

Black Holes in $\mathcal{N} = 4$ Super-Yang-Mills

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Quantum Field Theory Meets Gravity
DESY Hamburg, 25 September 2019

in collaboration with Paolo Milan, arXiv: 1811.04107

arXiv: 1812.09613

Quantum? gravity

- ★ For gravity in asymptotically AdS space:
AdS/CFT \Rightarrow non-perturbative definition
in terms of boundary ordinary QFT

Parameter map

- Large AdS_D compared with Planck scale \Rightarrow QFT with large “central charge” (large N)

$$\frac{\ell_{\text{AdS}}^{D-2}}{G_N} \sim \text{“c.c.”}$$

[Brown, Henneaux 86]

- Large AdS_D compared with higher derivative corrections to Einstein gravity (e.g., massive string or higher-spin modes) \Rightarrow QFT is strongly coupled

E.g., in string theory:

$$\frac{\ell_{\text{AdS}}^4}{\alpha'^2} \sim \lambda$$

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PROBLEM!

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! Take advantage of modern non-perturbative methods !

Black holes have an entropy

$$S_{\text{BH}} = \frac{\text{Area}}{4G_N \hbar / c^3}$$

[Bekenstein 72, 73, 74; Hawking 74, 75]

$$\text{Black hole} = \text{Ensemble of states in quantum gravity} = \text{Ensemble of states in boundary QFT}$$

$$S_{\text{micro}} = \log N_{\text{micro}} = \frac{\text{Area}}{4G_N} + \log \text{Area} + \dots \quad (\text{pert. and non-pert.})$$

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$$S_{\text{micro}} = \log N_{\text{micro}} = \frac{\text{Area}}{4G_N} + \log \text{Area} + \dots \quad (\text{pert. and non-pert.})$$

- Can we reproduce the Bekenstein-Hawking entropy?
- Can we go beyond and compute corrections?
- Can we determine the *exact integer number* N_{micro} ?

Black holes in flat space

- ★ String theory reproduces the Bekenstein-Hawking entropy [Strominger, Vafa 96] of BPS black holes in **asymptotically flat** spacetime

[G. Compere, A. Dabholkar, J. David, F. Denef, R. Dijkgraaf, D. Gaiotto, J. Gomes, J. A. Harvey, M. Henneaux, D. Jafferis, I. Klebanov, F. Larsen, J. Maldacena, G. Moore, S. Murthy, B. Pioline, V. Reys, A. Sen, A. Strominger, E. Verlinde, H. Verlinde, E. Witten, D. Zagier, ...]

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Black holes in $\text{AdS}_{\geq 4}$

- ★ Dual QFT reproduces the Bekenstein-Hawking entropy of magnetically-charged (dyonic) BPS black holes in **asymptotically AdS_4** space

[FB, Hristov, Zaffaroni 15]

[F. Azzurli, N. Bobev, A. Cabo-Bizet, M. Crichigno, D. Gang, M. Hosseini, I. Jeon, S. Lal, J. Liu, V.S. Min, A. Nedelin, L. Pando Zayas, A. Passias, B. Willett, I. Yaakov, A. Zaffaroni, ...]

BPS black holes in AdS_5

Setup:

$$\begin{array}{ccc} \text{Type IIB string theory} & \longleftrightarrow & \text{4d } SU(N) \\ \text{on } \text{AdS}_5 \times S^5 & & \mathcal{N} = 4 \text{ Super-Yang-Mills} \end{array}$$

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BPS black hole solutions can be constructed in 5d gauged SUGRA in AdS_5

(see Dario Martelli's talk for details)

- ★ Does $\mathcal{N} = 4$ SYM contain BPS states that reproduce the **black hole entropy**?

This has remained a puzzle for a long time. . .

[Kinney, Maldacena, Minwalla, Raju 05]

Rotating & electrically-charged $\frac{1}{16}$ -BPS black holes in AdS₅ [Gutowski, Reall 04] [Chong, Cvetic, Lu, Pope 05; Kunduri, Lucietti, Reall 06]

- Two angular momenta: J_1, J_2 (here $J_1 = J_2$)

Three electric charges $U(1)^3 \subset SO(6)$: R_1, R_2, R_3

- Extremal, 1 complex supercharge \mathcal{Q}

BPS relation: $2M = 2J_1 + 2J_2 + R_1 + R_2 + R_3$

Large smooth horizon: non-linear relation among 5 charges \rightarrow 4 parameters

- Near horizon: fibration $AdS_2 \rightarrow$ squashed S^3

B-H entropy:

$$S_{BH} = \frac{\text{Area}}{4G_N} = \pi \sqrt{R_1 R_2 + R_1 R_3 + R_2 R_3 - 2N^2(J_1 + J_2)}$$

- Angular momenta, charges and entropy scale $\sim N^2$

Superconformal index

[Romelsberger 05; Kinney, Maldacena, Minwalla, Raju 05]

- ★ Counts (with sign) **BPS states** on S^3 = protected operators on flat space

Index of $\mathcal{N} = 4$ SYM:

$$\mathcal{I}(p, q, y_1, y_2) = \text{Tr} (-1)^F e^{-\beta \{\mathcal{Q}, \mathcal{Q}^\dagger\}} p^{J_1 + \frac{1}{2}R_3} q^{J_2 + \frac{1}{2}R_3} y_1^{\frac{1}{2}(R_1 - R_3)} y_2^{\frac{1}{2}(R_2 - R_3)}$$

Write: $p = e^{2\pi i \tau}$ $q = e^{2\pi i \sigma}$ $y_a = e^{2\pi i \Delta_a}$ $F = R_3 = 2J_1 = 2J_2 \pmod{2}$

SUSY \Rightarrow at most 4 independent fugacities

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- ★ *Exact integral formula:*

$$\mathcal{I} = \kappa_N \oint_{\mathbb{T}^{\text{rk}(G)}} \prod_{i=1}^{\text{rk}(G)} \frac{dz_i}{2\pi i z_i} \times \frac{\prod_{a=1}^3 \prod_{\rho \in \mathfrak{R}_{\text{adj}}} \tilde{\Gamma}(\rho(u) + \Delta_a; \tau, \sigma)}{\prod_{\alpha \in \mathfrak{g}} \tilde{\Gamma}(\alpha(u); \tau, \sigma)}$$

The index encodes (weighted) degeneracies:

$$\mathcal{I} = 1 + \#y + \#y^2 + \dots + d(Q) y^Q + \dots$$

To extract the degeneracies:

$$d(Q) = \frac{1}{2\pi i} \oint \frac{dy}{y^{Q+1}} \mathcal{I}(y) = \oint d\Delta \ e^{\log \mathcal{I}(\Delta) - 2\pi i Q \Delta}$$

Assuming large degeneracies, saddle-point approximation → [Legendre transform](#)

$$\log d(Q) \simeq \log \mathcal{I}(\Delta) - 2\pi i Q \Delta \Big|_{\Delta = \text{extremum}}$$

- We are interested in $Q \sim N^2$

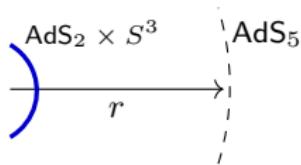
Log Index = Black hole Entropy

[FB, Hristov, Zaffaroni 16]

- ★ Black hole solution is an holographic RG flow $4d \rightarrow 1d$

Near-horizon AdS_2 : superconformal Quantum Mechanics

$$\mathfrak{su}(1, 1|1) \supset \mathfrak{sl}(2, \mathbb{R}) \times \mathfrak{u}(1)_{R_{sc}}$$



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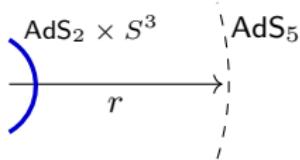
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Superconformal index \rightarrow Witten index of QM, with respect to “trial” R-charge

$$\mathcal{I}(\Delta) = \text{Tr} (-1)^{R_{\text{trial}}(\Re \Delta)} e^{-2\pi \sum \Im \Delta \cdot Q} e^{-\beta \underbrace{\{Q, Q^\dagger\}}_{H_{\text{near horizon}}}}$$
$$R_{\text{trial}} = R_3 + 2 \sum \Re \Delta \cdot Q$$

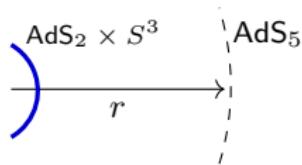


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Inputs from holography (large N):

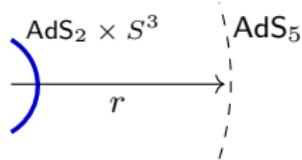
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Inputs from holography (large N):

- $\text{AdS}_2 \Rightarrow R_{sc} = 0$. At $\hat{\Delta}$ all states contribute with + sign
- Single-center black hole in microcanonical ensemble: all states have charge Q

$$\frac{\partial \log \mathcal{I}}{\partial \Delta} \Big|_{\hat{\Delta}} = i \langle Q \rangle \quad S_{\text{BH}} = \Re \left[\log \mathcal{I} - 2\pi i \sum \Delta Q \right]_{\hat{\Delta}}$$

Assuming s.c. black hole dominates $\Rightarrow \mathcal{I}$ captures the entropy [similar to Sen 09]

Three recent approaches

- Entropy from on-shell action

[Cabo-Bizet, Cassani, Martelli, Murthy 18]

- Cardy limit

[Choi, J. Kim, S. Kim, Nahmgoong 18]

[M. Honda 19; Ardehali 19]

[J. Kim, S. Kim, Song 19; Cabo-Bizet, Cassani, Martelli, Murthy 19]

- Large N limit

[FB, Milan 18]

[see also Murthy, Cabo-Bizet 19]

Bethe Ansatz formula for the superconformal index

For $\frac{\tau}{\sigma} = \frac{a}{b} \in \mathbb{Q}_+$ (i.e., $\tau = a\omega$, $\sigma = b\omega$) alternative formula: [Closset, Kim, Willett 17]

[FB, Milan 18]

$$\mathcal{I} = \kappa_N \sum_{\hat{u} \in \mathfrak{M}_{\text{BAE}}} \mathcal{Z}_{\text{tot}}(\hat{u}; \xi, \tau, \sigma) H(\hat{u}; \xi, \omega)^{-1}$$

- ① $\mathfrak{M}_{\text{BAE}}$ are solutions to “Bethe Ansatz Equations” for $\text{rk}(G)$ complexified holonomies $[\hat{u}_i]$ living on a complex torus T_ω^2 of modular parameter ω :

$$Q_i = \prod_{\rho_a \in \mathfrak{R}} P\left(\rho_a(u) + \omega_a(\xi) + r_a \frac{\tau+\sigma}{2}; \omega\right)^{\rho_a^i} \quad P(u; \omega) = \frac{e^{-\pi i \frac{u^2}{\omega} + \pi i u}}{\theta_0(u; \omega)}$$

$$\mathfrak{M}_{\text{BAE}} = \left\{ [\hat{u}_i] \in T_\omega^2 \mid Q_i(u) = 1, \quad w \cdot [\hat{u}] \neq [\hat{u}] \quad \forall w \in \mathcal{W}_G \right\}$$

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- ② κ_N and \mathcal{Z} are the same prefactor and integrand as in the integral formula,

$$\mathcal{Z}_{\text{tot}}(u; \dots) = \sum_{\{m_i\}=1}^{ab} \mathcal{Z}(u - m\omega; \dots)$$

- ③ H is a Jacobian:
$$H = \det_{ij} \left(\frac{\partial Q_i}{\partial u_j} \right)$$

Bethe Ansatz Equations for $\mathcal{N} = 4$ SYM

Specialize to 4d $SU(N)$ $\mathcal{N} = 4$ SYM, and $\tau = \sigma$ (i.e. $J_1 = J_2$). BAEs:

$$1 = Q_i = e^{2\pi i(\lambda + 3 \sum_j u_{ij})} \prod_{j=1}^N \prod_{\Delta \in \{\Delta_1, \Delta_2, -\Delta_1 - \Delta_2\}} \frac{\theta_0(u_{ji} + \Delta; \tau)}{\theta_0(u_{ij} + \Delta; \tau)}$$

Equations are defined on T_τ^2 and are invariant under $SL(2, \mathbb{Z})$

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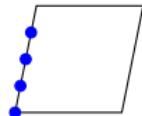
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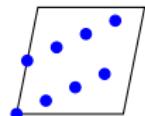
★ Class of exact solutions at finite N :

[Hosseini, Nedelin, Zaffaroni 16; Hong, Liu 18]

- BASIC SOLUTION: $u_{ij} = \frac{\tau}{N}(j - i)$



- T-TRANSFORMED SOL's: $u_{ij} = \frac{\tau+r}{N}(j - i)$ with $0 \leq r < N$

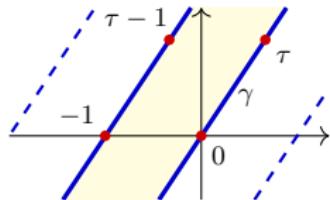


- Many other solutions — most related by $SL(2, \mathbb{Z})$

(This class does not exhaust all solutions)

Contribution of BASIC SOLUTION at large N

Define $[\Delta]_\tau \equiv \Delta + n$ s.t. $\in \text{STRIP}$



Contribution of the BASIC SOLUTION at large N :

$$\lim_{N \rightarrow \infty} \log \mathcal{I} \Big|_{\substack{\text{BASIC} \\ \text{SOLUTION}}} = -i\pi N^2 \Theta(\Delta_1, \Delta_2, \tau)$$

$$\Theta = \begin{cases} \frac{[\Delta_1]_\tau [\Delta_2]_\tau (2\tau - 1 - [\Delta_1]_\tau - [\Delta_2]_\tau)}{\tau^2} & \text{if } [\Delta_1]_\tau + [\Delta_2]_\tau \in \text{STRIP} \\ \frac{([\Delta_1]_\tau + 1)([\Delta_2]_\tau + 1)(2\tau - 1 - [\Delta_1]_\tau - [\Delta_2]_\tau)}{\tau^2} & \text{if } [\Delta_1]_\tau + [\Delta_2]_\tau + 1 \in \text{STRIP} \end{cases}$$

This limit is a discontinuous analytic function: Stokes phenomenon

Black hole entropy

Extract entropy from $\log \mathcal{I} \Big|_{\text{BASIC SOLUTION}}$

- Caveat: the theory has 5 charges, but the index only 4 fugacities

$$\int d\tau d\sigma d\Delta_1 d\Delta_2 \mathcal{I}(\tau, \sigma, \Delta_1, \Delta_2) p^{-J_1} q^{-J_2} \prod_{a=1}^3 y_a^{-\frac{R_a}{2}} = \sum_{R_3} d(J, R) \Big|_{\substack{\text{other charges} \\ \text{fixed}}}$$

SUGRA: at most one s.c. black hole contributes to the sum

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- ★ Set $X_1 = [\Delta_1]_\tau$ $X_2 = [\Delta_2]_\tau$. Obtain “entropy function”:

$$\boxed{\log \mathcal{I} = -i\pi N^2 \frac{X_1 X_2 X_3}{\tau^2}}$$

$$\text{with} \quad \sum_{a=1}^3 X_a - 2\tau + 1 = 0$$

Its (constrained) Legendre transform *exactly* gives the **black hole entropy**:

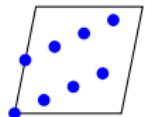
[Hosseini, Hristov, Zaffaroni 17]

$$S_{\text{BH}} = \log \mathcal{I} - 2\pi i \left(\sum X_a \frac{R_a}{2} + 2\tau J \right) \Big|_{\text{constrained extremum}}$$

Extract X, τ from R, J and check that satisfy strip inequality \Rightarrow self-consistency

What about other solutions? They play the role of multiple “saddle points”

- ★ T-TRANSFORMED SOL's with $-\frac{N}{2} \lesssim r \lesssim \frac{N}{2}$



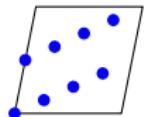
All contributions are of order N^2 : the one with largest real part dominates \mathcal{I}

$$\lim_{N \rightarrow \infty} \log \mathcal{I} \Big|_{\text{T-TRANSF}} = \widetilde{\max}_{r \in \mathbb{Z}} \left(-i\pi N^2 \Theta(\Delta_1, \Delta_2, \tau + r) \right)$$

This ensures periodicity under $\tau \rightarrow \tau + 1$

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★ Stokes phenomenon

In the limit, multiple exponential contributions compete (as in phase transitions)

$$\lim = e^{a_1(\Delta, \tau)N^2} + e^{a_2(\Delta, \tau)N^2} + \dots$$

→ Different regions with different analytic limits,
separated by (real-codimension-1) “Stokes lines”

Universal black holes

Special case:

$$\begin{array}{ccc} J_1 = J_2 & \leftrightarrow & \tau = \sigma \\ R_1 = R_2 = R_3 & & \Delta_1 = \Delta_2 = \Delta_3 \equiv \Delta = \frac{2\tau - 1}{3} \end{array}$$

Such black holes exist in 5d $\mathcal{N} = 1$ minimal gauged SUGRA

Uplift to any $\text{AdS}_5 \times \text{SE}_5$ dual to 4d $\mathcal{N} = 1$ SCFT

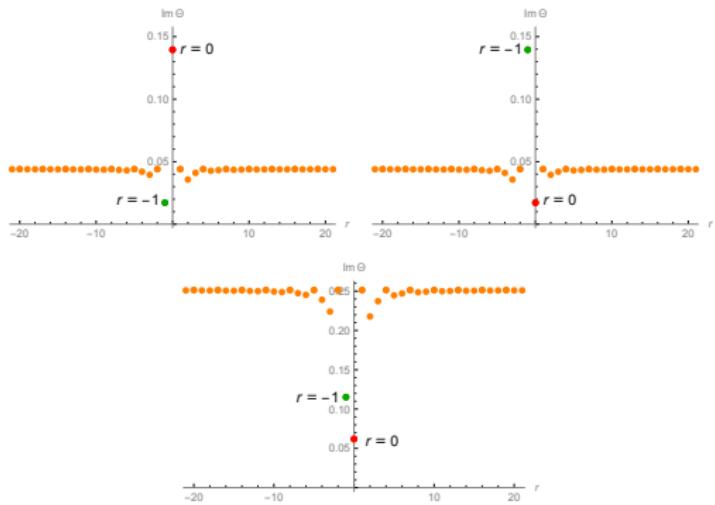
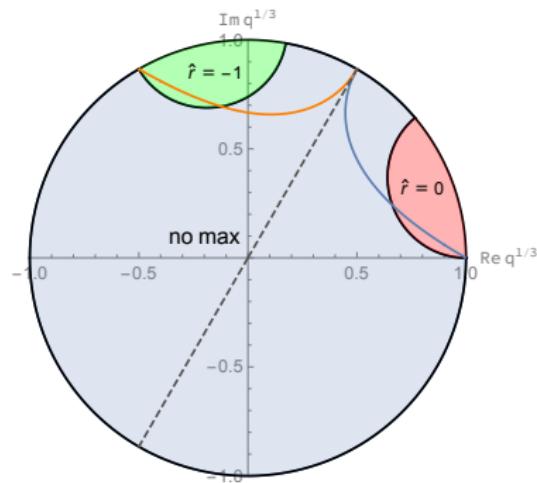
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Conclusions

Summary:

- Careful analysis of superconformal index of $\mathcal{N} = 4$ SYM, using an alternative [Bethe Ansatz formulation](#).
At large N , each Bethe Ansatz solution plays the role of a saddle point.
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Open questions:

- What do the other solutions represent?
- What is the nature of the phase transitions?
- Can we compute corrections?
- What signatures of quantum gravity emerge?