Hidden exceptional symmetry in the pure spinor superstring

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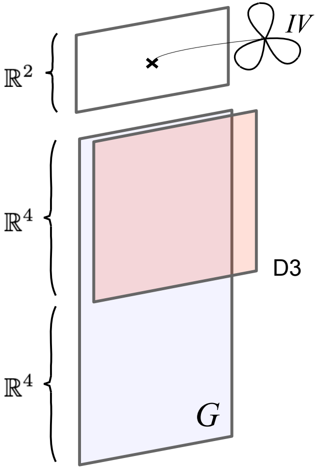
D3 branes probing F-theory sevenbranes

A $D3$ brane probing a sevenbrane is described by a $4d$ $\mathcal{N} = 2$ SCFT H_G (plus a c.o.m. hyper). The flavor symmetry G is determined by the Kodaira type of the F-theory elliptic fiber. The Higgs branch Higgs(H_G) coincides with the moduli space of one G-instanton, $\widetilde{\mathcal{M}}_{G,1}$.



Kodaira type	G
II	$-$
III	A_1
IV	A_2
I_0^*	D_4
IV^*	E_6
III^*	E_7
II^*	E_8

4d chiral algebra

4d $\mathcal{N} = 2$ SCFTs possess a chiral algebra which was discovered by [Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees]

For the D3 brane theories H_G,

$$\mathcal{A}_G^{4d} = \text{Kac-Moody algebra } G_{k^{4d}} \text{ at level } k^{4d} = -\frac{h_G^\vee}{6} - 1.$$

G	$-$	A_1	A_2	D_4	E_6	E_7	E_8
$\mathcal{A}_G^{4d}$	Yang-Lee	$A_1^{-4/3}$	$A_2^{-3/2}$	D_4^{-2}	E_6^{-3}	E_7^{-4}	E_8^{-6}

The vacuum module encodes the spectrum of Schur operators of H_G. Its character is a solution to a 2nd order modular linear differential equation (MLDE).

The second solution to the MLDE corresponds to a different module of the chiral algebra [Arakawa-Kawasetsu], which is conjectured to correspond to a surface operator $\mathfrak{S}_G$ of H_G [Beem-Rastelli]. Connection between surface operators and the chiral algebra has also been investigated by [Cordova-Gaiotto-Shao].

Twisting

We would like to compactify H_G on a two-sphere S^2, which requires performing a topological twist.

Spacetime rotations: $SO(4) \supset SO(2) \times SO(2)_J$.

R-symmetry group: $SU(2)_R \times U(1)_r$.

Choice of topological twist is labeled by an integer N [Cecotti-Song-Vafa-Yan]:

$$J \to J^{(N)} = J + R + \frac{1}{2}N(R - r).$$

Twisted compactification results in a 2d theory, which flows in the IR to a NLSM $\mathcal{T}_{2d}^{N}[H_G]$. Consistency requires that all fields take values in well-defined bundles over S^2, which for a given N restricts the allowed choice of G.

2d chiral algebra

The NLSMs $\mathcal{T}_{2d}^{N}[H_G]$ possess $\mathcal{N} = (0,2)$ supersymmetry; we will denote their chiral algebra by $\mathcal{A}_G^N$.

This class of NLSMs has noncompact target spaces; to cure this, one can turn on chemical potentials $\vec{\mu}$ for the global symmetries of $\mathcal{T}_{2d}^{N}[H_G]$, which lifts degeneracies in the spectrum.

Then the elliptic genus can still be expected to decompose as a sum of characters of the modules M of the chiral algebra:

$$\mathbb{E}(\vec{\mu}, \tau) = \sum_M n_M \chi_M(\vec{\mu}, \tau).$$

$N = -1$ twist and minuscule varieties/1

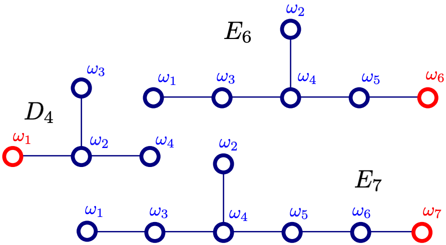
We will pick the $N = -1$ twist, $J^{(-1)} = J + \frac{1}{2}(R + r)$. We focus on $G = SO(8)$, E_6, E_7, for which the twist is allowed; one obtains a 2d NLSM with the same chiral algebra as in 4d:

$$\mathcal{A}_G^{(-1)} = G_{k_{4d}}, \qquad k_{4d} = -\frac{h_G^\vee}{6} - 1.$$

Modular invariance of the elliptic genus more or less determines it for us:

$$\mathbb{E}(\vec{\mu}, \tau) = -\widehat{\chi}_0^G(\vec{\mu}, \tau) + \widehat{\chi}_{\omega_R}^G(\vec{\mu}, \tau),$$

The vacuum character $\widehat{\chi}_0^G(\vec{\mu}, \tau)$ arises from 4d Schur operators. The non-vacuum character on the other hand corresponds to a Wallach representation of G, and can be computed explicitly in terms of Kazhdan-Lusztig polynomials.



G	D_4	E_6	E_7
k	-2	-3	-4
ω_R	$-2\,\omega_1$	$-3\,\omega_6$	$-4\,\omega_7$

$N=-1$ twist and minuscule varieties/2

The non-vacuum character $\widehat{\chi}_{\omega_R}(\vec{\mu},\tau)$ contains the ground states of the elliptic genus; these count to holomorphic functions on the target space of the NLSM.

This allows us to identify the target space of the NLSM as the minuscule variety $\widehat{X}_G$, which is a cone over a complex manifold X_G:

G	D_4	E_6	E_7
X_G	$Gr(2,4)$	$OG(5,10)$	$\mathbb{OP}^2$
$\dim_{\mathbb{C}}(X_G)$	4	10	16

The minuscule variety $\widehat{X}_G$ is a Lagrangian submanifold of $\widetilde{\mathcal{M}}_{G,1} = \mathrm{Higgs}(H_G)$. This is consistent with the expectation that the non-vacuum character is associated to a surface defect of H_G.

Moreover, the chiral algebra of the NLSM can be identified with a curved $\beta\gamma$ system on $\widehat{X}_G$ [Costello, Gorbunov-Gwilliam-Williams].

$N = -1$ twist and minuscule varieties/3

Naively, the NLSM only sees a maximal subalgebra $G^\flat$ of G, corresponding to the isometries of the target space:

G	D_4	E_6	E_7
X_G	$Gr(2,4)$	$OG(5,10)$	$\mathbb{OP}^2$
$G^\flat$	$SU(4) \times U(1)$	$SO(10) \times U(1)$	$E_6 \times U(1)$

This is ultimately due to the stress tensor of the NLSM being

$$T = T^G_{Sugawara} + \partial \mathcal{J},$$

where $\mathcal{J}$ is the $U(1)$ current.

With respect to this stress tensor, only the currents for $G^\flat$ have conformal dimension 1. The remaining currents acquire dimension 0 or 2.

Nevertheless, the chiral algebra is the larger $G_{-k_{4d}}$, which plays the role of a hidden symmetry of the theory. In particular, the spectrum organizes in representations of $\mathcal{A}_G^{(-1)}$. For $X_G = Gr(2,4)$, enhancement to $G = SO(8)$ was already noticed in [Dedushenko-Gukov].

Pure spinor superstring and hidden E_6 symmetry

Surprisingly, the case $G = E_6$ is intimately related to Berkovits' pure spinor formulation of the superstring! Indeed,

$$X_{E_6} = OG(5, 10) = \{\text{10d spinors } \lambda \,|\, \lambda\gamma^\mu\lambda = 0\},$$

At the level of global symmetries, we find

$$G^\flat = SO(10) \times U(1) = (\text{spacetime rotations}) \times (\text{ghost symmetry}) \to E_6.$$

Moreover, it is known that the curved $\beta\gamma$ system on the minuscule variety $\widehat{X}_{E_6}$ (i.e. the chiral algebra $\mathcal{A}_{E_6}^{(-1)}$) captures the ghost degrees of freedom of the pure spinor superstring [Nekrasov, Aisaka-Arroyo-Berkovits-Nekrasov].

Pure spinor partition function

We can check the existence of a hidden E_6 symmetry at the level of partition function. $Z_{\text{p.s.}} = \mathrm{Tr}(-1)^F q^{L_0} \ldots$ was computed in [Aisaka-Arroyo-Berkovits-Nekrasov] up to $\mathcal{O}(q^6)$. We find:

$$Z_{\text{p.s.}} = \mathbb{E}(\vec{\mu}, \tau) = -\widehat{\chi}_0^G(\vec{\mu}, \tau) + \widehat{\chi}_{-3\omega_6}^G(\vec{\mu}, \tau)!$$

Here $\widehat{\chi}_{-3\omega_6}^G(\vec{\mu}, \tau)$ captures the states built purely out of the pure spinor variable λ, while $\widehat{\chi}_0^G(\vec{\mu}, \tau)$ encodes the states that also involve the so-called b ghost composite operator.

We were also able to find an all-order expression for $Z_{\text{p.s.}}$ in terms of E_6 theta functions:

$$Z_{\text{10d pure spinors}} = \frac{\Theta_{\omega_6}^{E_6}(\vec{m}, \tau) - \Theta_{\omega_1}^{E_6}(\vec{m}, \tau)}{\prod_{i=1}^{16} \theta_1(m_i, \tau)/\eta(\tau)},$$

for a suitable choice of parameters $\vec{m}$ and m_i.

A remark: the existence of a hidden E_6 symmetry acting on 10d SYM (i.e. on the ground states of the pure spinor system) was first conjectured by Pioline and Waldran. Our results extends their observation to the massive states as well.