# Can high-scale axion models have a viable cosmological history?

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Work in progress

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## The aim:

Report **progress in understanding the cosmological history of high-scale axion models** in which the axion is embedded into the inflaton, which are **thought to be ruled out due to isocurvature constraints**

## The novelty:

We **discard invalid assumptions**

**Detailed simulations** of the evolution of axion isocurvature modes, including nonperturbative effects with **lattice** computations

## The plan:

Motivation of high-scale axion models

The trouble with isocurvature fluctuations

Improved calculations

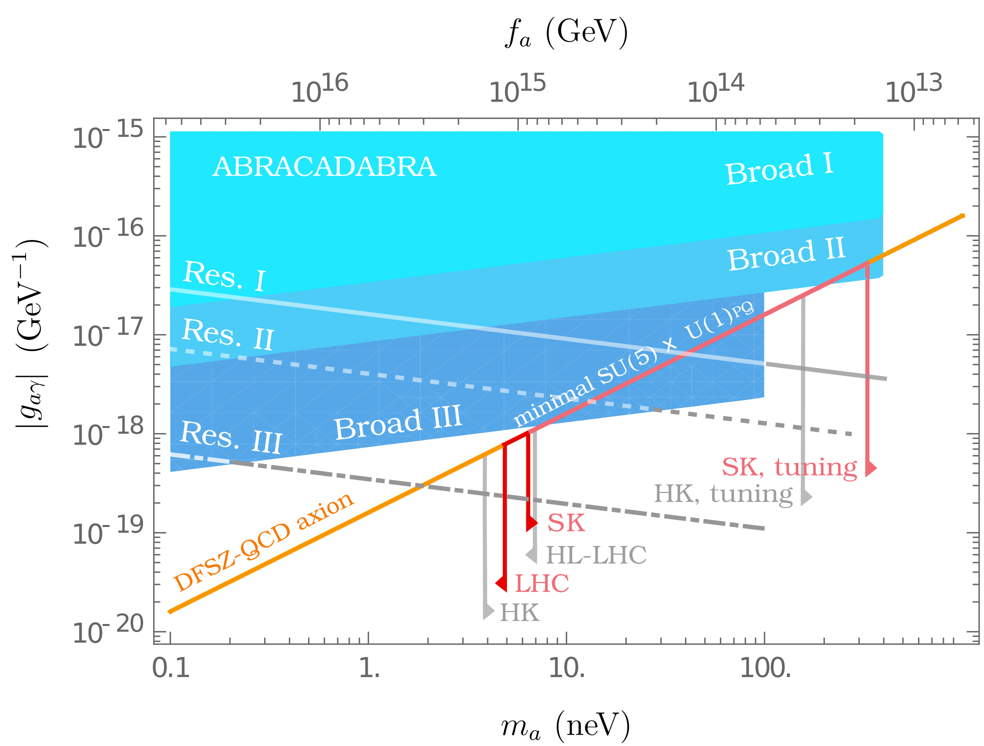
# Motivating high-scale axion models

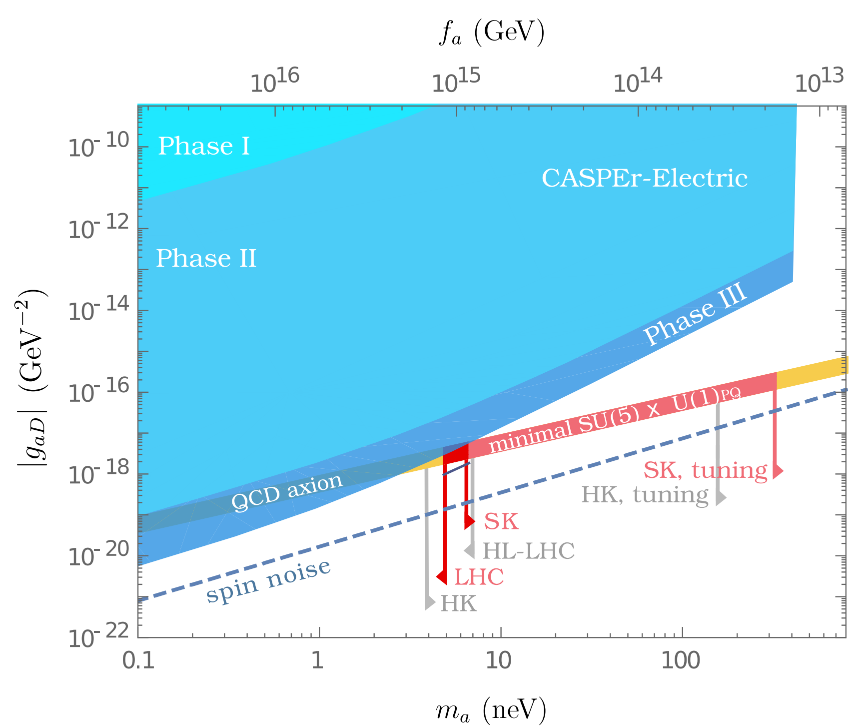
# Axion GUTs

Axion GUTs can offer an intriguing **connection** between the **axion scale $f_A$** and the **grand unification scale**.

This can result in **complementarities** between **axion searches** and **proton decay** experiments.

Motivates **targets for low-mass axion experiments** like ABRACADABRA and CASPER.




[Di Luzio, Ringwald, CT]

# Axion dark matter in GUTs

Axion field ($\sim$ phase of complex scalar x dimensionful scale) provides a dark matter candidate to GUTs that lack it, as typical SU(5) models

For $f_A \sim M_{\rm GUT}$ the axion can be dark matter in scenarios of pre-inflationary breaking of the U(1) symmetry associated with the axion: misalignment mechanism [Preskill et all, Abbott and Sikivie, Dine and Fischler]

Our universe comes from a patch with a homogeneous initial phase $\theta_I$

When the Hubble friction drops, the axion field starts to oscillate freely

This sources a stress-energy tensor which mimics a pressureless fluid

$$\theta_I^2 = 3.4 \times 10^{-7} \left(\frac{\Omega_{DM} h^2}{0.12}\right) \left(\frac{f_A}{3 \times 10^{17}\mathrm{GeV}}\right)^{-1.165}$$

[Borsanyi et al]

# The trouble with isocurvature

# Axion isocurvature perturbations

Isocurvature perturbations do not change the total energy density. With the universe comprising axions and radiations, one can define the **isocurvature perturbation**

$$S_{A\gamma} = \frac{\delta n_A}{n_A} - \frac{\delta n_{\rm rad}}{n_{\rm rad}}\bigg|_{\delta\rho=0} = \frac{\delta n_A}{n_A} - 3\frac{\delta T}{T}\bigg|_{\delta\rho=0}, \quad n_A \propto \theta^2$$

This is to be contrasted with the **density or curvature perturbation** $R$.

One can define **power spectra** and an **isocurvature fraction**

$$\langle S_{A\gamma}(\mathbf{k})\mathbf{S_{A\gamma}}(\mathbf{k}')\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\mathbf{P_{A\gamma}}(|\mathbf{k}|)$$
$$\langle R(\mathbf{k})\mathbf{R}(\mathbf{k}')\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\mathbf{P_R}(|\mathbf{k}|)$$

$$P_X(|\mathbf{k}|) \equiv \frac{\mathbf{2\pi^2}}{|\mathbf{k}|^\mathbf{3}}\mathbf{\Delta_X}(|\mathbf{k}|)$$

$$\beta_{\rm iso}(|\mathbf{k}|) = \frac{\mathbf{P_{A\gamma}}(|\mathbf{k}|)}{\mathbf{P_{A\gamma}}(|\mathbf{k}|) + \mathbf{P_R}(|\mathbf{k}|)} = \frac{\mathbf{\Delta_{A\gamma}}(|\mathbf{k}|)}{\mathbf{\Delta_{A\gamma}}(|\mathbf{k}|) + \mathbf{\Delta_R}(|\mathbf{k}|)}$$

Both types of perturbations affect the CMB differently, which allows to set bounds on the isocurvature fraction

$$\beta_{\rm iso}(0.002{\rm Mpc}^{-1}) < 0.035$$

[Planck 2018]

# Dark matter has dark consequences

Deep in the radiation era, the isocurvature is dominated by the axion

$$\rho_{\rm rad} \gg \rho_A \Rightarrow S_{A\gamma} \approx \frac{\delta n_A}{n_A};\; \theta = \langle\theta\rangle + \delta\theta \Rightarrow \langle S_{A\gamma} S_{A\gamma}\rangle \approx \frac{4}{\langle\theta\rangle^2}\langle\delta\theta\delta\theta\rangle = \frac{4}{\theta_I^2}\langle\delta\theta\delta\theta\rangle$$

If the axion is a massless field in De Sitter, its power spectrum freezes at horizon crossing:

$$\Delta_{\delta\theta} = \frac{1}{\rho^2}\Delta_{\delta A} = \frac{1}{\rho^2}\left(\frac{H}{2\pi}\right)^2$$

Given an inflationary model, fixing the DM relic abundance fixes the isocurvature fraction

$$\Delta_{A\gamma} \approx \frac{4}{\theta_I^2}\Delta_{\delta\theta} = \frac{4}{\rho^2\theta_I^2}\left(\frac{H}{2\pi}\right)^2$$

More suppression if the modulus $\rho$ is the inflaton! In simple inflationary models

$$\boxed{f_A < 1.4\times 10^{14}\mathrm{GeV}}$$

incompatible with a GUT-scale-sized $f_A$!

[Fairbairn & Marsh, Ballesteros & Redondo & Ringwald &CT]

# Invalid assumptions?

The previous bound assumed

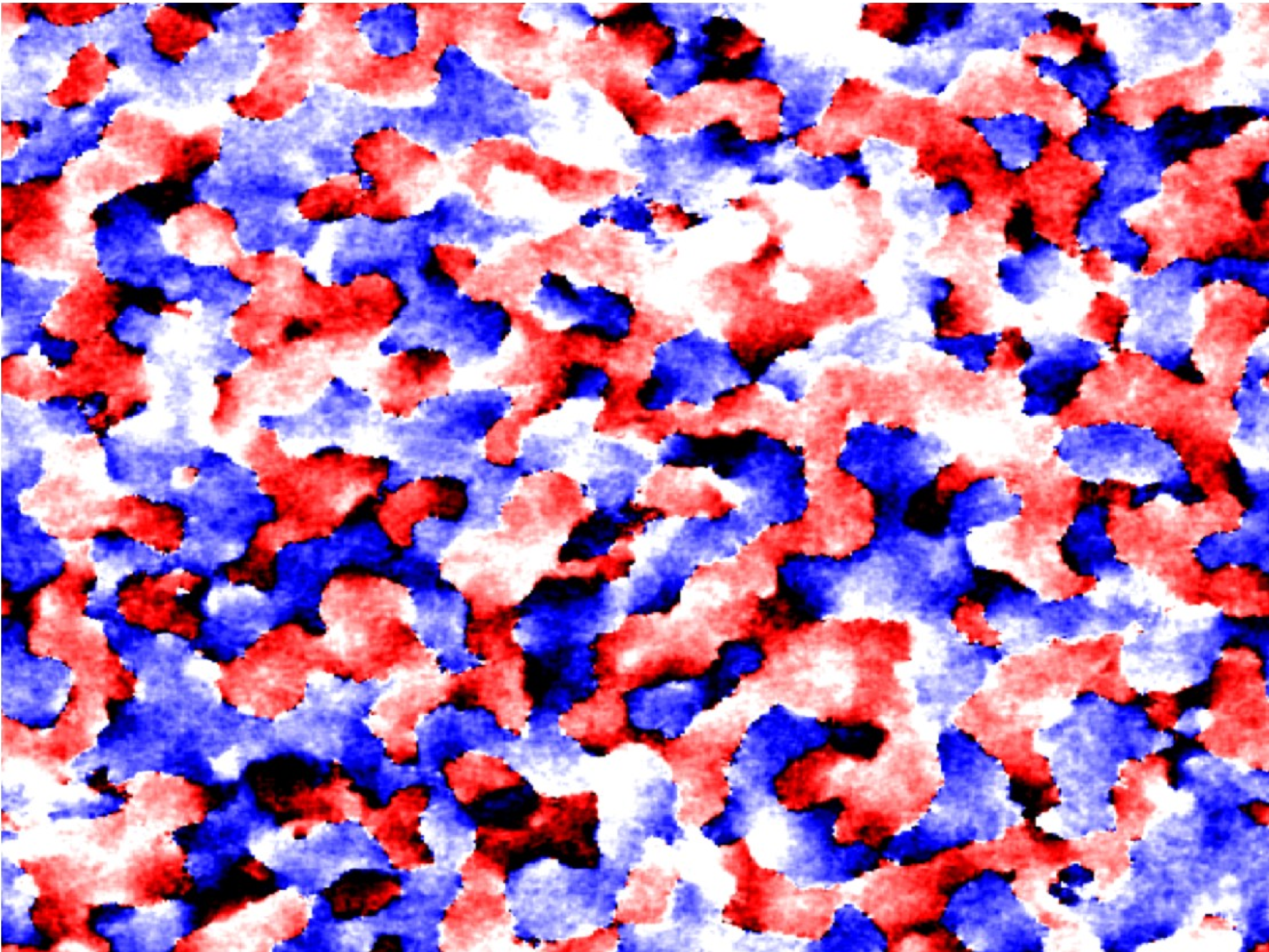
Axion field is massless during inflation

If axion is embedded into inflaton, the field is not at its minimum during inflation: **Goldstone theorem does not apply and the axion is massive**

Primordial angular perturbations fixed after horizon crossing during inflation

Not true for massive fields!

**Non-perturbative phenomena during reheating are known to affect angular perturbations**



# To-do list

Within an inflationary **model in which the axion is embedded into a complex inflaton, which couples to the Higgs**:

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Solve numerically the **evolution of inflationary perturbations**, accounting for nonzero masses

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Use the above as initial conditions for **lattice simulations with 3 real scalars,** (complex inflaton + Higgs) including effects of **Higgs decays**

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**Derive power spectra** for isocurvature perturbation and extrapolate to CMB scales

# Improved calculations

# Model setup

$$\sqrt{-g}\mathcal{L} \supset \sqrt{-g}\left\{\left(\frac{M_P^2}{2} + \xi\left(\phi^\dagger\phi - \frac{v_\phi^2}{2}\right)\right)R - \lambda_\phi\left(\phi^\dagger\phi - \frac{v_\phi^2}{2}\right) - \lambda_h\left(h^2 - \frac{v_h^2}{2}\right) - 2\lambda_{h\phi}\left(\phi^\dagger\phi - \frac{v_\phi^2}{2}\right)\left(h^2 - \frac{v_h^2}{2}\right)\right\}$$

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$\phi = \phi_1 + i\phi_2$ **Inflaton field, containing axion**

$$\phi = \frac{1}{\sqrt{2}}(v_\phi + \rho(x))\exp\left(\frac{iA(x)}{v_\phi + \rho(x)}\right)$$

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$h:$ **Higgs field**

# Evolution of perturbations during inflation

With $\delta\phi = \delta\phi_1 + i\delta\phi_2$ we separate the fluctuations components into

$\delta\phi_{\parallel}$ aligned with infationary background

$\delta\phi_{\perp}$ orthogonal to inflationary background

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Potential admits **straight inflationary trajectories**, with **misalignment angle** $\theta_I$

$$\delta\phi_{\parallel} = \cos\theta_I \delta\phi_1 + \sin\theta_I \delta\phi_2, \quad \delta\phi_{\perp} = \cos\theta_I \delta\phi_2 - \sin\theta_I \delta\phi_1$$

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For straight trajectories, perturbations in flat spatial curvature gauge satisfy [Gordon et al]

$$\delta\ddot{\phi}_{\parallel} + 3H\delta\dot{\phi}_{\parallel} + \left(\frac{\mathbf{k^2}}{a^2} + m_{\parallel}^2 - \frac{1}{M_P^2 a^3}\frac{d}{dt}\left(\frac{a^3\dot{\phi}_{\parallel}^2}{H}\right)\right)\delta\phi_{\parallel} = 0$$

$$\delta\ddot{\phi}_{\perp} + 3H\delta\dot{\phi}_{\perp} + \left(\frac{\mathbf{k^2}}{a^2} + m_{\perp}^2\right)\delta\phi_{\perp} = 0$$

# Power spectra at the end of inflation

Boundary conditions: match result in in the local Minkowski frame well below the horizon:

$$\phi_{i,\mathbf{k}}(t) \to \frac{e^{-i\omega\tau(t)}}{(2\omega)^{1/2}}, \; t \to -\infty, \quad \omega = \sqrt{\mathbf{k}^2 + a(t)^2 m_i^2}$$

These solutions with this normalization are associated with quantized fields

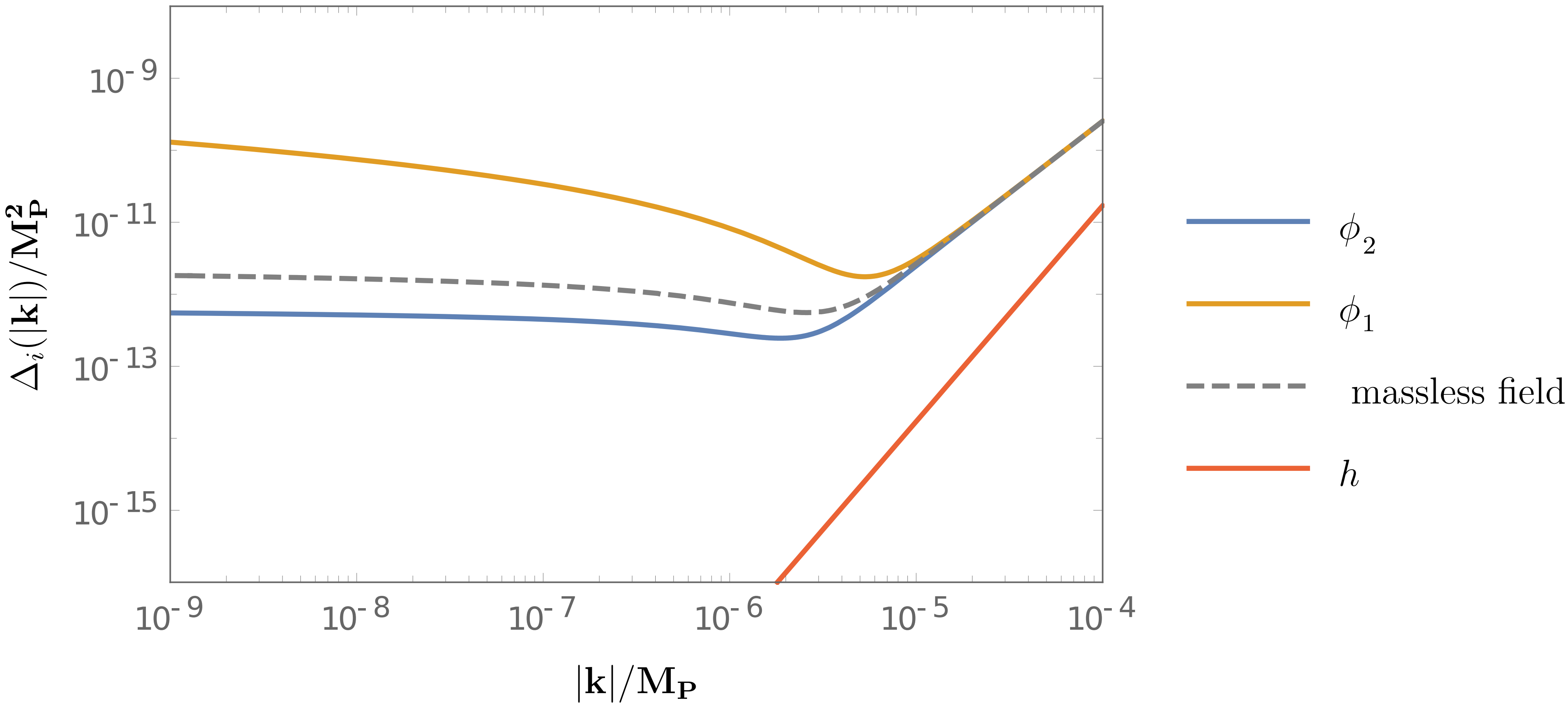
$$\hat{\phi}_i(\mathbf{k},\mathbf{t}) = \phi_{\mathbf{i},\mathbf{k}}(\mathbf{t})\mathbf{a}_{\mathbf{k}} + \phi_{\mathbf{i},\mathbf{k}}(\mathbf{t})^* \mathbf{a}^\dagger_{\mathbf{k}}, \quad [\mathbf{a}_{\mathbf{ik}}, \mathbf{a}^\dagger_{\mathbf{ik}'}] = (\mathbf{2}\pi)^{\mathbf{3}} \delta(\mathbf{k} - \mathbf{k}')$$

such that the corresponding power spectra are determined by the $\phi_{i,\mathbf{k}}(t)$

$$\left\langle \hat{\phi}_i(\mathbf{k},\mathbf{t})\hat{\phi}_{\mathbf{i}}(\mathbf{k}'.\mathbf{t}) \right\rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')\mathbf{P}_{\phi_{\mathbf{i}}}(|\mathbf{k}|.\mathbf{t}), \quad \mathbf{P}_{\phi_{\mathbf{i}}}(|\mathbf{k}|.\mathbf{t}) = |\phi_{\mathbf{i},\mathbf{k}}(\mathbf{t})|^{\mathbf{2}}$$

We can infer power spectra from properly normalized solutions of the equations of motion

# Power spectra at the end of inflation




Assumed: $\xi = 0.1, \quad \lambda_{h\phi} = 10^{-6}$

# Isocurvature power spectrum at the end of inflation

From the previous result we can estimate the power spectrum of the isocurvature perturbation

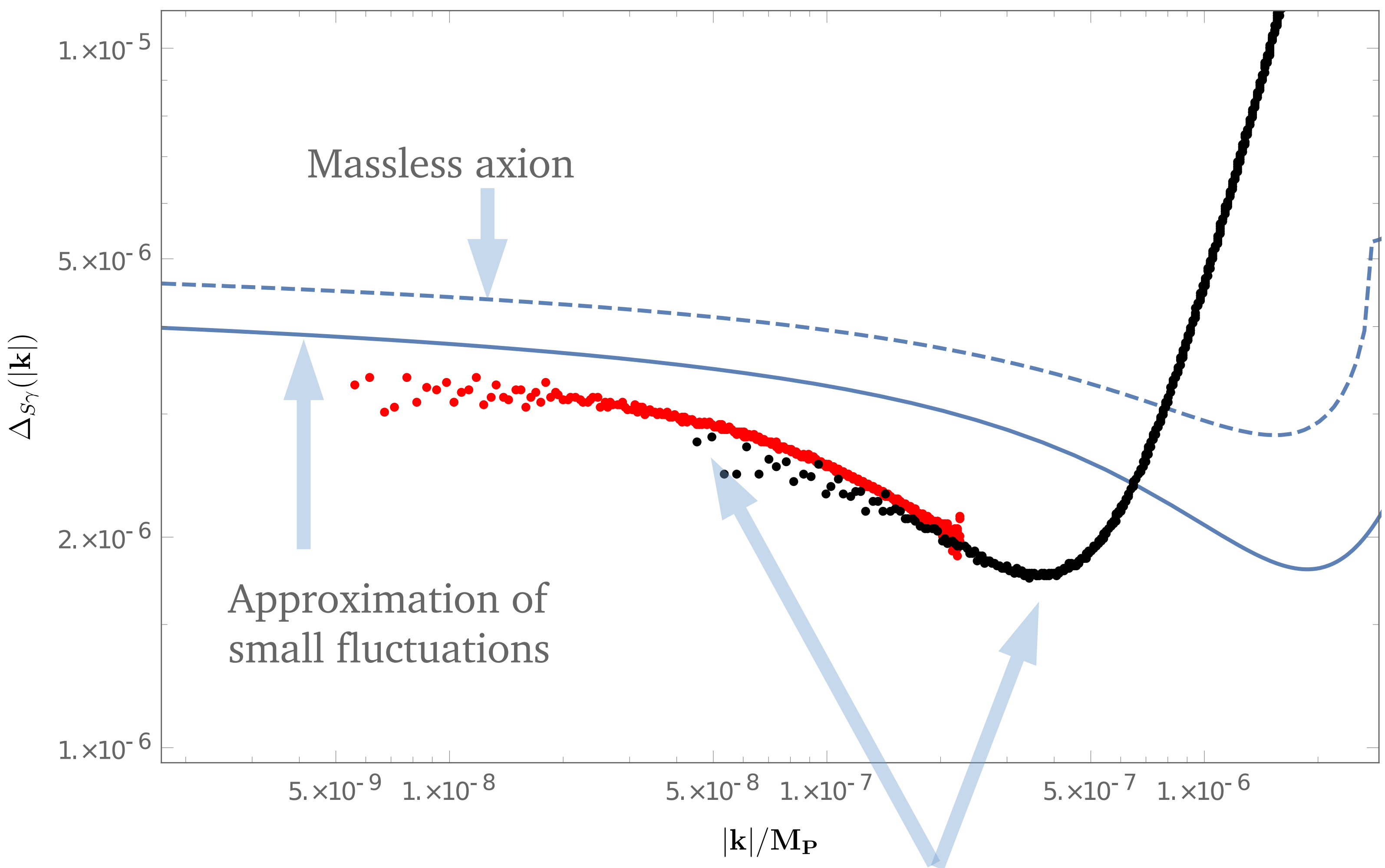
$$S_{A\gamma}(x) \approx \frac{\theta^2(x) - \langle\theta^2\rangle}{\langle\theta^2\rangle}, \quad \theta(x) = \arctan\frac{\phi_2(x)}{\phi_1(x)} \rightarrow P_{A\gamma}(|\mathbf{k}|) = |\mathbf{S_{A\gamma}}(|\mathbf{k}|)|^{\mathbf{2}}$$

Can either:

Assume small perturbations, $\delta\theta(x) \approx \dfrac{\delta\phi_2(x)}{\langle\phi_1(x)\rangle} \Rightarrow \langle\delta\theta^2\rangle = \dfrac{\langle\delta\phi_2^2\rangle}{\langle\phi_1\rangle^2}$

Interpret the power spectrum of the $\phi_i$ as the variance of a gaussian probability distribution for the modes in Fourier space, so that generating modes in a lattice we can directly compute $|S_{A\gamma}(|\mathbf{k}|)|$

# Isocurvature power spectrum at the end of inflation




Assumed: $f_A = 5\times10^{17}\mathrm{GeV}, \quad \xi = 0.1$

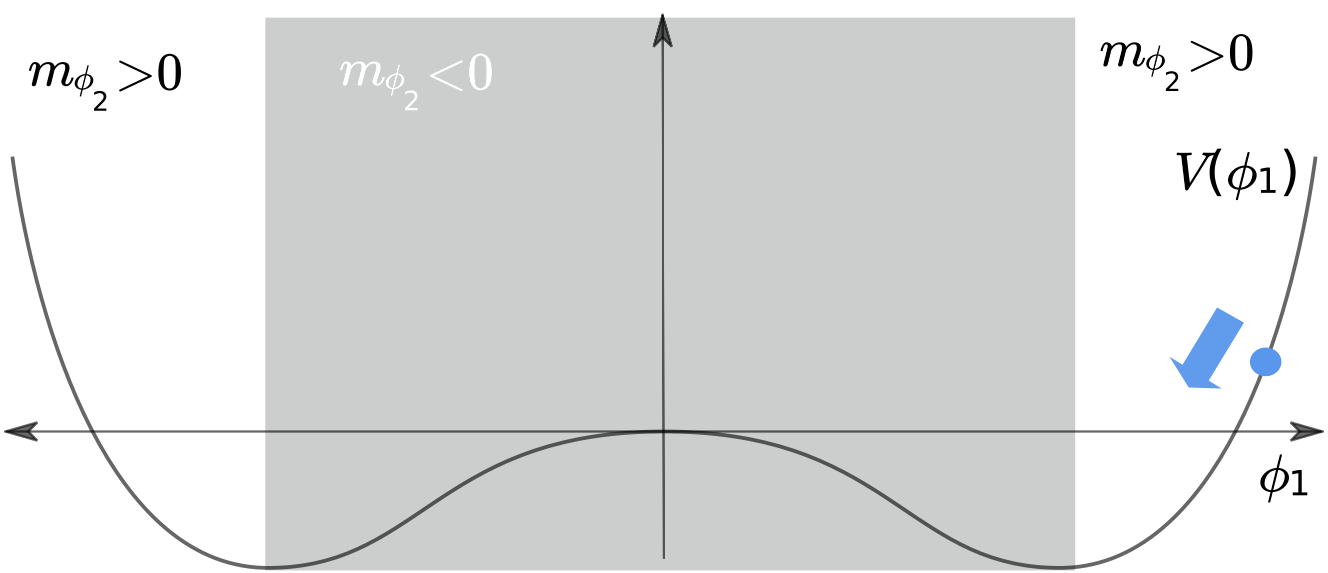
2 different lattices with $512^3$ points

# Isocurvature power spectrum at the end of inflation

The previous results show that the **(small) axion mass suppresses the power spectrum at super-horizon scales**

**Do the isocurvature modes decay further after inflation?**

What happens when the background crosses the origin and the $\phi_2$ **mass becomes negative**? Is there any **instability/growth**?



# Lattice simulations

We simulate **3 real scalars** $\phi_1(t,\mathbf{x}), \phi_2(t,\mathbf{x}), h(t,\mathbf{x})$ with $h$ decaying into a **relativistic bath of SM particles** with density $\rho_{SM}(t)$, in an **expanding universe** with scale factor $a(t)$

$$\ddot{\phi}_k + 3\frac{\dot{a}}{a}\dot{\phi}_k - \frac{1}{a^2}\vec{\nabla}^2\phi_k + \frac{\partial V(\phi_l)}{\partial \phi_k} = 0, \;\; k = 1,2,$$

$$\ddot{h} + 3\frac{\dot{a}}{a}\dot{h} - \frac{1}{a^2}\vec{\nabla}^2 h + \frac{\partial V(\phi_l)}{\partial h} + \Gamma_h \dot{h} = 0,$$

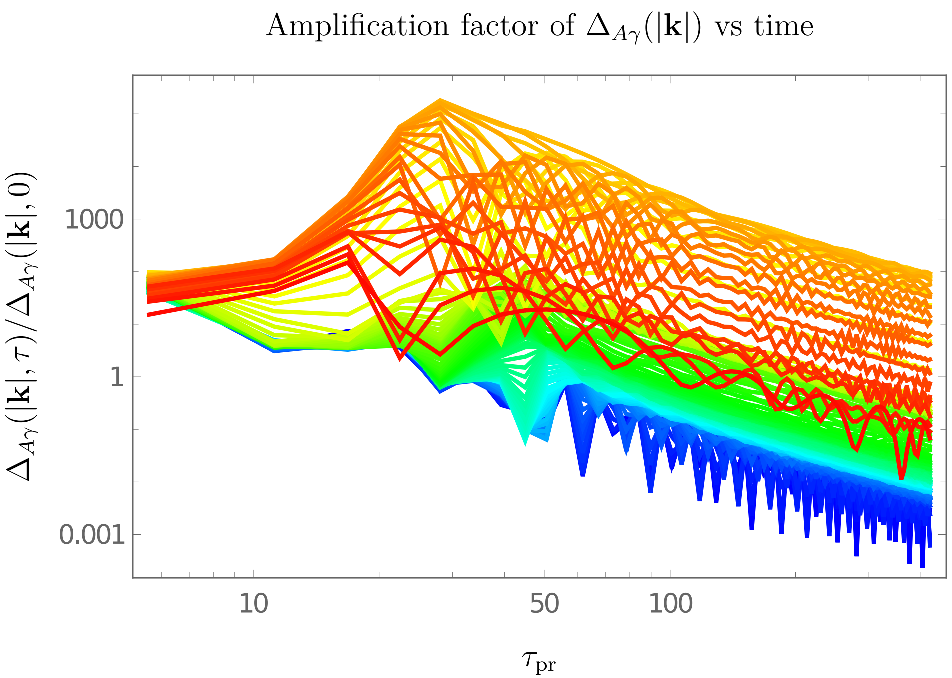
$$\dot{\rho}_{SM} + 4\frac{a}{a}\rho_{SM} = \Gamma_3 \dot{h}^2,$$

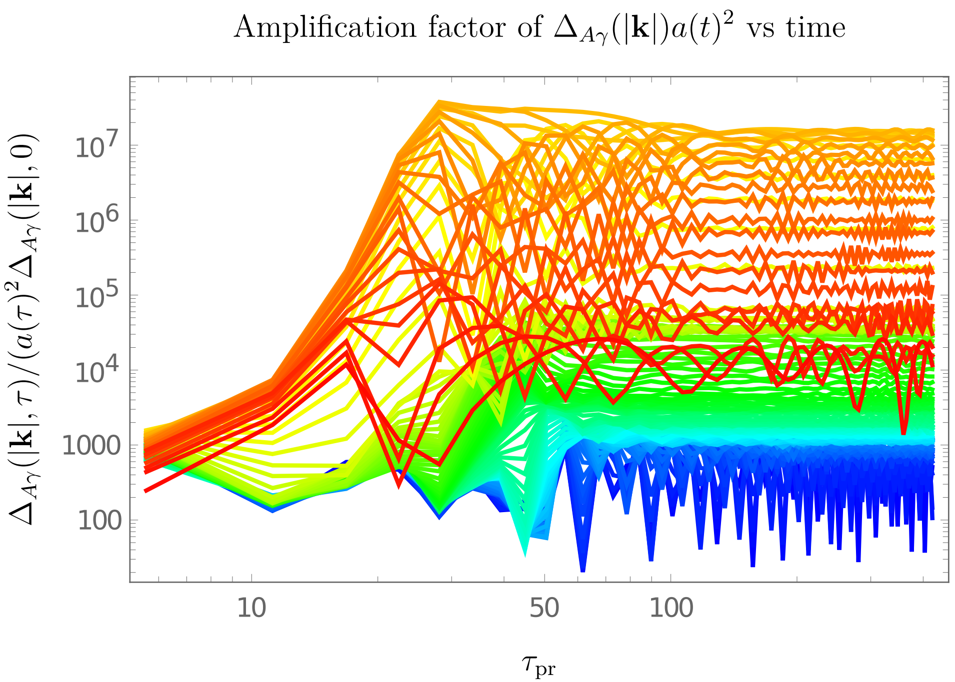
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_P^2}\left(\rho_{SM} + V + \frac{1}{2}(\dot{\phi}_1^2 + \dot{\phi}_2^2 + \dot{h}^2) + \frac{1}{2a^2}\left((\nabla\phi_1)^2 + (\nabla\phi_2)^2 + (\nabla h)^2\right)\right)$$

We use a modified version of “latticeeasy” [Felder,Tkachev]. Aside from highlighted changes we implement the initial conditions derived from the mode equations during inflation –the **default initial conditions in lattice easy are not valid for super-horizon modes**!

# Lattice results

Super-horizon isocurvature modes have an initial exponential growth but then decay as $1/a(t)^2$ !




Lower momenta in red, higher momenta in blue
Notice big initial growth for intermediate momenta (a resonance band)

# …but it could not be so easy, could it?

The **initial exponential growth is larger for lower modes**

Power spectrum at inflationary scales and CMB times will be a result of the **competition between initial amplification and later decay**

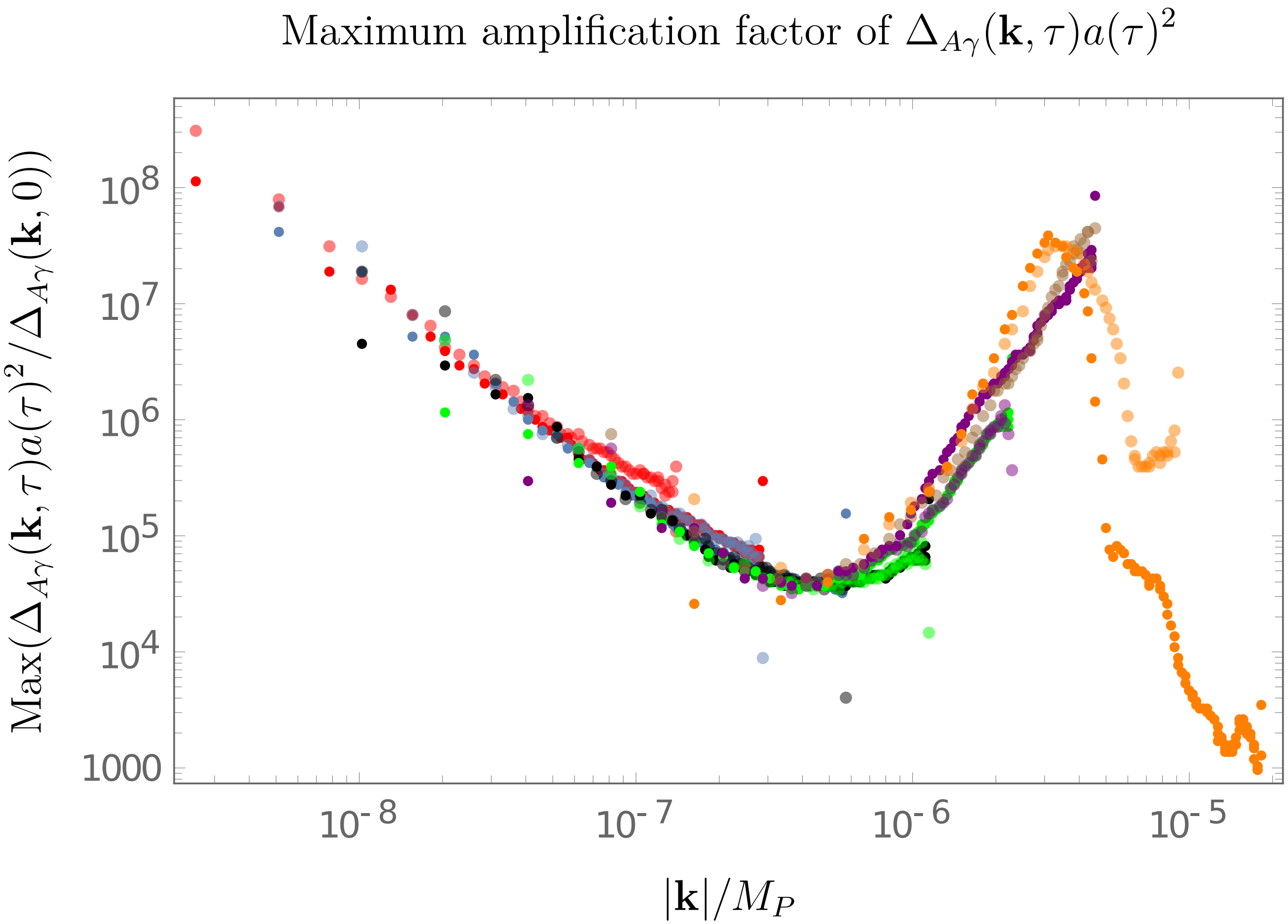
As $\Delta_{A\gamma}(|\mathbf{k}|,\tau)a(\tau)^2/\Delta_{A\gamma}(|\mathbf{k}|,0)$ seems to always peak and then oscillate around a constant value, we can estimate a bound

$$\Delta_{A\gamma}(|\mathbf{k}|,\tau) \lesssim \underbrace{\frac{\Delta_{A\gamma}(|\mathbf{k}|,0)}{a(\tau)^2}}_{\text{From initial conditions}} \underbrace{\mathrm{Max}\left(\frac{\Delta_{A\gamma}(|\mathbf{k}|,\tau)a(\tau)^2}{\Delta_{A\gamma}(|\mathbf{k}|,0)}\right)}_{\text{Amplification factor estimated from lattice simulations}}$$

From initial conditions

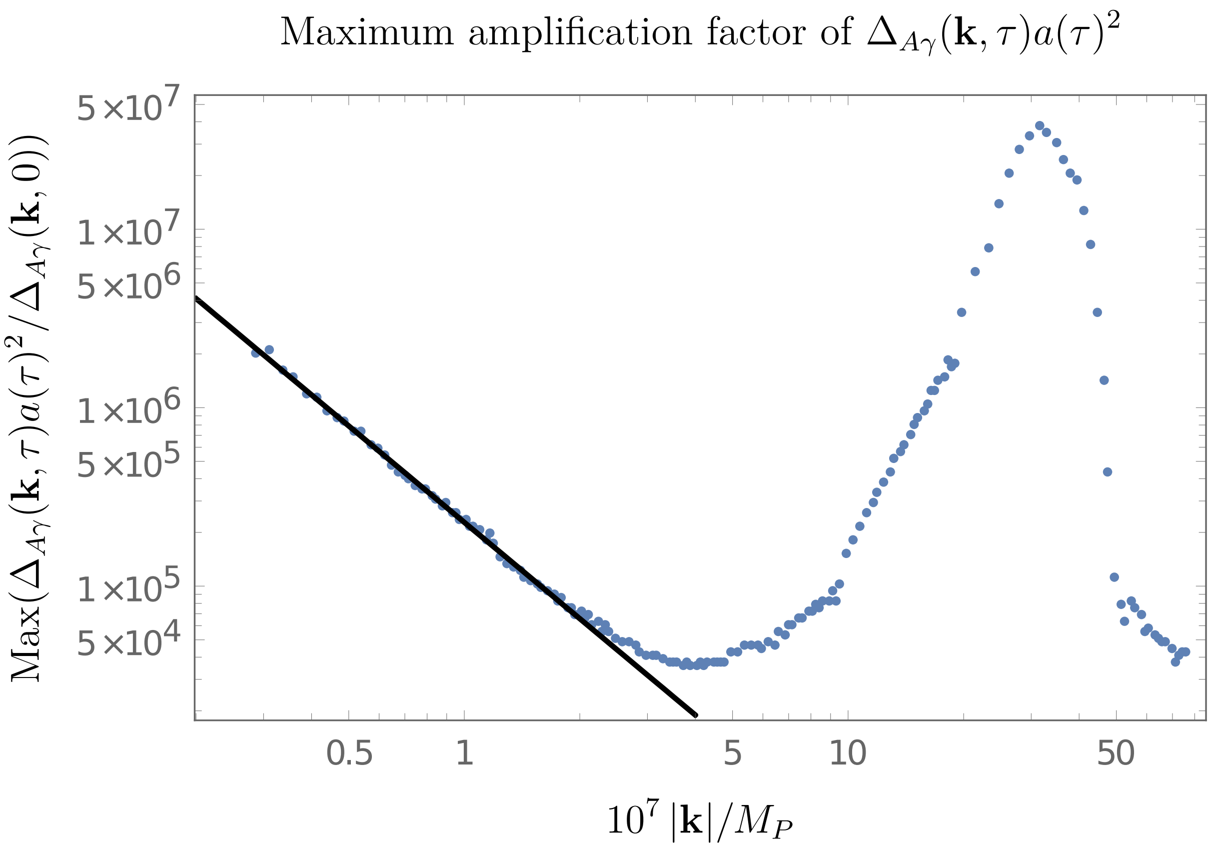
Amplification factor
estimated from lattice simulations

# ...but it could not be so easy, could it?




13 lattice simulations with different grid and box sizes..
Now we only have to extrapolate along 54 orders of magnitude to reach CMB scales :)!

# …but it could not be so easy, could it?




Cleanup: retain only larger grids, remove high and low momenta of each simulation, average over simulations with overlapping momenta.

For the chosen parameters this **gives upper bound on isocurvature power spectrum at CMB scales of O(1) x (value at end of inflation). Ruled out by Planck!**

# Conclusions

The life of an axion isocurvature mode is richer than it was thought.

Traditional arguments for computing axion isocurvature perturbations are based on invalid assumptions

**The axion isocurvature perturbation can evolve rather dramatically after inflation, with competing early time growth and late time decay, the latter suggesting the possibility of viable models**

Still need to explore parameter space for high-scale axion models $(\xi, f_A, \lambda_{\phi h})$

Perhaps low-scale axion-inflaton models with pre-inflationary U(1) breaking should be revisited in view of the complex post-inflationary evolution