3D effects in dielectric haloscopes and dish antennas

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Introduction

The precise estimate of axion haloscopes sensitivity requires the calculation of the 3D E-fields in axion electrodynamics. Full 3D finite element method (FEM) solutions for large setups are computationally expensive. We present and compare two effective methods [1] to elude a full 3D FEM computation. Exemplary the effect of a finite cold dark matter velocity is investigated for a dish antenna and a dielectric haloscope [2]. Our two effective methods are furthermore used to quantify the effects of diffraction and disk tilt tolerances in dielectric haloscopes.

Axion-Maxwell equations and solution techniques

First order axion-Maxwell equations [1] in the axion photon coupling $g_{a\gamma}$ for E-field:

$$\nabla \times (\mu^{-1} \nabla \times E) - \omega^2 \epsilon E = -m_a^2 E_a. \quad (1)$$

- external B-field $B^{(0)}$ and no external E-field.
- linear media $D = \epsilon E$, $H = \mu^{-1} B$ and no material losses.
- $E_a(x) \equiv -g_{a\gamma} B^{(0)}(x) a_0$, a_0 is the axion cold dark matter (CDM) field.
- Solution of (1) computationally very expensive with 3D finite element method (FEM). We present two methods [1] to elude a full 3D FEM solution.

2D3D FEM approach:

Radial symmetric geometry $\longrightarrow$ reduce the problem by one dimension, even though external E_a-field / external B-field is linear polarized. Decompose:

$$E_a(\rho, z) = E_a^+(\rho, \phi, z) + E_a^-(\rho, \phi, z), \quad (2)$$

with $m = \pm 1$:

$$E_a^m = \tilde{E}_a^m e^{im\phi} = \frac{E_a(\rho, z)}{2}(\hat{e}_\rho + im\hat{e}_\phi)e^{im\phi}. \quad (3)$$

Solve:

$$\nabla \times (\nabla \times E^m) - k_0^2 \epsilon E^m = -k_0^2 E_a^m. \quad (4)$$

With ansatz $E^m = \tilde{E}^m(\rho, \phi, z)e^{im\phi}$:

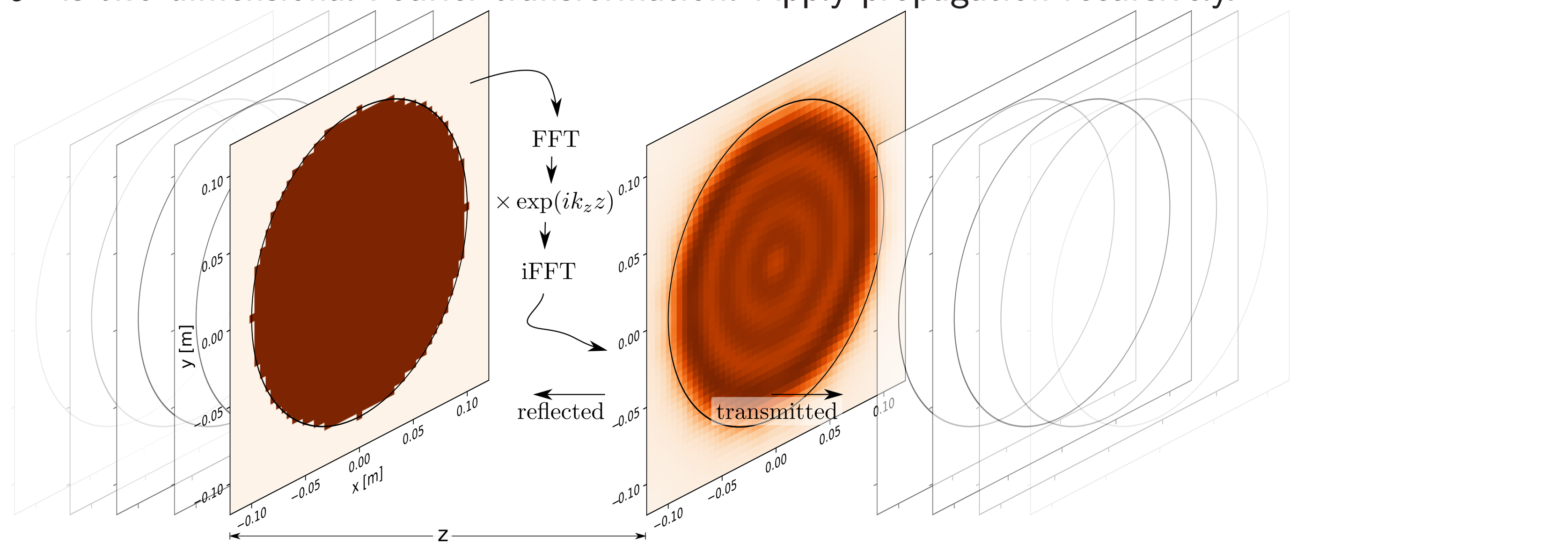
$$\tilde{E}^m = \tilde{E}_\rho^m(\rho, z)\hat{e}_\rho + \tilde{E}_\phi^m(\rho, z)\hat{e}_\phi + \tilde{E}_z^m(\rho, z)\hat{e}_z, \quad (5)$$

Recursive Fourier propagation approach:

Axion induced field E_a leads to propagating fields from interfaces with different refractive index n due to interface conditions for E and B-fields. Describe the emitted radiation with a scalar diffraction theory (neglects near fields):

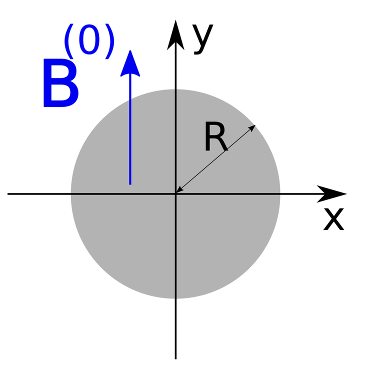
$$E(x) = \int_{\mathbb{R}^2} \frac{dk_x dk_y}{(2\pi)^2} \mathcal{F}(E)(k_x, k_y) e^{i|z|\sqrt{(\omega n)^2 - k_x^2 - k_y^2}} e^{ik_x x} e^{ik_y y}, \quad (6)$$

$\mathcal{F}$ is two dimensional Fourier transformation. Apply propagation recursively.

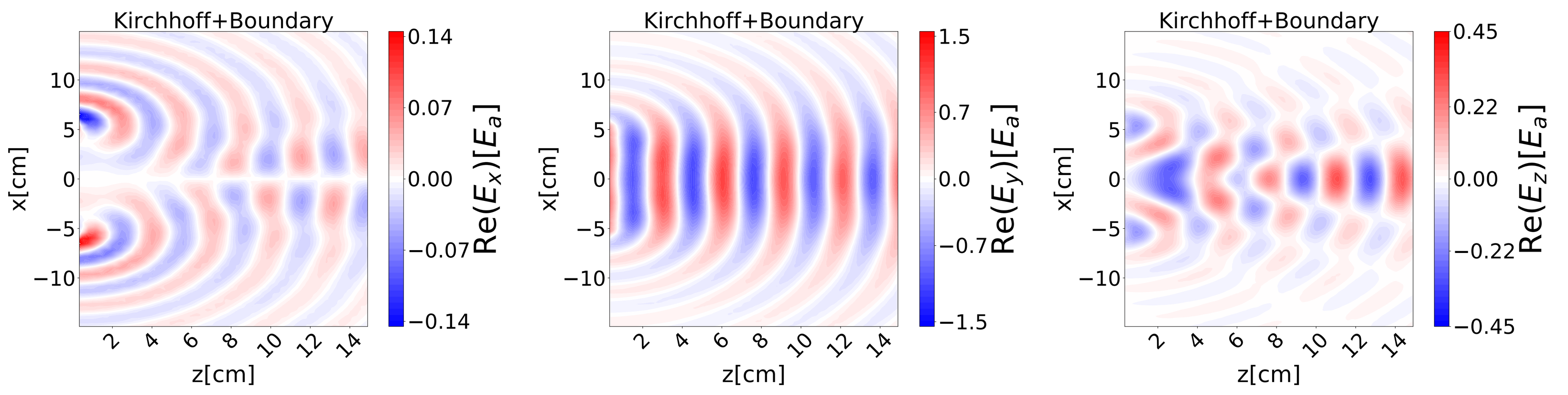


Dish antenna

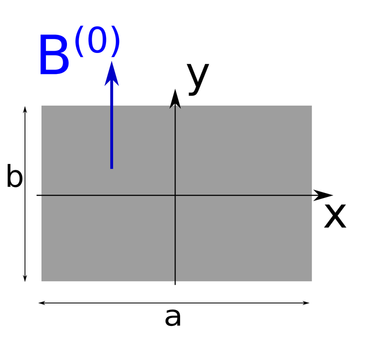
Radiating field shape of circular dish antenna:

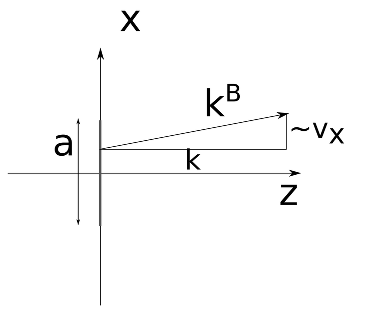


- Field patterns can be analytically described with Kirchhoff diffraction theory plus a line boundary charge $\sigma_L \sim \sin\phi$ and a line current density $K_L \sim \cos\phi \hat{e}_\phi$.
- After setting the magnitudes of K_L and σ_L the field patterns agree with the FEM solution.
- Diffraction losses decrease for larger disk size and axion masses.

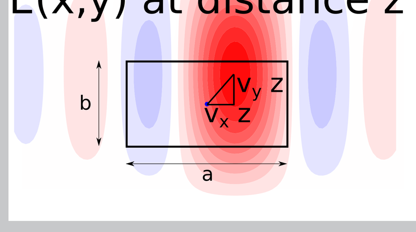


Axion velocity effects in rectangular dish antenna:





E(x,y) at distance z



Diffraction theory by Kirchhoff and Rayleigh:

$$E(x) = \frac{k}{2\pi i} \int_S dA' \frac{e^{ikD}}{D}\left(1 + \frac{i}{kD}\right)\frac{n' \cdot D}{D} E_S$$

$E_S = E_a e^{ik^B \cdot x}$ is E-field at the surface S of the dish antenna. $D = |x - x'|$, n' normal vector. Applicable if $\lambda \ll \min(a, b)$.

$\tan\alpha = \frac{k_x^B}{k} \approx v_x, \tan\beta = \frac{k_y^B}{k} \approx v_y \lesssim 10^{-3}$. In the far field:

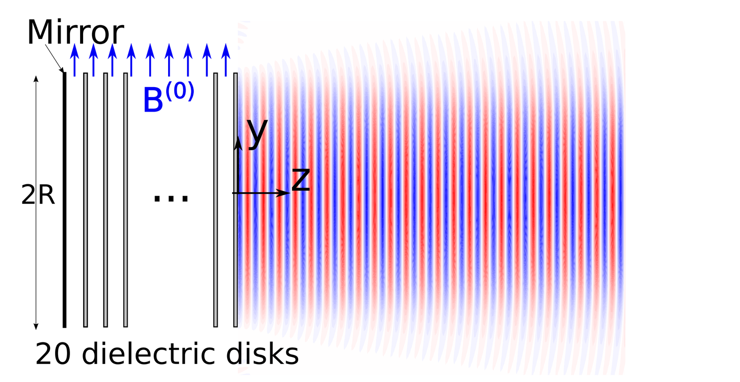
$$\frac{E(x)}{E_a} \sim \mathrm{sinc}\left(\frac{ka}{2}\chi_x\right)\mathrm{sinc}\left(\frac{kb}{2}\chi_y\right),$$

$$\chi_x = v_x - \frac{x}{z}, \quad \chi_y = v_y - \frac{y}{z}$$

Shift of the diffraction maximum at distance z :

$$x = v_x z, \qquad y = v_y z$$

20 disk dielectric haloscope



We consider a dielectric haloscope with 20 dielectric discs ($\epsilon = 24$ and thickness 1 mm). The disk radius is $R = 15$ cm.

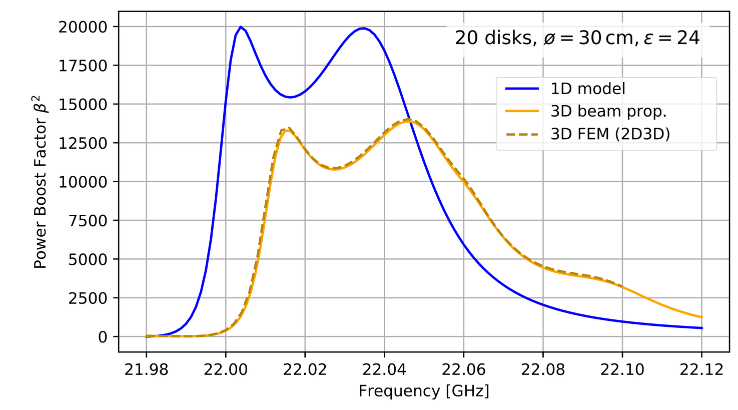
Power boost factor of setup:

$\beta^2 = \frac{\text{Power emitted by setup}}{\text{Power emitted by dish antenna of same size}}$.

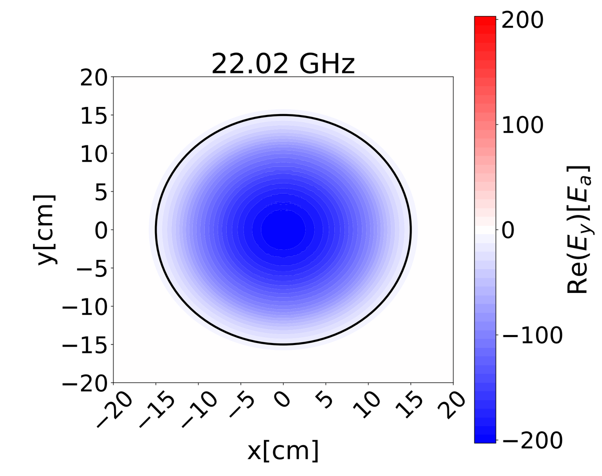
Optimizing the disk positions gives a large power enhancement in a certain frequency interval.

Distance of the disks around $\lambda/2$. Study motivated by MADMAX prototype booster ($18 - 25$ GHz)

Beam shapes and power boost:

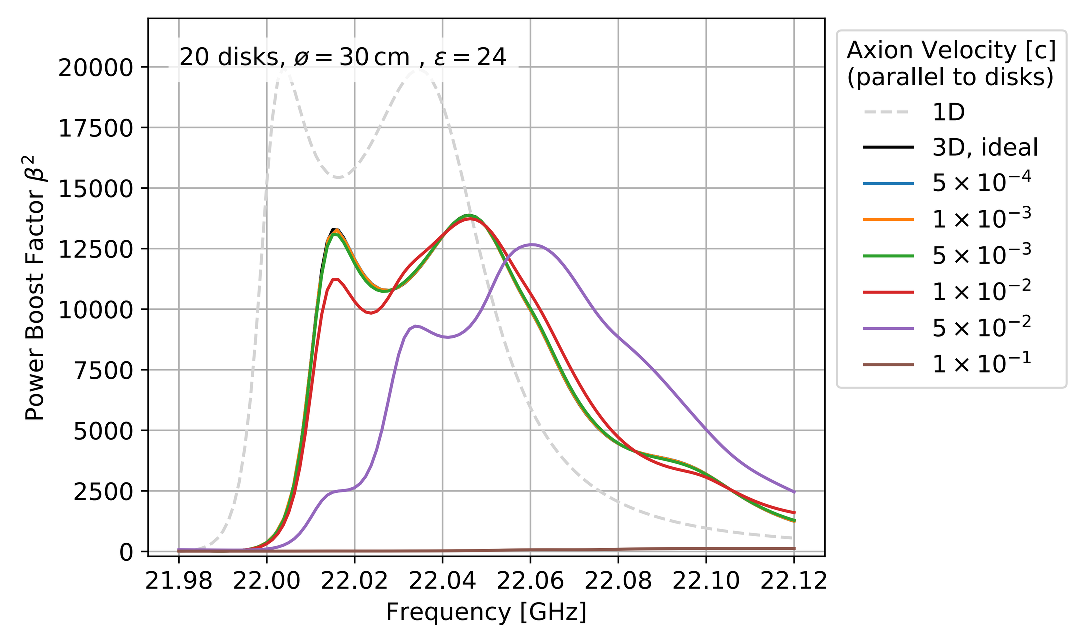


2D3D and Fourier propagation agree (near-fields negligible)



$z = 15$ cm, $\lambda \approx 1.6$ cm.

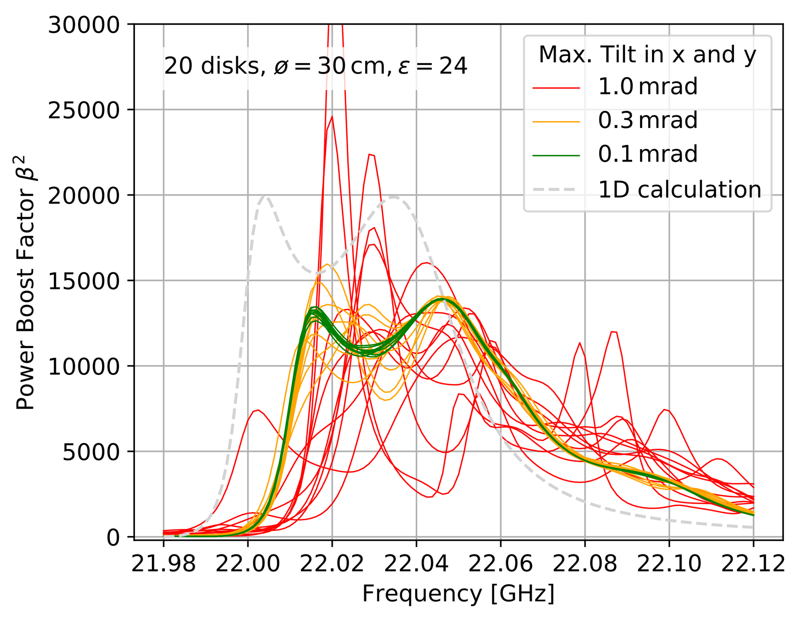
Axion velocity effects:



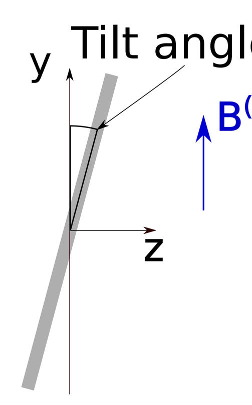
- Quality factor $\tilde{Q}$ number of reflections inside system.
- Distance of the interfaces is around $\lambda/2$. Total propagation distance $\tilde{Q}\lambda/2$.
- $\tilde{Q}\lambda/2v \overset{!}{\ll} R$.
- CDM velocities parallel to the disks smaller than 10^{-3} do not significantly change β^2.

Method: Recursive Fourier propagation approach

Disk tilt effects:



- Tilts larger than 0.1 mrad have significant influence on boost factor.
- Tilt angle for the disks which are smaller than 0.1 mrad are experimentally feasible.

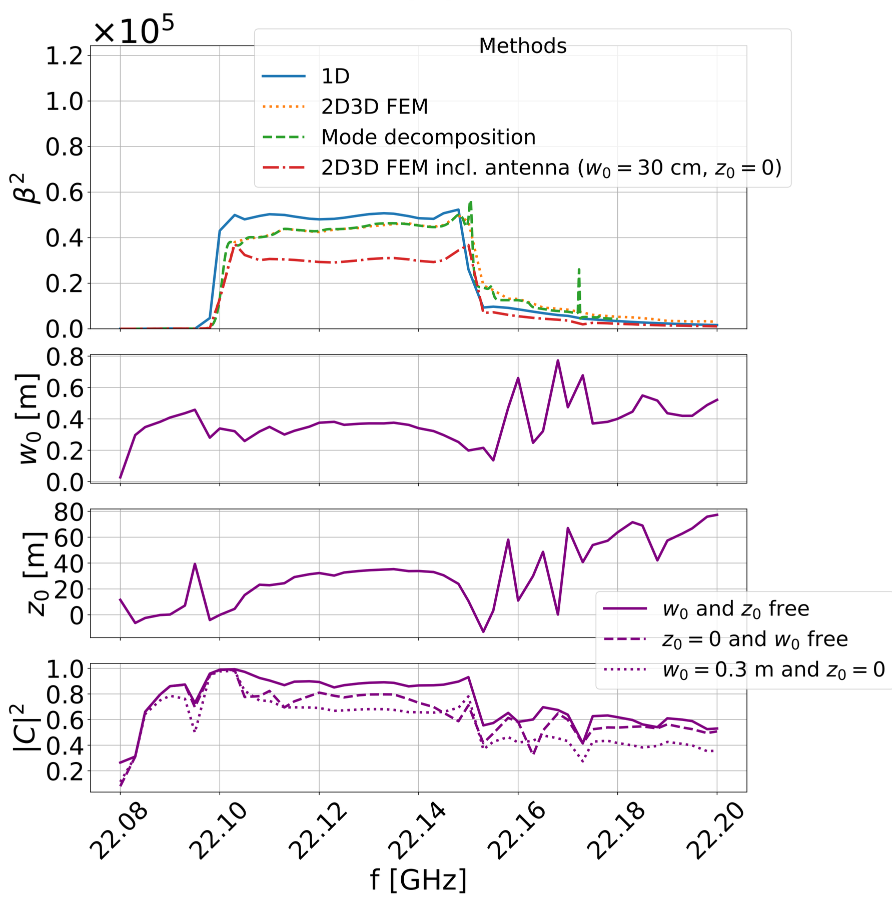


Random distributions of tilts for 20 disks

Method: Recursive Fourier propagation approach

80 disk dielectric haloscope

We consider a dielectric haloscope with 80 dielectric discs ($\epsilon = 24$ and thickness 1 mm). The disk radius is $R = 0.5$ m. The values are currently aimed at by MADMAX.



- Upper panel: β^2 is reduced due to diffraction (by $10\% - 20\%$). 3D calculation cross checked with 2D3D method and a mode decomposition. The 3D β^2 is reduced by coupling to Gaussian antenna around $10 - 20\%$.
- Lower panels: Fitting beam waist w_0 and waist position z_0. Almost constant w_0 over frequency interval where β^2 is large. Rule of thumb $w_0 = \frac{2}{3}R$. Fixing $z_0 = 0$ to the last disk does not reduce the power coupling $|C|^2$ significantly.

Conclusion

- 3D E-fields for open axion haloscopes are necessary for precise sensitivity prediction.
- Two methods are developed and validated. 3D effects can change the 1D results.
- CDM velocity effects are computed: negligible for 20 disk dielectric haloscopes / shift of diffraction maximum in dish antenna
- Sensitivity for dielectric haloscopes is quantified with 3D fields:
  - Diffraction losses are around $10\% - 20\%$ with respect to 1D calculations.
  - Losses due to the coupling to antenna are around $10\% - 20\%$.
  - Disk tilts < 0.1 mrad are acceptable.

References

S. Knirck, J. Schütte-Engel, A. Millar, J. Redondo, O. Reimann, A. Ringwald, F. Steffen *A First Look On 3D Effects in Open Axion Haloscopes*, arxiv:1906.

A.J. Millar, G. G. Raffelt, J. Redondo, F. Steffen, *Dielectric Haloscopes to Search for Axion Dark Matter: Theoretical Foundations*, JCAP **1701** (2017) 061

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