Seiberg-like Dualities for 2d Gauge Theories with a Boundary

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Outline

Introduction to Gauged Linear Sigma Models (GLSMs)

Dualities of Non-Abelian GLSMs

Extending Dualities to GLSMs with a Boundary

An Example

Summary and Outlook

Introduction to GLSMs

GLSMs are $\mathcal{N} = (2,2)$ gauge theories in 2d with $U(1)$ R-symmetry
[Witten '93]

Gauge group $\mathrm{G} = \left(U(1)^{\ell} \times K\right)/\Gamma$
(K: Product of semi-simple groups, Γ: Discrete group)

Spectrum : 1. Chiral Field (Φ) in a repr. of G

2. Vector Field (V) in the adjoint repr. of G

GLSM data can be modelled s.t. the RG flow to the infrared results in an $\mathcal{N} = (2,2)$ SCFT.

Dualities of Non-Abelian GLSMs

Amongst certain non-abelian GLSMs, there exist non-trivial dualities [Hori,Tong '07], [Hori '13]

Example: ∘ GLSM with G = SU(k) and N fundamentals Φ_i

Gauge invariant d.o.f.:

$$B_{i_1 \dots i_k} = \epsilon_{\alpha_1 \dots \alpha_k} \Phi^{\alpha_1}_{i_1} \dots \Phi^{\alpha_k}_{i_k} \qquad \# : \binom{N}{k}$$

∘ $\widetilde{\text{GLSM}}$ with G = SU($N-k$) and N fundamentals $\widetilde{\Phi}_i$

Gauge invariant d.o.f.:

$$\widetilde{B}_{i_1 \dots i_{N-k}} = \epsilon_{\alpha_1 \dots \alpha_{N-k}} \widetilde{\Phi}^{\alpha_1}_{i_1} \dots \widetilde{\Phi}^{\alpha_k}_{i_{N-k}} \qquad \# : \binom{N}{N-k}$$

Check: Sphere partition function of dual theories coincide [Benini, Cremonesi '14]

Remarks:
1. Similar to $\mathcal{N}=1$ 4d Seiberg Duality [Seiberg '94]
2. Dualities also exist for O(k) and USp($2k$) gauge groups

A 'Geometric' Duality

We consider an example of a duality motivated by the target spaces of the GLSMs being geometric duals:

$$\mathrm{Gr}(k, N) \simeq \mathrm{Gr}(N-k, N)$$

Recall: $\mathrm{Gr}(k, N)$ is the space of complex k-planes in an N-dimensional ambient space; $\dim_{\mathbb{C}}(\mathrm{Gr}(k, N)) = k \cdot (N-k)$

The duality is understood by associating a complex k-plane in $\mathbb{C}^N$ to its orthogonal complement, i.e. an $(N-k)$-plane.

Duality: ◦ $\mathrm{G} = \mathrm{U}(k)$ and N flavours Φ_i in the repr. $\square_1$

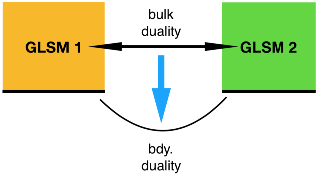
Target Space: $\mathrm{Gr}(k, N)$

◦ $\mathrm{G} = \mathrm{U}(N-k)$ and N flavours $\widetilde{\Phi}_i$ in the repr. $\square_1$

Target Space: $\mathrm{Gr}(N-k, N)$

Dualities of GLSMs with a Boundary

Duality discussion so far has centred on GLSMs without boundary. What is the effect of duality on boundaries of GLSMs?



GLSM on a Riemann surface with a boundary generically preserves only half of the susy (A- or B-type).

A supersymmetric action is typically invariant under a susy transformation upto a total derivative term Δ.

Δ non-vanishing for theory with a boundary $\Rightarrow$ 'Warner Problem'

[Warner '95]

The Warner Problem

$$\Delta = \delta_{\text{susy}} \cdot \int_D d^2z \; d^2\theta \; W(\Phi) = \int_{\partial D} d\sigma \; d\theta \; W(\Phi|_{\partial D}) \; .$$

To restore susy a term is added at the boundary of the form:

$$S_{\partial D} = -\int_{\partial D} d\sigma \; d\theta \, (\pi \cdot J(\Phi|_{\partial D})) \; .$$

where π are fermions at the bdy that satisfy: $\delta_{\text{susy}} \cdot \pi = E(\Phi|_{\partial D})$ and J, E are functions s.t. $J(\Phi) \cdot E(\Phi) = W(\Phi)$.
Then

$$\delta_{\text{susy}} S_{\partial D} = -\int_{\partial D} d\sigma \; d\theta \; W(\Phi|_{\partial D}) = -\Delta \; . \quad \checkmark \; \text{(susy restored)}$$

The boundary d.o.f. can be represented as:

$$Q(\Phi) = J(\Phi)\pi + E(\Phi)\bar{\pi} \; , s.t. \; Q^2(\Phi) = J(\Phi) \cdot E(\Phi) = W(\Phi) \cdot$$

Defects between GLSMs

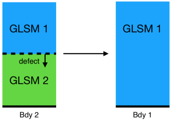
Check of boundary duality: Compare Hemisphere Partition functions:

$$Z_{H^2}(\mathrm{GLSM}_Q) = Z_{H^2}(\widetilde{\mathrm{GLSM}}_{\widetilde{Q}})$$

[Hori,Romo '13]
[Honda,Okuda '13]

$\rightarrow$ We aim for a stronger check: Categories of B-type boundary conditions on dual GLSMs must be equivalent.

In doing so, we first separate the two GLSMs with a 'defect'.



Corresponding to the defect, we construct an explicit map FM,

$$\mathrm{FM} : \mathrm{Bdy}\ 2 \rightarrow \mathrm{Bdy}\ 1$$

This map is known as the *Fourier-Mukai Kernel*.

The Fourier-Mukai Kernel

Fourier-Mukai Kernel from theory B to A is a map s.t.,

$$\mathrm{FM} : \text{Bdys of B} \to \text{Bdys of A}$$
$$\mathscr{B} \mapsto \mathscr{A}$$

More explicitly, consider the product of dual boundary theories and respective projections to individual theories:

$$\begin{tikzcd} & \mathbf{A} \times \mathbf{B} \arrow[dl, "p_1"'] \arrow[dr, "p_2"] & \\ \mathbf{A} & & \mathbf{B} \end{tikzcd}$$

Then the map FM corresponds to an object $\varphi_{\mathrm{FM}} \in$ Bdys of $(A \times B)$,

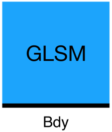
$$p_{1*}(\varphi_{\mathrm{FM}} \otimes p_2^*(\mathscr{B})) = \mathscr{A} \ .$$

$\Rightarrow$ FM acts as an integration kernel that maps bdys from B to A.

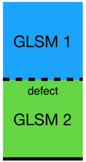
The Fourier-Mukai Kernel II: Application

How does the narrative of defects between dual GLSMs connect to their boundary d.o.f.?

Recall, boundary d.o.f of a GLSM $\equiv Q(x)$, with $Q^2(x) = W(x)$.



Since the defect is just a boundary in the product theory:

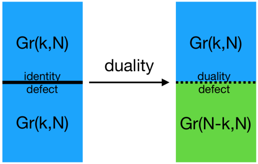


$\Rightarrow$ Defect d.o.f. between GLSM_1 and $\mathrm{GLSM}_2 \equiv Q(x_1, x_2)$, with

$$Q^2(x_1, x_2) = W(x_1) - W_2(x_2).$$

Example: Grassmannian Duality

For the Grassmannian duality, $\mathrm{Gr}(k, N) \simeq \mathrm{Gr}(N-k, N)$, we consider the duality transformation on the defect:



Matter	$(U(k) \times U(k))$ Rep.
x_i	$(\square_1, 1_0)$
y_i	$(1_0, \square_1)$
$\mathcal{M}$	$(\square_1, \square_{-1})$
π	$(\square_{-1}, 1_0)$

$$Q_1 = (x + y\mathcal{M})\,\pi$$
$$Q_1^2 = 0$$

Matter	$(U(k) \times U(N-k))$ Rep.
x_i	$(\square_1, 1_0)$
$\widetilde{y}_i$	$(1_0, \square_1)$
π'	$(\square_{-1}, \square_{-1})$

$$Q_2 = (x \cdot \widetilde{y})\,\pi'$$
$$Q_2^2 = 0$$

Categories of bdy conditions are shown to be equivalent.

Summary and Outlook

Summary

GLSMs and Seiberg-like dualities thereof

Extension of dualities to GLSMs w/ boundary: Fourier-Mukai Kernel

Outlook

Analyse dualities in quiver gauge theories leading to cluster algebras. Strengthen the cluster algebra proposal by an extension to theories with a boundary. **[Benini et al '14]**

Extend dualities to boundary GLSMs with other gauge groups that are Seiberg-like-dual, *eg.* USp($2k$).

Realise derived equivalences of target spaces relevant from a string compactification point of view.

Thank You

Questions/Comments?