1+n}\, k_{(1)}^{-d-dn+D_{\mathrm{div}}} \quad \text{or} \quad f(\omega_{(1)}, k_{(1)}) = \omega_{(1)}^{-1-n}\, k_{(1)}^{-d+dn+D_{\mathrm{div}}}$$

In relativistic case $f(\omega, k) = f(p)$ depends only on combination $p^2 = \omega^2 + k^2$

Regular propagators: scaling $[\langle \phi_1(t, \mathbf{x}), \phi_2(0) \rangle] = r_1 + r_2$ [Anselmi (2009)]

$$\langle \phi_1, \phi_2 \rangle = \sum \frac{P(\omega, \mathbf{k})}{D(\omega, \mathbf{k})}, \quad D = \prod_{m=1}^{M} [A_m\, \omega^2 + B_m\, k^{2d} + \ldots], \quad A_m, B_m > 0,$$

$P(\omega, k)$ polynomial in ω and k with scaling $[P] = r_1 + r_2 + 2(M-1)d$

$$\mathcal{P}_{\mathrm{s}}(p) = \left[\omega^2 + 4\,\mu\, \frac{1-\lambda}{1-2\,\lambda}\, k^4\right]^{-1}$$