Large radiative effects on dark matter annihilation resummed

Martin Vollmann
Lehrstuhl für Theoretische Elementarteilchenphysik (T-31)

Based on
Beneke, Broggio, Hasner, Urban, MV
arXiv:1903.08702
Beneke, Broggio, Hasner, MV
arXiv:1805.07367

Outline

- Motivation
- Gamma rays from DM annihilation
- Sommerfeld and Sudakov resummation
- Factorization formulas for the wino model
- Conclusions

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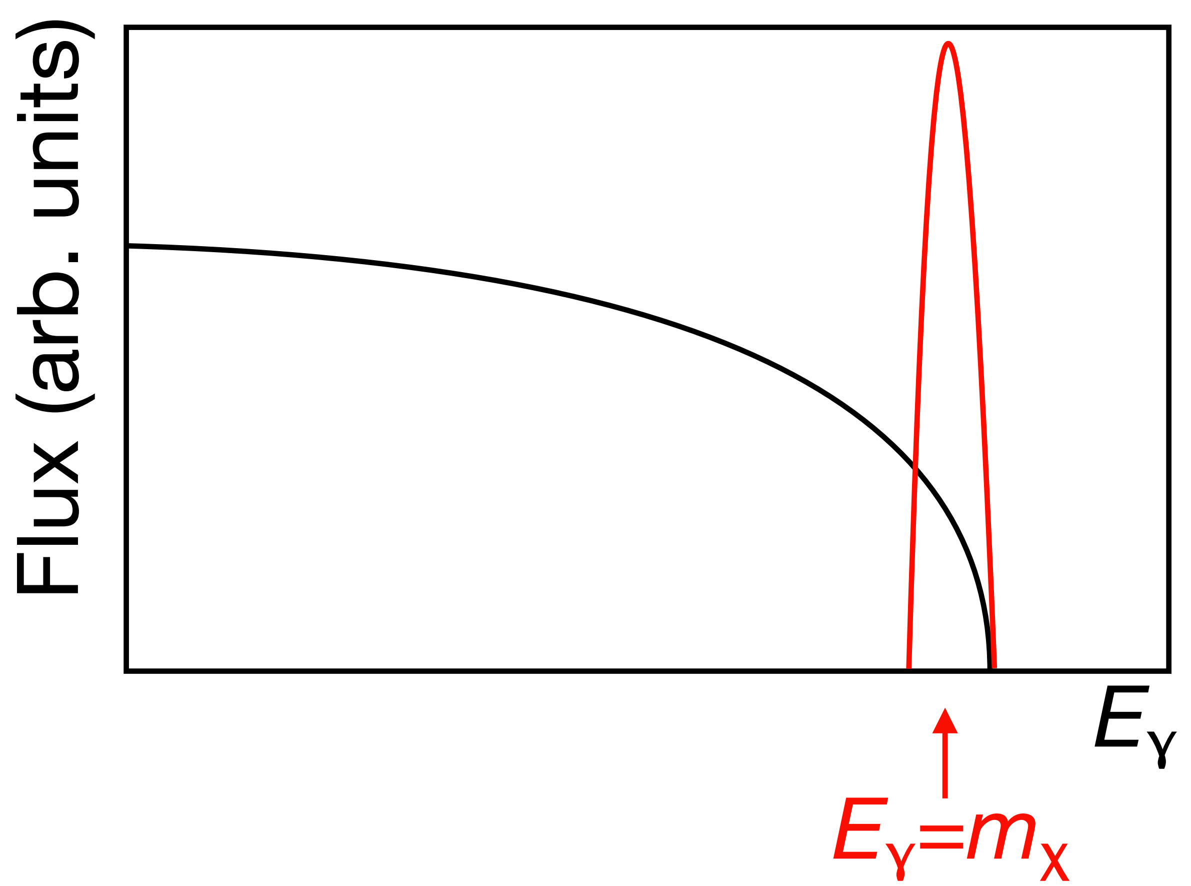
Motivation

- WIMP DM paradigm is very well motivated and scrutinized
- No discovery so far $\Rrightarrow$ $\mathcal{O}$(1-100GeV) wimp models are subject to stringent constraints
- Above-TeV wimps ‘start’ to become attractive

Motivation

- Heavy ($\mathcal{O}$(1-100TeV)) DM → Indirect detection
- Spectral-line feature in gamma ray spectrum is a smoking-gun signature of WIMP DM annihilation

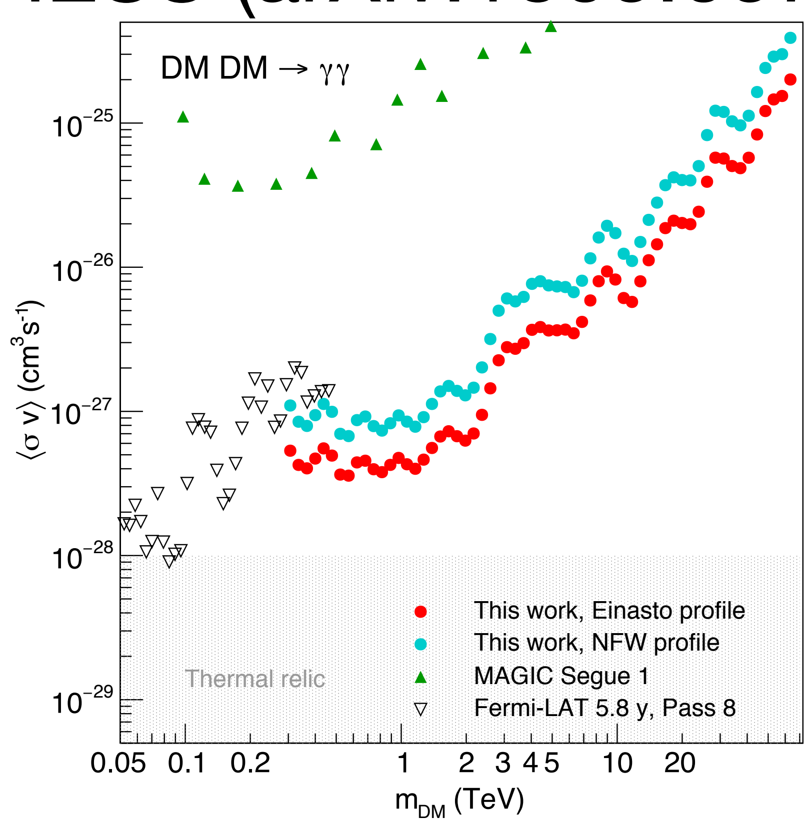


Motivation

- Current- and next-generation gamma-ray telescopes will search for such spectral lines
- Particularly promising is the Cherenkov Telescope Array (**CTA**) with ~1 order of magnitude improved sensitivity w.r.t. current technology

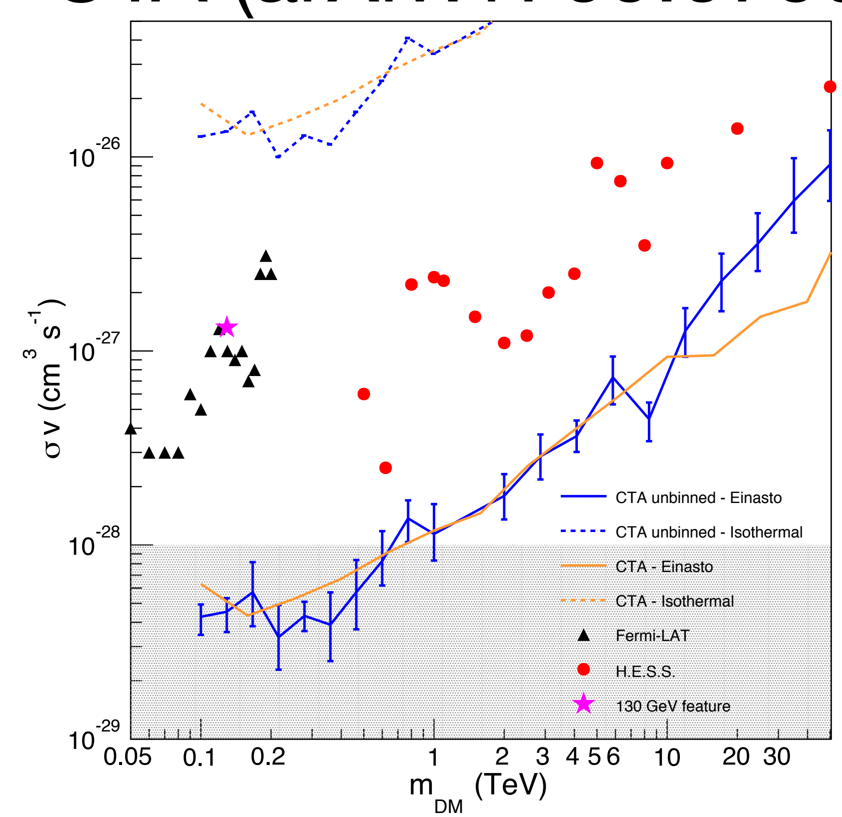
State of the art

HESS (arXiv:1805.05741)



Projected (500hrs)

CTA (arXiv:1709.07997)



Motivation

- Annihilation cross section computations for heavy wimps can be intricate
- Non-perturbative effects such as the *Sommerfeld* effect play a major role in their determination
- On top of this, large electroweak *Sudakov* double logarithms invalidate the perturbative expansion and need to be resummed
- In this talk I focus on the latter (see Kai Urban's talk on the *Sommerfeld* effect!)

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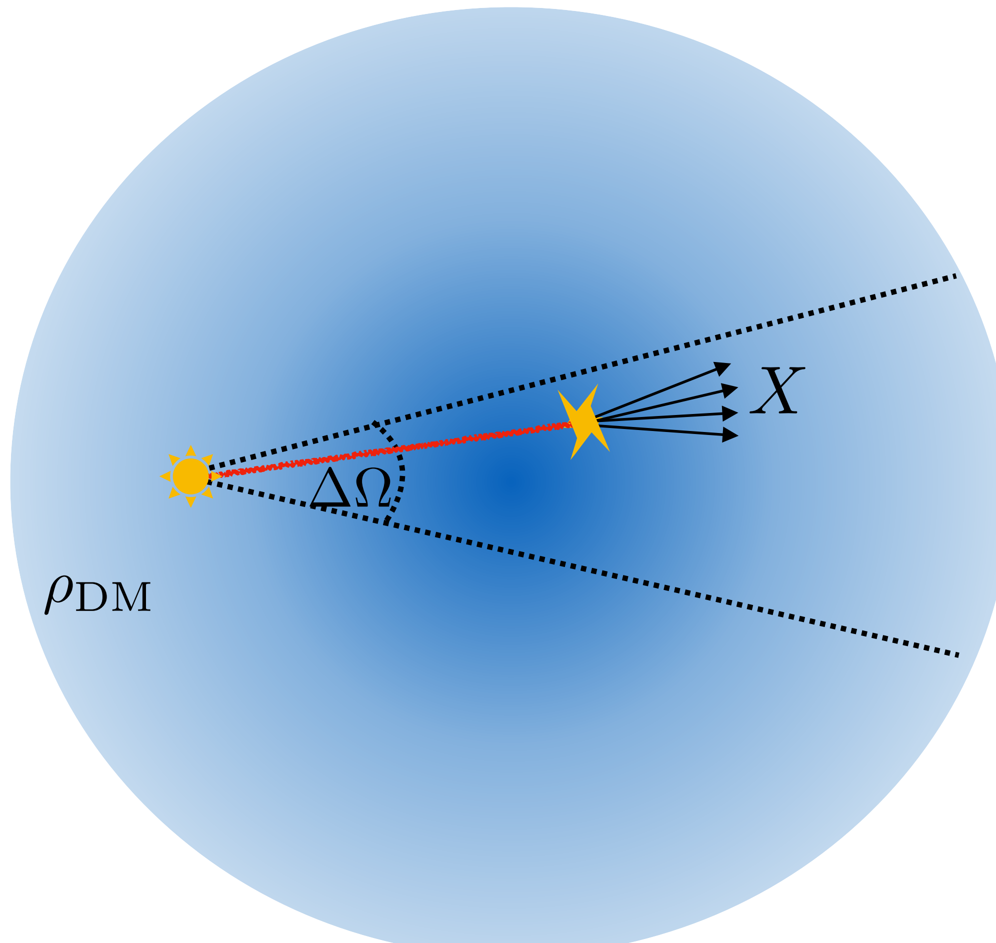
Gamma rays from dark matter annihilation



$$\Phi(E_\gamma) = \frac{1}{8\pi m_{\rm DM}^2} \int_{\Delta\Omega} \mathrm{d}\Omega \int_{\rm l.o.s.} \mathrm{d}s \rho_{\rm DM}^2(\boldsymbol{r}(s)) \frac{\mathrm{d}}{\mathrm{d}E_\gamma} [\sigma v]_{\gamma+X}$$

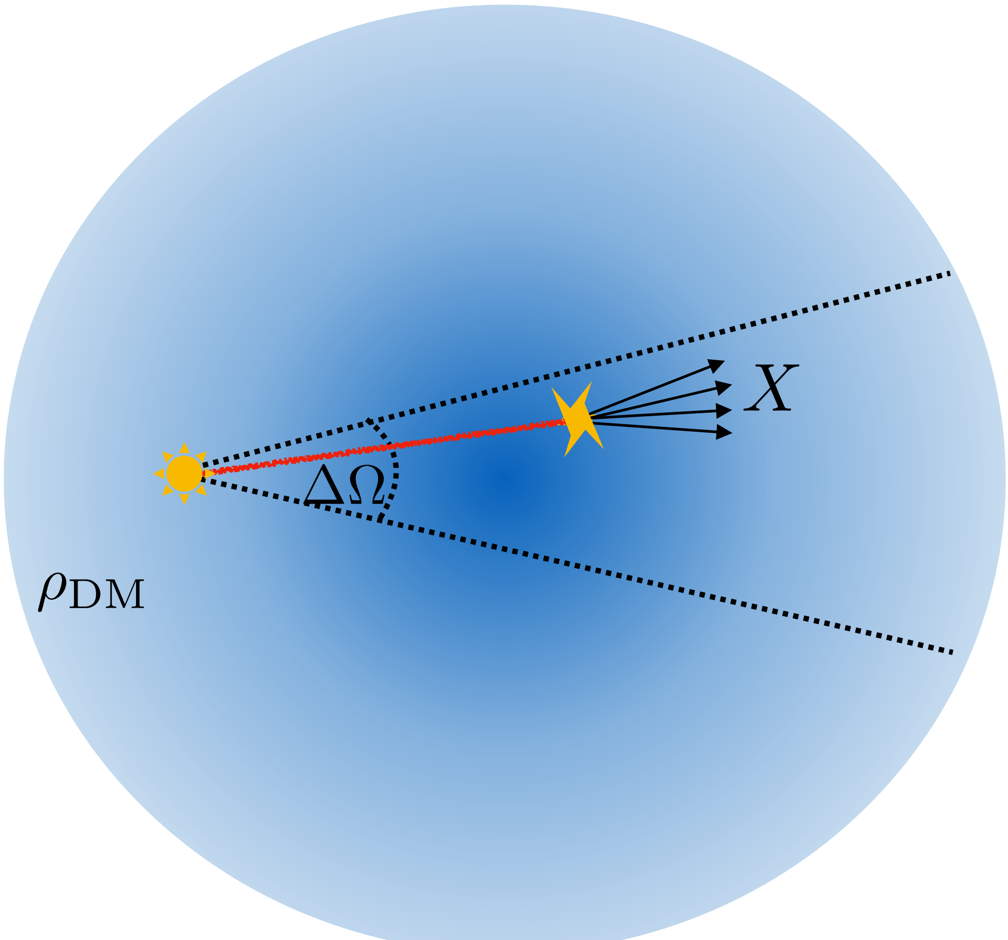
γ rays from dark matter annihilation. Multi-scale problem

$R_\odot \Delta\theta_{\mathrm{obs}}$ $\qquad$ $\lambda_{\mathrm{soft}\,\gamma} \sim (\Delta E^{\gamma}_{\mathrm{obs}})^{-1}$

$10^{20}\,\mathrm{m}$ $\quad$ $10^{17}\,\mathrm{m}$ $\qquad$ $10^{-17}\,\mathrm{m}$ $\quad$ $10^{-20}\,\mathrm{m}$

$r_s,\, R_\odot$ $\qquad$ $\lambda_{\mathrm{DM}} \sim m_{\mathrm{DM}}^{-1}$

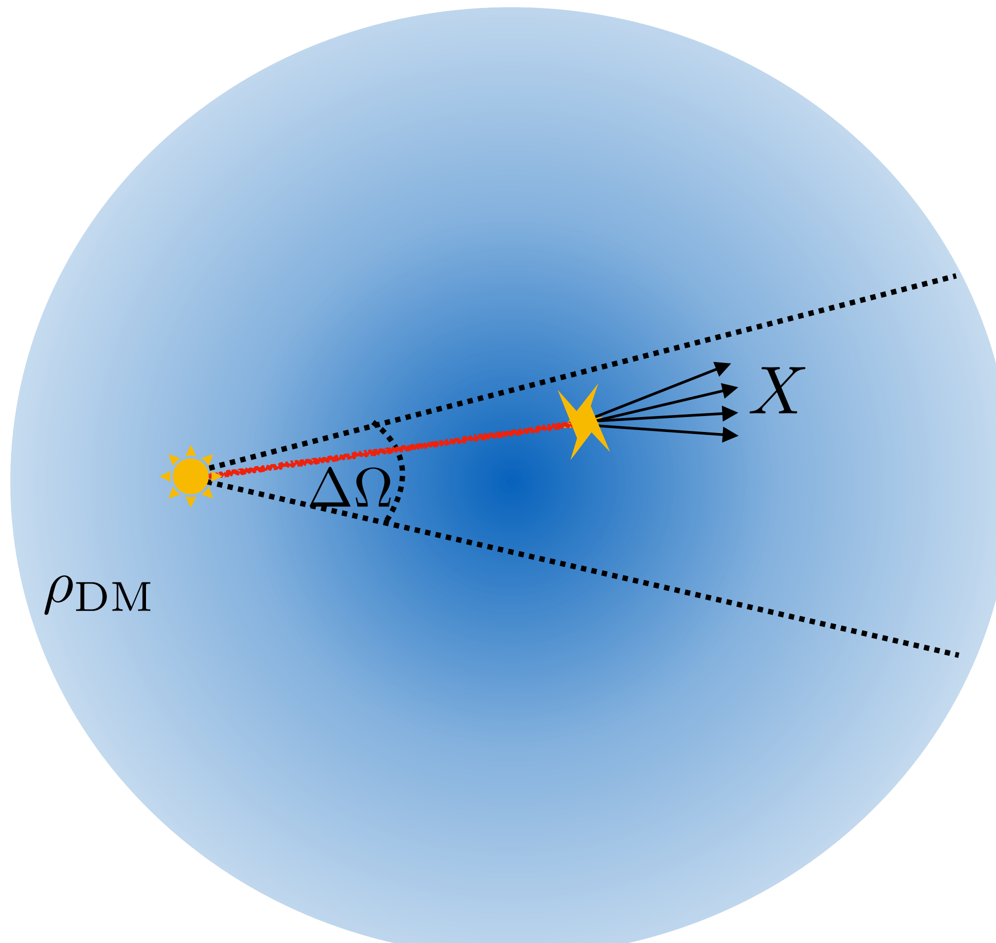
γ rays from dark matter annihilation. 1st factorization



$$\Phi(E_\gamma) = \frac{1}{8\pi m_{\mathrm{DM}}^2} \underbrace{\int_{\Delta\Omega} \mathrm{d}\Omega \int_{\mathrm{l.o.s.}} \mathrm{d}s\, \rho_{\mathrm{DM}}^2(\boldsymbol{r}(s))}_{J(\Delta\Omega)} \frac{\mathrm{d}}{\mathrm{d}E_\gamma}[\sigma v]_{\gamma+X}$$

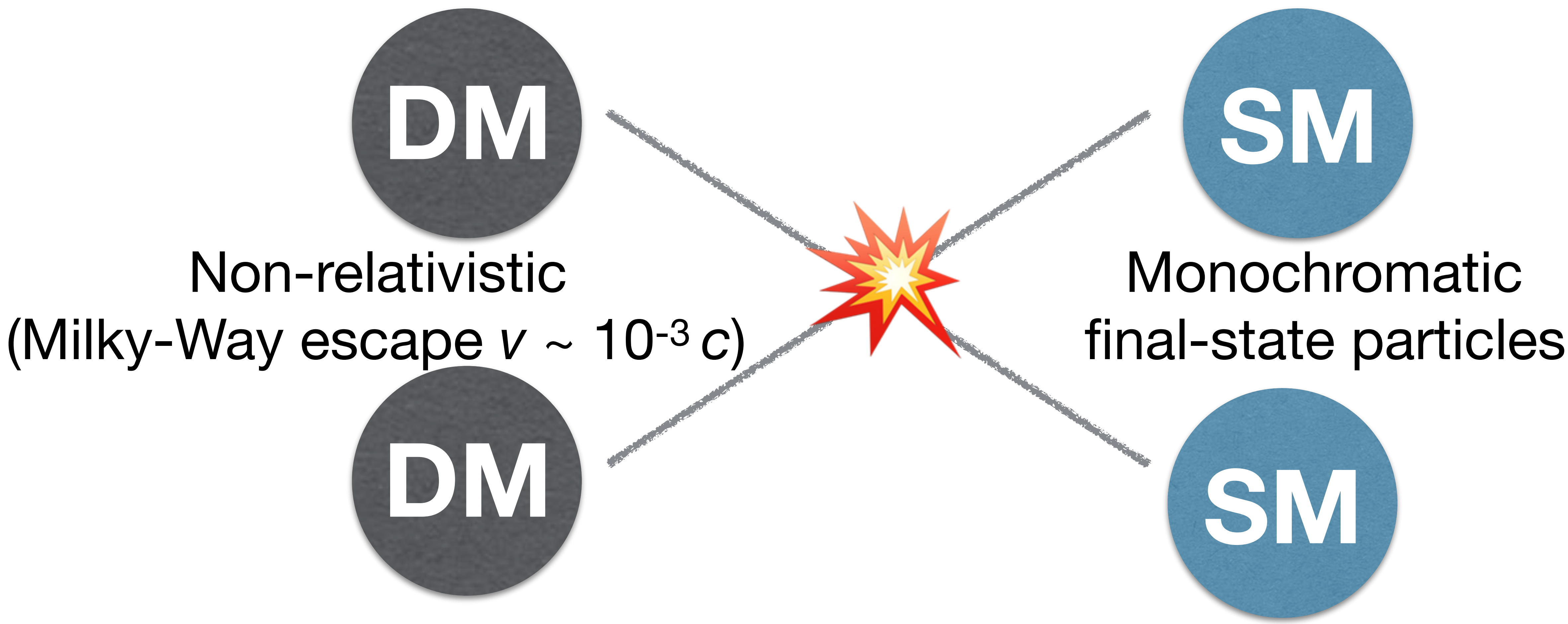
Astrophysical “*J*” factor
independent of gamma-ray energy

γ rays from dark matter annihilation. PP term



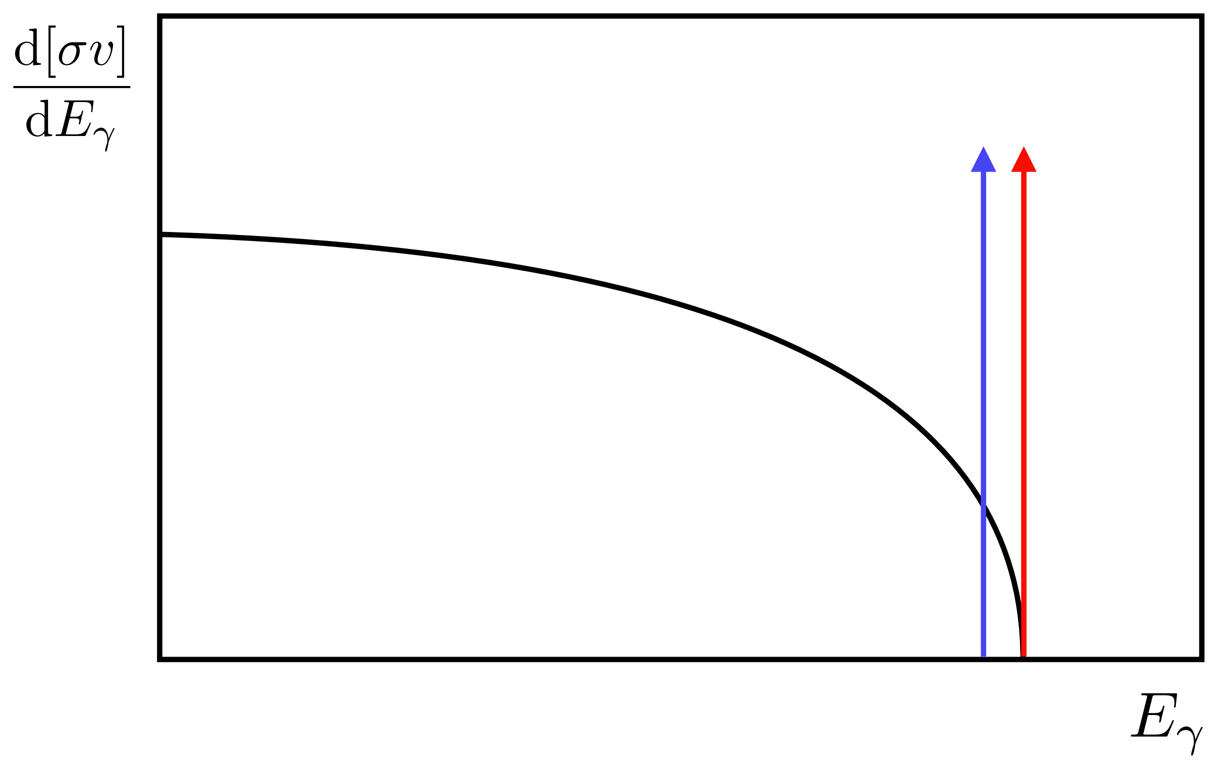
$$\Phi(E_\gamma) = \frac{1}{8\pi m_{\mathrm{DM}}^2} \underbrace{\int_{\Delta\Omega} \mathrm{d}\Omega \int_{\mathrm{l.o.s.}} \mathrm{d}s\, \rho_{\mathrm{DM}}^2(\boldsymbol{r}(s))}_{J(\Delta\Omega)} \frac{\mathrm{d}}{\mathrm{d}E_\gamma}[\sigma v]_{\gamma+X}$$

γ rays from dark matter annihilation. Endpoint spectrum



γ rays from dark matter annihilation. Endpoint spectrum

$$\frac{\mathrm{d}}{\mathrm{d}E_\gamma}[\sigma v]_{\gamma+X} = 2[\sigma v]_{\gamma\gamma}\delta(E_\gamma - m_{\mathrm{DM}}) + [\sigma v]_{\gamma Z}\delta(E_\gamma - E_0^{\gamma Z}) + \\ + \frac{\mathrm{d}}{\mathrm{d}E_\gamma}[\sigma v]_{\gamma+N\geq 2-\mathrm{bodies}}$$

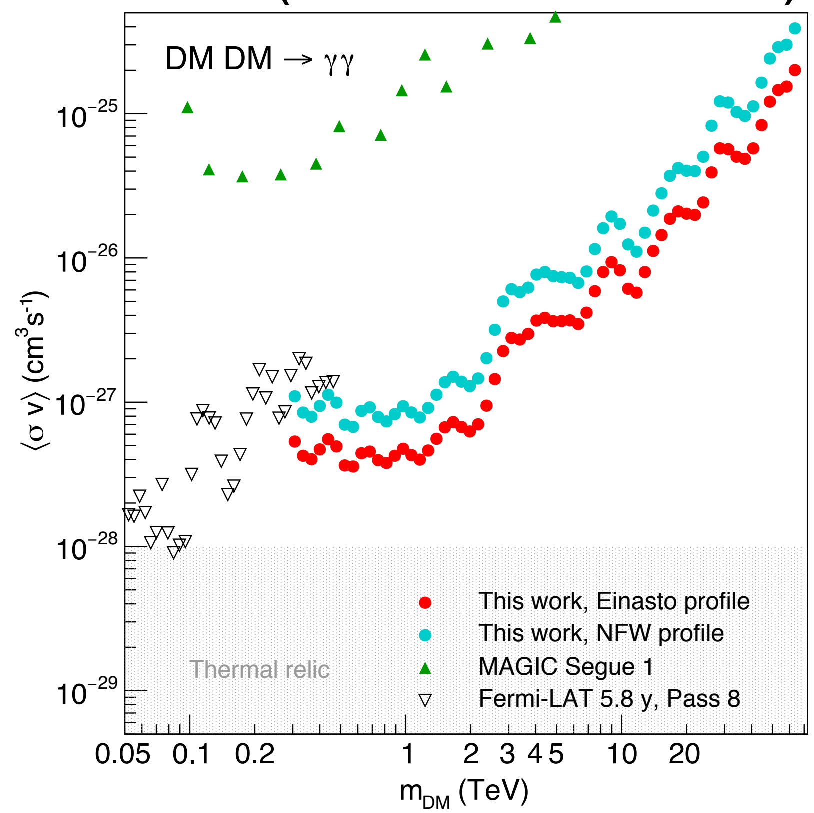


γ rays from dark matter annihilation. Endpoint spectrum

$$\frac{\mathrm{d}}{\mathrm{d}E_\gamma}[\sigma v]_{\gamma+X} = 2[\sigma v]_{\gamma\gamma}\delta(E_\gamma - m_{\mathrm{DM}})$$

State of the art

HESS (arXiv:1805.05741)



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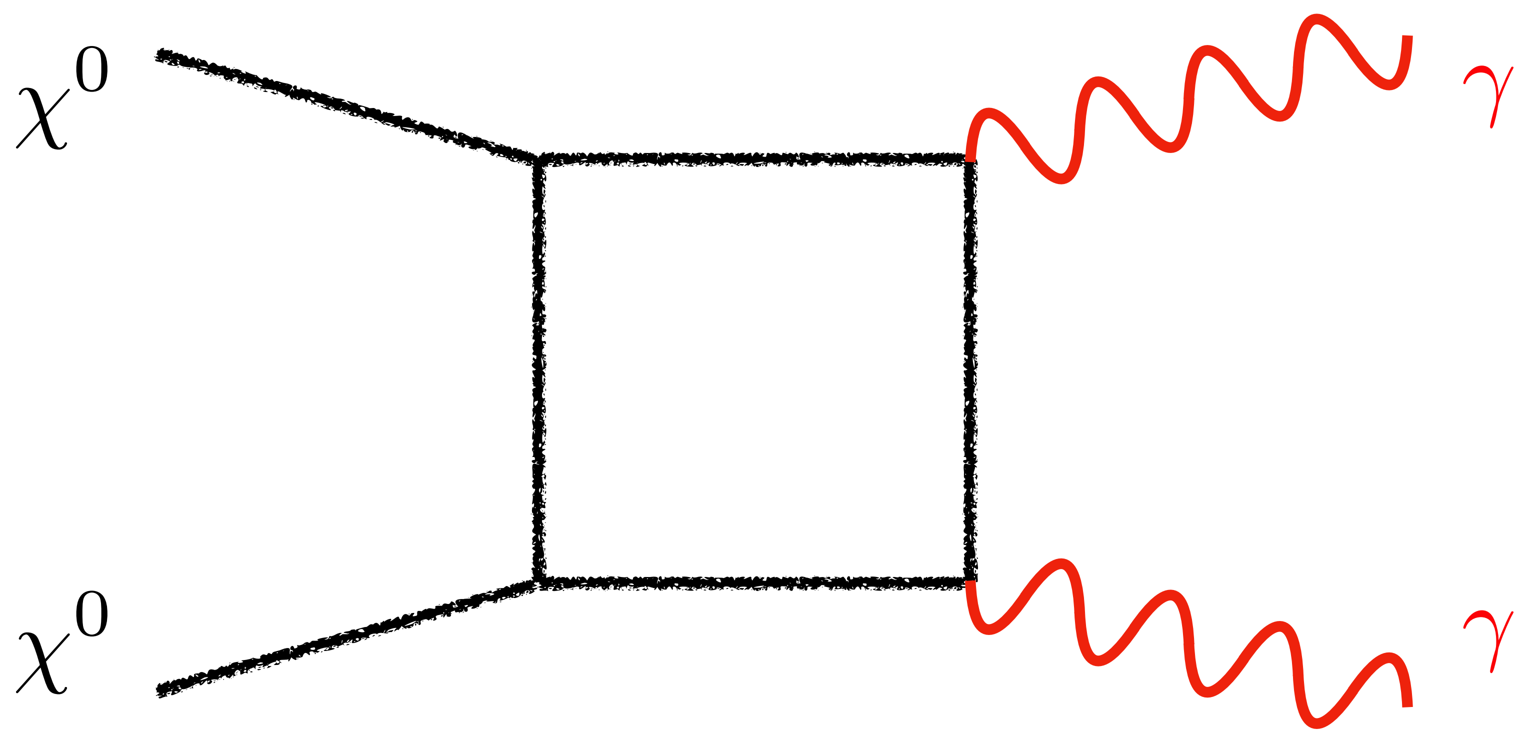
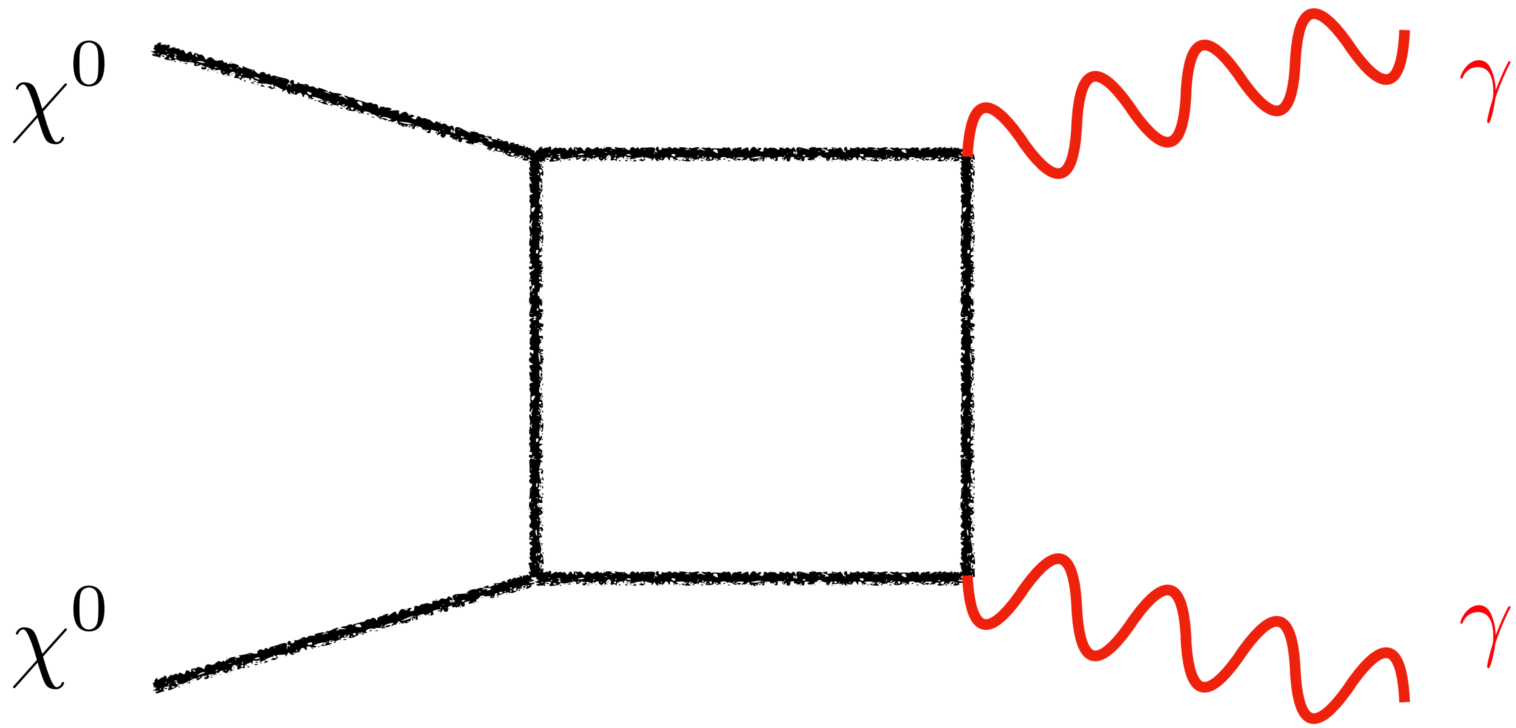
Naive computation of $\sigma v_{\gamma\gamma}$



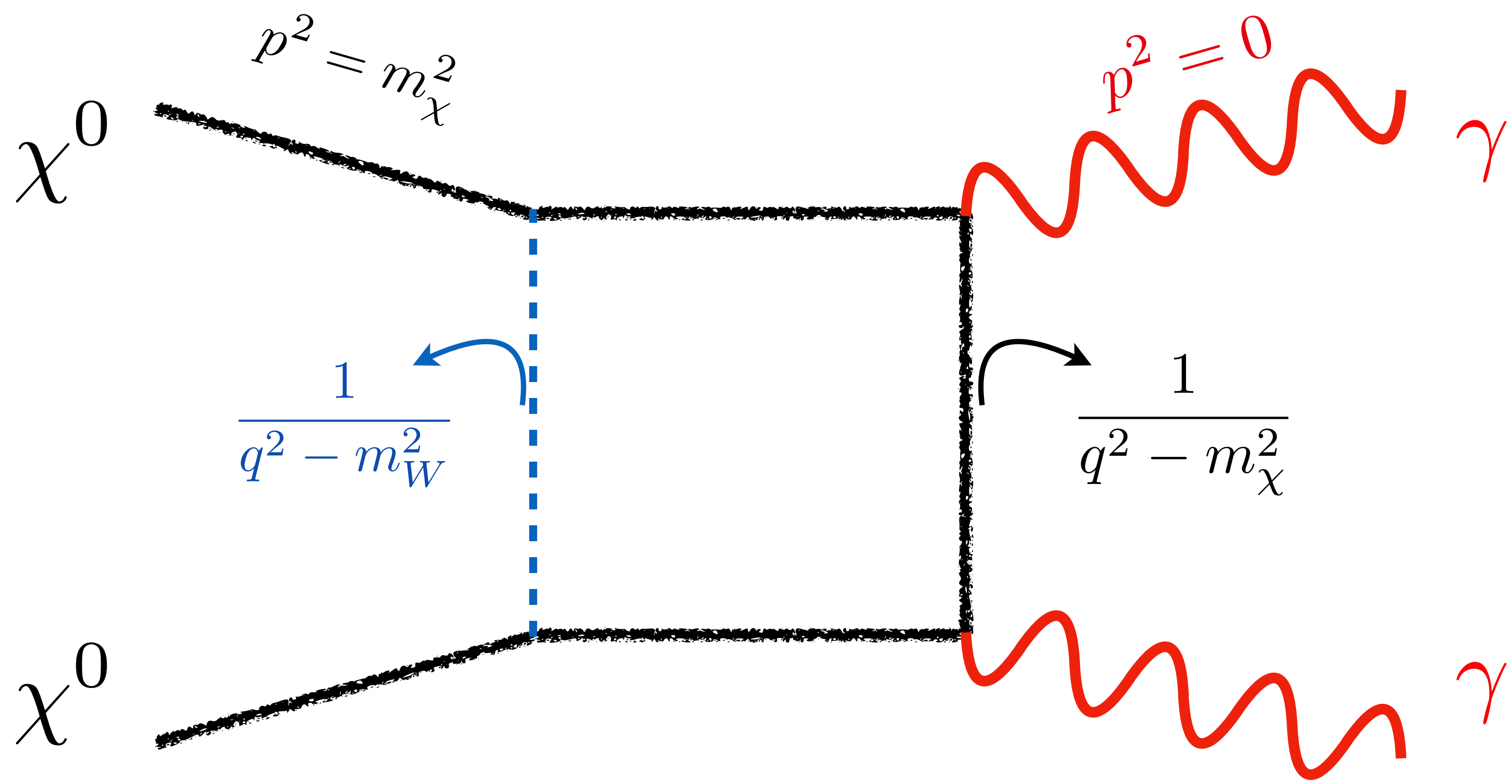
Naive computation of $\sigma v_{\gamma\gamma}$



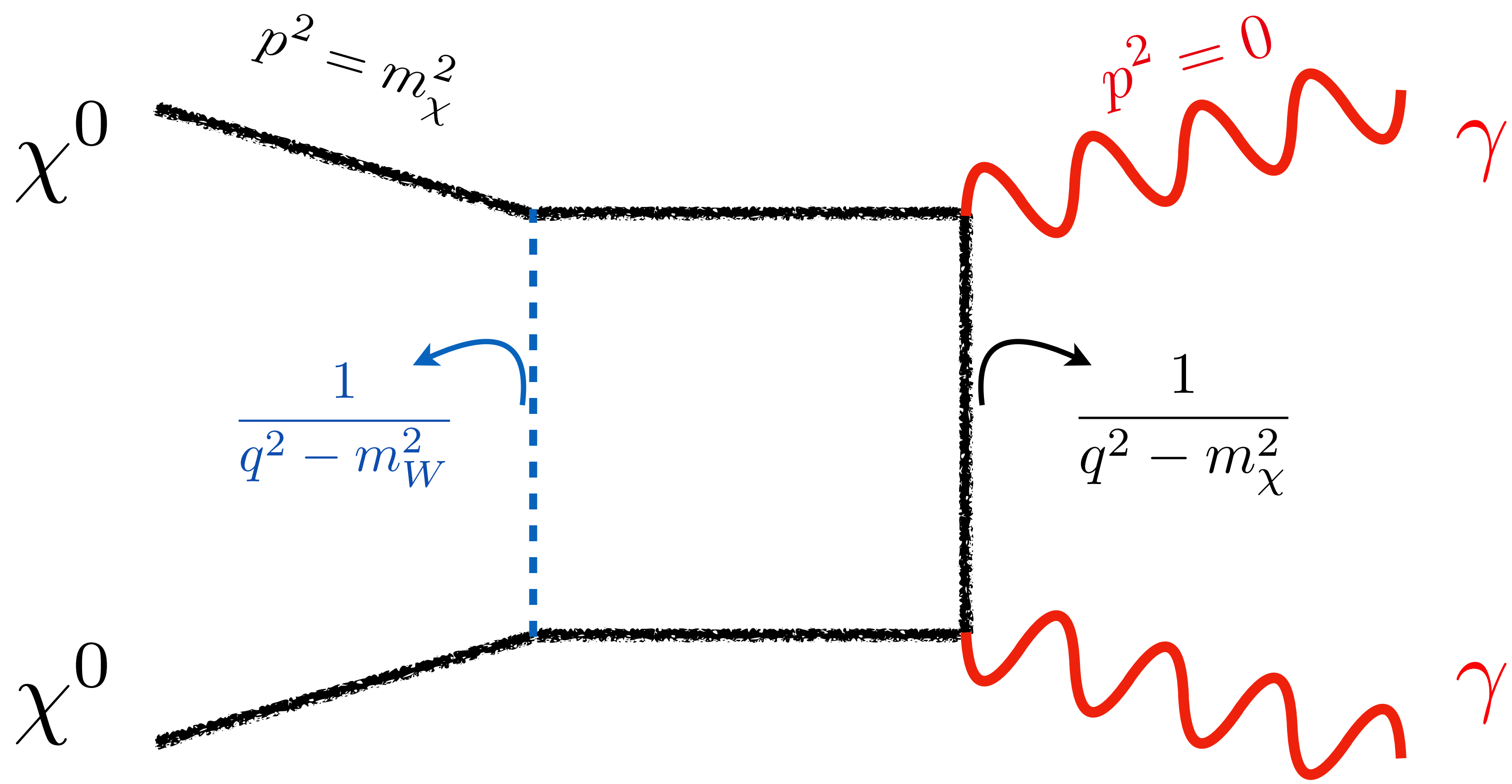
(naively) loop and mass suppressed → $\Phi(E_\gamma) \sim \left(\frac{\alpha_{\rm EW}}{m_{\rm DM}}\right)^4$

(do not give up yet!!)

Naive computation of $\sigma v_{\gamma\gamma}$



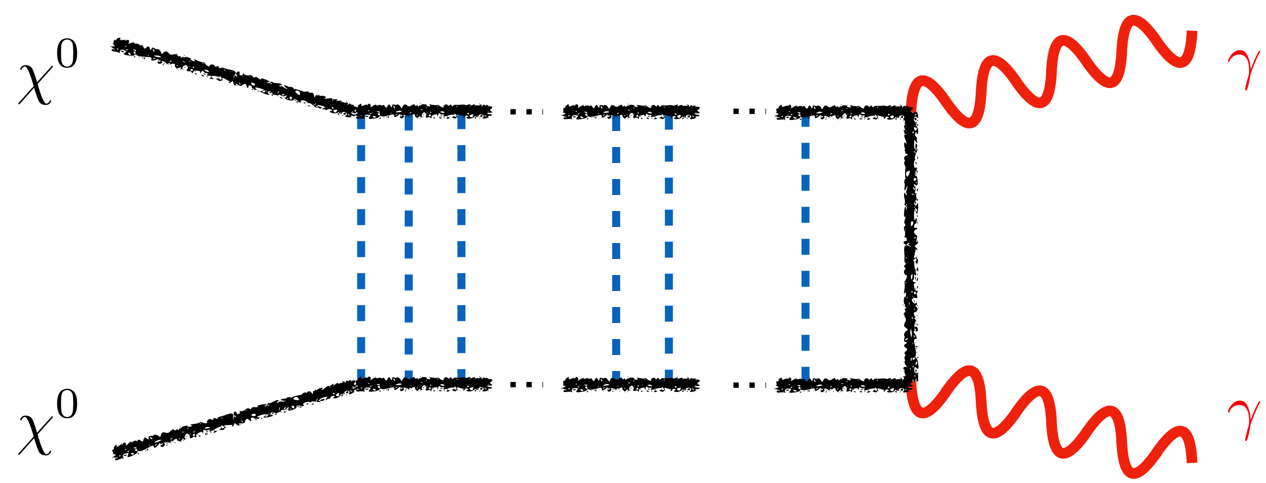
Naive computation of $\sigma v_{\gamma\gamma}$



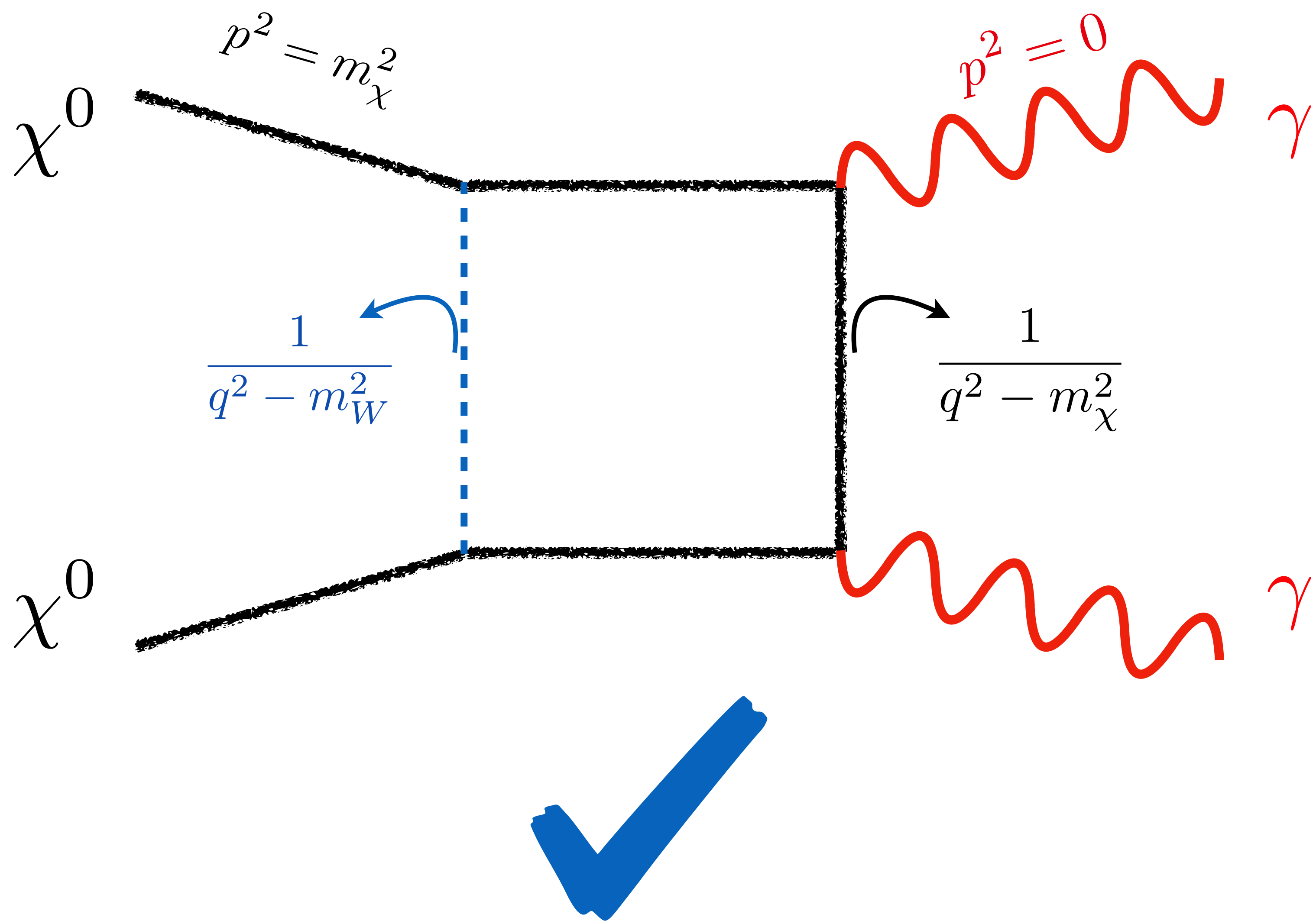
$$\mathcal{M}_{\mathrm{So}} \sim \frac{g^4 m_\chi^2}{m_W^2} \gg g^2$$

Naive computation of $\sigma v_{\gamma\gamma}$

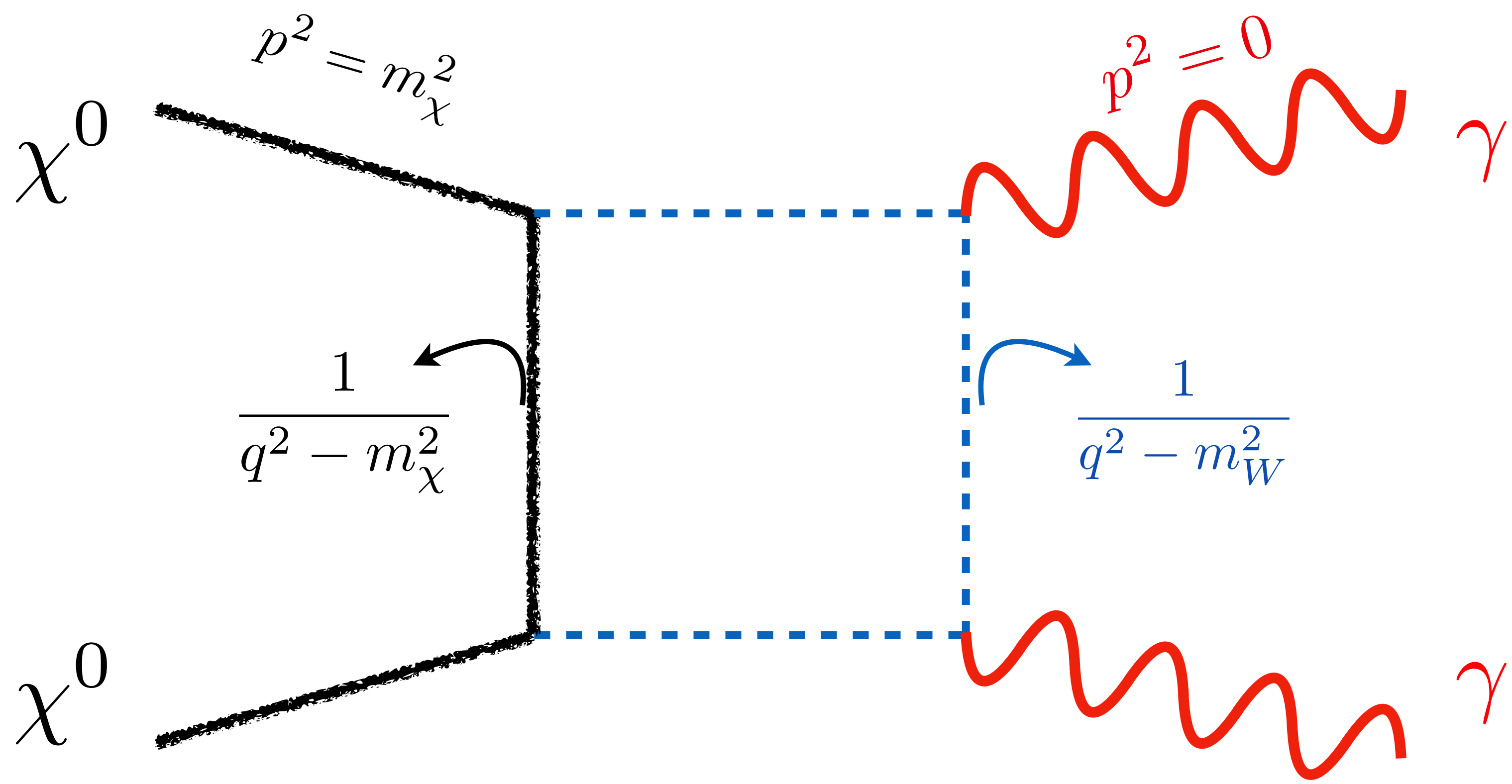
Solution: resum all ladder-like diagrams by matching onto a non-relativistic effective theory



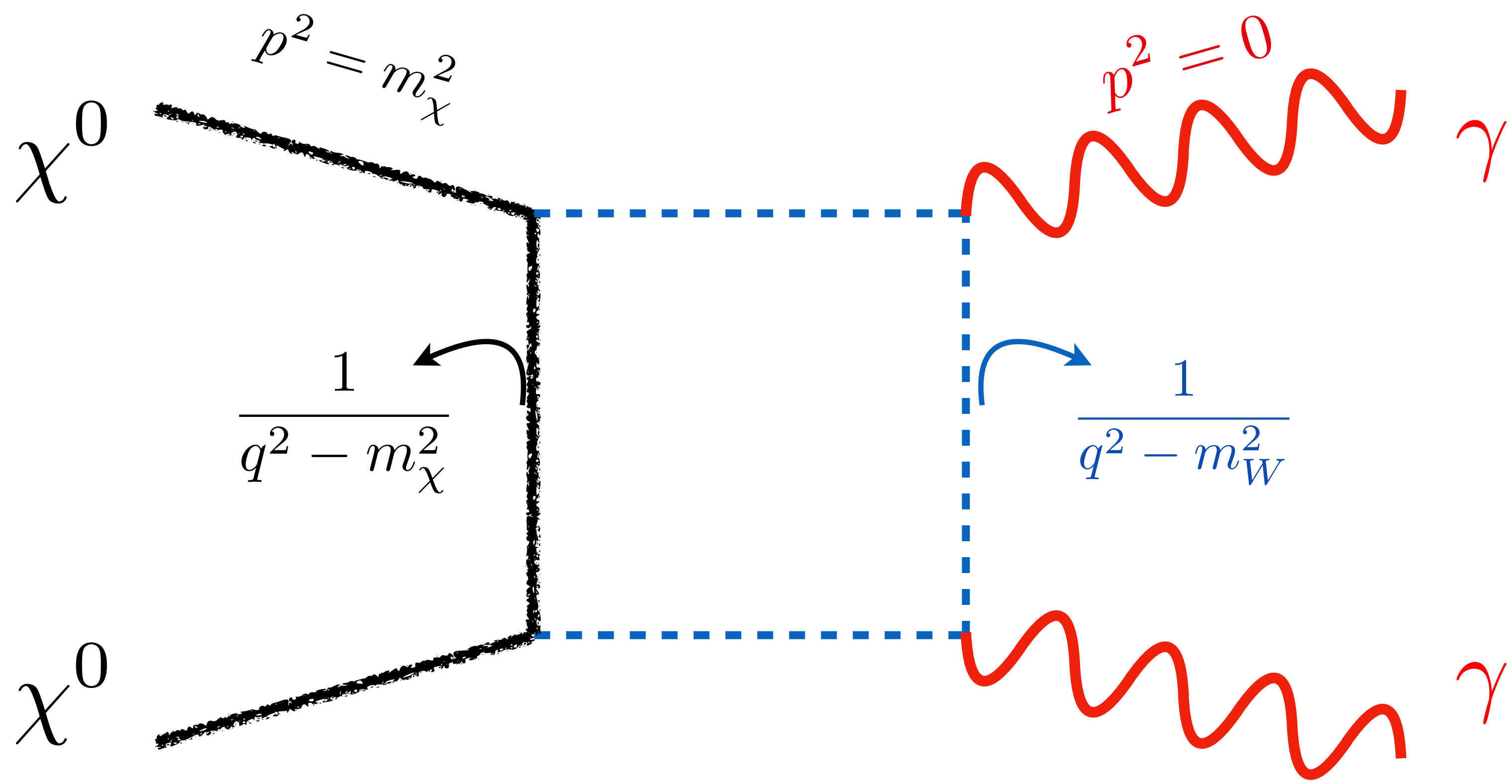
Got interested?
Look forward to the next talk by Kai Urban!!



Naive computation of $\sigma v_{\gamma\gamma}$



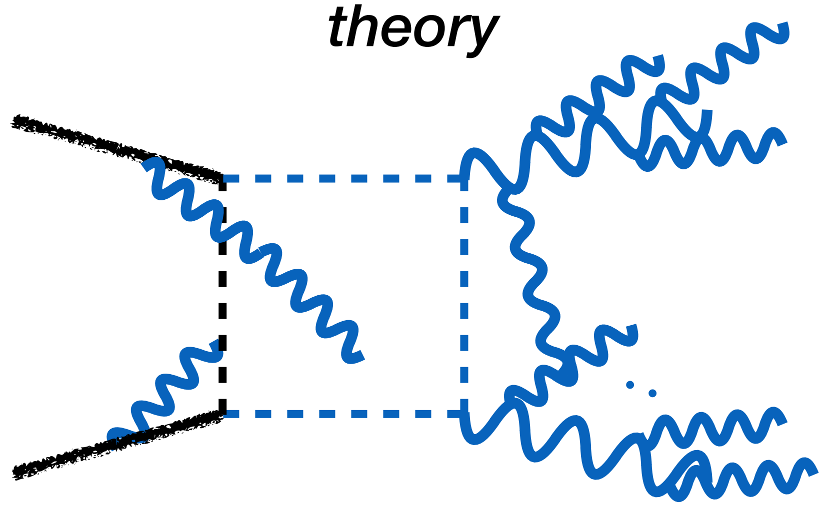
Sudakov double logarithms



$$\mathcal{M} \sim g^4 \log^2 \frac{4m_\chi^2}{m_W^2} \gg g^2$$

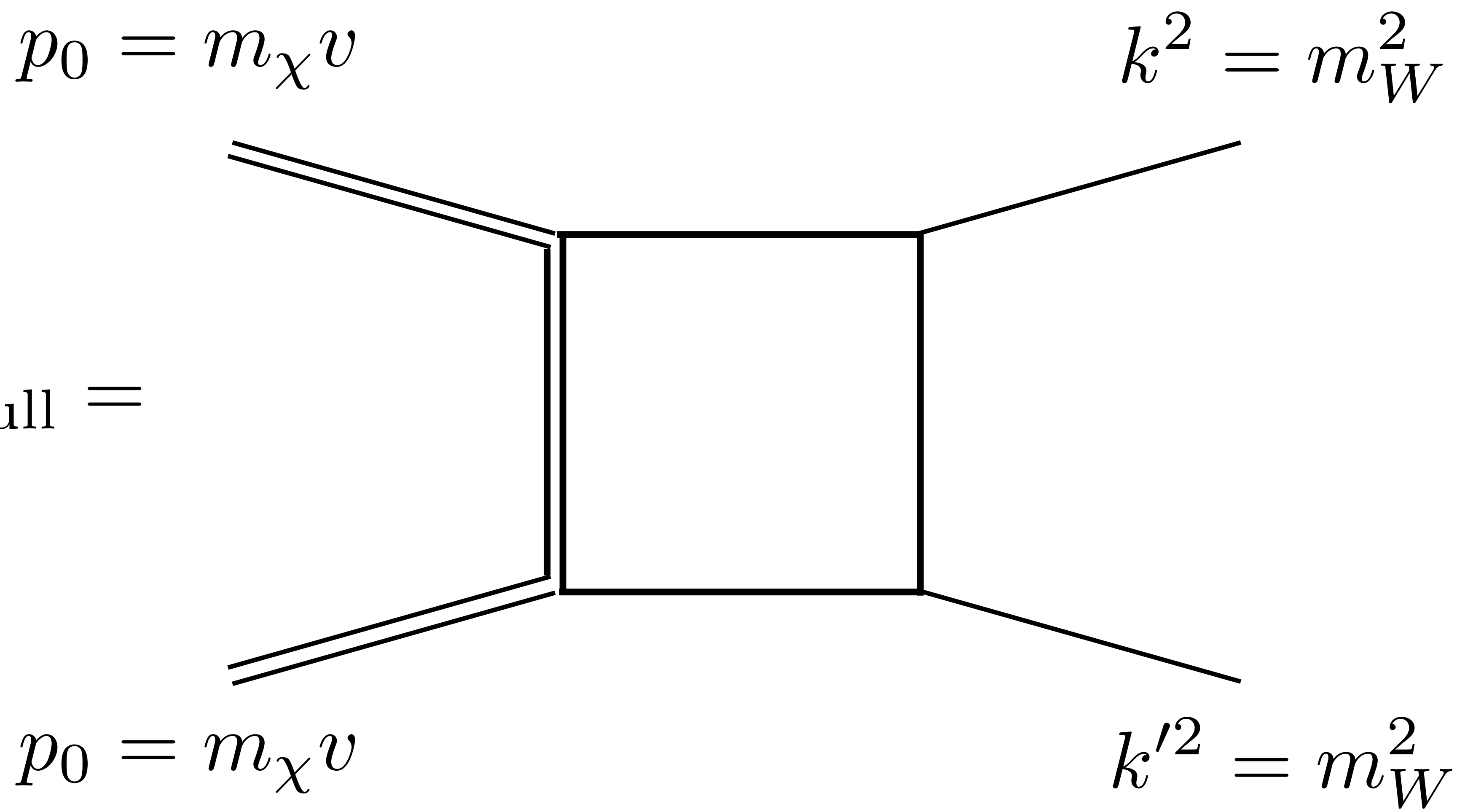
Sudakov-log resummation

Standard solution: resum soft virtual and real emissions by solving renormalization group eqs. in a *(soft-collinear) effective field theory*

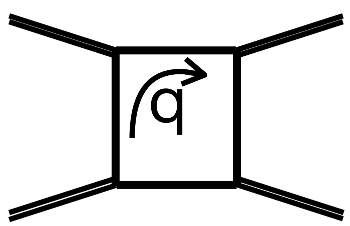


Soft-collinear effective theory (SCET). Method of regions

$I_{\text{full}} =$



SCET. Momentum regions

$I_{\text{full}} =$  $= \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{(q+k-p_0)^2 - m_\chi^2} \frac{1}{(q+k)^2} \frac{1}{q^2} \frac{1}{(q-k')^2}\Big|_{k^2, k'^2 \sim m_W^2 \ll m_\chi^2}$

Light-cone coordinates $\quad q = q_c n + q_{\bar{c}} \bar{n} + q_\perp \longrightarrow (q_c, q_{\bar{c}}, q_\perp)$

Momentum modes

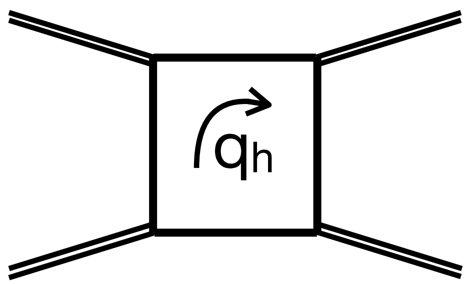
$$q_h \sim m_\chi (1,1,1) \qquad q_s \sim m_W (1,1,1)$$

$$q_{hc} \sim (m_W, m_\chi, \sqrt{m_\chi m_W}) \qquad q_{h\bar{c}} \sim (m_\chi, m_W, \sqrt{m_\chi m_W})$$

$$q_c \sim \left(\frac{m_W^2}{m_\chi}, m_\chi, m_W\right) \qquad q_{\bar{c}} \sim \left(m_\chi, \frac{m_W^2}{m_\chi}, m_W\right)$$

SCET. Momentum regions

$I_{\text{full}} =$



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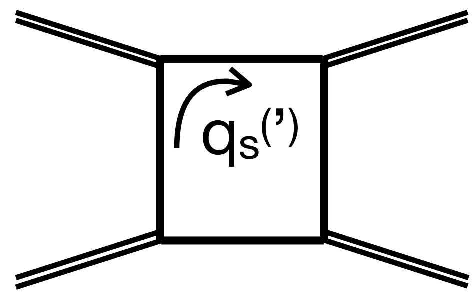

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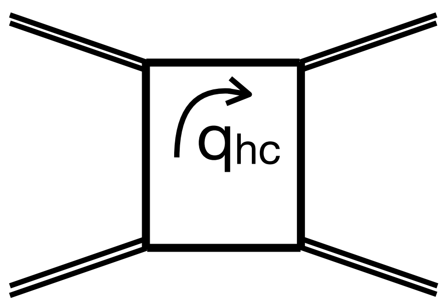

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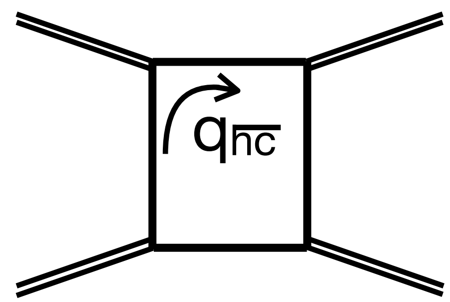
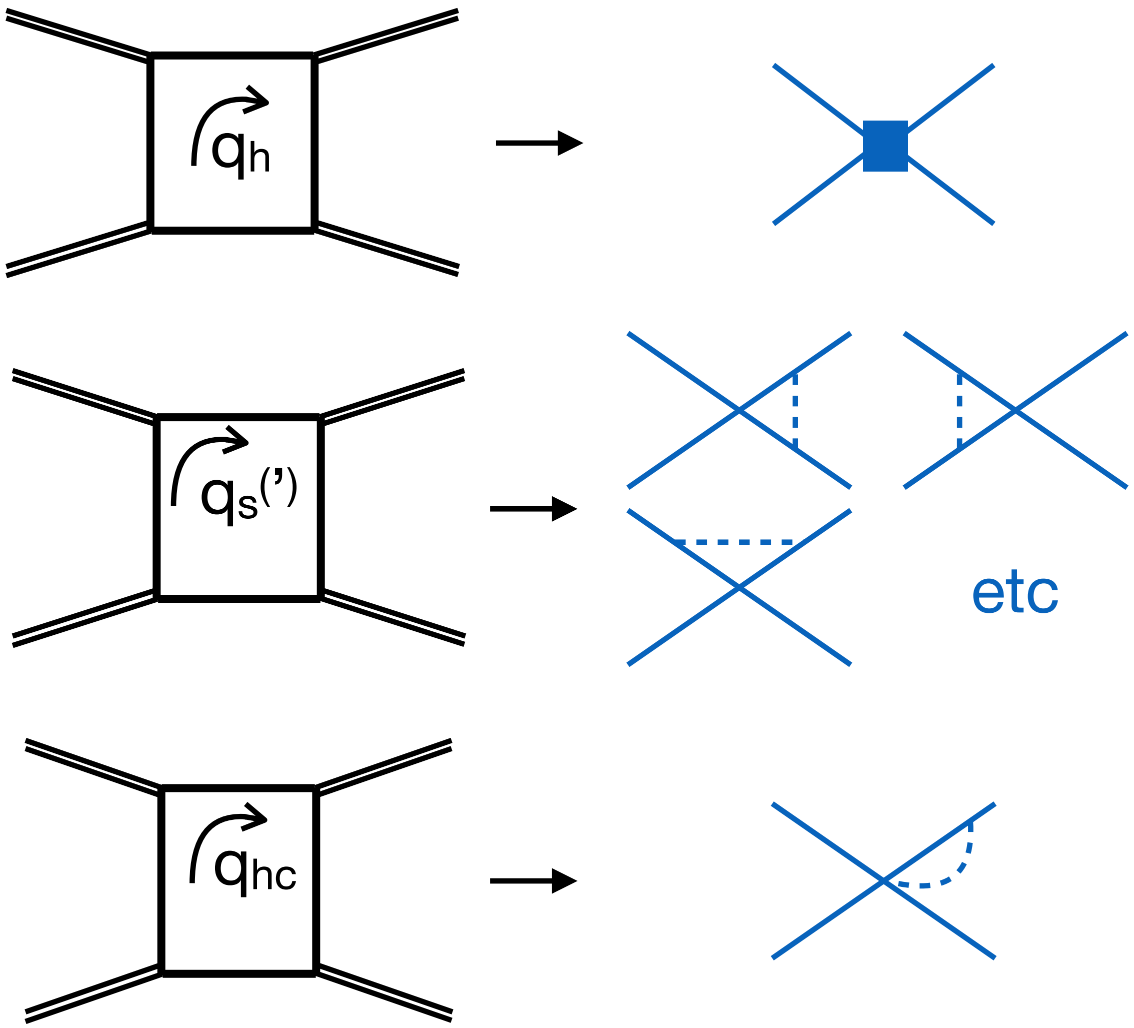


$\Big|_{k^2=0}$

+ power corrections

SCET. Factorization (narrow resolution)

Interpret each expansion as a Feynman diagram of the SCET



Factorization (after including all diagrams)

Wilson coefficients	**only depend on m_χ**
Soft functions	**only depend on m_W**
Jet functions	**only depends on m_W**

SCET-II (narrow resolution)

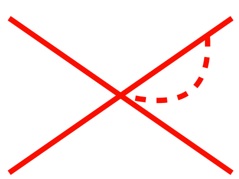
Interpret each expansion as a Feynman diagram of the SCET

Factorization (after including all diagrams)

Breakdown of the factorization

- **can be cured by introducing a regulator**

e. g. rapidity regulator:

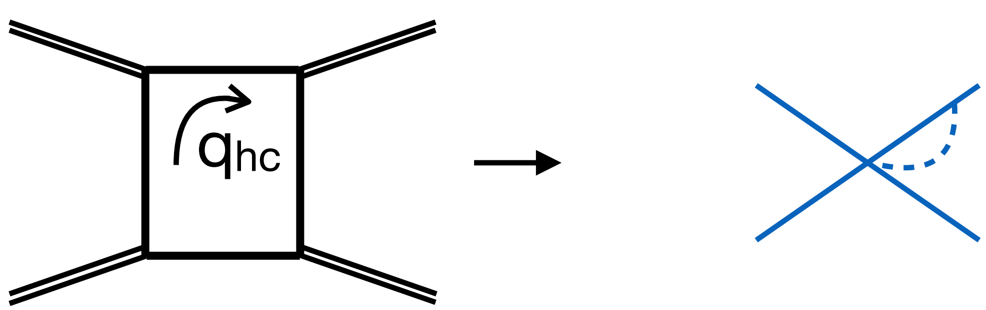


$$\rightarrow \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{(q^2 - m_W^2)[(k-q)^2 - m_W^2] n \cdot q} \frac{\nu^\eta}{|n \cdot q|^\eta}$$

s

only depend on m_χ

ideally would only depend on m_W



Jet functions

NRDM×SCET for DM annihilation

After several steps one can prove that:

$$\frac{\mathrm{d}}{\mathrm{d}E_\gamma}[\sigma v] = |\psi(0)|^2 \times |C|^2(\mu) \times Z_\gamma(\mu,\nu) \times J(\mu,\nu) \otimes W(\mu,\nu)$$

Resummation is achieved by solving

- an appropriate Schrödinger equation
- μ and ν renormalization group equations for every piece of the factorization formula

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The wino-like/MDM triplet model

SM + Majorana SU(2) triplet

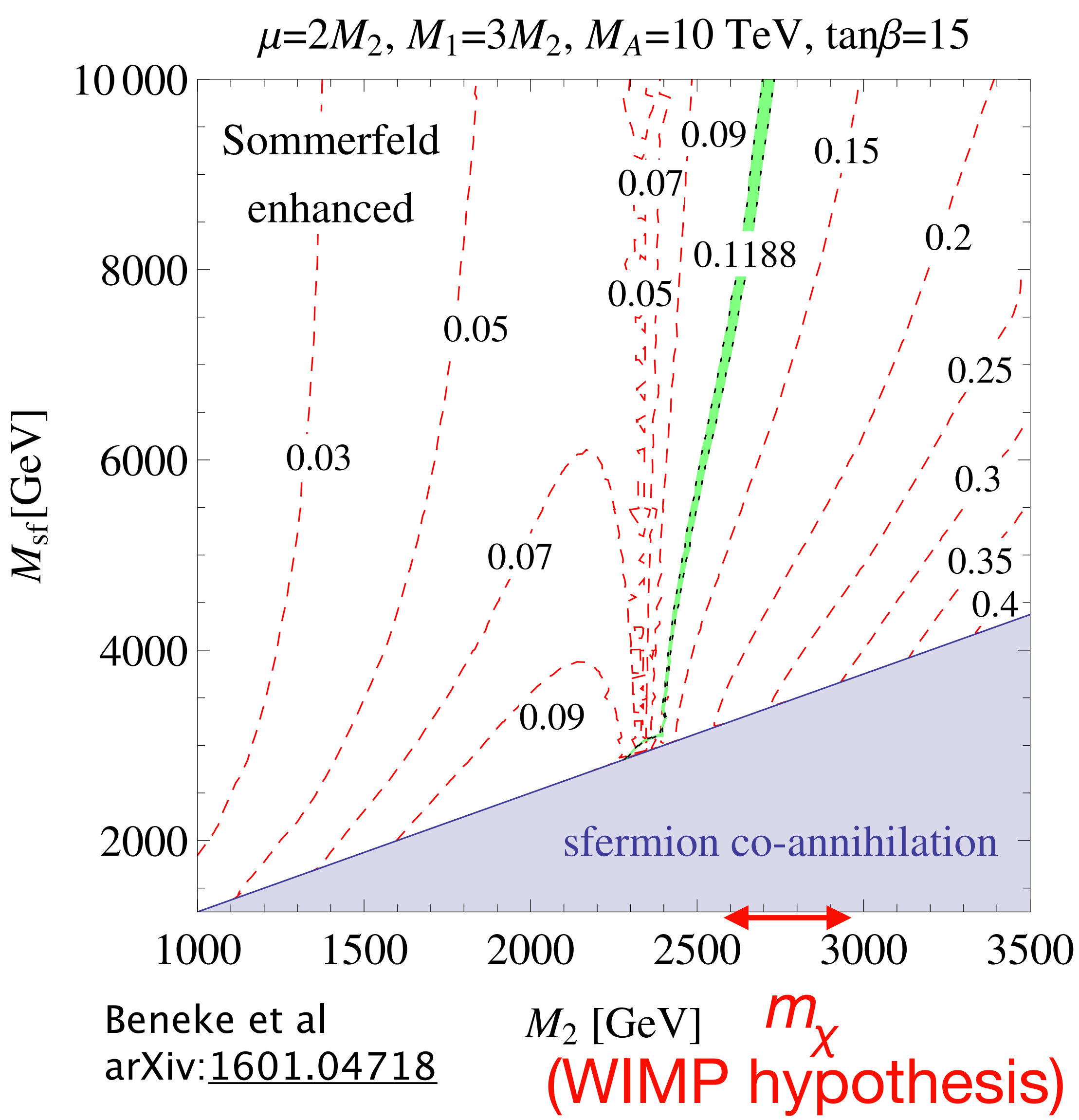
$$\delta\mathcal{L}_{\rm Wino} = \frac{1}{2}\bar{\chi}(i\gamma^\mu \mathrm{D}_\mu - m_\chi)\chi$$

$\xrightarrow{\cancel{\text{EW}}}$

Q=0 *Majorana* DM
Q=1 *Dirac* chargino

- $m_{\chi^\pm}$-$m_{\chi^0} \approx$ 164MeV
- DM stable through a Z_2 symmetry
- Suitable WIMP for $m_{\chi^0} \lesssim$ 3TeV
- Super-partner of the SU(2) gauge bosons in SUSY

The wino-like/MDM triplet model



Beneke et al
arXiv:1601.04718

- suppressed direct-detection cross sections (below the so-called neutrino floor)
- too heavy for the LHC

Factorization theorem. Sommerfeld effect

$$\frac{d(\sigma v_{\mathrm{rel}})}{dE_\gamma} = \sum_{I,J} S_{IJ}\,\Gamma_{IJ}(E_\gamma)$$

Sommerfeld matrix
I, J = (χ0χ0) or (χ+χ-)

Next talk

$$V(r) = \begin{pmatrix} 0 & -\sqrt{2}\alpha_2 \frac{\mathrm{e}^{-m_W r}}{r} \\ -\sqrt{2}\alpha_2 \frac{\mathrm{e}^{-m_W r}}{r} & -\frac{\alpha}{r} - \alpha_2 c_W^2 \frac{\mathrm{e}^{-m_Z r}}{r} \end{pmatrix}$$

see e.g. Beneke et al arXiv: 1411.6924
Hisano arXiv: hep-ph/0412403

Factorization theorem. Sommerfeld effect

$$\frac{d(\sigma v_{\rm rel})}{dE_\gamma} = \sum_{I,J} S_{IJ}\,\Gamma_{IJ}(E_\gamma)$$

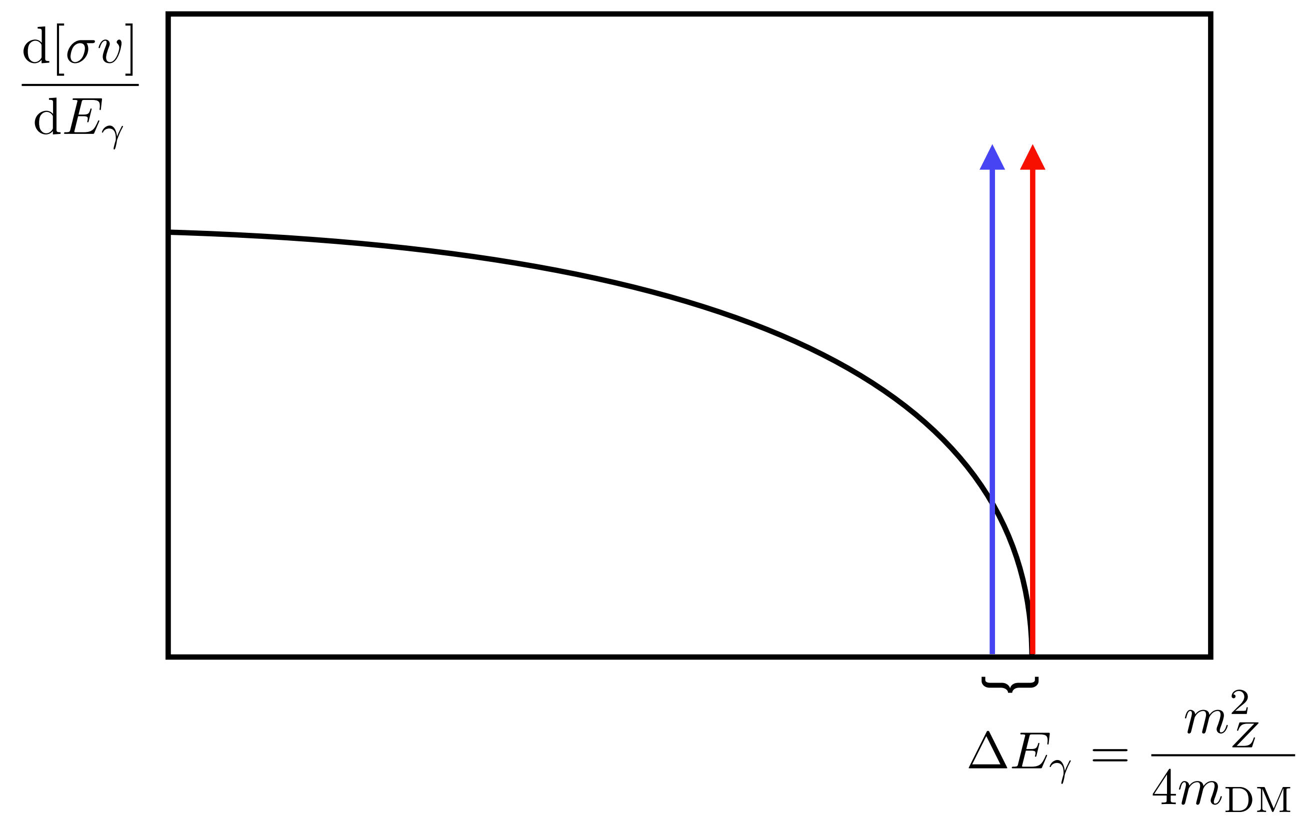
Assumptions on the energy resolutions

The variable $E_{\mathrm{res}} = m_\chi - E_\gamma$ plays a decisive role in the factorization problem

We investigated two situations

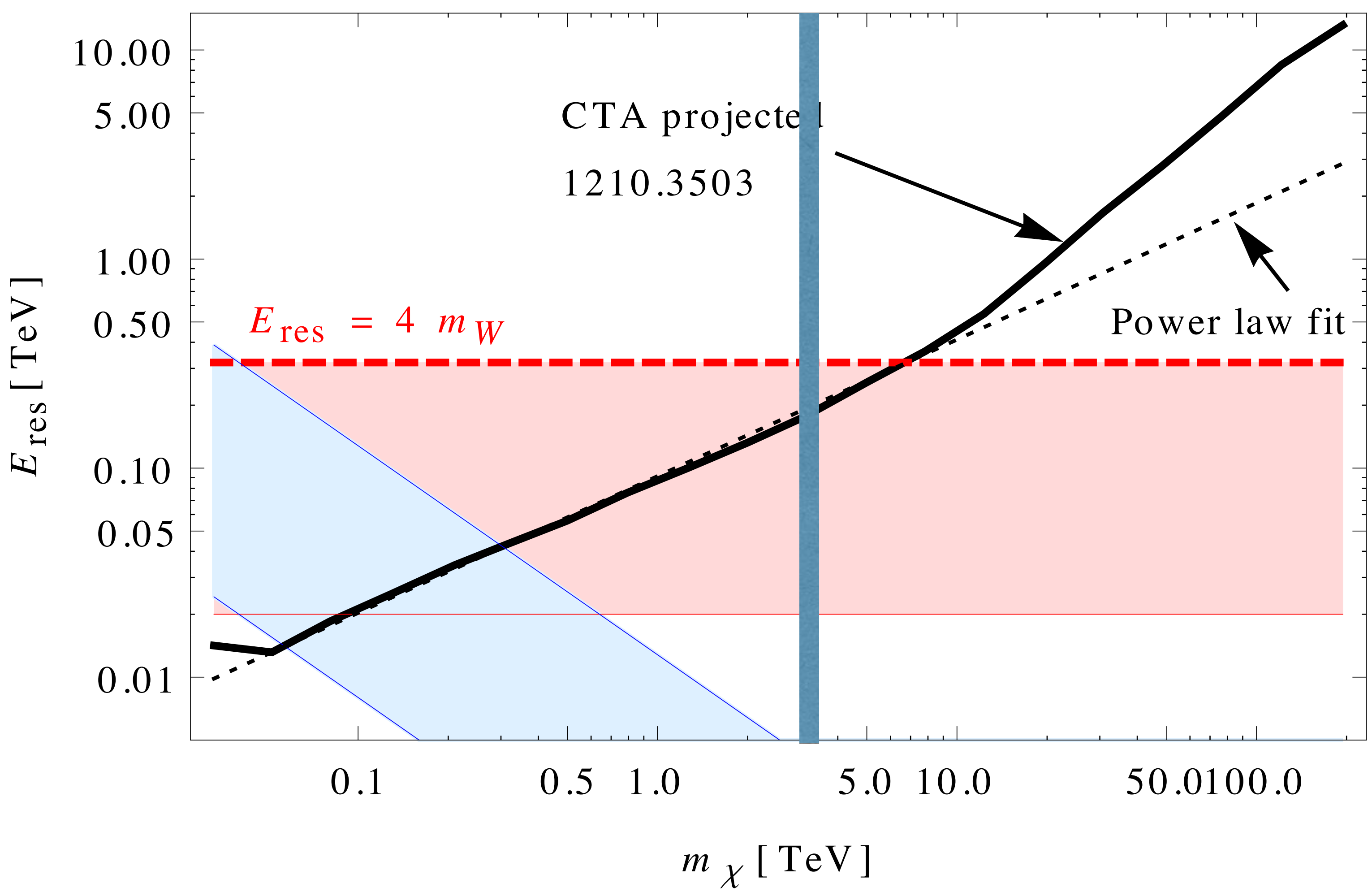
$E_{\mathrm{res}} \sim m_W^2/m_\chi$ (1805.07367)

$E_{\mathrm{res}} \sim m_W$ (1903.08702)



See also **Baumgart et al** (1712.07656 and 1808.08956) for the $E_{\mathrm{res}} \gg m_W$ case

Energy resolution

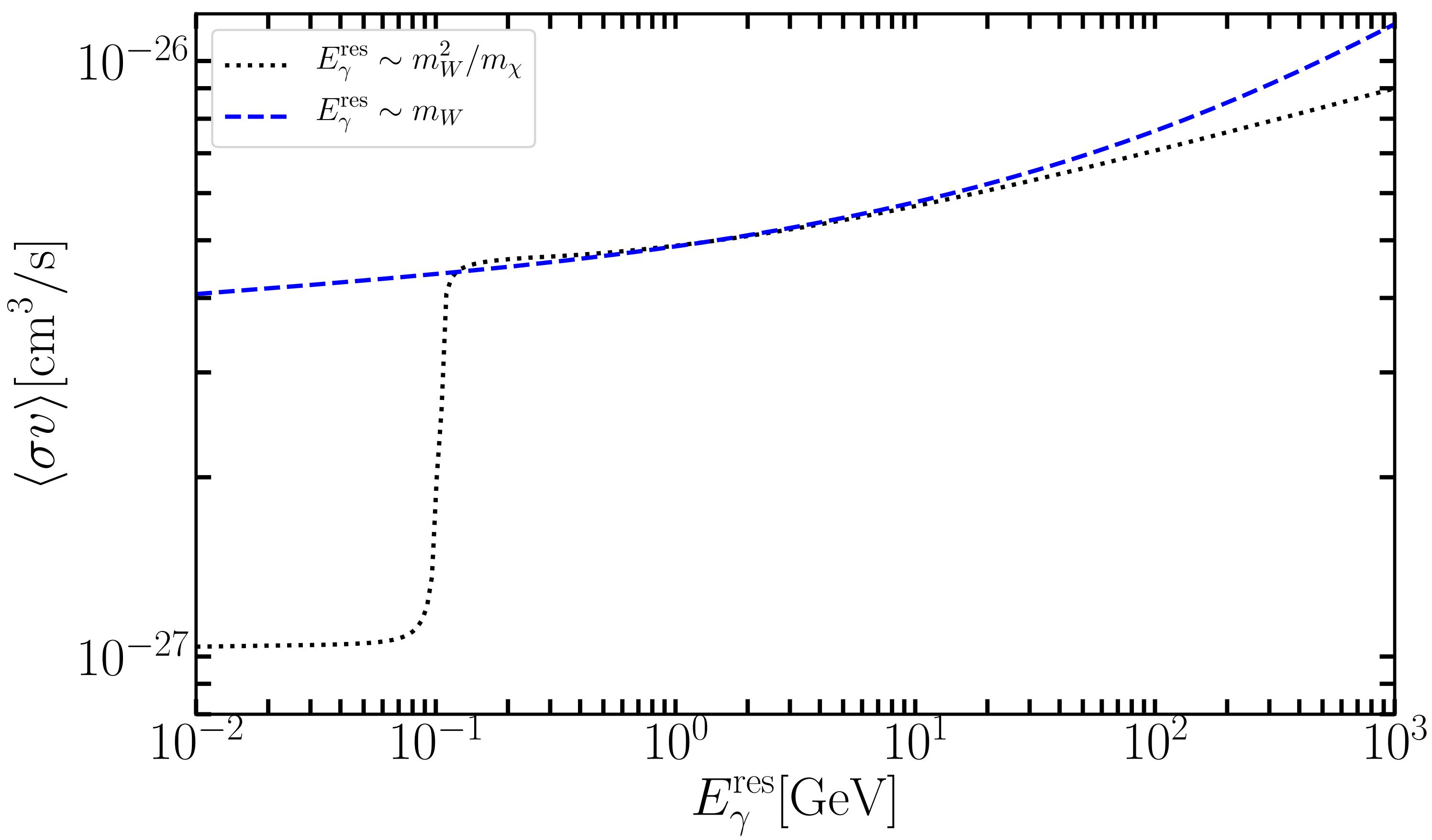


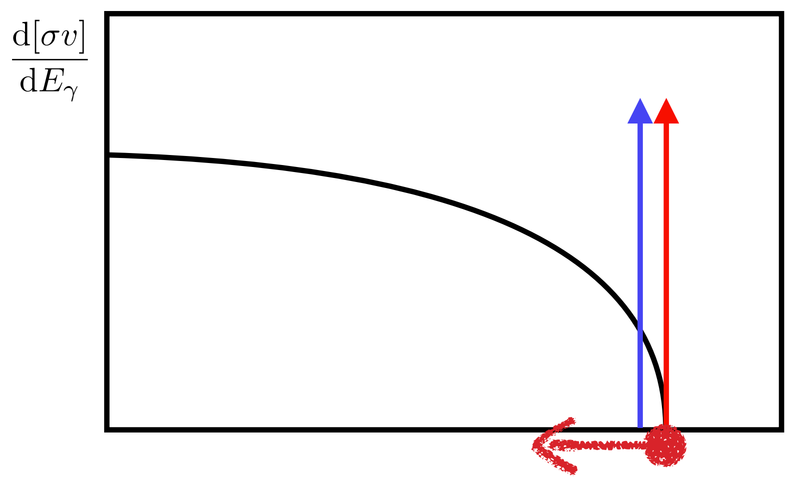
Factorization theorem. Exclusive Wino $\chi\chi \to \gamma + X$ annihilation

$$\frac{d(\sigma v_{\rm rel})}{dE_\gamma} = \sum_{I,J} S_{IJ}\, \Gamma_{IJ}(E_\gamma)$$

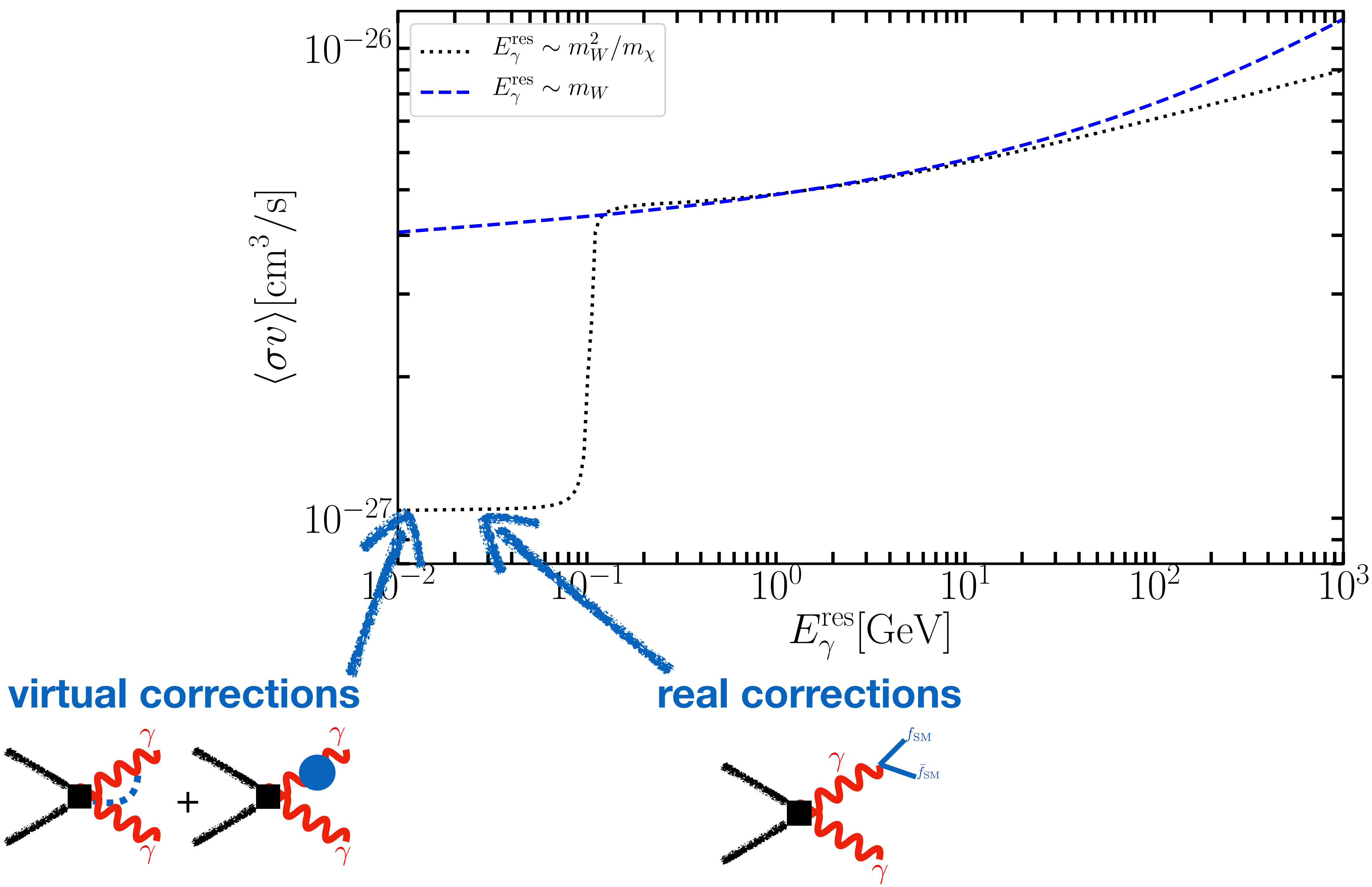
$$\Gamma_{IJ}(E_\gamma) = \frac{1}{4}\frac{2}{\pi m_\chi} \sum_{i,j=1,2} C_j^*(\mu_W) C_i(\mu_W) Z_\gamma(\mu_W, \nu_W)$$
$$\times \int J\left(4m_\chi(m_\chi - E_\gamma - \omega/2), \mu_W\right) W_{IJ}^{ij}(\omega, \mu_W, \nu_W)$$

Energy-integrated cross section

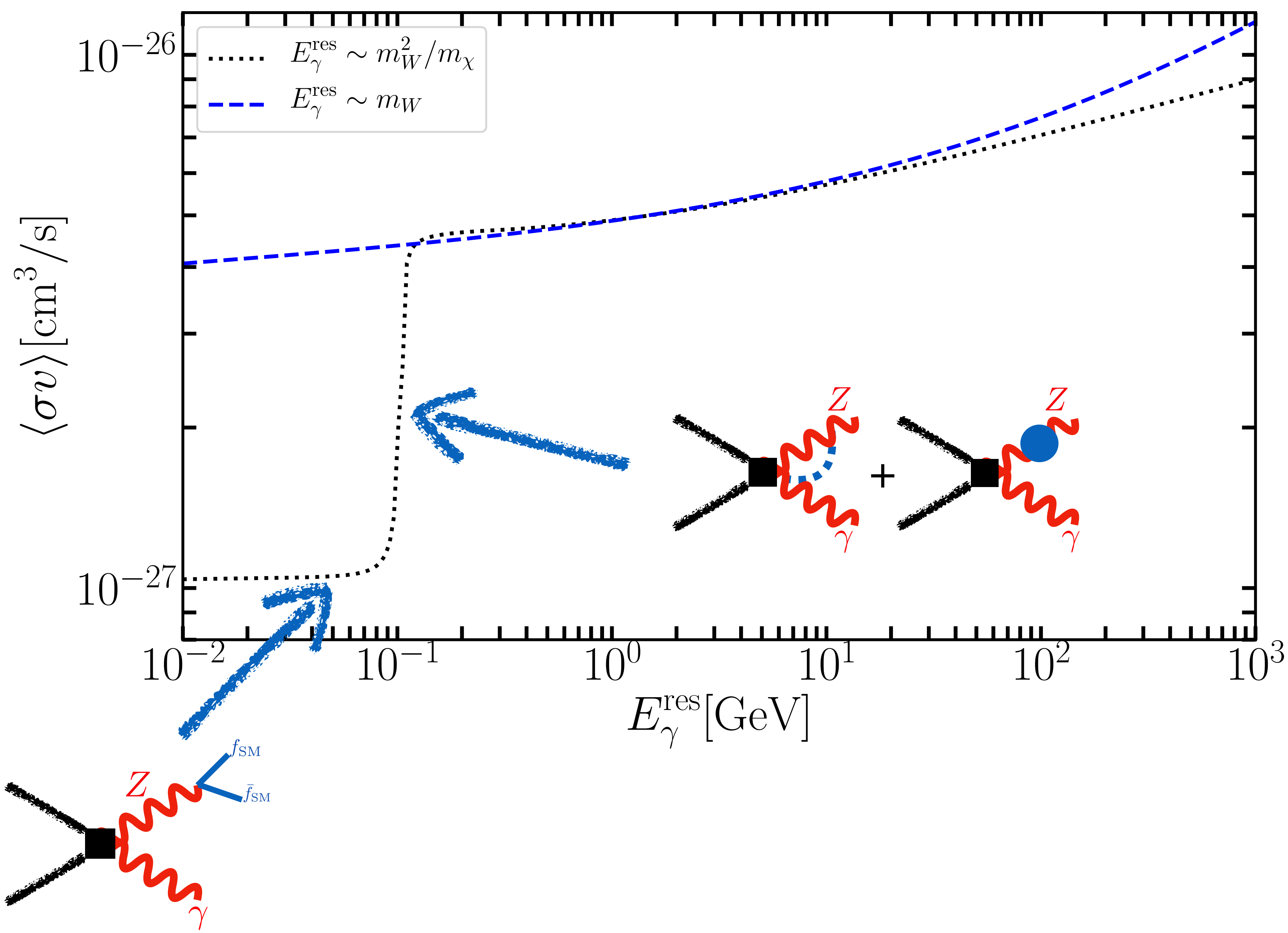




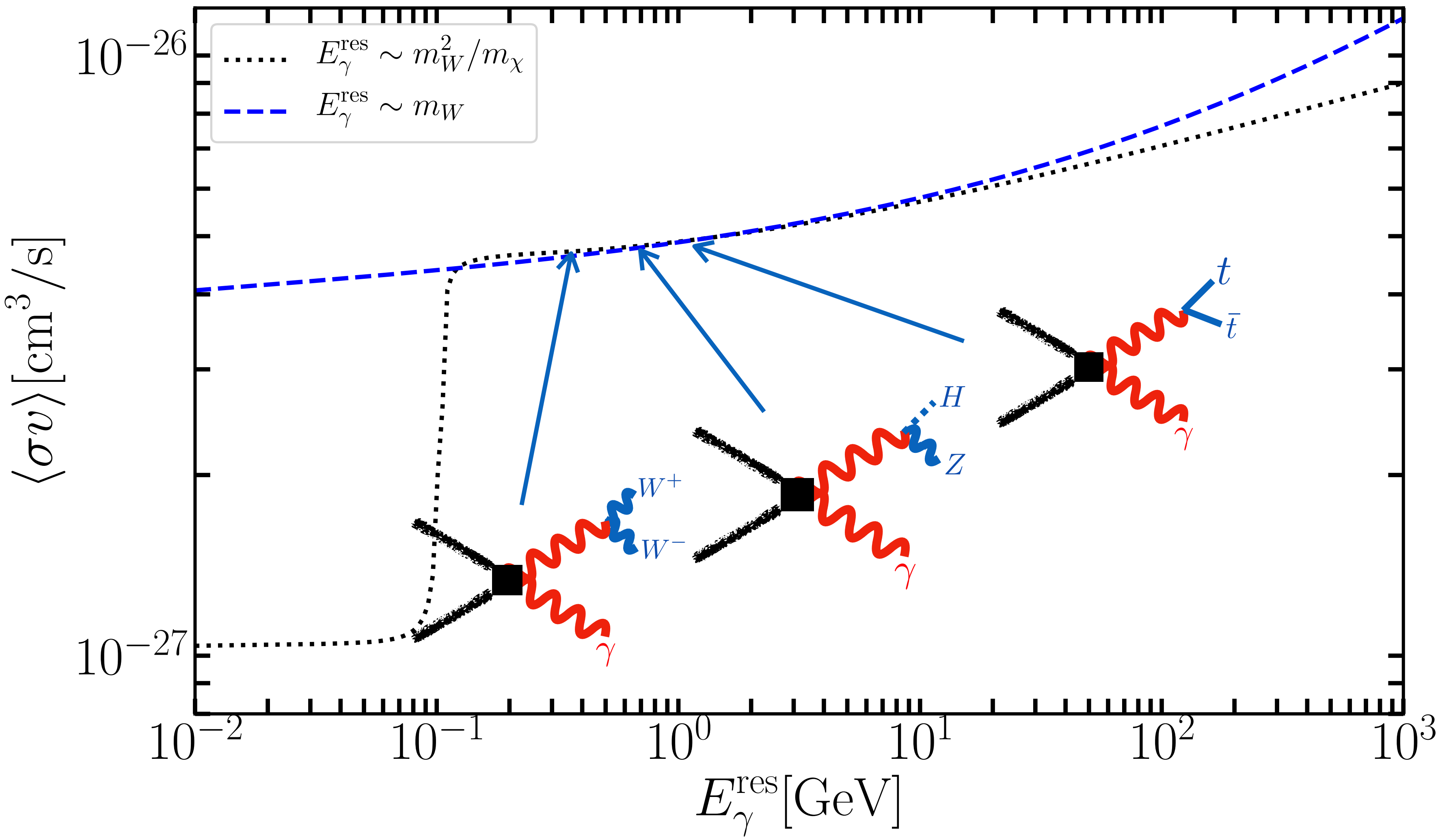
Energy-integrated cross section



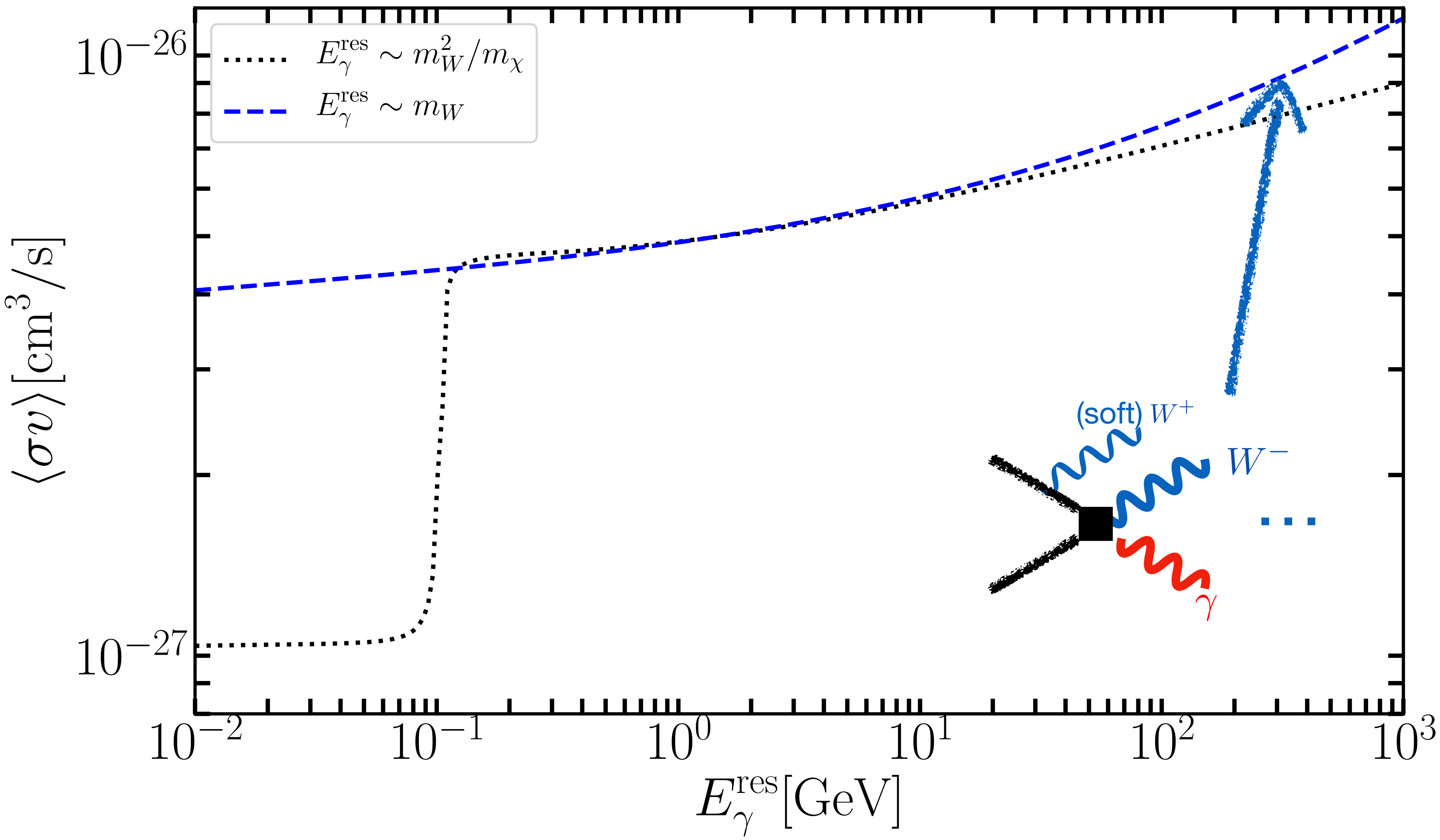
Energy-integrated cross section



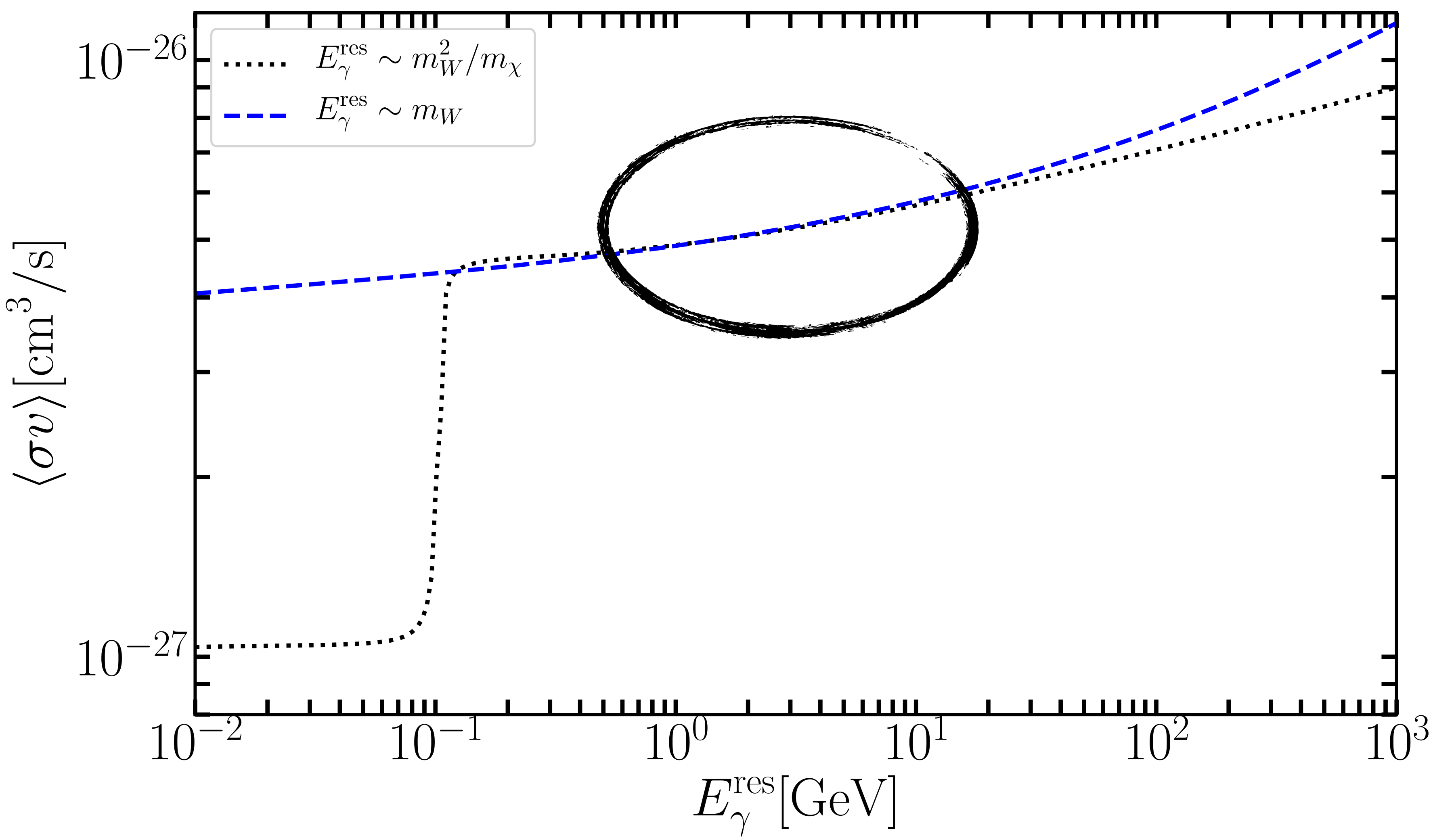
Energy-integrated cross section



Energy-integrated cross section



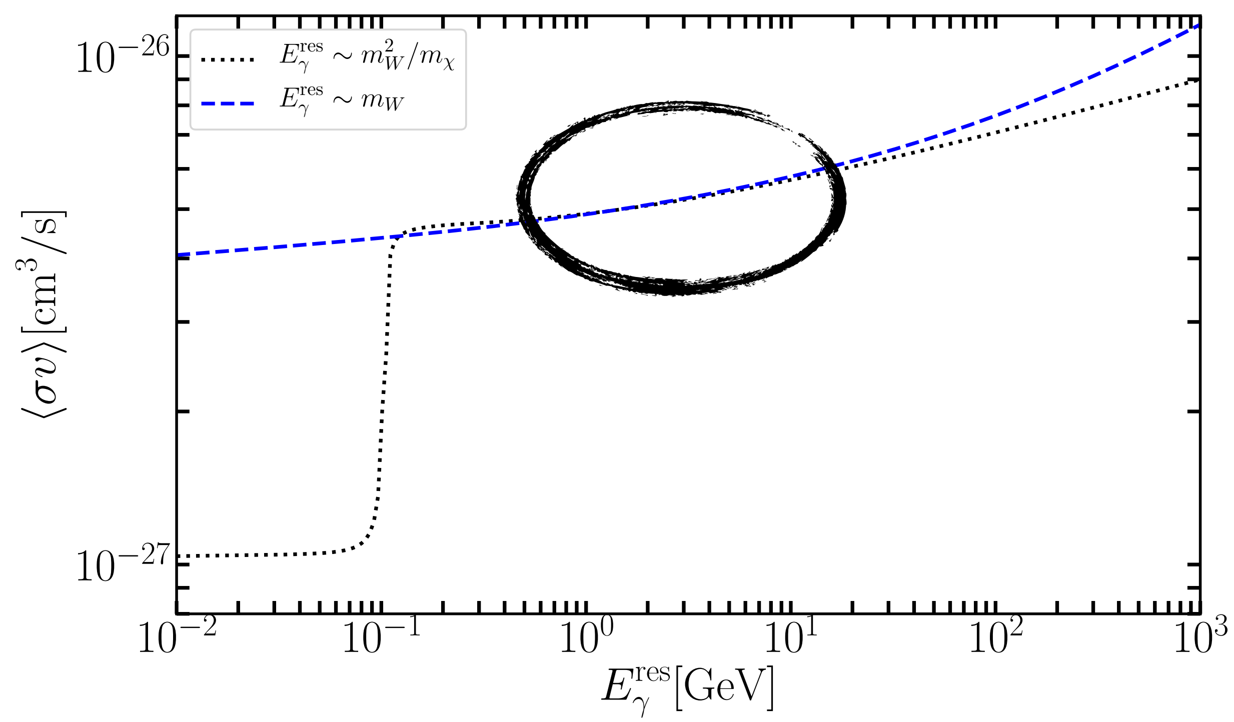
Energy-integrated cross section. Remarkable matching



Energy-integrated cross section. Remarkable matching

Not an obvious result

Can be understood by expanding our factorization formulas at fixed orders



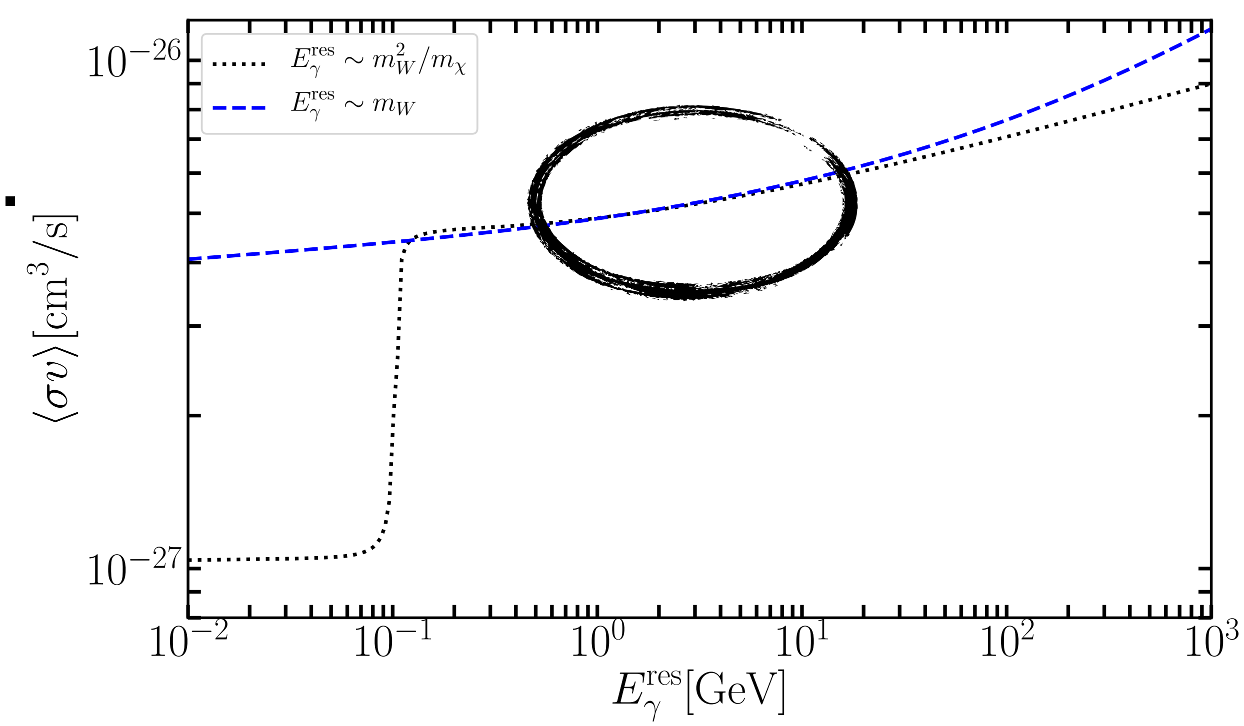
$$[\sigma v]_{IJ}(E_\gamma) = \frac{2\pi\hat{\alpha}_2(\mu)\hat{s}_W(\mu)}{\sqrt{2}^{n_{\rm id}} m_\chi^2} \sum_{n=0}^{\infty}\sum_{m=0}^{2n} c_{IJ}^{(n,m)}(E_\gamma,\mu) \left(\frac{\hat{\alpha}_2(\mu)}{\pi}\right)^n \ln^m \frac{4m_\chi^2}{m_W^2}$$

Energy-integrated cross section. Remarkable matching

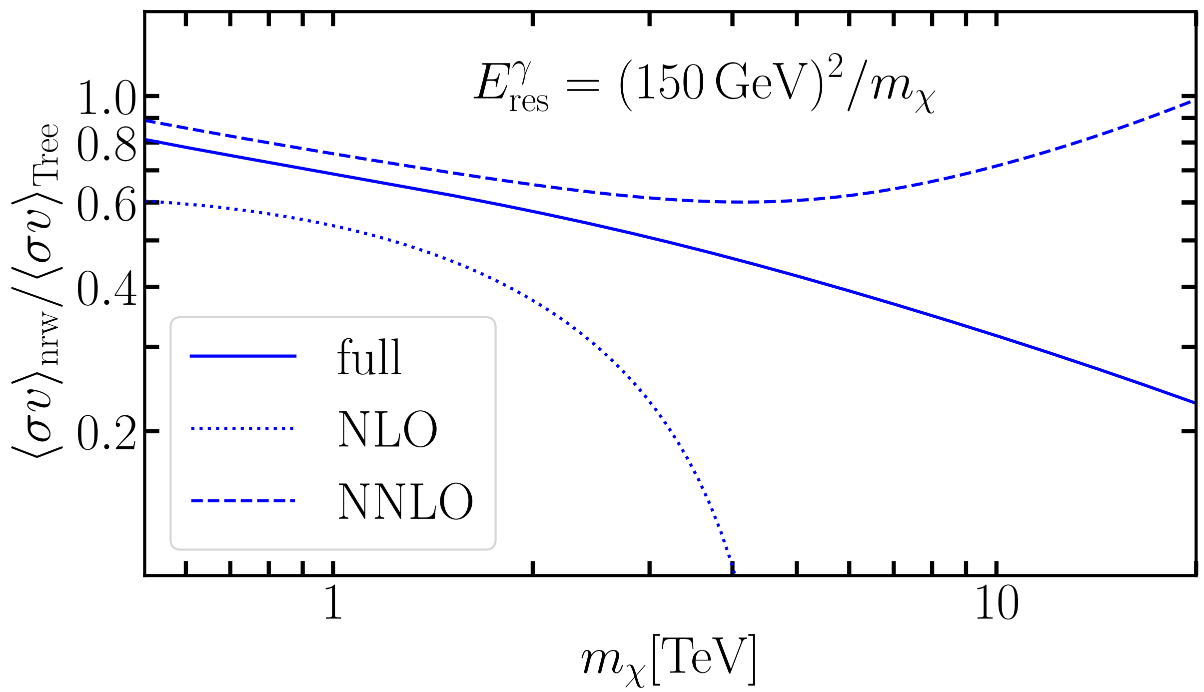
$$[\sigma v]_{IJ}(E_\gamma) = \frac{2\pi\hat{\alpha}_2(\mu)\hat{s}_W(\mu)}{\sqrt{2}^{n_{\rm id}} m_\chi^2} \sum_{n=0}^{\infty}\sum_{m=0}^{2n} c_{IJ}^{(n,m)}(E_\gamma,\mu) \left(\frac{\hat{\alpha}_2(\mu)}{\pi}\right)^n \ln^m \frac{4m_\chi^2}{m_W^2}$$

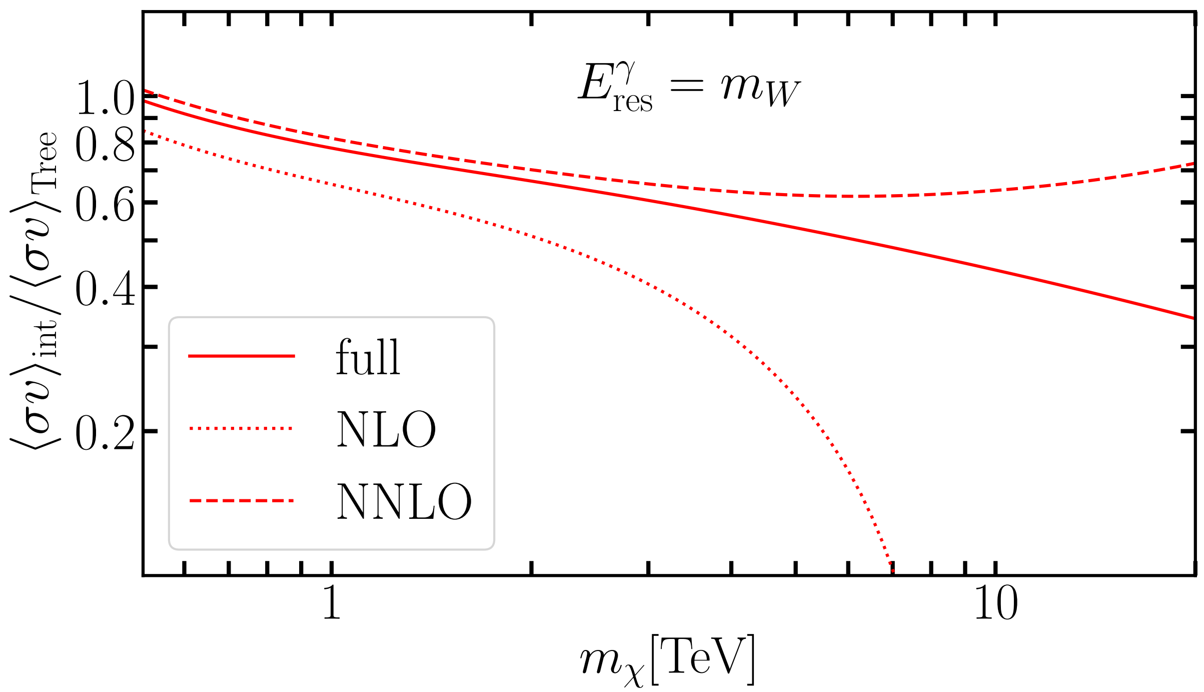
Factorization-theorem dependent

- Coefficients are (by definition) of $\mathcal{O}(1)$ but dependent on E_γ
- When evaluated outside the range of validity of the Fact. Th. they can become large
  - (and contribute to the (m+1) term instead)

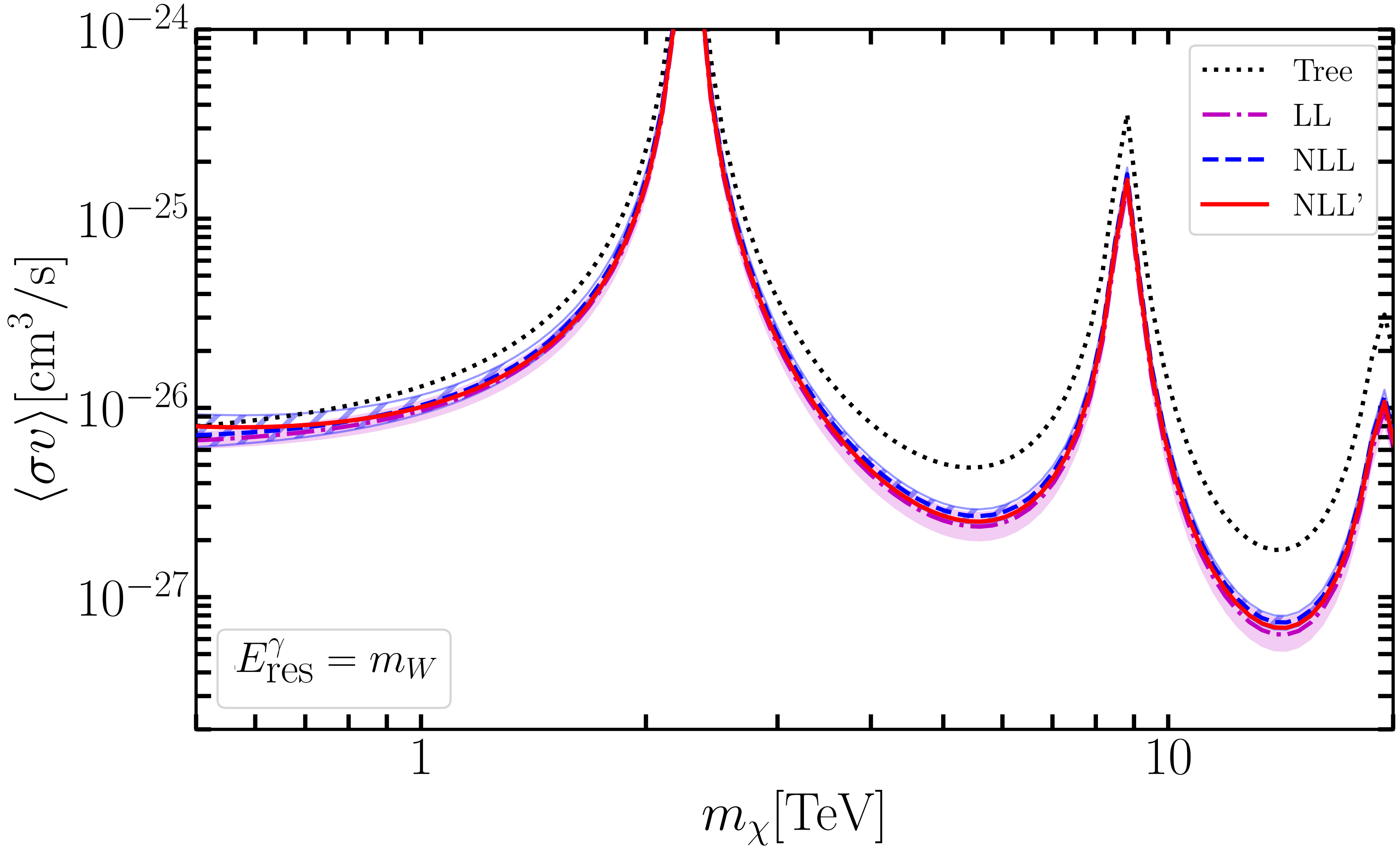


Further applications of our fixed-order expansions

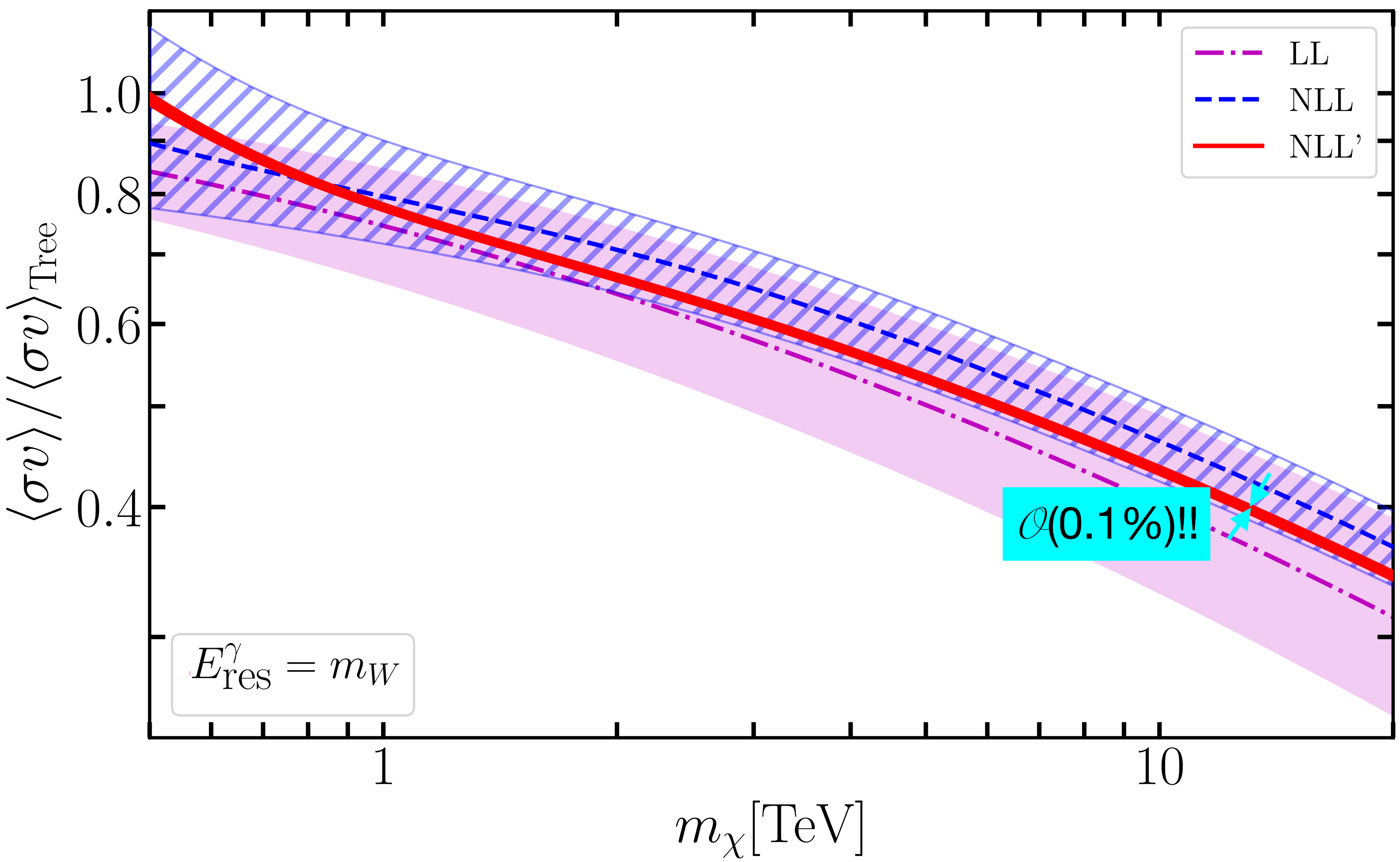




$\chi\chi\to\gamma$+X cross sections (E_{res} ~ m_W)



Effect of the Sudakov resummation



Summary

Sudakov is the new Sommerfeld for indirect DM detection

$$\mathrm{Flux} \simeq \frac{1}{8\pi m_\chi^2} \times J \times S_{(+-)(+-)}\, \mathrm{e}^{-\frac{\alpha_2}{\pi}\frac{3}{4}\ln^2\frac{4m_\chi^2}{m_W^2}} \frac{2\pi\hbar^2 c^3 \alpha_2^2 \sin^2\theta_W}{m_\chi^2}\delta(E_\gamma - m_\chi)$$

Conclusions

- Heavy WIMP region will be probed by indirect detection observations in the near future.
- Learned how to handle the technically/conceptually involved problem of correctly predicting cross sections that are relevant for spectral multi-TeV γ-ray line searches in two regimes: *narrow* and *intermediate* energy resolution
- Employed these methods on wino DM (**higgsino DM** coming soon!)
  - Reduced theoretical uncertainties down to the **permille** level for the (intermediate) *energy resolutions of the CTA* at the interesting mass range of 1-10 TeV
  - Observed a remarkable matching of the two factorization formulas in the "transition" region (narrow <—> intermediate resolutions)