Towards a first principles treatment of warm inflation

Gilles Buldgen (CP3, UCLouvain, Belgium)

Collaborators: Marco Drewes, Jin U Kang and Ui Ri Mun

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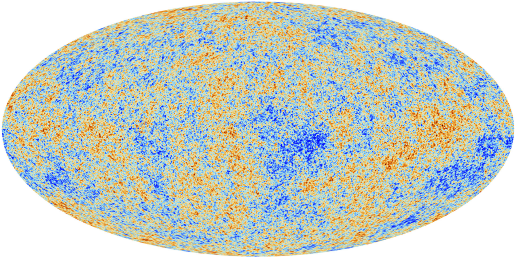
UCLouvain
Institut de recherche
en mathématique et physique

Cosmic inflation

- Exponentially accelerated expansion of the early observable universe
  - + Solves homogeneity, isotropy, flatness and horizon problems



$$T = 2.726 \pm 10^{-5} K$$

(Planck 2013 results - PLANCK collaboration)

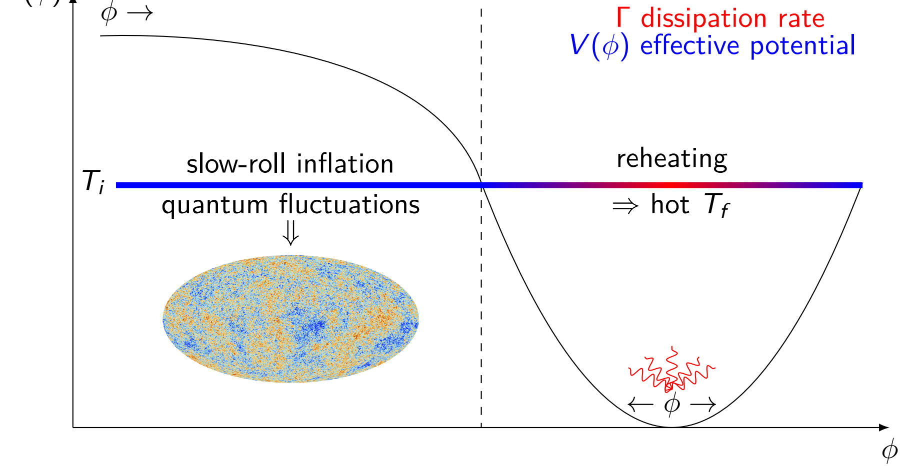
- Universe content dominated by the potential energy of a scalar field ϕ

$$\frac{P}{\rho} \sim -1 \ \ (V(\phi) \gg \dot{\phi}^2)$$

Effective potential driving standard inflation

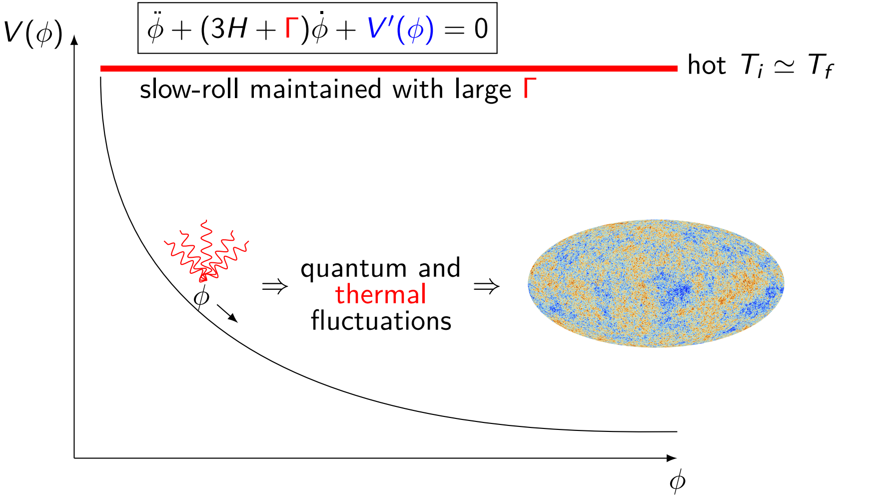
$$\boxed{\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V'(\phi) = 0}$$

Γ dissipation rate
$V(\phi)$ effective potential



Effective potential driving warm inflation

A.Berera Phys.Rev.Lett. 75 (1995) 3218-3221



Effective action for out-of-eq. interacting scalar fields

So far, Γ and V' from questionable assumptions

JCAP 1301 (2013) 016

- equilibrium propagators
- small ϕ-expansions
- Minkowski spacetime

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- Minkowski spacetime

Classical action for the inflaton φ coupled to a thermal bath made of χ

$$S_{\rm cl}[\varphi,\chi] = \int d^4x \Big[-\frac{1}{2}\varphi(\Box + m_\varphi^2)\varphi - \frac{\lambda_\varphi}{4!}\varphi^4 \\ -\frac{1}{2}\chi(\Box + m_\chi^2)\chi - \frac{\lambda}{4!}\chi^4 - \frac{h}{4}\chi^2\varphi^2 - V_\chi \Big]$$

$$\phi(x) = \langle \varphi(x) \rangle_{\rho_0} \qquad \Delta(x,y) = \langle \varphi(x)\varphi(y) \rangle_{\rho_0}$$

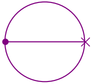
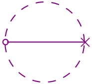
The 2 Particle Irreducible (2PI) quantum effective action is

$$S_{\rm eff}^{\rm 2PI}[\phi,\Delta] = S_{\rm cl}[\phi,0] + \underbrace{\frac{i}{2}\mathrm{Tr}\ln\left[\Delta^{-1}\right] + \frac{i}{2}\mathrm{Tr}\left[\Delta\Delta_0^{-1}[\varphi]\right]}_{\text{1-loop Coleman-Weinberg contribs.}} + \underbrace{S_2[\phi,\Delta]}_{\geq 2 \text{ loops}}$$

Inflation-like and Schwinger-Dyson equations (SDEs)

• As for the background field $\phi(x)$, we showed the EOM to be written in the general form (integrating out the χ-field of vanishing "VEVs")

$$\left(\Box_x + m_\varphi^2\right)\phi(x) = \int_y \Pi_\phi(x,y)\phi(y)$$

$$\Pi_\phi(x,y) = -\frac{\lambda_\phi}{3!}\phi(x)^2\delta(x-y) \; +$$  $+$  $+$  $+$ 

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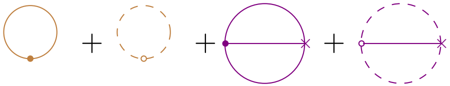


- Upon defining $\Pi = 2i\frac{\delta S_2}{\delta \Delta}$, the EOM for the propagators rewrites

$$\Delta^{-1} = \Delta_0^{-1} - \Pi \Rightarrow$$

Strategy: Analytically solve the SDEs and insert the **resummed nonequilibrium** propagators to calculate the corrections to the background field dynamics.

Analytic results

$$\boxed{\ddot{\phi}(t) + \Gamma_\phi \dot{\phi}(t) + \partial_\phi \mathcal{V} = 0,}$$

2-loop results:

$$\partial_\phi \mathcal{V} = m_\varphi^2 \phi(t) + \frac{\lambda_\varphi}{3!}\phi(t)^3 + (\lambda_\phi + h)\frac{T^2}{24}\phi(t) + \partial_\varphi \mathcal{V}_{\text{sun}}$$

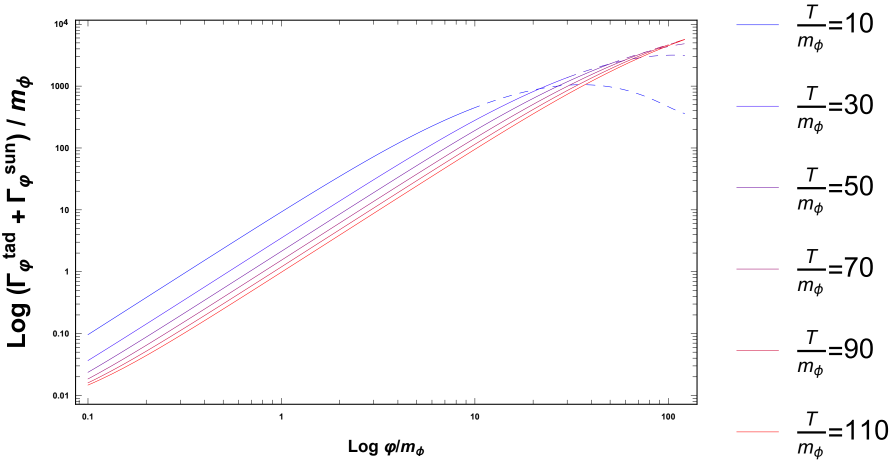
$$\Gamma_\phi = \Gamma_\phi^{\text{tad}} + \Gamma_\phi^{\text{sun}}$$

$$\Gamma_\phi^{\text{tad}} = \frac{h^2}{(4\pi)^2}\frac{\phi(t)^2}{T}\int \frac{p^2\,dp}{\omega_\chi^2 \Gamma_\chi(\cosh(\omega_\chi/T) - 1)}$$

$$\Gamma_\phi^{\text{sun}} = \frac{h^2}{(4\pi)^3}\frac{T^2}{M_\varphi}\log\frac{M_\varphi}{M_\chi}$$

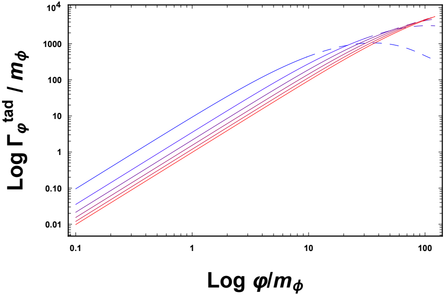
$$\omega_a^2 = p^2 + M_a^2[\phi, T] = p^2 + m_a^2 + \frac{g_a}{2}\phi(t)^2 + (\lambda_a + h)T^2/24$$

Understanding the damping rate Γ_φ



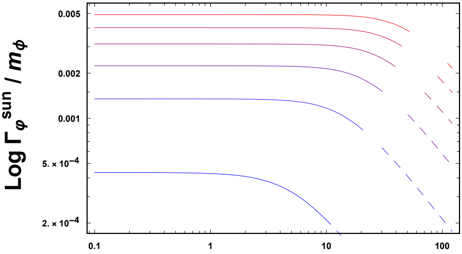
Why does the damping rate decrease with the temperature?

Equilibrium vs. out-of-equilibrium QFT



$$\Gamma_\phi^{\rm tad} = \frac{h^2}{(4\pi)^2}\frac{\phi(t)^2}{T}\int \frac{p^2 dp}{\omega_\chi^2 \Gamma_\chi (\cosh(\omega_\chi/T)-1)}$$

- scales like ϕ^2/T
- resonant enhancement suppressed by increasing Γ_χ width



$$\Gamma_\phi^{\rm sun} = \frac{h^2}{(4\pi)^3}\frac{T^2}{M_\varphi}\log\frac{M_\varphi}{M_\chi}$$

- $\Gamma_\phi^{\rm sun} \ll \Gamma_\phi^{\rm tad}$
- scales like T^2/M_φ

Take home messages

- Main contribution to Γ_ϕ comes from resummed out-of-eq. QFT and comes from a local diagram!
- Main contribution to Γ_ϕ decreases with T!
- This is only a first step!
  - $\rightarrow$ Curved space-time out-of-eq. QFT
  - $\rightarrow$ Derivation + Study of slow-roll parameters

Warm Inflation: Pros and Cons

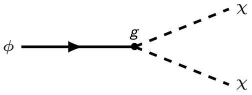
Warm inflation selling points:

+ overdamped inflaton motion $\Rightarrow$ steeper potential allowed
+ no "trans-Planckian effects"
+ no reheating phase needed and smooth evolution towards radiation era

Warm inflation downsides: $\boxed{\text{?? DAMPING vs SCREENING ??}}$

– $\nexists$ clear smoking-guns (tensor-to-scalar ratio??)

– Yokoyama and Linde (Phys.Rev.D60:083509, 1999) ruled out simplest direct coupling models



– State-of-the-art calculations (see e.g. JCAP 1301 (2013) 016) employ questionable methods (Minkowskian, equilibrium propagators, small ϕ-expansions)

Closed-Time-Path Formalism: Why do we need it?

The out-of-equilibrium dynamics of interacting quantum fields is an initial value problem where only the initial density matrix $\hat{\rho}_0$ and its expectation values $\rho_0^n = \langle n|\hat{\rho}_0|n\rangle$ are known.

A. **Quantities of interest:** $\rightarrow$ Statistically weighed quantum averages

$$\langle \hat{\mathcal{O}} \rangle_{\rho_0} \equiv \mathrm{Tr}[\hat{\rho}_0 \hat{\mathcal{O}}] = \sum_n \rho_0^n \langle n|\hat{\mathcal{O}}|n\rangle.$$

B. **Standard methods are not suitable:**

(i) The S-matrix formalism out-states are ill-defined

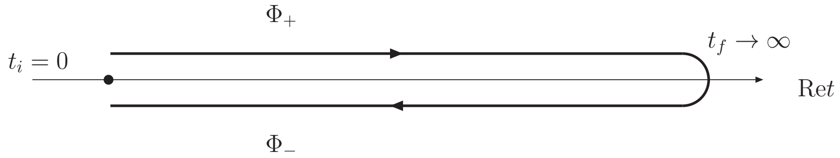
(ii) The S-matrix asymptotically free states also are ill-defined as well, states and particles are never free in high density media

(iii) $\nexists$ Gell-Man/Low formula for non vacuum states $\Rightarrow$ same goes for $\langle \hat{\mathcal{O}} \rangle_{\rho_0}$

$\vdots$

The Schwinger-Keldysh/in-in/CTP formalism

Allowing time arguments to belong to the closed time contour (orientation matters!) perturbation theory is shown to be consistently formulated, avoiding the previously mentioned issues.



Price to pay: $\delta(x^0 - y^0) \to \delta_{\mathcal{C}}(x^0 - y^0)$; $\theta(x^0 - y^0) \to \theta_{\mathcal{C}}(x^0 - y^0)$

$$\varphi(x) \to \begin{pmatrix} \varphi_+(x_+) \\ \varphi_-(x_-) \end{pmatrix} \; ; \; \Delta_F(x,y) \to \begin{pmatrix} \Delta_F(x_+, y_+) & \Delta_{+-}(x_+, y_-) \\ \Delta_{-+}(x_-, y_+) & \Delta_{\bar{F}}(x_-, y_-) \end{pmatrix}$$

$\ldots$

The 2PI Effective Action Definition

$$Z[J,R] = \int \mathcal{D}\phi \exp\left[i\left(S[\phi] + \int_x J_a(x)\phi_a(x) + \frac{1}{2}\int_{xy} R_{ab}(x,y)\phi_a(x)\phi_b(y)\right)\right]$$

$$Z[J,R] = \exp\left[iW[J,R]\right] \quad \Leftrightarrow \quad W[J,R] = -i\ln Z[J,R]$$

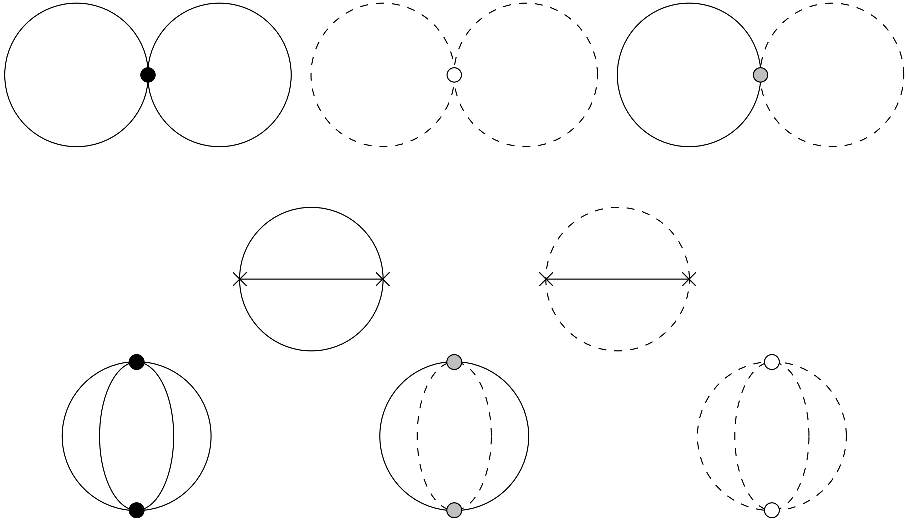
$$\Gamma^R_{1PI}[\varphi] = W[J,R] - \int_x J_a(x)\varphi_a(x).$$

$$\Gamma[\varphi,\Delta] = \Gamma^R_{1PI}[\varphi] - \int_{xy}\frac{\delta\Gamma^R_{1PI}[\varphi]}{\delta R_{ab}(x,y)}R_{ab}(x,y).$$

$$
\begin{aligned}
S_2^{(3)}[\varphi,\Delta] \;=\;& -\frac{i}{2^2}\Bigg[\frac{(-i\lambda_\phi)}{2}\int_x \Delta^2_{\eta\eta}(x,x) + \\
& + \sum_{i=1}^{N_\chi}\left(\frac{(-i\lambda_i)}{2}\int_x \Delta^2_{\chi_i\chi_i}(x,x) + (-ih_i)\int_x \Delta_{\eta\eta}(x)\Delta_{\chi_i\chi_i}(x)\right)\Bigg] \\
& - \frac{i}{2\cdot 3!}\int_{x,y}\left(-i\lambda_\phi\varphi(x)\right)\left(-i\lambda_\phi\varphi(y)\right)\Delta^3_{\eta\eta}(x,y) \\
& - \frac{i}{2\cdot 2}\sum_{i=1}^{N_\chi}\int_{x,y}\left(-ih_i\varphi(x)\right)\left(-ih_i\varphi(y)\right)\Delta^2_{\chi_i\chi_i}(x,y)\Delta_{\eta\eta}(x,y) \\
& - \frac{i}{4!2}\left[(-i\lambda_\phi)^2\int_{x,y}\Delta^4_{\eta\eta}(x,y) + \sum_{i=1}^{N_\chi}(-i\lambda_i)^2\int_{x,y}\Delta^4_{\chi_i\chi_i}(x,y)\right] \\
& - \frac{i}{2^3}\sum_{i=1}^{N_\chi}\int_{x,y}(-ih_i)^2\Delta^2_{\eta\eta}(x,y)\Delta^2_{\chi_i\chi_i}(x,y)\,.
\end{aligned}
$$

2PI Diagrams contributing to $\Pi_\phi(x, y)$

The 2PI diagrams (up to 3 loops) associated with this theory are



Kadanoff-Baym Equations (KBEs)

In terms of the statistical (+) and spectral (-) components, the Feynman propagator decomposes (same goes for the selfenergies)

$$\Delta_{ab}(x,y) = \Delta^{+}_{ab}(x,y) - \frac{i}{2}\Delta^{-}_{ab}(x,y)\,\mathrm{sign}_{\mathcal{C}}(x^0 - y^0)\,,$$

and the CTP-SDEs rewrite

$$\begin{aligned}
\sum_c \Big((\Box + (M_a^{\mathrm{tree}}(x))^2)\delta_{ac} + \Pi^0_{ac}(x,x) \Big) \Delta^{-}_{cb}(x,y) \\
= -\sum_c \int_{y_0}^{x_0} dz \Pi^{-}_{ac}(x,z)\ \Delta^{-}_{cb}(z,y)\,, \\
\sum_c \Big((\Box + (M_a^{\mathrm{tree}}(x))^2)\delta_{ac} + \Pi^0_{ac}(x,x) \Big) \Delta^{+}_{cb}(x,y) \\
= -\sum_c \int_{t_i}^{x_0} dz \Pi^{-}_{ac}(x,z)\Delta^{+}_{cb}(z,y) \\
+ \sum_c \int_{t_i}^{y_0} dz \Pi^{+}_{ac}(x,z)\Delta^{-}_{cb}(z,y)\,,
\end{aligned}$$

KBEs for Z_2-symmetric, homogeneous and isotropic scalars

For Z_2-symmetric scalar, we showed the KBEs to be diagonal at all orders. Fourier transforming the translation invariant spatial part, we get

$$\begin{aligned}
\left(\partial_{t_1}^2 + \omega_a^2(t_1;\mathbf{p})\right)\Delta^-_{aa}(t_1,t_2;\mathbf{p}) &= -\int_{t_2}^{t_1} dt'\, \Pi^-_{aa}(t_1,t';\mathbf{p})\Delta^-_{aa}(t',t_2;\mathbf{p})\,, \\
\left(\partial_{t_1}^2 + \omega_a^2(t_1;\mathbf{p})\right)\Delta^+_{aa}(t_1,t_2;\mathbf{p}) &= -\int_{t_i}^{t_1} dt'\, \Pi^-_{aa}(t_1,t';\mathbf{p})\Delta^+_{aa}(t',t_2;\mathbf{p}) \\
&\quad + \int_{t_i}^{t_2} dt'\, \Pi^+_{aa}(t_1,t';\mathbf{p})\Delta^-_{aa}(t',t_2;\mathbf{p})\,.
\end{aligned}$$

where

$$\omega_a(t_1;\mathbf{p}) \equiv \sqrt{\mathbf{p}^2 + (M_a^{\mathrm{tree}}(t_1))^2 + \int \frac{d^3\mathbf{q}}{(2\pi)^3}\,\Pi^0_{aa}(t_1,t_1;\mathbf{q})} \equiv \sqrt{\mathbf{p}^2 + M_a^2(t_1)}$$

with $M_a(t_1) \equiv \sqrt{(M_a^{\mathrm{tree}}(t_1))^2 + \int \frac{d^3\mathbf{q}}{(2\pi)^3}\,\Pi^0_{aa}(t_1,t_1;\mathbf{q})}$.

WKB solutions to KBEs

Defining

$$\Omega_{a;t} = \sqrt{\omega_a^2(t;\mathbf{p}) + \mathrm{Re}\tilde{\Pi}^-_{aa}(t,\hat{\Omega}_{a;t};\mathbf{p})}\,,$$
$$\Gamma_{a;t} = -\frac{\mathrm{Im}\tilde{\Pi}^-_{aa}(t,\hat{\Omega}_{a;t};\mathbf{p})}{\Omega_{a;t}}\,.$$

We employ the following assumptions (separation of time scales):

1) adiabaticity, i.e. $|\dot{\Omega}_{a;t}/\Omega^2_{a;t}| \ll 1$ and $|\dot{\Gamma}_{a;t}/\Gamma^2_{a;t}| \ll 1$
2) weak damping, i.e. $|\Gamma_{a;t}/\Omega_{a;t}| \ll 1$ due to weak coupling.
3) $\Pi^-_{aa}(t_1,t_2;\mathbf{p})$ and $\Pi^+_{aa}(t_1,t_2;\mathbf{p})$ have a finite support, i.e., they quickly approach zero for $|t_1 - t_2| \gtrsim \tau_{\mathrm{int}}$ with τ_{int} being some characteristic time, which can be defined as an interaction time or duration of e.g. scattering events.
4) $\Omega_{a;t}$ and $\Gamma_{a;t}$ can be approximately regarded as constant over the support of $\Pi^-_{aa}(t_1,t_2;\mathbf{p})$ and $\Pi^+_{aa}(t_1,t_2;\mathbf{p})$.

WKB solutions to KBEs

$$\Delta^-_{aa}(t_1, t_2; \mathbf{p}) \simeq \frac{\sin\left(\int_{t_2}^{t_1} dt' \Omega_{a;t'}\right) e^{-\frac{1}{2}\left|\int_{t_2}^{t_1} dt' \Gamma_{a;t'}\right|}}{\sqrt{\Omega_{a;t_1}\Omega_{a;t_2}}},$$

$$\Delta^+_{aa}(t_1, t_2; \mathbf{p}) \simeq \frac{-\cos\left(\int_{t_2}^{t_1} dt' \Omega_{a;t'}\right) e^{-\frac{1}{2}\left|\int_{t_2}^{t_1} dt' \Gamma_{a;t'}\right|}}{\sqrt{\Omega_{a;t_1}\Omega_{a;t_2}}} \times$$

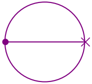
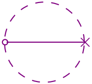
$$\times \int_{-\infty}^{t_B} d\tau \, \frac{\mathrm{Re}\tilde{\Pi}^+_{aa}(\tau, \hat{\Omega}^*_{a;\tau}; \mathbf{p})}{\Omega_{a;\tau}} \, e^{-\int_\tau^{t_B} dt' \Gamma_{a;t'}} \, .$$

Homogeneous, isotropic, slowly rolling background field

From the 2PI effective action

$$\left(\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu) + m_\varphi^2\right)\phi(x) = \int_y \Pi_\phi(x,y)\phi(y)$$

where

$\Pi_\phi(x,y) = -\frac{\lambda_\phi}{3!}\phi(x)^2\delta(x-y)$ +  +  +  + 

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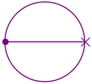
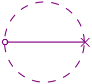
$$\Pi_\phi(x,y) = -\tfrac{\lambda_\phi}{3!}\phi(x)^2\delta(x-y) \; +$$  $+$  $+$  $+$ 

Under (**i**) **homogeneous** and (**ii**) **slow-rolling** assumptions, namely

$$(i)\ \phi(x) \simeq \phi(t) \qquad\qquad (ii)\ \phi(t') \simeq \phi(t) + (t'-t)\dot{\phi}(t),$$

the EOM for the background field rewrites (FLRW metric)

$$\left(\partial_t^2 + 3H\partial_t + m_\varphi^2\right)\phi(t) + \dot{\phi}(t)\int_y (t-t')\Pi_\phi(t,y) = \phi(t)\int_y \Pi_\phi(t,y).$$

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$$\left(\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu) + m_\varphi^2\right)\phi(x) = \int_y \Pi_\phi(x,y)\phi(y)$$

  where quantum and thermal corrections are contained in Π_ϕ. The damping term and the effective potential are there to be found as well.

- Upon defining $\Pi = 2i\frac{\delta S_2}{\delta \Delta}$, the EOM for the propagators rewrites

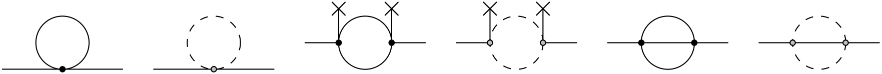
$$\Delta^{-1} = \Delta_0^{-1} - \Pi \Rightarrow$$

Idea: Analytically solve the SDEs and insert the **resummed** nonequilibrium propagators to calculate the corrections to the background field dynamics.

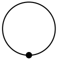
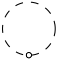
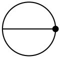
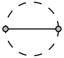
Diagrammatics of the EOMs

Having calculated the effective action at three-loop order $S^{(3)}_{\text{eff}}[\phi, \Delta]$, one can determine the diagrammatic expansion of the EOMs.

- Taking the functional derivative w.r.t. Δ cuts internal lines open:



- Taking the functional derivative w.r.t. ϕ, only diagrams with ϕ-couplings contribute, as well as the Coleman/Weinberg term:

  $\Pi_\phi(x, y) \supset \lambda_\phi \phi(x)^2 +$  $+$  $+$  $+$ 

  where resummed propagators run in the loops.