

Interface Flows in D1/D5 Holography

Christian Northe

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Julius-Maximilians-Universität Würzburg

Work in progress with J. Erdmenger and C. Melby-Thompson

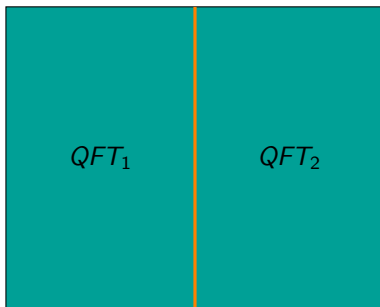
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Kondo Effect as Interface RG Flow



Interfaces provide mappings between possibly distinct physical systems.

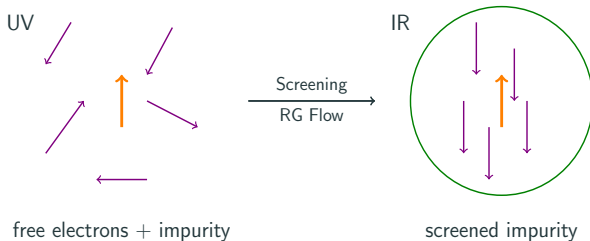
Study interfaces under renormalization group flows.

The Kondo Effect

Heavy **magnetic impurity** interacts with conduction electrons

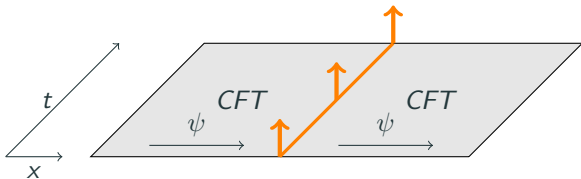
Ultraviolet. Free electrons with mild antiferromagnetic coupling to spin

Infrared. Impurity is screened through binding with conduction electrons



$$H = \psi^\dagger i\nabla\psi + \lambda \delta(\vec{r}) \vec{S} \cdot \vec{J}, \quad \vec{J} = \frac{1}{2} \psi^\dagger \vec{\tau} \psi$$

Kondo Impurity \longrightarrow Interface between 2d CFTs



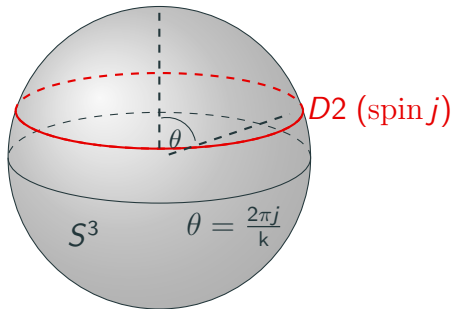
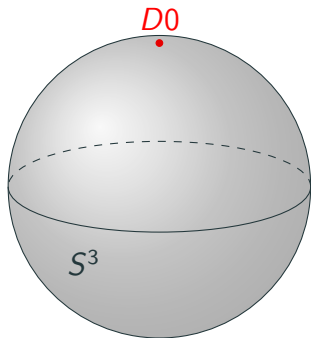
System described by action

$$\mathcal{I} = \mathcal{I}_{\text{WZW}}(\hat{\mathfrak{su}}(2)_k) + \lambda \int_{\partial\Sigma} dt \, \vec{S} \cdot \vec{J}(t),$$

where \vec{S} is in **spin-S** irreducible representation of $\mathfrak{su}(2)$.

Conformally Invariant Boundary Conditions of $\hat{\mathfrak{su}}(2)_k$

- Labeled by set of primaries $j = 0, \underbrace{\frac{1}{2}, \dots, \frac{k}{2}}_{k+1}$
- Correspond to discrete set of conjugacy classes



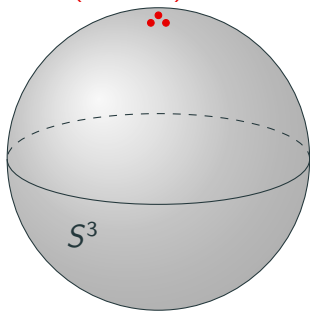
Both types of branes preserve $SO(3)$!

'Absorption of Boundary Spin' Principle Affleck & Ludwig 1991

Non-abelian polarization

$(2S + 1)$ pointlike Branes (spin-0) \rightarrow 1 brane of spin S

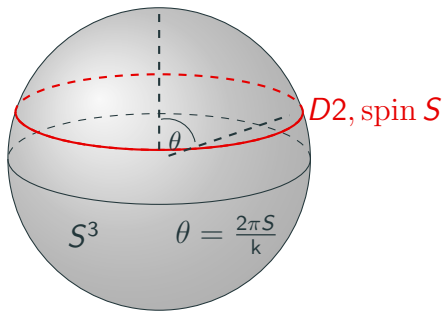
$(2S + 1) D0s$



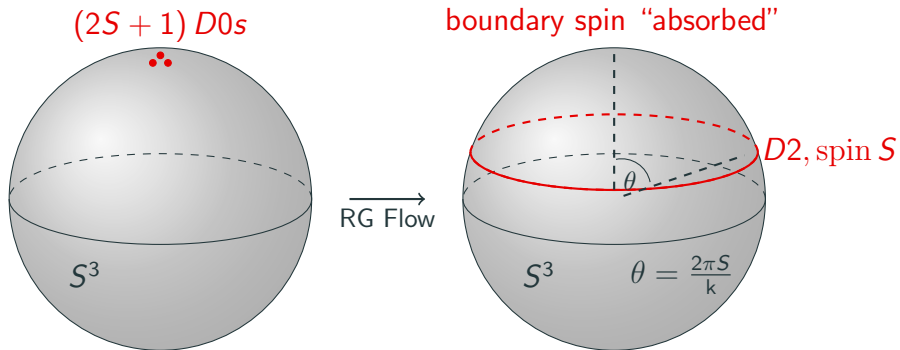
$\hat{\mathfrak{su}}(2)_k$

RG Flow \rightarrow

boundary spin "absorbed"



$S = 0, \frac{1}{2}, \dots, \frac{k}{2}$



Geometric implementation in AdS/CFT:

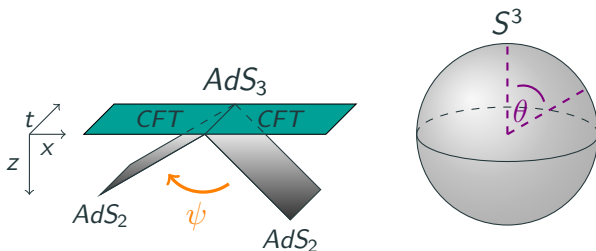
Kondo-like defect flows in D1/D5 system via non-abelian polarization.

Interface RG Flows in Holography

D1/D5 Holographic System

Embed D5-branes into Type IIB string theory and dissolve D1-branes inside.

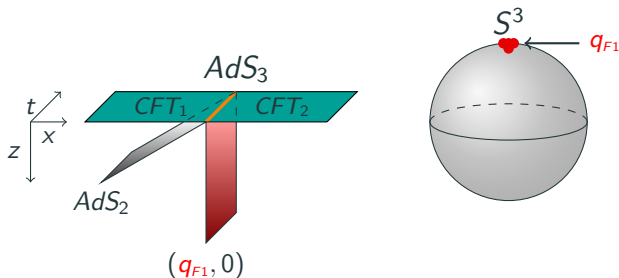
Gravity. Weakly coupled IIB string theory on $AdS_3 \times S^3 \times M_4$.



CFT. Two-dimensional, strongly coupled, lives on ∂AdS_3

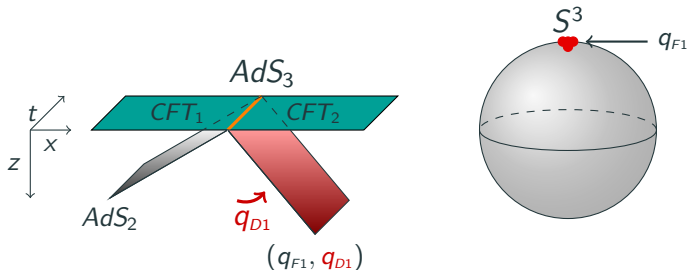
- $\mathcal{N} = (4, 4)$ small superconformal algebra.
- Bosonic part: $\mathfrak{so}(2, 2) \times \mathfrak{so}(4)$.

q_{F1} units of fundamental string charge



- Interface preserves $\mathfrak{so}(2, 1) \times \mathfrak{su}(2)$ & 8 superconformal charges
- $CFT_1 \neq CFT_2$, but their central charges coincide

q_{F1} = fundamental string charge, q_{D1} = D1-brane charge



- Interface preserves $\mathfrak{so}(2,1) \times \mathfrak{su}(2)$ & 8 superconformal charges
- $CFT_1 \neq CFT_2$, and their central charges differ

Flows from brane polarization

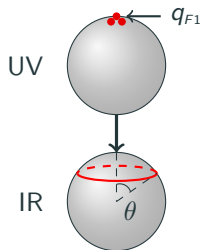
Deform branes by **non-abelian polarization**: Myers '99

Coordinates on S^3 become non-commutative \Rightarrow fuzzy S^2 inside S^3 .

Alekseev, Recknagel, Schomerus '00

- (q_{F1}, q_{D1}) strings puff up into D3 branes
- **BPS flow solutions** for general (q_{F1}, q_{D1}) obtained from κ symmetry projector (along lines of Gomis et al. '99).
- Flow preserves 4 supercharges
- Radial coordinate z in AdS_3 is energy scale

$$\Rightarrow \theta = \theta(z)$$



Flows from brane polarization

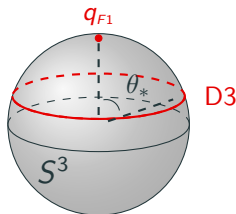
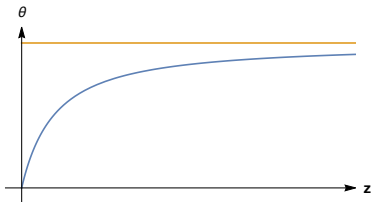
In D3-brane description, $\theta = \theta(z)$

$$I = T_{D3} \int d^4 \xi e^{-\Phi} \sqrt{-\det(\hat{g} + F)} + T_{D3} \int (C^{(2)} \wedge F + \frac{1}{2} F \wedge F)$$

Evolution of $\theta(z)$ under RG flow determined by

$$z = z_0 \frac{\sin \theta}{\theta_* - \theta} \quad \theta_* = \pi \frac{q_{F1}}{N_5}$$

where z is the radial coordinate in AdS_3 .



- Probe brane description includes effect of interface on CFT, but not how CFT affects the interface
- Brane annihilation processes, correlators,...

- Probe brane description includes effect of interface on CFT, but not how CFT affects the interface
- Brane annihilation processes, correlators,...

Construct fully backreacted supergravity solutions with symmetries
 $SO(2,1) \times SO(3)$, $\frac{1}{2}$ -BPS (8 SUSYs)

Constructed asymptotically $AdS_3 \times S^3 \times M_4$ $\frac{1}{2}$ -BPS solutions for

- (p, q) -strings
- $D3_{(p,q)}$ -brane charged under F1- and D1-charge
- Connect these solutions via our RG flow

Computed interface entropies.

- They depend crucially on backreaction
- Confirmed g -theorem

Summary and Outlook

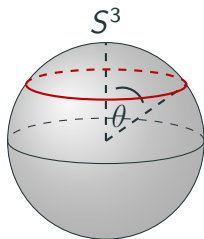
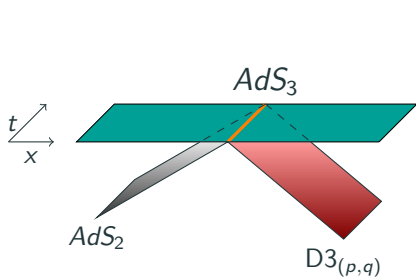
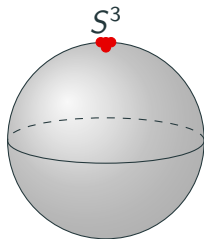
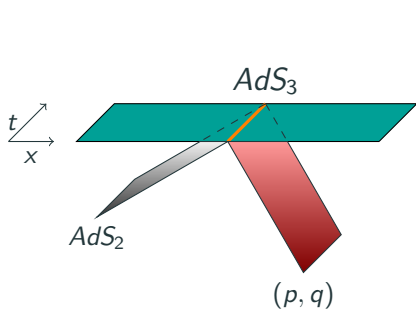
- Studied holographic duals of interface RG flows in the D1/D5 theory
- Probe brane limit: BPS RG flows for general (p, q) string defects
- Classical IIB Supergravity description representing backreaction for fixed points
- g -factor, including CFT contributions, in semi-classical limit of gravity

- More detailed study from CFT point of view \rightsquigarrow deformation
- Interfaces carrying D5/NS5 charges
- Generalizations to other top-down theories, especially

$$\text{AdS}_3 \times S^3 \times S^3 \times S^1$$

Thank you for your attention!

Interface Solutions and RG Flow



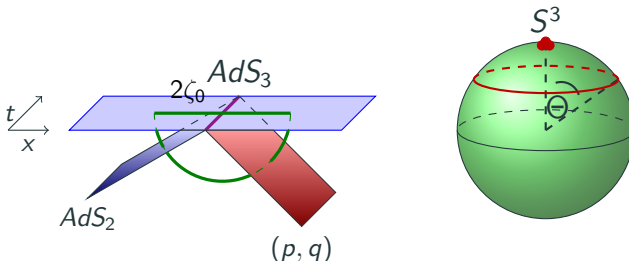
Interface Entropy

Interface Entropy from Holography Chiodaroli, Gutperle, Hung '10

- Boundary entropy $s = \log g \xleftrightarrow{\text{fold}}$ interface entropy.

$$\text{BCFT: } g = \langle 0 | \mathcal{B} \rangle$$

- Compute s as interface contribution to entanglement entropy
Calabrese, Cardy '04
- Gravity dual is semi-classical \implies use Ryu-Takayanagi formula
Ryu, Takayanagi '06



(p, q) g -Theorem

Simplest case: pure F1 interfaces $(p, 0)$

$$\log g = \frac{c}{6} \left(\log \kappa + 1 - \frac{1}{\kappa} \right)$$

$$(p, 0) : \quad \kappa = \frac{T(4N_1, p) + T(0, p)}{T(4N_1, p) - T(0, p)}$$

$$D3_{(p,0)} : \quad \kappa = \frac{T(4N_1, p \frac{\sin \theta}{\theta}) + T(0, p \frac{\sin \theta}{\theta})}{T(4N_1, p \frac{\sin \theta}{\theta}) - T(0, p \frac{\sin \theta}{\theta})}$$

- g -theorem satisfied for all (p, q) interfaces
- g -factor contains contribution not visible in the probe brane limit

Interfaces in the D1/D5 CFT

Brief review of D1/D5 CFT

Type IIB on $M_{10} = \mathbb{R}^{1,1} \times \mathbb{R}^4 \times M_4$ (with $M_4 = K3$ or T^4):

	0	1	2	3	4	5	6	7	8	9
D5 (N_5)	•	•					•	•	•	•
D1 (N_1)	•	•								

Gauge theory description. $U(N_1) \times U(N_5)$ gauge theory with bifundamental hypermultiplet. Consider Higgs branch. Gives:

Instanton description. D5 brane has a coupling $\int C^{(2)} \wedge \text{Tr}(F \wedge F)$.
 \Rightarrow D1 branes can be dissolved as $U(N_5)$ gauge instantons on M_4 .

Low energy dynamics. 2d $\mathcal{N} = (4,4)$ SCFT: Non-linear sigma model on the moduli space of instantons on M_4 . Strominger & Vafa '96

Interfaces in D1/D5 CFT

Type IIB on $M_{10} = \mathbb{R}^{1,1} \times \mathbb{R}^4 \times M_4$:

	0	1	2	3	4	5	6	7	8	9
D5 (N_5)	•	•					•	•	•	•
D1 (N_1)	•	•								
F1 (p)	•		•							

- preserves $\mathcal{N} = 4$, $d = 1$ supersymmetry
- realized in gauge theory as Wilson line. Sources jump in background electric field, changing the CFT on one side while preserving the central charge. This case is an **interface**, not a defect.



Wilson line interfaces in D1/D5 CFT

- **Wilson line** \leftrightarrow **long string** connecting distant D3 brane to D1/D5 system.
- After mixing, **lowest-lying fermions** have Lagrangian **Tong & Wong '14**

$$L_\eta = \eta^\dagger (i\partial_0 + \Omega_A \partial_t Z^A) \eta$$

where η is in the fundamental of $U(N_5)$, Z^A is the coordinate on \mathcal{M} , and Ω_A is a **$U(N_5)$ connection** on $M_4 \times \mathcal{M}$.

- This can be rewritten as the insertion of

$$W = \text{Tr}_F \mathcal{P} \exp \left(i \int dt \partial_t Z^A \Omega_A(y_0, Z) \right)$$

with y_0 the location of the Wilson line in M_4 .