Baryon Number Violating Scatterings in the Lab

Henry Tye

Hong Kong University of Science and Technology
and
Cornell University

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Sphaleron and Baryon Number Violation

This talk is based on work done with :

Sam Wong : arXiv : 1505.03690
"Bloch Wave Function for the Periodic Sphaleron Potential and Unsuppressed Baryon and Lepton Number Violating Processes," in PRD

arXiv : 1601.00418
"Chern-Simons Number as a Dynamical Variable"
in Annals of Mathematical Sciences and Applications

and arXiv : 1710.07223
"Comment on the Baryon Number Violating Processes in the Laboratory" in PRD

and **Yu-Cheng Qiu** : arXiv : 1812.07181
"The Role of Bloch Waves in Baryon-Number Violating Processes"

- A brief review.
- A step by step justification of our Bloch Wave Approach to the problem.
- Corrections to the idealized Bloch wave approximation.
- A brief summary.

Instanton in the Electroweak Theory

Keeping only the important parts in the $SU(2)$ electroweak theory,

$$\mathcal{L} = -\frac{1}{2} Tr[F_{\mu\nu}F^{\mu\nu}] + \frac{1}{2}(D_\mu \Phi)^\dagger D^\mu \Phi - \frac{\lambda}{4}\left(\Phi^\dagger \Phi - v^2\right)^2 + i\bar{\Psi}_L^{(i)}\gamma^\mu D_\mu \Psi_L^{(i)}$$

$$J_L^{i\mu} = \bar{\Psi}_L^{(i)}\gamma^\mu \Psi_L^{(i)} \qquad i = 1, 2, 3, ...12.$$

Belavin, Polyakov, Schwartz and Tyupkin (1975) constructed topological soliton solutions (instantons) in 4-dimensional Euclidean SU(2) theory:

$$N = \frac{g^2}{16\pi^2}\int d^4x Tr\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

where the topological (Chern-Pontryagin) index N takes only integer values.

Anomaly in Electroweak Theory

- A quantity that is conserved classically may not be conserved quantum mechanically, due to "anomaly".

$$\partial_\mu J_L^{i\mu} = \frac{g^2}{16\pi^2}\mathrm{Tr}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right] = \partial_\mu K^\mu$$

- We can define a new current that is conserved:

$$\partial_\mu \bar{J}^{i,\mu} = \partial_\mu (K^\mu - J_L^{i,\mu}) = 0$$

$$\rightarrow \quad \mathcal{Q}^i = \int d^3x \bar{J}^{i,0} = N - Q_F^i$$

- Electric charge is always conserved. So are $(B - L)$ and energy-momentum, but not $(B + L)$, as pointed out by 'tHooft (1976).

Examples :

3 hydrogen atoms $\rightarrow \nu_e \nu_e \bar{\nu}_\mu \bar{\nu}_\tau$

A single carbon C^{12} atom

Detection in cosmic rays, Ice Cube etc. have been propsed.

LHC or next pp collider :

$$q_L + q_L \rightarrow \bar{l}_e \bar{l}_\mu \bar{l}_\tau \bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q} + ...$$

Chern Character to Chern-Simons Number

Instead of going to Euclidean space, we can describe this phenomenon in our usual $3 + 1$ space-time.

Manton (1983) and Klinkhamer and Manton (1984)[1] constructed the "sphaleron", i.e., the unstable extremal static solution in the SU(2) part of the electroweak theory.

Its energy (barrier height) $E = 9.0$ TeV. Turning off the $U(1)$ coupling raises it to 9.1 TeV.

We can introduce Chern-Simons number n: $|n\rangle \to |n + N\rangle$. Treating n as a dynamical variable, we calculated its kinetic term. Conservation of the "new" fermion currents yields

$$|n\rangle \to |n + N + 3N(B + L)\rangle$$

[1] also Dashen, Hasslacher and Neveu, and Boguta

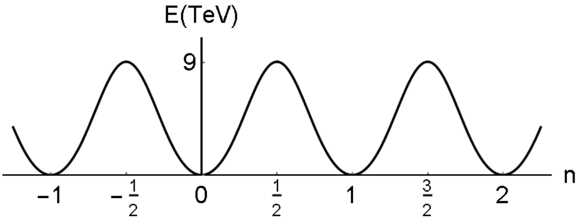
One-dimensional Schrödinger Equation

Mass m =17.1 TeV with coordinate $Q = \mu/m_W$, where $n\pi = \mu - \sin(2\mu)/2$,

$$L = \frac{1}{2} m\dot{Q}^2 - V(Q)$$

Next we quantize this system with Q (or n) as the coordinate to obtain the one-dimensional time-independent Schrödinger equation:

$$\left(-\frac{1}{2m}\frac{\partial^2}{\partial Q^2} + V(Q)\right)\Psi(Q) = E\Psi(Q)$$



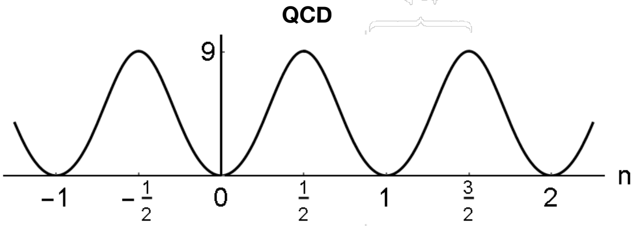
Recall QCD :

The presence of instantons generates a periodic potential.
So the vacuum is described by θ,

$$|\theta\rangle = \sum_n \exp(in\theta)\,|n\rangle$$

Note that, for $\alpha_{QCD} \sim 1/6$, the tunneling rate is

$$\Gamma \sim e^{-4\pi/\alpha_{QCD}} \sim 10^{-30}$$

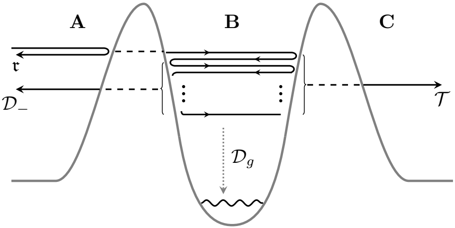


Resonant Tunneling

Simple QM property :

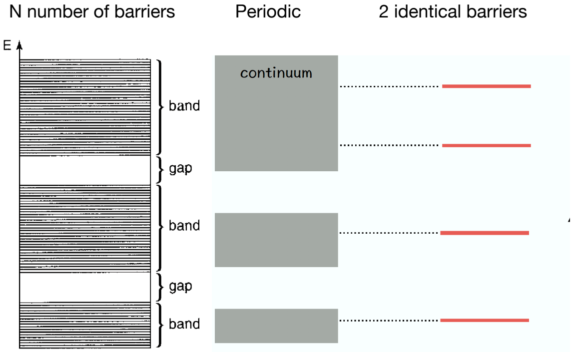
For identical barriers, at the appropriate (resonant) energy E, incident wave from the left at A will end in C at the right with the transmission coefficient $T \to 1$.

That is, at appropriate energies, tunneling through the double barriers is unsuppressed.



Lessons from QCD

For finite number of identical barriers, we have bands of discrete E states where transmission is unsuppressed, separated by gaps.
As the number of barriers $\to \infty$, i.e., a periodic potential, we have continuous (conducting) bands (Bloch wave bands) separated by gaps.



Bloch wave bands ?

• The $|\theta\rangle$ in QCD, $|\theta\rangle = \sum_n \exp(in\theta)\,|n\rangle$, tells us that

(1) we cannot ignore the periodic barriers even if the tunneling through a single barrier is exponentially suppressed.

(2) the resonant tunneling phenomenon happens in QCD.

• Same for electroweak theory, where the barrier height of 9.0 TeV is simply the sphaleron mass.

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• Same for electroweak theory, where the barrier height of 9.0 TeV is simply the sphaleron mass.

• However, no one talks about Bloch wave bands in QCD. Does QCD has Bloch wave bands ?

• The answer is **NO**, but the electroweak theory has.

Consider a particle moving in a one-dimensional periodic potential with period 2π, $V(\mu) = V(\mu + 2n\pi)$, $n \in \mathbf{Z}$, with action

$$S = \int dt\, L = \int dt \left[\frac{1}{2} M \dot{\mu}^2 - V(\mu) - \theta \dot{\mu}\right]$$

where the topological term plays the role of the θ term in the $SU(N)$ gauge theory.

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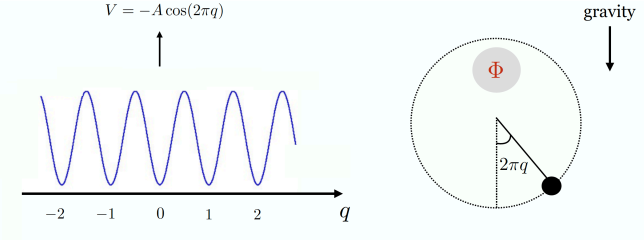
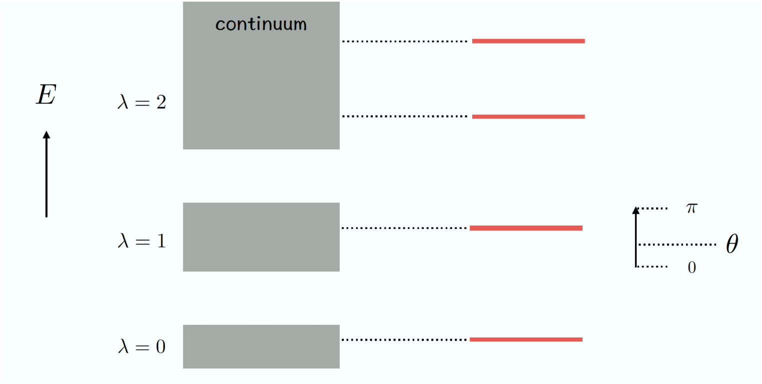
However, the model is not fully specified : the translational symmetry is local (gauged) or global ?

Bachas-Tomaras:



If it is gauged (local), μ plays the role of the angle of a circle, and $V(\mu)$ has a unique ground state and there is no continuous bands of solutions.



Bloch wave bands **No bands**

Global **Gauged**

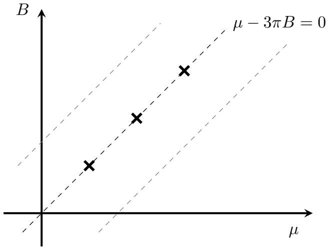
Difference between QCD and the Electroweak Theory

'tHooft (1976)

$$\partial_\mu \bar{J}^{i,\mu} = \partial_\mu (K^\mu - J_L^{i,\mu}) = 0$$

$$\rightarrow \quad \mathcal{Q}^i = \int d^3x \bar{J}^{i,0} = N - Q_F^i$$

$$\rightarrow \quad n - 3(B+L) = 0$$

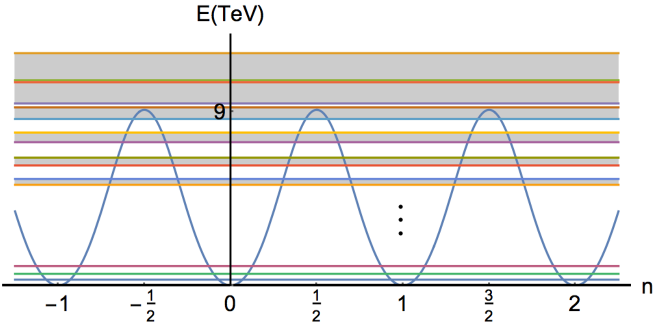


Where is the difference between QCD and the EW theory ?

In electroweak theory, $SU(2)$ gauge fields couple to the fermions via left-handed currents, and J^i_μ are NOT conserved: $|n+1\rangle$ has 3(B+L) more than $|n\rangle$. So the periodic sphaleron potential is global and continuous Bloch wave bands exist.

In QCD, gauge fields couple the fermions via vector currents, which are conserved. So $|n+1\rangle$ is a gauge-transformed $|n\rangle$.

It is now straightforward to solve the one-dim. Schrödinger Equation using the Bloch Theorem to get the conducting (pass) band structure (one-dimensional Brillouin zone) :



Here n is the Chern-Simons number.

- There are about 150 bands.
- The lowest conducting or pass band is at about 35 GeV, with band width $\sim 10^{-177}$ GeV. The next one is about 70 GeV higher. That is, the band gap is about 70 GeV, decreasing to about 50 GeV towards the band just below the sphaleron energy.

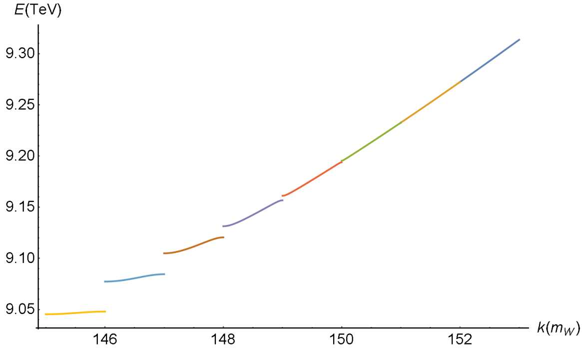
- There are about 150 bands.
- The lowest conducting or pass band is at about 35 GeV, with band width $\sim 10^{-177}$ GeV. The next one is about 70 GeV higher. That is, the band gap is about 70 GeV, decreasing to about 50 GeV towards the band just below the sphaleron energy.
- If the wavefunction energy is spread over a few GeV, then only 10^{-177} fraction would pass through the lowest band unsuppressed.

$$\sigma(\Delta n \neq 0) \simeq \frac{\text{bandwidth}}{\text{bandgap}} \sim 10^{-177}$$

- This is how the instanton tunneling suppression [$\exp(-4\pi/\alpha_{EW}) \sim 10^{-164}$] is reproduced in this approach (with $\alpha_{EW} \simeq 1/30$).

Extended Brillouin Zone

The higher bands have larger band widths.



$$\mathcal{E} = E_{qq} - 4.1\,\mathrm{TeV} = k^2/2m$$

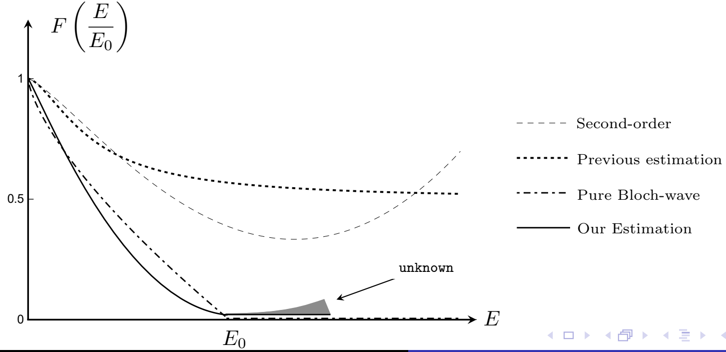
for $E_{qq} > E_{sph} = 9.1$ TeV.

Comparing with Earlier Results

$$\sigma_{\Delta n\neq 0}(2 \to any) \sim \exp\left[-\frac{4\pi}{\alpha_W} F(E/E_0)\right]$$

$$F(E/E_0) = 1 - \frac{9}{8}(E/E_0)^{4/3} + \frac{9}{16}(E/E_0)^2 - \dots$$

For low energies, $F \simeq 1$, $\sigma \sim e^{-4\pi/\alpha_W} \sim 10^{-164}$

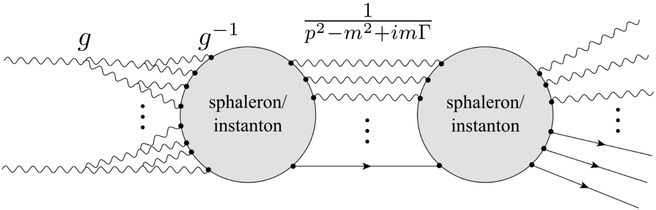


Comparing with Earlier Results

The "few" to "many" argument: $\sigma_{\Delta n \neq 0}(2 \to any) \propto \alpha_{EW}^{1/\alpha_{EW}}$

However, Voloshin (1994) PRD e.g., $F \to 0.16$, not $F \to 0.5$.

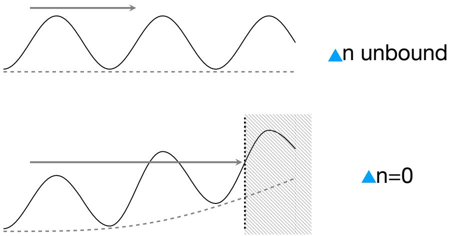
Ringwald (1990) and Espinosa (1990), Khoze & Ringwald . . . .



Each propagator : $1/\Gamma \sim g^{-2}$, so each resonant gauge field propagating between two sphalerons contributes a factor of g^{-4} !

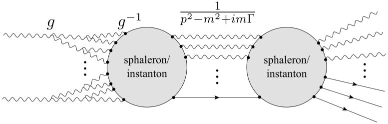
Improving the idealized Bloch wave picture

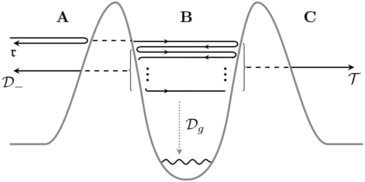
The Bloch wave solution is the idealized case. It is the zero-th order approximation. There are 2 important corrections that have to be included: Tilt and Decay.



Starting from $|n = 0\rangle$ with zero energy, producing 3(B+L) at $|n + 1\rangle$ raises the energy of the state $|n + 1\rangle$.

Decay



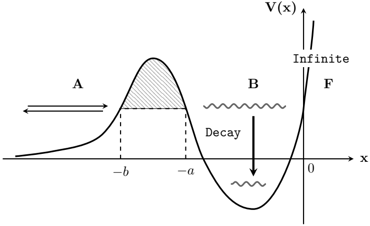


Some energy along the sphaleron direction is lost to the (B+L)-conserving direction.

Propagating along a higher band drops to a lower band after passing through a sphaleron.

Now we have to take into account both the tilt of the potential and the decay.

A simple Case



We consider the N-barrier case with both tilt and decay. It turns out that a single barrier with a wall captures the key feature of decay and resonant tunneling.

The probability to stay in region B is

$$|\mathcal{G}|^2 \equiv 1 - |\tilde{\mathcal{R}}|^2 = \frac{(\frac{1}{\beta} - \beta)(\frac{1}{t} - t)}{\frac{1}{t\beta} + t\beta + 2\cos(2L)}$$

where $\beta = e^{-2\Delta}$, where Δ measures the decay amplitude, $t = \tanh S$ measures the tunneling amplitude, and L measures the phase coming from traveling in region B.
$\beta = 1$ means no decay and $t = 1$ means tunneling is suppressed.

Details :

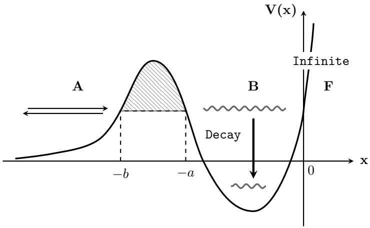
$$t = \tanh S, \qquad S_i = \int_{a_i}^{b_i} \sqrt{2m(V(x) - E)}dx + \ln 2$$

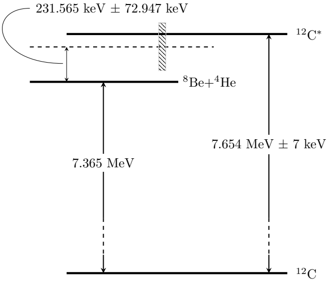
$$L_i \ \rightarrow \ \frac{1}{\hbar}\int_{b_i}^{a_{i+1}} \sqrt{2m\left(E - V(x) + i\frac{\gamma}{2}\right)}\mathrm{dx}$$

We can simply let $L = L_r + i\Delta$, where L_r represents the real part of the integral. Usually, $\gamma \ll E$, so we can obtain approximate expression for the decay amplitude Δ, and

$$\beta = e^{-2\Delta}$$

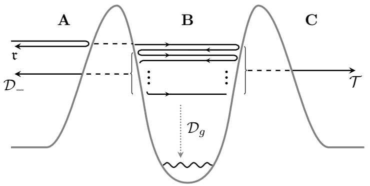
The above resonant tunneling process has been applied to the triple α process to create carbon-12 in stars. Based on the existence of carbon and higher elements in nature, the resonant state was predicted by Salpeter and Hoyle in 1953 and quickly confirmed in experiment.

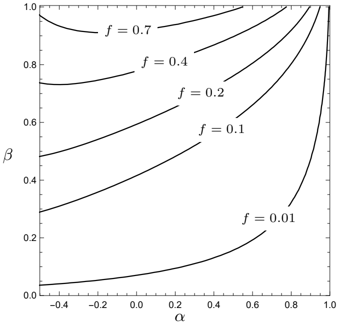




$$^4\text{He} +^4 \text{He} +^4 \text{He} \quad \rightarrow \quad ^4\text{He} +^8 \text{Be} \quad \rightarrow \quad ^{12}\text{C}^*$$

where $^{12}\text{C}^*(B)$ is a resonance state in carbon-12.





$$|\tilde{\mathcal{T}}|^2 = \frac{\beta(1-t_1^2)(1-t_2^2)}{1+(t_1 t_2 \beta)^2 + 2 t_1 t_2 \beta \cos 2L}$$

$$\cos 2L = -1, \qquad \alpha = \tanh(\delta S) < 1$$

$$f(\alpha,\beta) \equiv \max |\tilde{\mathcal{T}}|^2 = \frac{\beta(1-{t_1}^2)^2(1-\alpha^2)}{(1-\beta {t_1}^2 + (1-\beta) t_1 \alpha)^2}$$

Conclusion :

• Ideal case : Δn is unbounded.

• Including only the tilt of the $V(Q)$: $\Delta n = 0$

• Including decay (energy lost to the $(B+L)$- conserving direction) : $\Delta n = \pm 1$ should dominate.

$E_{qq} \gtrsim 9$ TeV has to be shared by $(B+L)$-violating $E_{||}$ and $(B+L)$-conserving $E_{\perp}$.

We need $E_{||} \gtrsim 9$ TeV, so phase space suppression due to $(1 - E_{||}/E_{qq})^p$ is guesstimated to be of order-of-magnitude 10^{-4} to 10^{-3}.

Phenomenology : LHC and Beyond

In the coming LHC run : $\Delta n = +1$ process:

$$u_L + d_L \to e^- \mu^- \tau^- bbtccsuuuud + ...$$

so a single pp (i.e., quark-quark) collision can produce 3 same sign charged leptons plus baryons.

Similarly, one has $\Delta n = -1$ process:

$$q_L + q_L \to \bar{l}_e \bar{l}_\mu \bar{l}_\tau \bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q} + ...$$

For $E_{pp} = 14$ TeV, parton distribution suppression is 10^{-6} for $E_{qq} \gtrsim 9$ TeV. In addition, there is the $(1 - E_{||}/E_{qq})^p$ phase space suppression factor of 10^{-4}. (For $E_{pp} = 13$ TeV, one loses another factor of 10^{-3} or worse.)

HE-LHC at 28 TeV will erase most of this 10^{-10} suppression.

Summary

There is a 70 order-of-magnitude discrepancy in the estimate of the rate of baryon-number violating processes in the well-known electroweak theory !!!!!

The previous prediction relies on an extrapolation to higher energies that the theorists themselves agree is unreliable.

The Bloch wave approach is clean and going to higher energies is straightforward and more reliable.

We hope that the $(B + L)$-violating events will be observed in LHC 14 TeV run.

We are confident that they will be observed if one goes to higher energies, like HE-LHC, FCC-pp or SppC.

THANKS

The CS number N_{CS} is closely related to the gauge invariant Chern-Pontryagin CP number N. It is also related to the winding number W and the Hopf invariant H:

$$H = W = N_{CS} = N = \Delta B/3 = \Delta L/3$$

via Gauge field	$N : \pi_3(S^3)$
Higgs field	$W : \pi_3(S^3)$
Higgs field	$H : \pi_3(S^2)$

The topological invariants can be extended from integers to include half-integers as well.
Generalizing these indices to dynamical variables $\mu(t)$,

$$H = W = N_{CS} = N \to n = \mu/\pi$$

Taking $f(14\text{TeV}, 9\text{TeV}) \sim 10^{-8}$ and an integrated luminosity of $L_{pp} = 3000\ fb^{-1}$ (inverse femtobarns), we guess the number of $(B+L)$-violating events with 3 same sign charged leptons to be:

$$\text{Integrated event number in the LHC 14 TeV run}$$
$$\sim \sigma(pp) \cdot f(14,9) \cdot F_{EW} \cdot \kappa \cdot F_3 \cdot L_{pp}$$
$$\sim (80 \times 10^{-3} b)(10^{-8})(10^{-2})(10^{-2}/3)(\frac{1}{8})(3000 \times 10^{15} b^{-1})$$
$$\sim 10^4 \to 10^{4\pm 2}$$

where we naively take $F_{EW} \sim 10^{-2}$ and the fraction $\kappa \sim 10^{-2}/3$. Increasing the pp energy $E(pp)$ by just a few TeV above 14 TeV will increase the event rate by a few orders of magnitude.

Phenomenology

$(B+L)$-violating events will look like fireballs.
Ellis and Sakurai (arXiv: 1601.03654) claim that experiments at 100 TeV collider can reach

$$\kappa \sim 10^{-10}$$

Brooijmans, Schichtel and Spannowsky (arXiv: 1602.00647) claim that detection is possible in cosmic ray experiments like that at Pierre AUGER Observatory with cosmic ray energies reaching $E \sim 10^{11}$ GeV, so events reach the equivalent of COM energies

$$E \sim 500\,\mathrm{TeV}$$

Searches in neutrino experiments like ICE CUBE was also discussed (arXiv: 1603.06573).